Costly Self-Control and Limited Willpower

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October 26, 2017 (NTU Seminar)

Two main studies on Willpower in Psychology

- Stanford marshmallow experiment
  - was a series of studies on delayed gratification in the late 1960s and early 1970s led by psychologist Walter Mischel
  - Willpower is a good predictor of your life success.
- Is Willpower a Limited Resource?
  - The lab of Roy Baumeister, Since that work was published in 1998, numerous studies have built a case for willpower depletion.
  - Kathleen Vohs

Preferences on Packing food for one hour hike

- Prizes: Z= {**w**ater, **c**oke, **b**anana}
- Lottery: $\Delta(Z) = \{p\mathbf{w}+q\mathbf{c}+(1-p-q)\mathbf{b}: p, q \in [0,1], p+q \leq 1\}$
- Menu:{w},{c},{b}, {w, c},{w, b},{b, c}, {w, c, b},…
- Cold stage: packing (not thirsty)
- Hot stage: hiking (thirsty)
- Suppose at hot stage, you have unit demand (only consume one good).

Preferences over menus at packing stage

- Assumption: perfect foresight
  - no hot-cold empathy gap,
  - in general this is not true, see Loewenstein (2000)
  - We are interested in characterizing the utilities at hot stage when this gap approaches zero.
  - Our decision maker (DM) knows his own consumption preferences during hiking when he packs his bag.

vNM-rationality at hot stage

Suppose the temptation driven by thirsty at hot stage is measured by v, and v(c)> v(w)> v (b) for DM1, DM2 and DM3

However, their personality are quite different. Hence, they have different normative utility rankings at hot stage. We use u_1, u_2 and u_3 to measure their normative preference for these three prizes at hot stage.

DM1 aims for health (very strict person, put zero weight on temptation)---ant

- $u_1(\text{w}) > u_1(\text{b}) > u_1(\text{c})$

DM2 lives for the moment, put zero weight on health—grasshopper

- $u_2(\text{c}) > u_2(\text{w}) > u_2(\text{b})$

DM3 is someone in between: he puts α weight on his health concern and (1- α) weight on his thirsty self.

- one case: $u_3(\text{w}) > u_3(\text{c}) > u_3(\text{b})$
- other ranks are also possible.

Using the ranking over menus at cold stage to get the trade off between u and v

- Note that for DM2 there is no conflict between his normative utility and temptation utility. Hence, there is no trade off involved.
- Let us take DM1's ranking.
- Suppose that DM1's ranking over his choice menus during hiking when he packs his bag at cold stage is as following:
- $\{w\} \sim \{w, b\} \succ \{w, c\} \sim \{w, b, c\} \succ \{b\} \succ \{b, c\} \sim \{c\}$

The reason that he ranks {w} ~ {w, b} is because b is not tempting when he has water available. Hence, having this worse alternative b in the choice menu has no effect on his utility during hot stage.

However, c is more tempting than water when he is thirsty. Hence, even DM1 picks w when facing {w, c}, he has to pay a resisting cost for his temptation on coke. Hence, {w} ≻ {w, c}.

{b, c} ~{c} is because c is too tempting to resist when there is only banana available. Hence, DM1 will end in choosing c instead of better normative alternative b. Hence, his utility at hot stage for this menu is the same as having the worse normative alternative c only. (we assume DM1 is not the type who suffers self-defeating feelings when he gives in to his temptation.)

Hence, if the only thing relevant to DM1's utility at hot stage is his normative utility and resisting cost for temptation. His ranking among menus should satisfy set-betweeness, i.e.,

$$A \subseteq A \cup B \subseteq B$$

We assume DM1 possess vNM-rationality at hot stage. Hence, follow the traditional construction for vNM, we get U to measure the utilities over menus. Then we use $U(\{w\})$-$U(\{w,c\})$ to recover $v(c)$.

