Uncovered Interest Rate Parity Puzzle and Sunspot Fluctuations*

Ippei Fujiwara† Yasuo Hirose‡

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Abstract

Empirical evidence has constantly rejected the uncovered interest rate parity (UIP) condition, well known as the UIP puzzle. We evaluate the empirical performance of a small open-economy model for Canada incorporating an endogenous risk premium on foreign bond holdings so that the resulting modified UIP condition can exhibit a negative relationship between expected exchange rate depreciation and interest rate differentials as the UIP puzzle suggests. Because the model is susceptible to equilibrium indeterminacy, we estimate it with Bayesian methods allowing for both determinacy and indeterminacy of equilibrium. Our results show that the data strongly favor indeterminacy over determinacy and that the estimated model can account for the UIP puzzle both unconditionally and conditionally. Variance decompositions demonstrate that a shock to the modified UIP condition is the main driving force of exchange rate fluctuations whereas sunspot shocks play a secondary role.

JEL codes: C62; E32; F31; F41

Keywords: UIP puzzle; Exchange rate; Indeterminacy; Sunspot shocks; Bayesian estimation

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†Faculty of Economics, Keio University; Crawford School of Public Policy, Australian National University. Email: `ippei.fujiwara@keio.jp`

‡Faculty of Economics, Keio University. Email: `yhirose@econ.keio.ac.jp`

1 Introduction

The uncovered interest rate parity (UIP) condition constitutes a fundamental building block in modern open-economy models. It states that countries with relatively high interest rates should expect subsequent currency depreciations to ensure zero expected excess returns from cross-border financial investments. However, as is well known since Fama (1984), a wide range of international data have empirically rejected this condition and often exhibit almost zero or negative correlations between expected exchange rate depreciation and interest rate differentials, giving rise to the UIP puzzle.

From a theoretical perspective, Beaudry and Lahiri (2019) indicate that a perfect-foresight equilibrium in a small open economy is indeterminate if the UIP condition fails and exhibits a negative relationship between expected exchange rate depreciation and interest rate differentials as the UIP puzzle suggests. Their finding is quite novel and relevant to the empirical regularities. However, their analysis is based on a small open-economy version of a simple (continuous-time) sticky-price model, where the modified UIP equation is specified in a reduced-form manner, and the empirical validity of their model is not examined formally through model estimation.

This paper explores the empirical performance of a fully specified dynamic stochastic general equilibrium (DSGE) model of the Canadian economy incorporating a modified UIP condition that shares the same spirit as the one considered in Beaudry and Lahiri (2019). To derive the modified UIP equation structurally, we introduce an endogenous risk premium on foreign bond holdings *à la* Adolfson et al. (2008) into a small open-economy version of the standard New Keynesian model so that the resulting modified UIP condition can exhibit a negative relationship between expected exchange rate depreciation and interest rate differentials. In the model, the endogenous risk premium is specified as a function of an expected change in the exchange rate, a net foreign asset position, and a shock. Then, the coefficient on interest rate differentials in the modified UIP equation can be negative, leading to equilibrium indeterminacy, if the parameter on the risk premium associated with exchange rate changes is sufficiently large.[1]

[1] Christiano et al. (2011) specify the risk premium as a function of interest rate differentials instead of the expected exchange rate changes and derive the modified UIP equation in the same form as ours so that its coefficient on the interest differentials can be negative. However, they estimate their model only in the

A notable feature of our analysis is that we estimate the model over the parameter space in which the coefficient on interest rate differentials in the modified UIP equation can be either positive or negative, leading to either determinacy or indeterminacy of equilibrium.[2] If we restricted our model's parameter space to one that leads only to equilibrium determinacy, the estimated model would be unlikely to replicate the observed pattern between the exchange rate and interest rate differentials. Therefore, we estimate the model using full-information Bayesian methods that allow for both determinacy and indeterminacy of equilibrium. To this end, following Lubik and Schorfheide (2004), we construct the model's likelihood function not only for the determinacy region of its parameter space but also for the indeterminacy region. While Lubik and Schorfheide (2004) conduct model estimation separately for each region, we estimate the model for both the determinacy and indeterminacy regions in one step by adopting a sequential Monte Carlo (SMC) algorithm, as implemented by Hirose et al. (2020). In contrast to the widely used Metropolis–Hastings algorithm, the SMC algorithm can deal with discontinuity in the likelihood function at the boundary of each region and help us find the entire posterior distribution of model parameters.

