Expanding Applications in College Admissions*

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Abstract

In college admissions, standardized tests typically consist of required main tests and optional subject tests. If a college uses an expanding strategy of not requiring students to take the subject tests, it will benefit from a caliber effect in a larger application pool but suffer from a mismatch effect due to the absence of measuring applicants' capacity in specific subjects. Our analysis provides conditions under which the expanding strategy is used in a centralized mechanism and a decentralized mechanism, respectively. Moreover, the mismatch effect can be partially eliminated in the decentralized mechanism by a conflicting strategy in which students' choices are restricted to selecting one college. As a result, combining the expanding and conflicting strategies can yield an efficient and stable equilibrium via students' choices.

JEL classification: C78; D82; I23

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1 Introduction

In most college admissions, students are required to take a standardized test. A standardized test typically consists of required main tests and optional subject tests, such as SAT

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I and SAT II, respectively, for the decentralized college admissions in the United States. Similarly, the centralized college admissions in Taiwan also allows colleges to decide the combination of main tests (math and languages) and subject tests (physics, history, and so on). In both cases, students taking subject tests must be a subset of those who take the main tests. If a college does not require students to take the subject tests, it actually expands its applicant pool, albeit at a cost, in that students' capacity in specific subjects is not measured by the tests. In this paper, we analyze both the centralized and the decentralized college admissions problem in which colleges could attract more desired students by expanding the applicant pool via a policy of not requiring the subject tests, called the expanding strategy. In the decentralized college admissions, the aforementioned cost might be partially eliminated by restricting applicants' choices, called the conflicting strategy. Moreover, we provides sufficient conditions such that using the two strategies together can yield an efficient and stable equilibrium.

In a centralized college admissions scenario, such as the case in Taiwan, the expanding strategy is commonly used by colleges or departments in the fields of humanities and social-science. Figure 1 shows the weighted average scores and the sum of subject weights for Taiwanese colleges in the centralized mechanism for each year from 2006 to 2013. The red-solid points are colleges or departments using the expanding strategy, and others are colleges or departments requiring both main tests and subject tests. High values of weighted average scores imply that students enrolled by those colleges have high test scores.[1] According to the pattern in Figure 1, colleges on average seem to attract better students in the test by using the expanding strategy. This raises a question of why the expanding strategy is not generally adopted in Taiwan.

Similarly, in a decentralized college admissions scenario, such as the case in the United States, we observe that Yale uses the expanding strategy of requiring only SAT I (main tests) but MIT requires both SAT I and SAT II (subject tests) in their 2018 testing policies.[2] Table 1 summarizes the testing policies, rankings, and provisions of early decision programs for some selective colleges in the United States in 2018. The subject

[1]For example, a department only requires main tests and sets the relative weights of 1, 1.5, and 1.5 for Chinese, English, and math B, respectively. After the admission process, the threshold of the weighted sum of scores for students enrolled by that department is announced. If the threshold is 360, then the weighted average score is 90 (=360/4). Appendix A provides more details about the Taiwanese admission process. Hsu (2018) analyzes the student placement of Balinski and Sönmez (1999) with weights, and shows that there is no stable mechanism to avoid weight manipulation in a centralized college admissions scenario.

[2]Yale's testing policy in 2018 states "SAT Subject Tests are recommended but not required. Applicants who do not take SAT Subject Tests will not be disadvantaged in the application process" (https://admissions.yale.edu/standardized-testing). In contrast, MIT's testing policy in 2018 states "We require two SAT Subject Tests: one in math (level 1 or 2), and one in science (physics, chemistry, or biology e/m)" (http://mitadmissions.org/apply/freshman/tests).

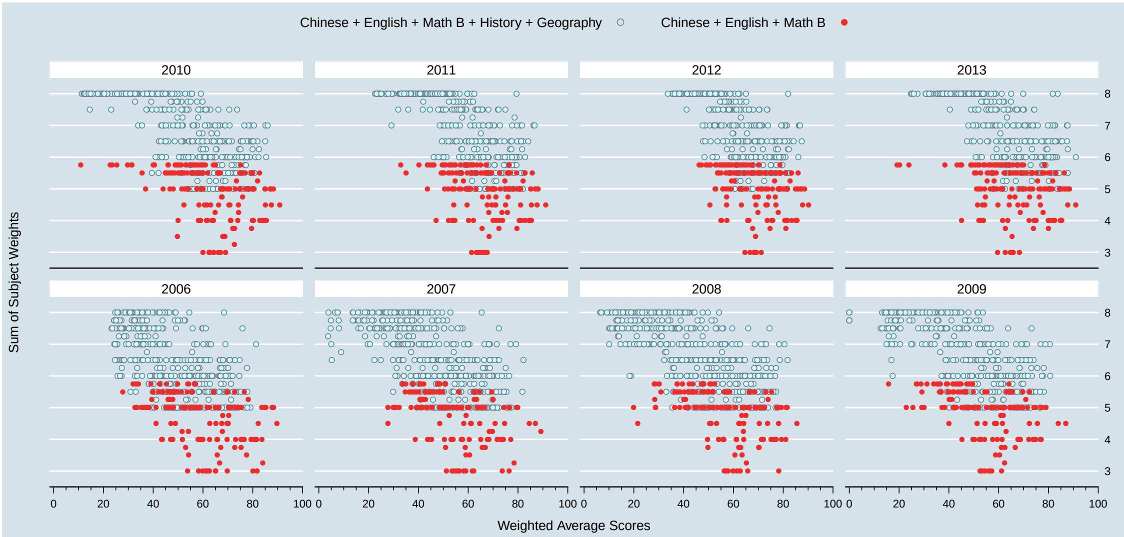

Figure 1: Distribution of student scores under different testing policies in Taiwan

tests are rarely required by those colleges, i.e., the expanding strategy is commonly used in practice. Moreover, more than half of those colleges using the expanding strategy also provide early decision programs, which is defined as using the conflicting strategy here since students are restricted to applying to one of them. This raises another question of why a bundle of the expanding strategy for more applications and the conflicting strategy for limiting students' choices is adopted by many selective colleges in the United States.

Our explanation for the two questions is as follows. If a college uses the expanding strategy, it will benefit from a caliber effect but suffer from a mismatch effect. The caliber effect comes from an expanding pool with more applicants, and some students who do not take a given subject test may have higher general caliber, measured by a better score in the main tests. The mismatch effect is due to the absence of measuring applicants' capacity in specific subjects. Some colleges may care about admitted students' capacity in science subjects and the mismatch effect is crucial for those colleges. In contrast, other colleges may emphasize a general high level of applicants' caliber in the main tests and hence the mismatch effect is minor for them. Our analysis shows that the expanding strategy is used if the caliber effect dominates the mismatch effect. Moreover, in a decentralized college admissions scenario, when colleges have similar levels of prestige, the mismatch effect can be partially eliminated by using a conflicting strategy, such as early decision programs or simultaneous exams. This is because students only can apply to one college under the conflicting strategy and this restriction plays a role resembling that of subject tests. That is, students choose their best-fit colleges when their choices are restricted by the conflicting strategy. As a result, compared to using the expanding strategy alone, the combination of the expanding and conflicting strategies can yield a medium-sized pool

Table 1: Testing policies and rankings for selective colleges in the US in 2018

College	U.S. News Rankings	Subject Tests	Early Decision
Princeton University	1	Recommended	No
Harvard University	2	Required	No
Columbia University	3	Considered	Yes
MIT	3	Required	No
University of Chicago	3	Considered	No
Yale University	3	Recommended	No
Stanford University	7	Considered	No
Duke University	8	Recommended	Yes
University of Pennsylvania	8	Considered	Yes
Johns Hopkins University	10	Considered	Yes
Northwestern University	10	Considered	Yes

Note: The testing policies and the provisions of early decision programs are from the admissions offices at those universities in 2018.

with a larger fraction of desired applicants.

The conflicting strategy is commonly used by a second-ranked college in practice since it may attract some students of high caliber who could have been admitted to the best college if their application choice had not been restricted. The conflicting strategy is observed in simultaneously entrance examinations in Asian countries (Avery, Lee, and Roth, 2014; Chen and Kao, 2014 and 2018; Kao and Lin, 2017) and is also observed in early decision programs in the United States (Avery, Fairbanks, and Zeckhauser, 2003; Lee, 2009; Avery and Levin, 2010; Kim, 2010). In particular, Chen, Chen, and Kao (2018) demonstrate that a second-ranked college could benefit from using the conflicting strategy under certain conditions; however, the best college would suffer from losing desired students.

In our model, both the best and second-ranked colleges can gain from using the conflicting strategy. Moreover, if both colleges use a combination of the conflicting strategy and the expanding strategy, the equilibrium outcome can be efficient and stable in a decentralized college admissions scenario.[3] The reason is as follows. There are two types of students in our model: good students and ordinary students. All students are divided into two groups: the science-stream students and the humanities-stream students. A test is an imperfect device to screen students' caliber, and inefficiency occurs when ordinary

[3]Note that the expanding strategy can be used in a centralized mechanism as well as a decentralized mechanism, but the conflicting strategy can only be used in a decentralized mechanism.

students perform better than good students. This inefficiency can be reduced by using the expanding strategy because there are more good students in a larger pool of applications. However, such a reduction can create unstable matching due to the mismatch effect. The mismatch effect can be partially eliminated by the conflicting strategy. As a result, both colleges can admit more desired students from a medium-sized pool formed by students' choices and then we obtain an efficient and stable equilibrium.