Given our DM1's ranking over menus at cold stage

$\{w\} \sim \{w, b\} \succ \{w, c\} \sim \{w, b, c\} \succ \{b\} \succ \{b, c\} \sim \{c\}$

	U	**u**	**v**
{w}	1	1	
{w, c}			
{b} ~{0.5w+0.5c}	0.5	0.5	
{c}	0	0	

Given our DM1's ranking over menus at cold stage

$$\{w\} \sim \{w, b\} \succ \{w, c\} \sim \{w, b, c\} \succ \{b\} \succ \{b, c\} \sim \{c\}$$

	U	u	v
{w}	1	1	
{w, c} $\sim$ {0.6w+0.4c}	0.6		
{b} $\sim$ {0.5w+0.5c}	0.5	0.5	
{c}	0	0	

Given our DM1's ranking over menus at cold stage

$\{w\} \sim \{w, b\} \succ \{w, c\} \sim \{w, b, c\} \succ \{b\} \succ \{b, c\} \sim \{c\}$

Using $\{w\} \succ \{w, c\} \succ \{c\}$ to find v(c) and v(w).

Resisting cost =v(c)-v(w)=U({w})-U({w,c})

	U	u	v
{w}	1	1	0
{w, c} ~{0.6w+0.4c}	0.6		
{b} ~{0.5w+0.5c}	0.5	0.5	
{c}	0	0	0.4

How to get v(b)?

How to get v(b)?

Need to find some lottery pb+(1-p)c

s.t.

$\{w\} \succ \{w, pb+(1-p)c\} \succ \{pb+(1-p)c\}$

Use $\{w\} \succ \{w, 0.5b+0.5c\} \succ \{0.5b+0.5c\}$ to get v(b)

	U	u	v
{w}	1	1	0
{w, 0.5b+0.5c} ~{0.9w+0.1c}	0.9		
{w, c} ~{0.6w+0.4c}	0.6		
{b} ~{0.5w+0.5c}	0.5	0.5	-0.2
{c}	0	0	0.4

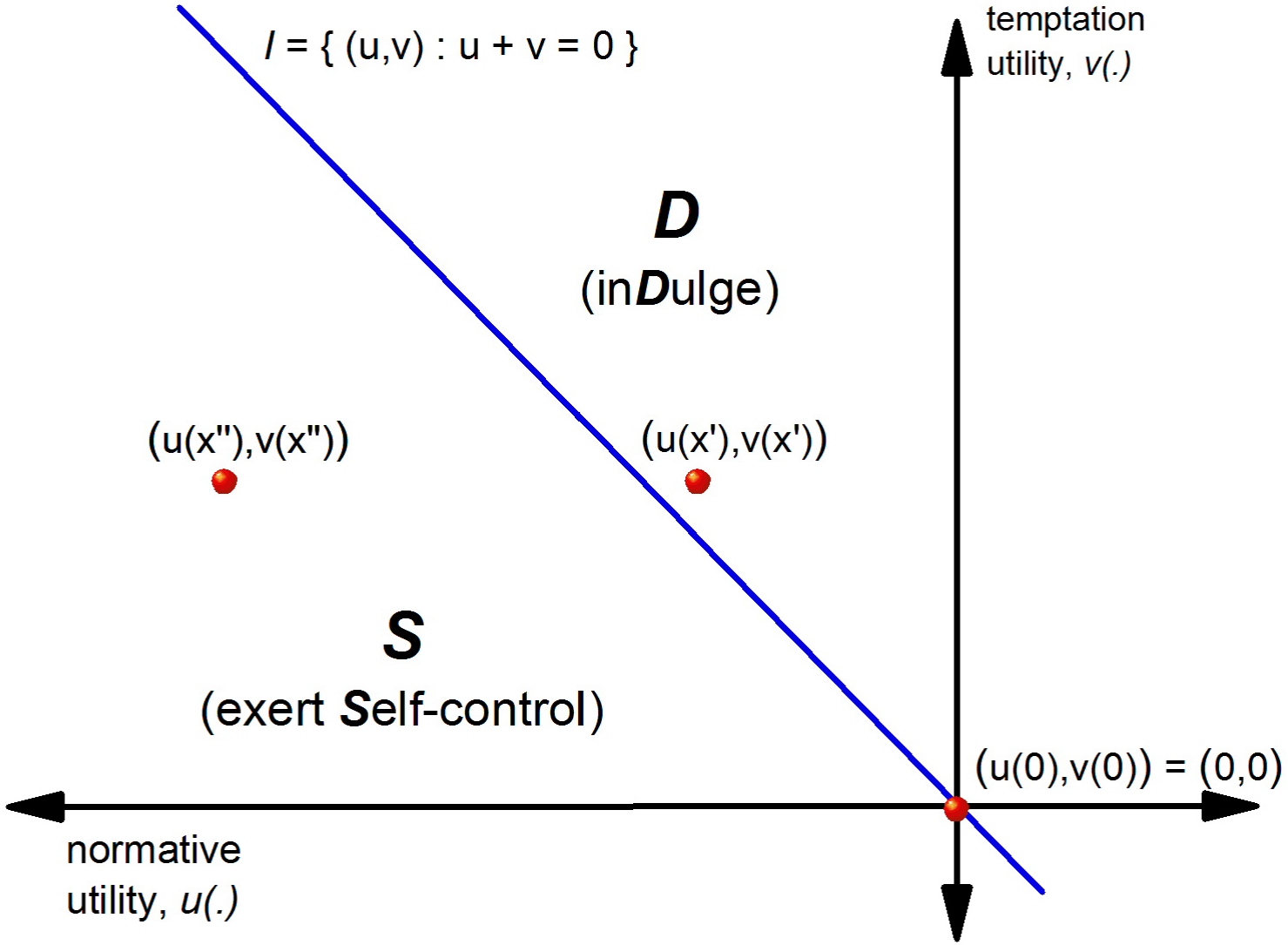
Resisting cost v(0.5b+0.5c)-v(w)=U(w)-U({w, 0.5b+0.5c}
Hence, 0.5v(b)+0.2=0.1, v(b)=-0.2