Based on the estimated model, we investigate the sources of macroeconomic fluctuations, in particular, Canada–US exchange rate dynamics. Lubik and Schorfheide (2006), Hirose (2013), and Chen et al. (2021) document that, because of the empirical failure of the UIP relationship, estimations of open-economy DSGE models typically find fluctuations in nominal exchange rates mostly unrelated to macroeconomic fundamentals and attributed to a wedge to the standard UIP condition, called a UIP shock. Our empirical analysis may alter this prevailing view because the modified UIP condition in our model can exhibit a negative relationship between the expected exchange rate depreciation and interest rate differentials as observed in the data. Moreover, because we allow for equilibrium indeterminacy, sunspot shocks, which are nonfundamental disturbances to agents' expectations, can affect the aggregate fluctuations. Under indeterminacy, the contribution of UIP shocks to the exchange rate may be partly replaced with that of sunspot shocks.

The main results of this paper are as follows. First, the posterior distributions of the

determinacy region of the parameter space without considering the possibility of indeterminacy.

[2]Equilibrium indeterminacy does not arise in the model with Adolfson et al. (2008)'s original specification of the endogenous risk premium on foreign bond holdings, in which the risk premium depends on expected *consecutive* currency depreciation (changes from $t-1$ to $t+1$).

model parameters indicate that the parameter on the risk premium on foreign bond holdings associated with the exchange rate is so large that the coefficient on interest rate differentials in the modified UIP equation is negative, leading to equilibrium indeterminacy. Comparing the baseline estimation results with those obtained by estimating the model only in the determinacy region of its parameter space, we find that the baseline model allowing for indeterminacy fits the data much better than the model estimated only under determinacy. We also show that the propagation of shocks can be remarkably different between the baseline model and its counterpart estimated only under determinacy. These differences are attributed to the estimated arbitrary components—which work as an equilibrium selection device—in the solution under indeterminacy.

Second, stochastic simulations demonstrate that the estimated baseline model can replicate the observed negative correlation between expected exchange rate depreciations and interest rate differentials, both unconditionally and conditionally. In other words, the model accounts for the UIP puzzle not only when the economy is driven by all shocks but also when it is driven by each single shock. Indeed, each of all shocks can generate a negative correlation in the baseline model, because of the negative slope coefficient in the modified UIP equation. In the model estimated only under determinacy, however, only the UIP shock can generate such a negative correlation, and the other shocks cannot.

Third, forecast error variance decompositions based on the estimated model show that the UIP shock is the main driving force of exchange rate fluctuations. Thus, conventional wisdom in the literature that indicates the importance of a direct shock to the UIP condition is not overturned. Moreover, we find that the UIP shock can account for fluctuations in other observed variables, as is consistent with the argument in Itskhoki and Mukhin (2021), who offer microfoundations for a direct shock to the UIP condition and demonstrate the importance of the shock in explaining aggregate variables including exchange rates. Our empirical analysis enhances their argument further, showing that the contributions of the UIP shock to inflation and the nominal interest rate are much larger under indeterminacy than under determinacy.

Finally, a novel finding in the variance decomposition analysis is that sunspot shocks play a secondary role in explaining exchange rate dynamics for all forecast horizons. We also find that sunspot shocks play a nonnegligible role in fluctuations in output growth, inflation, and the nominal interest rate. In particular, their contribution to the interest rate is remarkable for relatively short horizons.

The remainder of this paper proceeds as follows. Section 2 provides a brief overview of the related literature. Section 3 presents the small open-economy DSGE model used for our empirical analysis and analyzes the determinacy regions in the model's parameter space. Section 4 explains the estimation strategy and data. Section 5 presents and discusses the results of the empirical analysis. Section 6 conducts robustness analysis for various estimation settings. Section 7 concludes.

2 Related Literature

This paper is most closely related to the following three strands of literature in the field of international finance: the UIP puzzle, accounting for exchange rate dynamics, and equilibrium indeterminacy in open economies.

UIP puzzle When agents are risk-neutral and there are no frictions in international financial markets, a no-arbitrage condition for home and foreign bond holdings implies the UIP condition:

$$\log \mathbb{E}_t S_{t+1} - \log S_t = \log i_t - \log i_t^*,$$

where S_t denotes the nominal exchange rate (price of foreign currency in terms of domestic currency), and i_t and i_t^* are respectively the home and foreign (gross) nominal interest rates. Assuming the rational expectations, this UIP condition can be empirically tested by conducting the so-called UIP regression:

$$\log S_{t+1} - \log S_t = \alpha_0 + \alpha_1(\log i_t - \log i_t^*) + v_t,$$

with the null hypothesis $H_0 : \alpha_0 = 0$ and $\alpha_1 = 1$, where α_0 and α_1 are regression coefficients and v_t is an error term. Since the seminar paper by Fama (1984), numerous papers have rejected this null hypothesis using a wide rage of international data and have found the estimated slope coefficient α_1 to be significantly below unity, and often negative.