In a decentralized college admissions scenario, Chade, Lewis, and Smith (2014) and Che and Koh (2016) investigate college competition by setting different admission standards and conclude that the equilibrium outcome could be inefficient and unstable for colleges and students. Our finding is different from theirs because setting admission standards is a tool to screen students given an application pool, while the expanding and conflicting strategies are used to constitute a desired application pool in the admissions process. In other words, under certain conditions, colleges can attract more desired applicants ex ante by using a bundle of the expanding strategy and the conflicting strategy, and hence the equilibrium outcome can be efficient and stable.

The rest of the paper is organized as follows. Section 2 presents the model with many students and two colleges. Section 3 solves the equilibrium in the centralized and decentralized mechanisms. Section 4 shows that the equilibrium outcome can be efficient and stable in the decentralized mechanism. Section 5 concludes.

2 Model

There is a continuum of students S and two colleges. S is a measurable set with the measure denoted by $\|S\|$. A student's caliber can be one of two types: good and ordinary. Moreover, students in high school are divided into two groups: science-stream students and humanities-stream students. Let s_{A1} and s_{A2} be the set of good and ordinary science-stream students, and s_{B1} and s_{B2} be the set of good and ordinary humanities-stream students, respectively. The measures of good and ordinary students are denoted by $\|s_{A1}\| = \|s_{B1}\| = 1$ and $\|s_{A2}\| = \|s_{B2}\| = n$, where $n \geq 1$.

2.1 Colleges and Tests

Colleges cannot observe students' types directly, and they admit students according to entrance tests. There are three entrance tests: physics, history, and math, which are denoted by $\{p, h, m\}$, respectively. The science-stream students can take the tests of physics and math, and the humanities-stream students can take the tests of history and math. Hence, we call m and $\{p,h\}$ the main test and the subject tests, respectively.[4]

There are two selective colleges $\{A,B\}$ with corresponding prestige of v_A and v_B, and their capacities of students are $q_A \in (0,1]$ and $q_B \in (0,1]$, respectively. A is the best college and hence $v_A > v_B$. Moreover, A is a science college and it chooses either $\{p,m\}$ or m to screen students. In contrast, B is a humanities college and it chooses either $\{h,m\}$ or m to screen students. We simply use pm and hm to denote $\{p,m\}$ and $\{h,m\}$, respectively. Let $(\tilde{A},\tilde{B})$ be the profile of the colleges' strategies in choosing the test. Since all students take the math test, we call $\tilde{A}=m$ and $\tilde{B}=m$ the expanding strategy.

The entrance tests are imperfect devices to screen students' caliber. Let

$$T=\{t\in\mathbf{R}^+ : \underline{t}\le t\le \bar{t}\}$$

be a set of test scores. A test score of student s on a test k is a random variable t_{ks} with the probability density function (PDF) of $f_{ks}(t)$, the cumulative distribution function (CDF) of $F_{ks}(t)$, and the conditional CDF of $F_{ks|t_{k's}\le t'}(t)$ given a test score $t_{k's}$ in which the following assumptions are satisfied.

Assumption 1. The conditional CDF is a strictly increasing function of t such that

$$F_{ks|t_{k's}\le t'}(t)=F_{k's|t_{k's}\le t'}(t)$$

for $t,t'\in T$, $s\in S$, and $k,k'\in\{pm,hm,m\}$.

Assumption 2. There is a monotone likelihood ratio property (MLRP) such that for every $t<t'$ we have

$$\frac{f_{ks_{i2}}(t)}{f_{ks_{i1}}(t)}\le\frac{f_{ks_{i2}}(t')}{f_{ks_{i1}}(t')}$$

for $i\in\{A,B\}$ and $k\in\{pm,hm\}$; for the math test, we have

$$\frac{f_{ms_{i'2}}(t)}{f_{ms_{i1}}(t)}\le\frac{f_{ms_{i'2}}(t')}{f_{ms_{i1}}(t')}$$

for $i',i\in\{A,B\}$ and

$$\frac{f_{ms_{Bj}}(t)}{f_{ms_{Aj}}(t)}\le\frac{f_{ms_{Bj}}(t')}{f_{ms_{Aj}}(t')}$$

for $j\in\{1,2\}$.

[4]In practice, the main test consists of more than one subject. For example, in China, the main tests are math, Chinese, and foreign language, and the subject tests are physics, chemistry, biology, geography, history, and politics (Chen and Kesten, 2017). If a science college requires subject tests along with the main tests, the combination should be the three main tests plus the physics, chemistry, and biology tests. To simplify notation, we use m to denote the main tests and use p and h to denote the subject tests for science-stream and humanities-stream students, respectively.

Therefore, we say that students with a score of t perform better than students with a score of t' when $t < t'$. Assumption 1 implies that, given students' caliber, their test scores have the same distribution across the tests. For example, let $t' = \bar{t}$ and then we have $f_{ks}(t) = f_{k's}(t)$. Note that science-stream (humanities-stream) students do not take the history (physics) test and hence we cannot have $t_{hms_{Aj}}$ or $t_{pms_{Bj}}$ for $j \in \{1,2\}$. The MLRP in Assumption 2 implies that good students on average have scores closer to $\underline{t}$ than that of ordinary students; moreover, given students' caliber, science-stream students in the math test tend to perform better than humanities-stream students.

2.2 Preferences

In college admissions, a matching δ is a function from students to colleges: $S \to \{\emptyset, A, B\}$ such that $\|\delta^{-1}(i)\| \leq q_i$ for $i \in \{A,B\}$. Colleges' strict preference relations over students $\{R_i\}_{i \in \{A,B\}}$ are represented by the following utility function.

$$u_i\big(\delta^{-1}(i)\big) = \sum_{j \in \{1,2\}} v_j\big(\|\delta^{-1}(i) \cap s_{ij}\| + (1-\beta)\|\delta^{-1}(i) \cap s_{i'j}\|\big)$$

for $\beta \in [0,1)$, $i, i' \in \{A,B\}$, and $i \neq i'$, where $v_1 > v_2 > 0$ because colleges prefer good students. β represents the mismatch effect for A and B. That is, given students' caliber, the science (humanities) college prefers the science-stream (humanities-stream) students.

Let $\theta_{ki}^{k'}$ be the cutoff score of students admitted by college i who use $\tilde{i} = k$ while the other college i' use $\tilde{i}' = k'$. For example, if $(\tilde{A}, \tilde{B}) = (pm, hm)$, the application pools are disjoint for A and B, and we have the expected utility of colleges as follows:

$$E[u_A|(\tilde{A},\tilde{B}) = (pm,hm)] = v_1 F_{pms_{A1}}(\theta_{pmA}^{hm}) + n v_2 F_{pms_{A2}}(\theta_{pmA}^{hm}),$$
$$E[u_B|(\tilde{A},\tilde{B}) = (pm,hm)] = v_1 F_{hms_{B1}}(\theta_{hmB}^{pm}) + n v_2 F_{hms_{B2}}(\theta_{hmB}^{pm}).$$

Let $I_{\{x\}} \in \{0,1\}$ be an indicator function of event x. Students' strict preference relations over colleges $\{\succ_s\}_{s \in S}$ are represented by a utility function:

$$g_{s_{ij}}(i') = \begin{cases} v_{i'}\big(1 - \alpha I_{\{i \neq i'\}}\big) & \text{if } \delta(s_{ij}) = i', \\ 0 & \text{if } \delta(s_{ij}) \in \emptyset, \end{cases}$$

for $i, i' \in \{A,B\}$, $\alpha \in (0,1)$, and $j \in \{1,2\}$. Note that the indicator function takes the value of 1 when a humanities-stream (science-stream) student is enrolled by the science (humanities) college, and it takes the value of 0 when a humanities-stream (science-stream) student is enrolled by the humanities (science) college. Thus, α represents the mismatch effect for students.

3 Equilibrium

In this section, we solve the equilibrium in which the expanding strategy is used by colleges under two different mechanisms in the college admissions: a centralized mechanism and a decentralized mechanism. Moreover, we also model students' application strategy in the equilibrium under the decentralized mechanism.

3.1 Centralized Mechanism

In the centralized mechanism, students in different streams take their corresponding tests and then submit their preference lists of colleges. Colleges admit students according to the scores of the required tests, and a centralized clearinghouse assigns students to colleges via the deferred acceptance algorithm of Gale and Shapley (1962). In particular, when both colleges use the expanding strategy, i.e., $(\tilde{A},\tilde{B}) = (m,m)$, the matching becomes the induced simple serial dictatorship.[5]

Students' preference lists depend on colleges' relative prestige. When $v_A(1-\alpha) > v_B$, all students prefer A to B. When $v_A(1-\alpha) < v_B$, B is the first choice for humanities-stream students. If there is no mismatch effect, colleges generally prefer good students; however, if the effect is sufficiently large, we can have $s_{A2}R_As_{B1}$ or $s_{B2}R_Bs_{A1}$. Therefore, the equilibrium in the centralized mechanism depends on the values of colleges' prestige, v_A and v_B, as well as the mismatch effects, α and β for students and colleges, respectively. We focus on the equilibrium of $(\tilde{A},\tilde{B}) = (m,m)$ here. When $v_A(1-\alpha) > v_B$, we summarize the conditions for that equilibrium in the following proposition.