$$U(A) = \max_{x \in A} \{u(x) - (\max_{y \in A} v(y) - v(x))\}$$
$$= \max_{x \in A} \{u(x) + v(x)\} - \max_{y \in A} v(y)$$

We call the sum of normative utility and temptation utility,

$$u(x) + v(x)$$

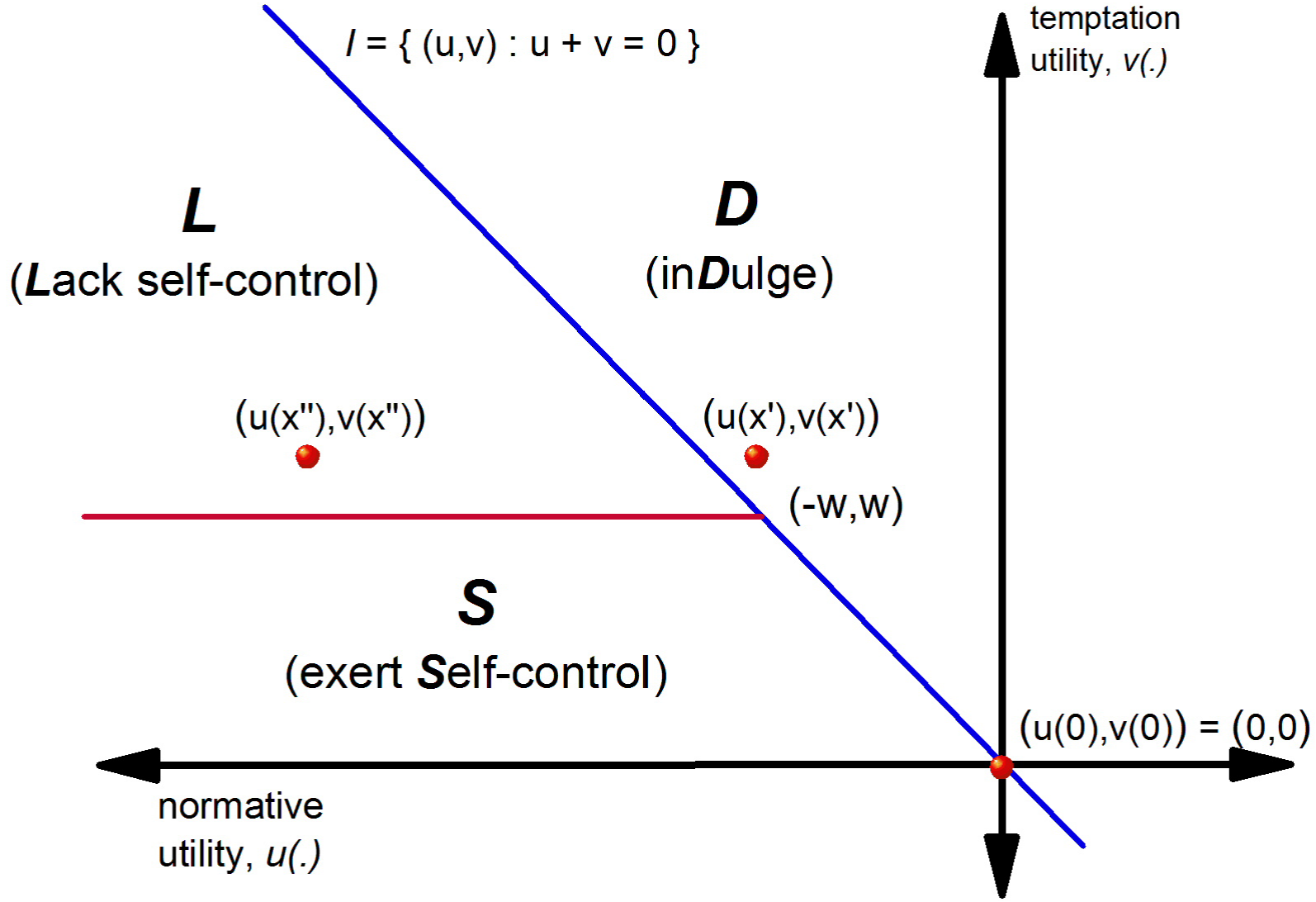
compromise utility, this is the utility that decide which alternative to be chosen within a menu in hot stage