In line with these findings, Lustig and Verdelhan (2007) and Burnside et al. (2008) find sizable gains from the *carry trade*, an investment strategy by investing in high interest rate currencies with funding from low interest rate currencies. Eichenbaum and Evans (1995) and Scholl and Uhlig (2008) provide the empirical pattern called the *delayed overshooting*: a country's currency tends to appreciate for a while after a positive monetary policy shock. Grilli and Roubini (1996) and Kim and Roubini (2000) find the same pattern for Canadian data.

There have been many attempts to replicate this failure of the UIP condition in a theoretical framework. Backus et al. (2001), Duarte and Stockman (2005), Verdelhan (2010), Colacito and Croce (2011), Bansal and Shaliastovich (2012), Benigno et al. (2011), Backus et al. (2010), Gourio et al. (2013), Engel (2016), and Chen et al. (2021) aim to solve the UIP puzzle through risk corrections, namely the covariance between the stochastic discount factor and payoffs. Bacchetta and van Wincoop (2021) show that delayed portfolio adjustment can account for the UIP puzzle. In these studies, structural or macroeconomic fundamental shocks can raise interest rates and appreciate the nominal exchange rates simultaneously. Gourinchas and Tornell (2004), Chakraborty and Evans (2008), Burnside et al. (2011), Ilut (2012), and Candian and Leo (2021) explain the UIP puzzle by deviating from the rational expectations.

Accounting for exchange rate dynamics Because the standard UIP condition fails to replicate the observed pattern between expected exchange rate depreciation and interest rate differentials, the literature finds that exchange rate dynamics are mostly explained by a wedge to the UIP condition, called a UIP shock. Itskhoki and Mukhin (2021) provide microfoundations for a direct shock to the UIP condition and demonstrate its importance in accounting for aggregate fluctuations including exchange rate dynamics.

Lubik and Schorfheide (2006) and Hirose (2013) estimate a two-country open-economy model for the U.S. and the Euro economies in a linear setting and find that a shock to the purchasing power parity (PPP) condition, which works in a similar way to the UIP shock, explains more than 80% of the exchange rate fluctuations. Chen et al. (2021) extend their analysis by allowing for stochastic volatilities in fundamental shocks and estimate their model approximated up to the third order. Albeit with the consideration of nonlinearity and risk components, they report that the UIP shock still plays a significant role in accounting for exchange rate dynamics.

Adolfson et al. (2008) introduce an endogenous risk premium on foreign bond holdings into a small open-economy model as a function of the aggregate net foreign asset position of domestic households and expected *consecutive* depreciation (changes from $t-1$ to $t+1$) and show that their estimated model can better replicate the observed properties of Swedish macroeconomic data including the exchange rate.

Indeterminacy in open economies While Kareken and Wallace (1981) discuss the steady-state indeterminacy in an open economy, our paper focuses on dynamic indeterminacy, *i.e.*, the possibility of multiple equilibrium paths toward a unique steady state.

In a small open-economy setting, Carlstrom and Fuerst (2002) report that determinacy conditions in a closed economy framework carry over to a small open economy, whereas De Fiore and Liu (2005) point out the importance of trade openness for a determinate equilibrium. In a multi-country setting, Bullard and Singh (2008) and Bullard and Schaling (2009) demonstrate that the worldwide equilibrium can be indeterminate when one country satisfies the determinacy condition but when one of the others does not. Hirose (2013) estimates a two-country model for the U.S. and the Euro area allowing for both determinacy and indeterminacy of equilibrium.

Beaudry and Lahiri (2019) is the most closely related paper to ours. They show that the perfect-foresight equilibrium is indeterminate if a modified interest rate parity condition exhibits a negative relationship between exchange rate depreciations and interest rate differentials in a small open-economy version of a simple (continuous-time) sticky-price model. In contrast, we demonstrate that the negative relationship is not a sufficient condition for equilibrium indeterminacy in a more general stochastic setting.