Proposition 1. In the centralized mechanism, when $v_A(1-\alpha) > v_B$, $(\tilde{A},\tilde{B}) = (m,m)$ is the Nash equilibrium if

$$\beta \leq 1 - \frac{\left(f_{ms_{B1}}(\theta^m_{mA}) + nf_{ms_{B2}}(\theta^m_{mA})\right)\left(v_1 f_{ms_{A1}}(\theta^m_{mA}) + v_2 n f_{ms_{A2}}(\theta^m_{mA})\right)}{\left(f_{ms_{A1}}(\theta^m_{mA}) + nf_{ms_{A2}}(\theta^m_{mA})\right)\left(v_1 f_{ms_{B1}}(\theta^m_{mA}) + v_2 n f_{ms_{B2}}(\theta^m_{mA})\right)} \tag{1}$$

and

$$\beta \leq 1 - \frac{\left(f_{ms_{A1}}(\theta^m_{mB}) + nf_{ms_{A2}}(\theta^m_{mB})\right)\left(v_1 f_{ms_{B1}}(\theta^m_{mB}) + v_2 n f_{ms_{B2}}(\theta^m_{mB})\right)}{\left(f_{ms_{B1}}(\theta^m_{mB}) + nf_{ms_{B2}}(\theta^m_{mB})\right)\left(v_1 f_{ms_{A1}}(\theta^m_{mB}) + v_2 n f_{ms_{A2}}(\theta^m_{mB})\right)} \tag{2}$$

are satisfied.

Proof. See Appendix B. □

[5] That is, students with higher scores are assigned to their first choice till the capacity is filled, and then students with lower scores are assigned to their second choice till the capacity is filled. In such a case, Balinski and Sönmez (1999) show that the induced simple serial dictatorship is the only fair and Pareto efficient matching.

Thus, the expanding strategy is used if the mismatch effect β is sufficiently small in the centralized mechanism. We simply call $(\tilde{A},\tilde{B}) = (m,m)$ the expanding strategy equilibrium. For B, we can rewrite (2) as

$$(1-\beta)\left(\frac{v_1 f_{ms_{A1}}(\theta^m_{mB}) + v_2 n f_{ms_{A2}}(\theta^m_{mB})}{f_{ms_{A1}}(\theta^m_{mB}) + n f_{ms_{A2}}(\theta^m_{mB})}\right) \geq \left(\frac{v_1 f_{ms_{B1}}(\theta^m_{mB}) + v_2 n f_{ms_{B2}}(\theta^m_{mB})}{f_{ms_{B1}}(\theta^m_{mB}) + n f_{ms_{B2}}(\theta^m_{mB})}\right), \quad (3)$$

which means the marginal benefit of using the expanding strategy, the left-hand part of (3), should not be lower than the marginal cost, the right-hand part of (3). When using the expanding strategy, the marginal benefit comes from having additional science students with a discount as the mismatch effect, and the marginal cost is due to losing some humanities students who could have been admitted if the expanding strategy had not been used. Since the college actually expands the application pool with a fixed capacity, we have $\theta^m_{mB} \leq \theta^m_{hmB}$. That is, the number of students who can apply to B is increasing when the expanding strategy is used by B; and there must be some science students who have $t_{ms} < \theta^m_{mB}$ being enrolled by B and then $\theta^m_{mB} < \theta^m_{hmB}$, except for the case where all those science students are enrolled by A. Compared to θ^m_{hmB}, the MLRP implies that the proportion of admitted good students increases under the threshold of θ^m_{mB} given (3) and $\theta^m_{mB} < \theta^m_{hmB}$. This gain is called the caliber effect, and it is summarized in the right-hand part of (2). If the caliber effect dominates the mismatch effect, the college can have more desired students by using the expanding strategy. We can rewrite and analyze the condition for A in the same fashion, but the marginal benefit and the marginal cost comes from having additional humanities students and losing some science students, respectively.

Nevertheless, it is possible that (1) and (2) are not satisfied even if $\beta = 0$. We state the conditions for the existence of the expanding strategy equilibrium of Proposition 1 in the following corollary.

Corollary 1. When $v_A(1-\alpha) > v_B$ and β is sufficiently small, we have the expanding strategy equilibrium if

$$\frac{f_{ms_{A2}}(\theta^m_{mA})}{f_{ms_{A1}}(\theta^m_{mA})} \geq \frac{f_{ms_{B2}}(\theta^m_{mA})}{f_{ms_{B1}}(\theta^m_{mA})} \quad (4)$$

and

$$\frac{f_{ms_{A2}}(\theta^m_{mB})}{f_{ms_{A1}}(\theta^m_{mB})} \leq \frac{f_{ms_{B2}}(\theta^m_{mB})}{f_{ms_{B1}}(\theta^m_{mB})} \quad (5)$$

for A and B, respectively.

Proof. See Appendix C. □

Thus, if the conditions in Corollary 1 hold, we can have the expanding strategy equilibrium as long as β is sufficiently small such that (1) and (2) are satisfied. For B, Corollary 1 says that in Proposition 1 the marginal benefit of using the expanding strategy is larger than the marginal cost, i.e., condition (2) holds, only if the density of good science-stream students is sufficiently large at θ_{mB}^m. Similarly, for A, condition (1) can be satisfied only if the density of good humanities-stream students at θ_{mA}^m is sufficiently large. Note that the likelihood ratios of $f_{ms_{A2}}(t)/f_{ms_{A1}}(t)$ and $f_{ms_{B2}}(t)/f_{ms_{B1}}(t)$ are increasing and the good student types in the source of the marginal benefit are symmetric for A and B. Therefore, in order to satisfy the conditions in Corollary 1, we must have $\theta_{mA}^m \neq \theta_{mB}^m$, and the values of the two likelihood ratios must be crossing at some $t^* \in (\underline{t}, \bar{t})$.

Figure 2 illustrates a single-crossing example implied by Corollary 1. We have $\theta_{mB}^m > \theta_{mA}^m$ because students with $t < \theta_{mA}^m$ will attend their best choice, A. Moreover, when (4) and (5) hold, we have $\theta_{mB}^m > t^*$ because the marginal benefit can be larger than the marginal cost only if $\theta_{mB}^m \in (t^*, \bar{t})$. Symmetrically, we have $\theta_{mA}^m \in (\underline{t}, t^*)$ for the same reason. Figure 2 also reveals that the slope of $f_{ms_{A2}}(t)/f_{ms_{A1}}(t)$ must be smaller than that of $f_{ms_{B2}}(t)/f_{ms_{B1}}(t)$ when there is a single-crossing point, which implies $f_{ms_{B2}}(t)/f_{ms_{B1}}(t) < f_{ms_{A2}}(t)/f_{ms_{A1}}(t)$ for $t \in (\underline{t}, t^*)$. That is, the math test has a higher validity to screen out good humanities-stream students when $t \in (\underline{t}, t^*)$, and therefore A's marginal benefit can be larger than the marginal cost at $\theta_{mA}^m \in (\underline{t}, t^*)$. On the other hand, the math test has a lower validity to identify the types of humanities-stream students when $t \in (t^*, \bar{t})$, and thus B can have a smaller marginal cost than the marginal benefit at $\theta_{mB}^m \in (t^*, \bar{t})$.

This analysis can be easily extended to the multiple-crossing cases, and the interpretation is the same. That is, in the expanding strategy equilibrium, we must have sufficiently large proportions of the good humanities-stream students and the good science-stream students around θ_{mA}^m and θ_{mB}^m, respectively, since those two types of students are the source of the marginal benefit for the colleges.

We now state the conditions for the expanding equilibrium when B is the best choice for all humanities-stream students in the following proposition.

Proposition 2. In the centralized mechanism, when $v_A(1-\alpha) < v_B$, $(\tilde{A}, \tilde{B}) = (m, m)$ is the Nash equilibrium if

$$\beta \leq 1 - \frac{\left(f_{ms_{B1}}(\theta_{mA}^m) + n f_{ms_{B2}}(\theta_{mA}^m)\right)\left(v_1 f_{ms_{A1}}(\theta_{mA}^m) + v_2 n f_{ms_{A2}}(\theta_{mA}^m)\right)}{\left(f_{ms_{A1}}(\theta_{mA}^m) + n f_{ms_{A2}}(\theta_{mA}^m)\right)\left(v_1 f_{ms_{B1}}(\theta_{mA}^m) + v_2 n f_{ms_{B2}}(\theta_{mA}^m)\right)}$$

and

$$\beta \leq 1 - \frac{\left(f_{ms_{A1}}(\theta_{mB}^m) + n f_{ms_{A2}}(\theta_{mB}^m)\right)\left(v_1 f_{ms_{B1}}(\theta_{mB}^m) + v_2 n f_{ms_{B2}}(\theta_{mB}^m)\right)}{\left(f_{ms_{B1}}(\theta_{mB}^m) + n f_{ms_{B2}}(\theta_{mB}^m)\right)\left(v_1 f_{ms_{A1}}(\theta_{mB}^m) + v_2 n f_{ms_{A2}}(\theta_{mB}^m)\right)}$$

are satisfied.