I = { (u,v) : u + v = 0 }
temptation
utility, v(.)
D
(inDulge)
(u(x''),v(x''))
(u(x'),v(x'))
S
(exert Self-control)
(u(0),v(0)) = (0,0)
normative
utility, u(.)

The idea of above construction is based on GP(EM2001).
The main difference is that we focus on the construction of hot stage utility and their paper's focus is on commitment preference at cold stage.
The second new feature is that we link our construction with the literature on limited willpower. Since there is a resisting cost, we construct our hot stage utility functions involving some upper bond on this resisting cost, which generates from ranking preferences over menus and is constant on all menus considered.

$$U(A) = \max_{x \in A}\{u(x)+v(x)\} - \max_{y \in A} v(y)$$

$$\text{s.t. } \max_{y \in A} v(y) - v(x) \leq w$$


I = { (u,v) : u + v = 0 }
temptation
utility, v(.)
L
(Lack self-control)
D
(inDulge)
(u(x''),v(x''))
(u(x'),v(x'))
(-w,w)
S
(exert Self-control)
(u(0),v(0)) = (0,0)
normative
utility, u(.)

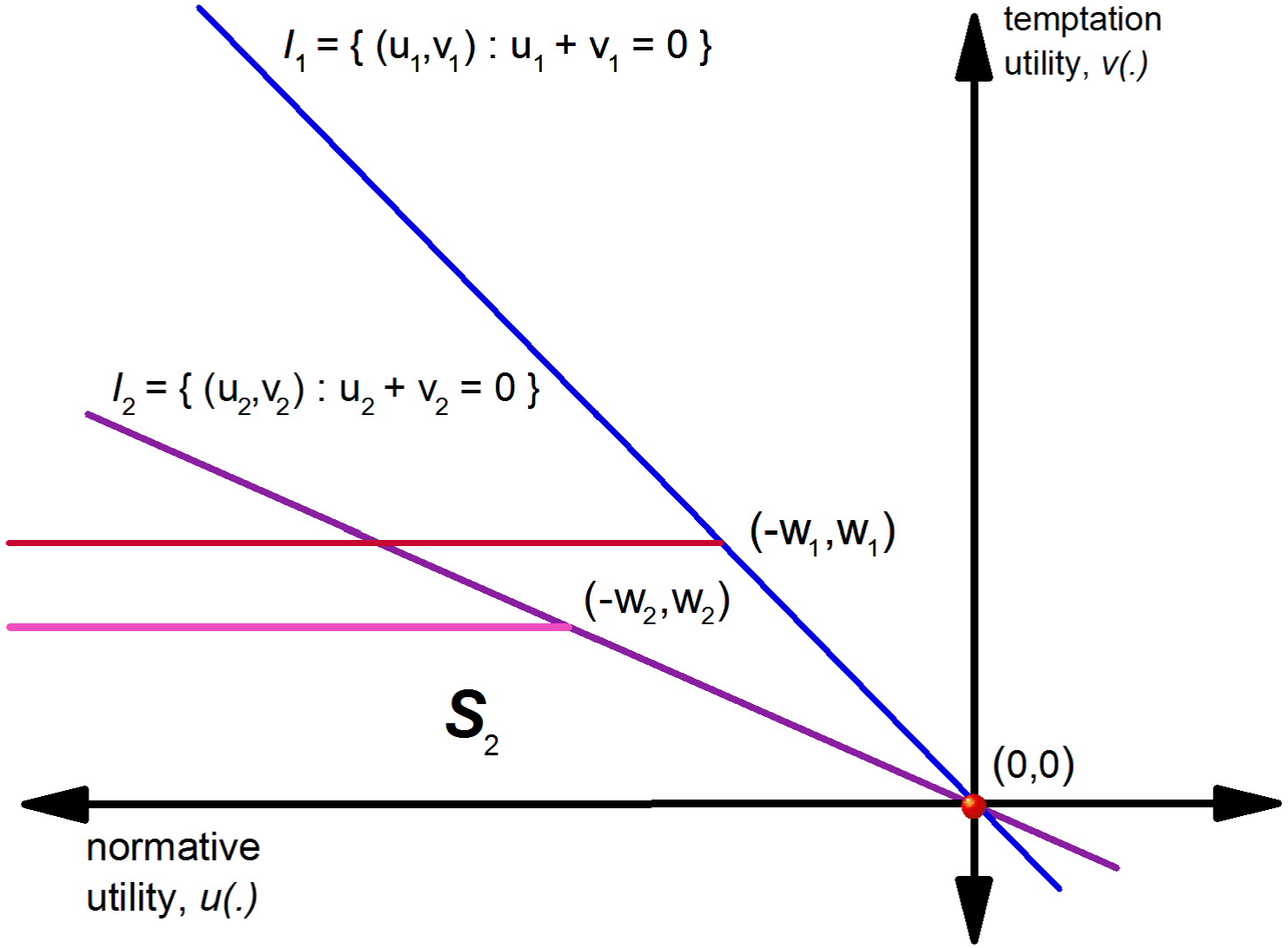
- Given in to temptation has two distinct situations.
  - Region D: the resisting cost is higher than the gain from normative utility. Hence, not worth to resist and no market for getting help to do self-control. For drug users in this region, they will not go to rehab center.
  - Region L, which means DM will pay to get help to resist his temptation, ex. go to rehab center.
- Note that paying for help in controlling self is different from the preference for commitment discussed in GP and Noor. In their characterizations, the only incentive that DM will pay is to avoid the temptation and save the resisting cost. There is no incentive to pay for help to control self and still have to pay the resisting cost.

- Instead of using given in to temptation to measure DM's willpower, we think the literature should use this bound *w* to measure willpower. Then we can discuss whether willpower depletes after using it or not. **In flow sense, there is a limit on willpower.** However, if willpower does not deplete, we do have unlimited willpower stock to use as long as DM doesn't pass this flow limit.
- Since DM in region L has incentive to be forced to resist temptation beyond his willpower flow limit w, it requires studies to better understand whether using outside help to control self will damage his future willpower flow limit.

Suppose DM3's normative utility is $0.8u_1 + 0.2v$

	u3	u1	v	u3+v	u1+v
{w}	0.8	1	0	0.8	1
{b} ~{0.5w+ 0.5c}	0.36	0.5	-0.2	0.16	0.3
{c}	0.08	0	0.4	0.48	0.4

Hence, DM3's level curve u3+v=0 is flatter than DM1


$I_1 = \{ (u_1,v_1) : u_1 + v_1 = 0 \}$
temptation utility, *v(.)*
$I_2 = \{ (u_2,v_2) : u_2 + v_2 = 0 \}$
$(-w_1,w_1)$
$(-w_2,w_2)$
S_2
(0,0)
normative utility, *u(.)*

- When DM1 gets tired after one difficult task and changes his normative utility to be more like DM3. Hence, for sure his level curve becomes flatter. Hence, he gives in more. If he becomes more energetic after one difficult task and move his level curve to be steeper, then he gives in less as long as all the resisting cost is below the flow limit of willpower. These could explain the controversial findings on willpower depletion in Psychology
- However, measuring the flow limit w is more difficult. Our characterization and Corollary 4.1 suggest that the discontinuity of preferences over menus along with violation of WARP for choices within menus could reveal the limit w of an individual's willpower in exercising costly self-control.