Proof. See Appendix D. □

Proposition 2 seems the same as Proposition 1, but their cutoff scores are different. For B, the cutoff score θ_{mB}^m in Proposition 2 should be smaller than that in Proposition

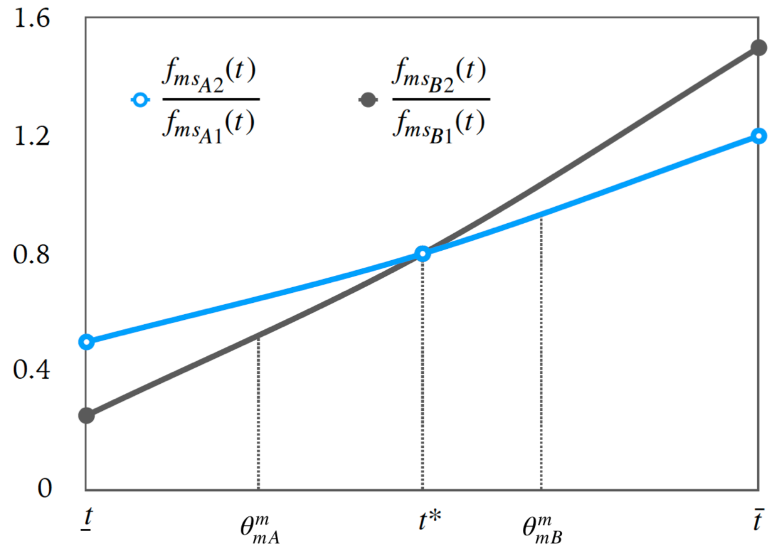

Figure 2: Likelihood ratios of test scores implied by Corollary 1

1, because some good humanities-stream students with scores lower than θ^m_{mA} will still take B as their best choice when $v_A(1-\alpha) < v_B$. On the other hand, the cutoff score θ^m_{mA} in Proposition 2 should be larger than that in Proposition 1 for the same reason. However, in the expanding strategy equilibrium, we still have $\theta^m_{mB} > \theta^m_{mA}$ when $v_A(1-\alpha) < v_B$. This is because all science students with $t \leq \theta^m_{mA}$ will prefer A, and hence B has no marginal benefit from using the expanding strategy when $\theta^m_{mB} < \theta^m_{mA}$. Therefore, given the corresponding cutoff scores, we also have the same conditions of (4) and (5) in Corollary 1 for the existence of the expanding strategy equilibrium in Proposition 2.

Corollary 2. When $v_A(1-\alpha) < v_B$ and β is sufficiently small, we have the expanding strategy equilibrium if

$$\frac{f_{ms_{A2}}(\theta^m_{mA})}{f_{ms_{A1}}(\theta^m_{mA})} \geq \frac{f_{ms_{B2}}(\theta^m_{mA})}{f_{ms_{B1}}(\theta^m_{mA})}$$

and

$$\frac{f_{ms_{A2}}(\theta^m_{mB})}{f_{ms_{A1}}(\theta^m_{mB})} \leq \frac{f_{ms_{B2}}(\theta^m_{mB})}{f_{ms_{B1}}(\theta^m_{mB})}$$

for A and B, respectively.

Proof. The same proof as Corollary 1. $\square$

In sum, the equilibrium in the centralized mechanism depends on the value of the mismatch effect, the relative numbers of good and ordinary students, and the cutoff scores. If the mismatch effect is minor, both colleges will use the expanding strategy of choosing the math test. If the mismatch effect is large, colleges care much about students' capacity in corresponding subjects and hence choose the *pm* and *hm* tests.

3.2 Decentralized Mechanism

In the decentralized mechanism, the admissions processes are individually conducted by colleges. In addition to choosing the tests, colleges also have to decide whether to use the conflicting strategy or not. When the conflicting strategy is used by colleges, students' choices are restricted to applying to one of them, and this is a scenario of simultaneous entrance examinations in Asian countries as well as early decision programs in the United States. Therefore, we need to consider students' application strategies when the conflicting strategy is used by colleges. For simplicity, we assume that students with a type of $i \in \{A, B\}$ and $j \in \{1, 2\}$ will use the same strategy in the equilibrium. Specifically, let $\tilde{s}_{ij} \in [0, 1]$ be the probability of applying to B for $i \in \{A, B\}$ and $j \in \{1, 2\}$ when the conflicting strategy is used in the decentralized mechanism. For example, $(\tilde{s}_{A1}, \tilde{s}_{A2}, \tilde{s}_{B1}, \tilde{s}_{B2}) = (0, 0, 1, 1)$ means that all science-stream students apply to A and all humanities-stream students apply to B.

In a discrete model with good and ordinary student types, Chen and Kao (2014) show that the conflicting strategy is used when the prestige of the second-best college is close to that of the best college in the decentralized mechanism. In the following analysis, we show a more general result in our model consisting of a continuum of students with four types. Specifically, we show conditions for the decentralized equilibrium in which colleges use the combination of the conflicting strategy and the expanding strategy. Note that if colleges use the expanding strategy but do not use the conflicting strategy in the decentralized mechanism, students can apply to two colleges, and the equilibrium outcome will be the same as the centralized one along with the expanding strategy. Moreover, if colleges do not use the expanding strategy in the decentralized mechanism, students cannot apply to another stream college, and then the equilibrium outcome will also be the same as the centralized one without the expanding strategy. Therefore, we focus on the equilibrium such that both colleges use the conflicting strategy as well as the expanding strategy.

Since the incentive for B to use the conflicting strategy is that the marginal benefit of attracting more good science-stream students is larger than the marginal cost, in the following lemma we prove the condition in which some good science-stream students will apply to B when both colleges use the conflicting strategy and the expanding strategy.

Lemma 1. In the decentralized mechanism, if both colleges use the conflicting strategy and the expanding strategy and

$$v_B > \frac{v_A F_{s_{A1}}(\bar{\theta}^m_{mA})}{(1-\alpha) F_{s_{A1}}(\bar{\theta}^m_{mB})}, \tag{6}$$

then good science-stream students will use a mixed strategy of $\tilde{s}_{A1} = \pi \in (0, 1)$, where

$\bar{\theta}_{mi}^{m}$ is the cutoff score for $i \in \{A, B\}$ when $(\tilde{s}_{A1}, \tilde{s}_{A2}, \tilde{s}_{B1}, \tilde{s}_{B2}) = (0, 1, 1, 1)$.

Proof. See Appendix E. $\square$

Lemma 1 says that if B's prestige v_B is sufficiently large, there could be some good science students applying to B. Given (6), we show the condition in which the marginal benefit is larger than the marginal cost such that colleges would use the conflicting strategy and the expanding strategy in the equilibrium.

Proposition 3. In the decentralized mechanism, when (6) holds and

$$\beta \leq 1 - \frac{\left(\pi f_{ms_{A1}}(\theta_{mB}^{m}) + n f_{ms_{A2}}(\theta_{mB}^{m})\right)\left(v_1 f_{ms_{B1}}(\theta_{mB}^{m}) + v_2 n f_{ms_{B2}}(\theta_{mB}^{m})\right)}{\left(f_{ms_{B1}}(\theta_{mB}^{m}) + n f_{ms_{B2}}(\theta_{mB}^{m})\right)\left(v_1 \pi f_{ms_{A1}}(\theta_{mB}^{m}) + v_2 n f_{ms_{A2}}(\theta_{mB}^{m})\right)}, \tag{7}$$

the Nash equilibrium is that A and B use the conflicting strategy as well as the expanding strategy and $(\tilde{s}_{A1}, \tilde{s}_{A2}, \tilde{s}_{B1}, \tilde{s}_{B2}) = (\pi, 1, 1, 1)$, where $\pi \in (0, 1)$.

Proof. See Appendix F. $\square$

Proposition 3 says that when B's prestige is sufficiently large and the mismatch effect β is sufficiently small, both colleges will use the combination of the conflicting strategy and the expanding strategy in the equilibrium. In the following section, we show that the result of Proposition 3 is stable and efficient.

4 Stability and Efficiency

The efficiency and stability are defined as follows. A matching δ is efficient if it is not Pareto dominated by another matching; that is, there is no matching σ such that $\| i \in \{\sigma(i) \succ_i \delta(i)\} \| > 0$ or $\sigma^{-1}(j) R_j \delta^{-1}(j)$ for some $j \in \{A, B\}$ without harming others' welfare. A matching δ is stable if there is no matching σ in which the measure of students such that $\sigma(i) \succ_i \delta(i)$, $i R_j i'$ for $j = \sigma(i)$, $i \notin \{\delta^{-1}(j)\}$, and $i' \in \{\delta^{-1}(j)\}$ for $j \in \{A, B\}$ is greater than zero.

According to Lemma 1 and Proposition 3, when college B's prestige is higher than the threshold as that in the right hand part of (6) and (7) holds, the conflicting strategy is used by the two colleges and the math test is also chosen by them, i.e., both colleges use the combination of the conflicting strategy and the expanding strategy. Moreover, the strategy of students is $(\tilde{s}_{A1}, \tilde{s}_{A2}, \tilde{s}_{B1}, \tilde{s}_{B2}) = (\pi, 1, 1, 1)$ in the equilibrium. Therefore, the science college can admit the good science-stream students for sure, and the humanities college also maximizes its expected utility in the equilibrium. Since college A admits its most desired students who also prefer A to B and there is no seat left for students, this matching outcome is efficient and stable. In this situation, the mismatch effect is partially eliminated by using the conflicting strategy since there are no humanities-stream students applying to A. In other words, students only can apply to one college under the conflicting strategy, and this restriction plays a role resembling that of subject tests. That is, only students who are good at the *pm* tests apply to college A, even though the subject tests are not required. As a result, combining the expanding strategy and the conflicting strategy can yield an efficient and stable matching.

This result is related to some labor markets in which firms compete for desired workers by offering early contracts in order to restrict workers' choices (Li and Rosen, 1998; Li and Suen, 2000 and 2004; Suen, 2000); this phenomenon is extensively studied and called unraveling in the literature (Sönmez, 1999; Ünver, 2001; Kagel and Roth, 2000; Ünver, 2005; Haruvy, Roth, and Ünver, 2006; Niederle and Roth, 2009; Niederle, Roth, and Ünver, 2013; Echenique and Pereyra, 2016; Pan, 2018). Inefficiency occurs in those markets because information about workers is not fully revealed, and this inefficiency could be reduced by a centralized clearinghouse (Roth and Xing, 1994). A successful clearinghouse could produce a stable matching (Niederle, McKinney, and Roth, 2005) and expand the market scope (Niederle and Roth, 2003). In our analysis, a parallel result is found in a decentralized college admissions scenario such that an efficient and stable matching could be obtained by restricting applicants' choices in an expanded pool of applications.

5 Concluding Remarks

In this study, we investigate a college admissions problem in which colleges may benefit from using the expanding strategy to enlarge their potential pool of applicants. However, a large pool also brings a mismatch cost such as a science college admitting some good students who actually are good at humanities subjects. We call such a benefit and a cost the caliber effect and the mismatch effect, respectively. Our analysis shows that in a centralized admissions process, the expanding strategy is used when the caliber effect is larger than the mismatch effect. Moreover, in a decentralized admissions process, the mismatch effect may be partially eliminated by using the conflicting strategy, e.g., the same examination date or an early decision program. Colleges could use the conflicting strategy to screen students because students have to choose their best-fit college before they apply. As a result, combining the expanding strategy and the conflicting strategy can yield an efficient and stable equilibrium.

There remain three important issues. First, if we can find a proxy to measure the mismatch effect, we can empirically test the model prediction. Second, in a decentralized admissions process, if the conflicting strategy is used by colleges, students' behavior in the application game can be tested by conducting an experimental study. Third, this study focuses on college admissions competition between two selective colleges, and a more general model consisting of three or more colleges may have additional implications about the competition among many colleges. Those issues are left for future research.

Appendix A. College Admissions in Taiwan

In Taiwan, there are two major mechanisms in college admissions: a centralized mechanism called the examination channel and a decentralized mechanism called the application channel (Li, Lee, and Lien, 2016; Luoh, 2018). In the centralized mechanism, students have to take a standardized test given by the college entrance examination center (CEEC). After the CEEC reports the test scores, students submit their preference lists of colleges and colleges enroll students via the deferred acceptance algorithm of Gale and Shapley (1962).

The decentralized mechanism consists of two steps. In the first step, colleges screen students based on application documents as well as scores from another standardized test given by the CEEC. In the second step, the invited students have to attend colleges for interviews or other individual exams implemented by the colleges. If two colleges strategically decide to have the same exam date in the second step, students only can apply to one of them. Since this strategy actually limits the applicant pool, we call it the conflicting strategy.

In those two mechanisms in Taiwan, colleges have to decide the combination of required subject tests and the relative weights of those subjects in a standardized test. The combination can be changed every year. For example, in 2010 the Department of Finance at Tamkang University changed the combination from main tests (Chinese, English, and math B) and subject tests (history and geography) to only main tests. This action potentially expands the applicant pool because the science-stream students in high schools who do not take the history and geography tests now can apply to that department. On the other hand, the humanities-stream students who take those subject tests still can apply to that department. Thus, we call this policy, only requiring students to take main tests, the expanding strategy in the paper.

Appendix B. Proof of Proposition 1

In this proof, we show that A and B will use the expanding strategy as a Nash equilibrium if the mismatch effect β is sufficiently small. Specifically, we provide sufficient conditions for A and B to satisfy $E[u_A|(\tilde{A},\tilde{B})=(m,m)] \geq E[u_A|(\tilde{A},\tilde{B})=(pm,m)]$ and $E[u_B|(\tilde{A},\tilde{B})=(m,m)] \geq E[u_B|(\tilde{A},\tilde{B})=(m,hm)]$, respectively.

Sufficient Conditions for *B*

If $(\tilde{A},\tilde{B})=(m,hm)$ and $v_A(1-\alpha)>v_B$, only humanities students can apply to B, but some of those students could be admitted by A if their math scores are larger or equal to θ_{mA}^{hm}. If $(\tilde{A},\tilde{B})=(m,m)$, all students can apply to A and B, but A is the first choice for students; in this situation B may enroll some science students by replacing some humanities students who would have been admitted when $(\tilde{A},\tilde{B})=(pm,hm)$. Therefore, given $\tilde{A}=m$ and $F_{ms_{Ai}|t_{ms_{Ai}}\leq\theta_{mB}^m}(\theta_{mA}^m)<1$ for $i\in\{1,2\}$, the marginal benefit for B to use the expanding strategy is

$$\begin{aligned}(1-\beta)\Big(&v_1F_{ms_{A1}}(\theta_{mB}^m)\big(1-F_{ms_{A1}|t_{ms_{A1}}\leq\theta_{mB}^m}(\theta_{mA}^m)\big)+\\&v_2nF_{ms_{A2}}(\theta_{mB}^m)\big(1-F_{ms_{A2}|t_{ms_{A2}}\leq\theta_{mB}^m}(\theta_{mA}^m)\big)\Big)\end{aligned}\tag{B1}$$

and the marginal cost is

$$\begin{aligned}&v_1\big(F_{hms_{B1}|t_{ms_{B1}}>\theta_{mA}^{hm}}(\theta_{hmB}^m)-F_{ms_{B1}|t_{ms_{B1}}>\theta_{mA}^m}(\theta_{mB}^m)\big)+\\&\quad v_2n\big(F_{hms_{B2}|t_{ms_{B2}}>\theta_{mA}^{hm}}(\theta_{hmB}^m)-F_{ms_{B2}|t_{ms_{B1}}>\theta_{mA}^m}(\theta_{mB}^m)\big),\end{aligned}$$

which is equal to

$$\begin{aligned}&v_1\big(F_{ms_{B1}|t_{ms_{B1}}>\theta_{mA}^m}(\theta_{hmB}^m)-F_{ms_{B1}|t_{ms_{B1}}>\theta_{mA}^m}(\theta_{mB}^m)\big)+\\&\quad v_2n\big(F_{ms_{B2}|t_{ms_{B2}}>\theta_{mA}^m}(\theta_{hmB}^m)-F_{ms_{B2}|t_{ms_{B1}}>\theta_{mA}^m}(\theta_{mB}^m)\big),\end{aligned}\tag{B2}$$

by Assumption 1 and the fact that A is the first choice for all students and hence $\theta_{mA}^{hm}=\theta_{mA}^m$ in this case. Note that if $F_{ms_{Ai}|t_{ms_{Ai}}\leq\theta_{mB}^m}(\theta_{mA}^m)=1$ for $i\in\{1,2\}$, the marginal benefit is zero and B is indifferent between $(\tilde{A},\tilde{B})=(m,m)$ and $(\tilde{A},\tilde{B})=(m,hm)$. We thus only need to solve conditions for (B1) $\geq$ (B2) in cases of $F_{ms_{Ai}|t_{ms_{Ai}}\leq\theta_{mB}^m}(\theta_{mA}^m)<1$ for $i\in\{1,2\}$. Since

$$F_{ms_{Ai}|t_{ms_{Ai}}\leq\theta_{mB}^m}(\theta_{mA}^m)=\frac{F_{ms_{Ai}}(\theta_{mA}^m)}{F_{ms_{Ai}}(\theta_{mB}^m)}$$

for $i\in\{1,2\}$, (B1) can be rewritten as

$$(1-\beta)\Big(v_1\big(F_{ms_{A1}}(\theta_{mB})-F_{ms_{A1}}(\theta_{mA})\big)+v_2n\big(F_{ms_{A2}}(\theta_{mB})-F_{ms_{A2}}(\theta_{mA})\big)\Big).\tag{B3}$$

We have $\theta_{mA} < \theta_{mB}$ because the marginal benefit (B3) is positive. Thus, (B3) is equal to

$$\begin{aligned}
&\int_{\theta_{mA}}^{\theta_{mB}} (1-\beta)\big(v_1 f_{ms_{A1}}(t) + v_2 n f_{ms_{A2}}(t)\big)\,dt \\
&= \int_{\theta_{mA}^m}^{\theta_{mB}^m} (1-\beta)\left(v_1 + v_2 - v_2 \times \frac{1+\frac{v_1 n f_{ms_{A2}}(t)}{v_2 f_{ms_{A1}}(t)}}{1+\frac{n f_{ms_{A2}}(t)}{f_{ms_{A1}}(t)}}\right)\big(f_{ms_{A1}}(t) + n f_{ms_{A2}}(t)\big)\,dt \\
&\geq \int_{\theta_{mA}^m}^{\theta_{mB}^m} (1-\beta)\left(v_1 + v_2 - v_2 \times \frac{1+\frac{v_1 n f_{ms_{A2}}(\theta_{mB}^m)}{v_2 f_{ms_{A1}}(\theta_{mB}^m)}}{1+\frac{n f_{ms_{A2}}(\theta_{mB}^m)}{f_{ms_{A1}}(\theta_{mB}^m)}}\right)\big(f_{ms_{A1}}(t) + n f_{ms_{A2}}(t)\big)\,dt \\
&= (1-\beta)\left(\frac{v_1 f_{ms_{A1}}(\theta_{mB}^m) + v_2 n f_{ms_{A2}}(\theta_{mB}^m)}{f_{ms_{A1}}(\theta_{mB}^m) + n f_{ms_{A2}}(\theta_{mB}^m)}\right)\int_{\theta_{mA}^m}^{\theta_{mB}^m} \big(f_{ms_{A1}}(t) + n f_{ms_{A2}}(t)\big)\,dt
\end{aligned}$$

because $f_{ms_{A2}}(t)/f_{ms_{A1}}(t)$ is increasing in t by the MLRP and $v_1 > v_2 > 0$. Similarly, by Assumptions 1 and 2, (B2) can be rewritten as

$$\begin{aligned}
&v_1\Big(\big(F_{ms_{B1}}(\theta_{hmB}^m) - F_{ms_{B1}}(\theta_{mA}^m)\big) - \big(F_{ms_{B1}}(\theta_{mB}^m) - F_{ms_{B1}}(\theta_{mA}^m)\big)\Big) \\
&+ v_2 n\Big(\big(F_{ms_{B2}}(\theta_{hmB}^m) - F_{ms_{B2}}(\theta_{mA}^m)\big) - \big(F_{ms_{B2}}(\theta_{mB}^m) - F_{ms_{B2}}(\theta_{mA}^m)\big)\Big) \\
&= v_1\big(F_{ms_{B1}}(\theta_{hmB}^m) - F_{ms_{B1}}(\theta_{mB}^m)\big) + v_2 n\big(F_{ms_{B2}}(\theta_{hmB}^m) - F_{ms_{B2}}(\theta_{mB}^m)\big) \\
&= \int_{\theta_{mB}^m}^{\theta_{hmB}^m} \big(v_1 f_{ms_{B1}}(t) + v_2 n f_{ms_{B2}}(t)\big)\,dt \\
&\leq \left(\frac{v_1 f_{ms_{B1}}(\theta_{mB}^m) + v_2 n f_{ms_{B2}}(\theta_{mB}^m)}{f_{ms_{B1}}(\theta_{mB}^m) + n f_{ms_{B2}}(\theta_{mB}^m)}\right)\int_{\theta_{mB}^m}^{\theta_{hmB}^m} \big(f_{ms_{B1}}(t) + n f_{ms_{B2}}(t)\big)\,dt.
\end{aligned}$$

The capacity constraint ensures that

$$\int_{\theta_{mA}^m}^{\theta_{mB}^m} \big(f_{ms_{A1}}(t) + n f_{ms_{A2}}(t)\big)dt = \int_{\theta_{mB}^m}^{\theta_{hmB}^m} \big(f_{ms_{B1}}(t) + n f_{ms_{B2}}(t)\big)dt.$$

Thus, the sufficient condition for (B1) $\geq$ (B2) is

$$(1-\beta)\left(\frac{v_1 f_{ms_{A1}}(\theta_{mB}^m) + v_2 n f_{ms_{A2}}(\theta_{mB}^m)}{f_{ms_{A1}}(\theta_{mB}^m) + n f_{ms_{A2}}(\theta_{mB}^m)}\right) \geq \left(\frac{v_1 f_{ms_{B1}}(\theta_{mB}^m) + v_2 n f_{ms_{B2}}(\theta_{mB}^m)}{f_{ms_{B1}}(\theta_{mB}^m) + n f_{ms_{B2}}(\theta_{mB}^m)}\right)$$

which can be rearranged as

$$\beta \leq 1 - \frac{\big(f_{ms_{A1}}(\theta_{mB}^m) + n f_{ms_{A2}}(\theta_{mB}^m)\big)\big(v_1 f_{ms_{B1}}(\theta_{mB}^m) + v_2 n f_{ms_{B2}}(\theta_{mB}^m)\big)}{\big(f_{ms_{B1}}(\theta_{mB}^m) + n f_{ms_{B2}}(\theta_{mB}^m)\big)\big(v_1 f_{ms_{A1}}(\theta_{mB}^m) + v_2 n f_{ms_{A2}}(\theta_{mB})\big)}. \tag{B4}$$

Sufficient Conditions for A

We use the same method to prove conditions for A. Since $v_A(1-\alpha) > v_B$, all humanities-stream students will apply to A and take it as the first choice when $\tilde{A} = m$. For A, the marginal benefit of using the expanding strategy is

$$(1-\beta)\big(v_1 F_{ms_{B1}}(\theta^m_{mA}) + v_2 n F_{ms_{B2}}(\theta^m_{mA})\big) \tag{B5}$$

and the marginal cost is

$$v_1\big(F_{pms_{A1}}(\theta^m_{pmA}) - F_{ms_{A1}}(\theta^m_{mA})\big) + v_2 n\big(F_{pms_{A2}}(\theta^m_{pmA}) - F_{ms_{A2}}(\theta^m_{mA})\big). \tag{B6}$$

By Assumptions 1 and 2, (B5) is equal to

$$\begin{aligned}
&\int_{\underline{t}}^{\theta^m_{mA}} (1-\beta)\big(v_1 f_{ms_{B1}}(t) + v_2 n f_{ms_{B2}}(t)\big)\, dt \\
&\geq (1-\beta)\left(\frac{v_1 f_{ms_{B1}}(\theta^m_{mA}) + v_2 n f_{ms_{B2}}(\theta^m_{mA})}{f_{ms_{B1}}(\theta^m_{mA}) + n f_{ms_{B2}}(\theta^m_{mA})}\right) \int_{\underline{t}}^{\theta^m_{mA}} \big(f_{ms_{B1}}(t) + n f_{ms_{B2}}(t)\big)\, dt
\end{aligned}$$

and (B6) is equal to

$$\begin{aligned}
&v_1\big(F_{ms_{A1}}(\theta^m_{pmA}) - F_{ms_{A1}}(\theta^m_{mA})\big) + v_2 n\big(F_{ms_{A2}}(\theta^m_{pmA}) - F_{ms_{A2}}(\theta^m_{mA})\big) \\
&= \int_{\theta^m_{mA}}^{\theta^m_{pmA}} \big(v_1 f_{ms_{A1}}(t) + v_2 n f_{ms_{A2}}(t)\big)\, dt \\
&\leq \left(\frac{v_1 f_{ms_{A1}}(\theta^m_{mA}) + v_2 n f_{ms_{A2}}(\theta^m_{mA})}{f_{ms_{A1}}(\theta^m_{mA}) + n f_{ms_{A2}}(\theta^m_{mA})}\right) \int_{\theta^m_{mA}}^{\theta^m_{pmA}} \big(f_{ms_{A1}}(t) + n f_{ms_{A2}}(t)\big)\, dt.
\end{aligned}$$

The capacity constraint ensures that

$$\int_{\underline{t}}^{\theta^m_{mA}} \big(f_{ms_{B1}}(t) + n f_{ms_{B2}}(t)\big) dt = \int_{\theta^m_{mA}}^{\theta^m_{pmA}} \big(f_{ms_{A1}}(t) + n f_{ms_{A2}}(t)\big) dt$$

Thus, the sufficient condition for (B5) $\geq$ (B6) is

$$(1-\beta)\left(\frac{v_1 f_{ms_{B1}}(\theta^m_{mA}) + v_2 n f_{ms_{B2}}(\theta^m_{mA})}{f_{ms_{B1}}(\theta^m_{mA}) + n f_{ms_{B2}}(\theta^m_{mA})}\right) \geq \left(\frac{v_1 f_{ms_{A1}}(\theta^m_{mA}) + v_2 n f_{ms_{A2}}(\theta^m_{mA})}{f_{ms_{A1}}(\theta^m_{mA}) + n f_{ms_{A2}}(\theta^m_{mA})}\right),$$

which can be rearranged as

$$\beta \leq 1 - \frac{\big(f_{ms_{B1}}(\theta^m_{mA}) + n f_{ms_{B2}}(\theta^m_{mA})\big)\big(v_1 f_{ms_{A1}}(\theta^m_{mA}) + v_2 n f_{ms_{A2}}(\theta^m_{mA})\big)}{\big(f_{ms_{A1}}(\theta^m_{mA}) + n f_{ms_{A2}}(\theta^m_{mA})\big)\big(v_1 f_{ms_{B1}}(\theta^m_{mA}) + v_2 n f_{ms_{B2}}(\theta^m_{mA})\big)}. \tag{B7}$$

Therefore, A and B will use the expanding strategy if (B4) and (B7) are satisfied. Q.E.D.

Appendix C. Proof of Corollary 1

We can rewrite (B4) as

$$(1-\beta)\left(v_1 + v_2 - v_2 \times \frac{1 + \frac{v_1 n f_{ms_{A2}}(\theta^m_{mB})}{v_2 f_{ms_{A1}}(\theta^m_{mB})}}{1 + \frac{n f_{ms_{A2}}(\theta^m_{mB})}{f_{ms_{A1}}(\theta^m_{mB})}}\right) \geq \left(v_1 + v_2 - v_2 \times \frac{1 + \frac{v_1 n f_{ms_{B2}}(\theta^m_{mB})}{v_2 f_{ms_{B1}}(\theta^m_{mB})}}{1 + \frac{n f_{ms_{B2}}(\theta^m_{mB})}{f_{ms_{B1}}(\theta^m_{mB})}}\right). \tag{C1}$$

Since v_1, v_2, and n are strictly positive, (C1) holds for a nonnegative β if

$$v_1 + v_2 - v_2 \times \frac{\frac{v_2}{v_1 n} + \frac{f_{ms_{A2}}(\theta^m_{mB})}{f_{ms_{A1}}(\theta^m_{mB})}}{\frac{1}{n} + \frac{f_{ms_{A2}}(\theta^m_{mB})}{f_{ms_{A1}}(\theta^m_{mB})}} \geq v_1 + v_2 - v_2 \times \frac{\frac{v_2}{v_1 n} + \frac{f_{ms_{B2}}(\theta^m_{mB})}{f_{ms_{B1}}(\theta^m_{mB})}}{\frac{1}{n} + \frac{f_{ms_{B2}}(\theta^m_{mB})}{f_{ms_{B1}}(\theta^m_{mB})}},$$

which is equivalent to

$$\frac{\frac{v_2}{v_1 n} + \frac{f_{ms_{A2}}(\theta^m_{mB})}{f_{ms_{A1}}(\theta^m_{mB})}}{\frac{1}{n} + \frac{f_{ms_{A2}}(\theta^m_{mB})}{f_{ms_{A1}}(\theta^m_{mB})}} \leq \frac{\frac{v_2}{v_1 n} + \frac{f_{ms_{B2}}(\theta^m_{mB})}{f_{ms_{B1}}(\theta^m_{mB})}}{\frac{1}{n} + \frac{f_{ms_{B2}}(\theta^m_{mB})}{f_{ms_{B1}}(\theta^m_{mB})}}. \tag{C2}$$

Using cross multiplication to (C2) yields

$$\frac{1}{n}\left(\frac{f_{ms_{A2}}(\theta^m_{mB})}{f_{ms_{A1}}(\theta^m_{mB})} - \frac{f_{ms_{B2}}(\theta^m_{mB})}{f_{ms_{B1}}(\theta^m_{mB})}\right) \leq \frac{v_2}{v_1 n}\left(\frac{f_{ms_{A2}}(\theta^m_{mB})}{f_{ms_{A1}}(\theta^m_{mB})} - \frac{f_{ms_{B2}}(\theta^m_{mB})}{f_{ms_{B1}}(\theta^m_{mB})}\right) \tag{C3}$$

Since $v_1 > v_2$ and $n > 0$, the necessary condition for (C3) becomes

$$\frac{f_{ms_{A2}}(\theta^m_{mB})}{f_{ms_{A1}}(\theta^m_{mB})} \leq \frac{f_{ms_{B2}}(\theta^m_{mB})}{f_{ms_{B1}}(\theta^m_{mB})}. \tag{C4}$$

Similarly, the necessary condition for (B7) is

$$\frac{f_{ms_{A2}}(\theta^m_{mA})}{f_{ms_{A1}}(\theta^m_{mA})} \geq \frac{f_{ms_{B2}}(\theta^m_{mA})}{f_{ms_{B1}}(\theta^m_{mA})}. \tag{C5}$$

Therefore, when (C4) and (C5) hold, we have the expanding strategy equilibrium as long as β is sufficiently small. Q.E.D.

Appendix D. Proof of Proposition 2

In this proof, we show that A and B will use the expanding strategy as a Nash equilibrium if the mismatch effect β is sufficiently small when $v_A(1-\alpha) < v_B$, i.e., B is the first choice for humanities-stream students.

Sufficient Conditions for *B*

Given $\tilde{A} = m$ and $v_A(1-\alpha) < v_B$, the marginal benefit and the marginal cost for B to use the expanding strategy are, respectively,

$$\begin{aligned}(1-\beta)\Big(& v_1 F_{ms_{A1}}(\theta^m_{mB})\big(1 - F_{ms_{A1}|t_{ms_{A1}} \leq \theta^m_{mB}}(\theta^m_{mA})\big) + \\ & v_2 n F_{ms_{A2}}(\theta^m_{mB})\big(1 - F_{ms_{A2}|t_{ms_{A2}} \leq \theta^m_{mB}}(\theta^m_{mA})\big)\Big)\end{aligned} \tag{D1}$$

and

$$v_1\big(F_{hms_{B1}}(\theta^m_{hmB})-F_{ms_{B1}}(\theta^m_{mB})\big)+v_2n\big(F_{hms_{B2}}(\theta^m_{hmB})-F_{ms_{B2}}(\theta^m_{mB})\big). \tag{D2}$$

Note that (D1) is the same as (B1), and hence it can be rewritten as

$$\begin{aligned}
&\int_{\theta_{mA}}^{\theta_{mB}}(1-\beta)\big(v_1f_{ms_{A1}}(t)+v_2nf_{ms_{A2}}(t)\big)\,dt\\
&\geq(1-\beta)\left(\frac{v_1f_{ms_{A1}}(\theta^m_{mB})+v_2nf_{ms_{A2}}(\theta^m_{mB})}{f_{ms_{A1}}(\theta^m_{mB})+nf_{ms_{A2}}(\theta^m_{mB})}\right)\int_{\theta^m_{mA}}^{\theta^m_{mB}}\big(f_{ms_{A1}}(t)+nf_{ms_{A2}}(t)\big)\,dt
\end{aligned}$$

Similarly, (D2) can be rewritten as

$$\begin{aligned}
&\int_{\theta^m_{mB}}^{\theta^m_{hmB}}\big(v_1f_{ms_{B1}}(t)+v_2nf_{ms_{B2}}(t)\big)\,dt\\
&\leq\left(\frac{v_1f_{ms_{B1}}(\theta^m_{mB})+v_2nf_{ms_{B2}}(\theta^m_{mB})}{f_{ms_{B1}}(\theta^m_{mB})+nf_{ms_{B2}}(\theta^m_{mB})}\right)\int_{\theta^m_{mB}}^{\theta^m_{hmB}}\big(f_{ms_{B1}}(t)+nf_{ms_{B2}}(t)\big)\,dt.
\end{aligned}$$

The capacity constraint ensures that

$$\int_{\theta^m_{mA}}^{\theta^m_{mB}}\big(f_{ms_{A1}}(t)+nf_{ms_{A2}}(t)\big)dt=\int_{\theta^m_{mB}}^{\theta^m_{hmB}}\big(f_{ms_{B1}}(t)+nf_{ms_{B2}}(t)\big)dt$$

Thus, the sufficient condition for $E[u_B|(\tilde{A},\tilde{B})=(m,m)]\geq E[u_B|(\tilde{A},\tilde{B})=(m,hm)]$ in this case is

$$(1-\beta)\left(\frac{v_1f_{ms_{A1}}(\theta^m_{mB})+v_2nf_{ms_{A2}}(\theta^m_{mB})}{f_{ms_{A1}}(\theta^m_{mB})+nf_{ms_{A2}}(\theta^m_{mB})}\right)\geq\left(\frac{v_1f_{ms_{B1}}(\theta^m_{mB})+v_2nf_{ms_{B2}}(\theta^m_{mB})}{f_{ms_{B1}}(\theta^m_{mB})+nf_{ms_{B2}}(\theta^m_{mB})}\right),$$

which can be rearranged as

$$\beta\leq1-\frac{\big(f_{ms_{A1}}(\theta^m_{mB})+nf_{ms_{A2}}(\theta^m_{mB})\big)\big(v_1f_{ms_{B1}}(\theta^m_{mB})+v_2nf_{ms_{B2}}(\theta^m_{mB})\big)}{\big(f_{ms_{B1}}(\theta^m_{mB})+nf_{ms_{B2}}(\theta^m_{mB})\big)\big(v_1f_{ms_{A1}}(\theta^m_{mB})+v_2nf_{ms_{A2}}(\theta_{mB})\big)}. \tag{D3}$$

Sufficient Conditions for A

Given $\tilde{B}=m$, the science-stream students can apply to A and B. However, A is the best choice for those students. In this case, the marginal benefit and the marginal cost for A to use the expanding strategy are exactly the same as (B5) and (B6), respectively. Thus, the sufficient condition for $E[u_A|(\tilde{A},\tilde{B})=(m,m)]\geq E[u_A|(\tilde{A},\tilde{B})=(pm,m)]$ in this case is also

$$\beta\leq1-\frac{\big(f_{ms_{B1}}(\theta^m_{mA})+nf_{ms_{B2}}(\theta^m_{mA})\big)\big(v_1f_{ms_{A1}}(\theta^m_{mA})+v_2nf_{ms_{A2}}(\theta^m_{mA})\big)}{\big(f_{ms_{A1}}(\theta^m_{mA})+nf_{ms_{A2}}(\theta^m_{mA})\big)\big(v_1f_{ms_{B1}}(\theta^m_{mA})+v_2nf_{ms_{B2}}(\theta^m_{mA})\big)}. \tag{D4}$$

Therefore, A and B will use the expanding strategy if (D3) and (D4) are satisfied. Q.E.D.

Appendix E. Proof of Lemma 1

Good and ordinary humanities-stream students will apply to B when

$$v_B F_{s_{B1}}(\theta^m_{mB}) > (1-\alpha)v_A F_{s_{B1}}(\theta^m_{mA}) \tag{E1}$$

and

$$v_B F_{s_{B2}}(\theta^m_{mB}) > (1-\alpha)v_A F_{s_{B2}}(\theta^m_{mA}) \tag{E2}$$

are satisfied, respectively. (E1) and (E2) can be rewritten as

$$v_B > \frac{(1-\alpha)v_A F_{s_{B1}}(\theta^m_{mA})}{F_{s_{B1}}(\theta^m_{mB})} \tag{E3}$$

and

$$v_B > \frac{(1-\alpha)v_A F_{s_{B2}}(\theta^m_{mA})}{F_{s_{B2}}(\theta^m_{mB})}. \tag{E4}$$

Symmetrically, good and ordinary science-stream students will apply to B when

$$v_B > \frac{v_A F_{s_{A1}}(\theta^m_{mA})}{(1-\alpha)F_{s_{A1}}(\theta^m_{mB})} \tag{E5}$$

and

$$v_B > \frac{v_A F_{s_{A2}}(\theta^m_{mA})}{(1-\alpha)F_{s_{A2}}(\theta^m_{mB})} \tag{E6}$$

are satisfied, respectively.

Given θ^m_{mA} and θ^m_{mB}, we show that (E3), (E4), and (E6) are implied by (E5). The MLRP in Assumption 2 implies that

$$\frac{f_{s_{A2}}(t)}{f_{s_{A1}}(t)} > \frac{F_{s_{A2}}(t)}{F_{s_{A1}}(t)},$$

which is equivalent to saying

$$\frac{d}{dt}\left(\frac{F_{s_{A2}}(t)}{F_{s_{A1}}(t)}\right) = \frac{f_{s_{A2}}(t)F_{s_{A1}}(t) - F_{s_{A2}}(t)f_{s_{A1}}(t)}{\left(F_{s_{A1}}(t)\right)^2} > 0.$$

Since $\theta^m_{mB} > \theta^m_{mA}$ and $\alpha \in (0,1)$, we thus have

$$\frac{v_A F_{s_{A1}}(\theta^m_{mA})}{(1-\alpha)F_{s_{A1}}(\theta^m_{mB})} > \frac{v_A F_{s_{A2}}(\theta^m_{mA})}{(1-\alpha)F_{s_{A2}}(\theta^m_{mB})}.$$

Similarly, we also have

$$\frac{v_A F_{s_{A1}}(\theta^m_{mA})}{(1-\alpha)F_{s_{A1}}(\theta^m_{mB})} > \frac{(1-\alpha)v_A F_{s_{Bj}}(\theta^m_{mA})}{F_{s_{Bj}}(\theta^m_{mB})}$$

and

$$\frac{F_{s_{Bj}}(\theta_{mB}^m)}{F_{s_{A1}}(\theta_{mB}^m)} > \frac{(1-\alpha)^2 F_{s_{Bj}}(\theta_{mA}^m)}{F_{s_{A1}}(\theta_{mA}^m)}$$

for $j \in \{1,2\}$ because of the MLRP and $\alpha \in (0,1)$.

Let $\bar{\theta}_{mi}^m$ and $\hat{\theta}_{mi}^m$ be the cutoff score for $i \in \{A,B\}$ when $(\tilde{s}_{A1}, \tilde{s}_{A2}, \tilde{s}_{B1}, \tilde{s}_{B2}) = (0,1,1,1)$ and $(\tilde{s}_{A1}, \tilde{s}_{A2}, \tilde{s}_{B1}, \tilde{s}_{B2}) = (1,1,1,1)$, respectively. If

$$v_B > \frac{v_A F_{s_{A1}}(\bar{\theta}_{mA}^m)}{(1-\alpha) F_{s_{A1}}(\bar{\theta}_{mB}^m)}, \tag{E7}$$

then good science-stream students have an incentive to deviate from $\tilde{s}_{A1} = 0$. Now suppose that v_B is large enough such that all students have an incentive to apply to B. Nevertheless, $(\tilde{s}_{A1}, \tilde{s}_{A2}, \tilde{s}_{B1}, \tilde{s}_{B2}) = (1,1,1,1)$ cannot be an equilibrium because any science student can be better off by switching to applying to A in this situation. Therefore, we must have

$$v_B < \frac{v_A F_{s_{A1}}(\hat{\theta}_{mA}^m)}{(1-\alpha) F_{s_{A1}}(\hat{\theta}_{mB}^m)}. \tag{E8}$$

Moreover, since $F_{s_{A1}}(\theta_{mA}^m)/F_{s_{A1}}(\theta_{mB}^m)$ is continuous and monotonically increasing in $\tilde{s}_{A1}$, we must have a unique $\tilde{s}_{A1} = \pi \in (0,1)$ such that

$$v_B = \frac{v_A F_{s_{A1}}(\theta_{mA}^m)}{(1-\alpha) F_{s_{A1}}(\theta_{mB}^m)} \tag{E9}$$

and the strategy profile will be $(\tilde{s}_{A1}, \tilde{s}_{A2}, \tilde{s}_{B1}, \tilde{s}_{B2}) = (\pi,1,1,1)$. Thus, if there are some good science-stream students applying to B when the conflicting strategy and the expanding strategy are used, it must be the case that (E7) holds and $(\tilde{s}_{A1}, \tilde{s}_{A2}, \tilde{s}_{B1}, \tilde{s}_{B2}) = (\pi,1,1,1)$. Q.E.D

Appendix F. Proof of Proposition 3

In Lemma 1, when (6) holds, the student strategy profile is $(\tilde{s}_{A1}, \tilde{s}_{A2}, \tilde{s}_{B1}, \tilde{s}_{B2}) = (\pi,1,1,1)$, where $\pi \in (0,1)$. Since only good science students apply to A, the result is the best outcome for A. However, when $\tilde{B} = hm$, the student strategy profile will be $(\tilde{s}_{A1}, \tilde{s}_{A2}, \tilde{s}_{B1}, \tilde{s}_{B2}) = (0,0,1,1)$, regardless of A's strategy. Hence the combination of the expanding strategy and the conflicting strategy, called the combination strategy in this appendix, is the weakly dominant strategy for A.

Given that A uses the combination strategy, students are restricted to applying to one of the two colleges. In this situation, B only has to compare the marginal benefit and the marginal cost of using the expanding strategy. When (6) holds, the marginal benefit is

$$
\begin{aligned}
&(1-\beta)\big(v_1\pi F_{ms_{A1}}(\theta^m_{mB})+v_2 nF_{ms_{A2}}(\theta^m_{mB})\big)\\
&=\int_{\underline{t}}^{\theta^m_{mB}}(1-\beta)\big(v_1\pi f_{ms_{A1}}(t)+v_2 nf_{ms_{A2}}(t)\big)\,dt\\
&=\int_{\underline{t}}^{\theta^m_{mB}}(1-\beta)\left(v_1+v_2-v_2\times\frac{1+\frac{v_1 nf_{ms_{A2}}(t)}{v_2\pi f_{ms_{A1}}(t)}}{1+\frac{nf_{ms_{A2}}(t)}{\pi f_{ms_{A1}}(t)}}\right)\big(\pi f_{ms_{A1}}(t)+nf_{ms_{A2}}(t)\big)\,dt\\
&\geq\int_{\underline{t}}^{\theta^m_{mB}}(1-\beta)\left(v_1+v_2-v_2\times\frac{1+\frac{v_1 nf_{ms_{A2}}(\theta^m_{mB})}{v_2\pi f_{ms_{A1}}(\theta^m_{mB})}}{1+\frac{nf_{ms_{A2}}(\theta^m_{mB})}{\pi f_{ms_{A1}}(\theta^m_{mB})}}\right)\big(\pi f_{ms_{A1}}(t)+nf_{ms_{A2}}(t)\big)\,dt\\
&=(1-\beta)\left(\frac{v_1\pi f_{ms_{A1}}(\theta^m_{mB})+v_2 nf_{ms_{A2}}(\theta^m_{mB})}{\pi f_{ms_{A1}}(\theta^m_{mB})+nf_{ms_{A2}}(\theta^m_{mB})}\right)\int_{\underline{t}}^{\theta^m_{mB}}\big(\pi f_{ms_{A1}}(t)+nf_{ms_{A2}}(t)\big)\,dt
\end{aligned}
$$

and the marginal cost is the same as (D2), i.e.,

$$
v_1\big(F_{hms_{B1}}(\theta^m_{hmB})-F_{ms_{B1}}(\theta^m_{mB})\big)+v_2 n\big(F_{hms_{B2}}(\theta^m_{hmB})-F_{ms_{B2}}(\theta^m_{mB})\big),
$$

which can also be rewritten as

$$
\begin{aligned}
&\int_{\theta^m_{mB}}^{\theta^m_{hmB}}\big(v_1 f_{ms_{B1}}(t)+v_2 nf_{ms_{B2}}(t)\big)\,dt\\
&\leq\left(\frac{v_1 f_{ms_{B1}}(\theta^m_{mB})+v_2 nf_{ms_{B2}}(\theta^m_{mB})}{f_{ms_{B1}}(\theta^m_{mB})+nf_{ms_{B2}}(\theta^m_{mB})}\right)\int_{\theta^m_{mB}}^{\theta^m_{hmB}}\big(f_{ms_{B1}}(t)+nf_{ms_{B2}}(t)\big)\,dt.
\end{aligned}
$$

The capacity constraint ensures that

$$
\int_{\underline{t}}^{\theta^m_{mB}}\big(\pi f_{ms_{A1}}(t)+nf_{ms_{A2}}(t)\big)dt=\int_{\theta^m_{mB}}^{\theta_{hmB}}\big(f_{ms_{B1}}(t)+nf_{ms_{B2}}(t)\big)dt.
$$

Thus, the sufficient condition for B to use the combination strategy is

$$
(1-\beta)\left(\frac{v_1\pi f_{ms_{A1}}(\theta^m_{mB})+v_2 nf_{ms_{A2}}(\theta^m_{mB})}{\pi f_{ms_{A1}}(\theta^m_{mB})+nf_{ms_{A2}}(\theta^m_{mB})}\right)\geq\left(\frac{v_1 f_{ms_{B1}}(\theta^m_{mB})+v_2 nf_{ms_{B2}}(\theta^m_{mB})}{f_{ms_{B1}}(\theta^m_{mB})+nf_{ms_{B2}}(\theta^m_{mB})}\right),
$$

which can be rearranged as

$$
\beta\leq 1-\frac{\big(\pi f_{ms_{A1}}(\theta^m_{mB})+nf_{ms_{A2}}(\theta^m_{mB})\big)\big(v_1 f_{ms_{B1}}(\theta^m_{mB})+v_2 nf_{ms_{B2}}(\theta^m_{mB})\big)}{\big(f_{ms_{B1}}(\theta^m_{mB})+nf_{ms_{B2}}(\theta^m_{mB})\big)\big(v_1\pi f_{ms_{A1}}(\theta^m_{mB})+v_2 nf_{ms_{A2}}(\theta^m_{mB})\big)}.
$$

Q.E.D.

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