3 The Model

The model estimated in this paper is a small open-economy version of the standard New Keynesian model, as in Galí and Monacelli (2005) but incorporates an endogenous risk premium on the foreign bond holdings *à la* Adolfson et al. (2008) so that the resulting modified UIP condition can exhibit a negative relationship between expected exchange rate depreciation and interest rate differentials as observed in the data. A representative household gains utility from aggregate consumption composed of home and foreign goods, and trades both home and foreign bonds in domestic and international asset markets. Monopolistically competitive firms produce differentiated goods, and are subject to a price adjustment cost. The central bank adjusts the nominal interest rate in response to inflation, output growth, and nominal exchange rate depreciation. For a better fit to the macroeconomic data, the model features habit persistence in consumption preferences, price indexation to past inflation, and monetary policy smoothing.

3.1 Household

A representative household in the home country maximizes the utility function:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\log\left(C_t - b\bar{C}_{t-1}\right) - \frac{h_t^{1+\eta}}{1+\eta} \right],$$

where C_t is aggregate consumption, $b\bar{C}_{t-1}$ is an external habit taken as given by the household, h_t is labor supply, β is the subjective discount factor, and η is the inverse of the labor supply elasticity. Aggregate consumption C_t is a composite of home- and foreign-produced goods, $C_{H,t}$ and $C_{F,t}$, given by

$$C_t = \left(\frac{C_{H,t}}{\lambda}\right)^{\lambda} \left(\frac{C_{F,t}}{1-\lambda}\right)^{1-\lambda},$$

with

$$C_{H,t} = \left[\int_0^1 C_{H,t}\left(i\right)^{1-\frac{1}{\epsilon}} \mathrm{d}i\right]^{\frac{\epsilon}{\epsilon-1}},$$

$$C_{F,t} = \left[\int_0^1 C_{F,t}\left(i^*\right)^{1-\frac{1}{\epsilon}} \mathrm{d}i^*\right]^{\frac{\epsilon}{\epsilon-1}},$$

where $C_{H,t}(i)$ and $C_{F,t}(i^*)$ are differentiated consumption goods produced by home and foreign firms, each of which is indexed by j and j^*, respectively. λ is the relative weight on the domestically produced goods in aggregate consumption, and ϵ is the elasticity of substitution among differentiated products in each country.

The household's utility maximization is subject to the budget constraint:

$$P_tC_t + A_t + S_tA_t^* = W_th_t + i_{t-1}A_{t-1} + S_t\Phi_{t-1}i_{t-1}^*A_{t-1}^* + T_t,$$

where P_t is the consumer price index, A_t and A_t^* are respectively the holding of home and foreign bonds, S_t denotes the nominal exchange rate (price of foreign currency in terms of domestic currency), W_t is the nominal wage, i_t and i_t^* are respectively the home and foreign nominal interest rates, Φ_t is a risk premium on the foreign bond holdings, and T_t is the net transfer from firms and the government.

Following Adolfson et al. (2008), the risk premium depends on the net foreign asset position of the domestic household, the expected change in the exchange rate, and a shock. Specifically, Φ_t is of the form

$$\Phi_t := \exp\left[-\phi_a\left(\frac{S_tA_t^*}{P_tZ_t} - a^*\right) - \phi_s\left(\frac{\mathbb{E}_tS_{t+1}}{S_t} - \frac{\pi}{\pi^*}\right) + \psi_t\right], \tag{1}$$

where Z_t is a nonstationary trend component explained below. a^*, π, and π^* are the steady-state values of detrended real net foreign assets in the home currency and home and foreign inflation. ϕ_a and ϕ_s are parameters. ψ_t is a shock to the risk premium. We call this shock a UIP shock because it will appear as a direct shock to the resulting modified UIP condition. The first term $-\phi_a\left(S_tA_t^*/(P_tZ_t) - a^*\right)$ is needed to ensure stationarity of the small-open economy model with incomplete asset markets. The second term $-\phi_s\left(\mathbb{E}_tS_{t+1}/S_t - \pi/\pi^*\right)$ is based on the empirical regularity that risk premia are negatively correlated with expected currency depreciations (e.g., Fama, 1984; Duarte and Stockman, 2005).[3]

3.2 Firms

In the home country, each firm, indexed by j, produces one kind of differentiated good $Y_t(j)$ by choosing a cost-minimizing labor input $h_t(j)$, given the wage, subject to the production

[3] Assuming that $\pi = \pi^*$, Adolfson et al. (2008) specify the second risk-premium term as $-\phi_s\left[(\mathbb{E}_tS_{t+1}/S_t)(S_t/S_{t-1}) - 1\right]$, in which the risk premium depends on expected *consecutive* depreciation, or expected changes in the exchange rate from two periods ago. Our specification is simpler than theirs but straightforwardly captures the empirical regularity found in the literature.

function:

$$Y_t(j) = \exp(z_t) Z_t h_t(j),$$

where z_t is a stationary technology shock, and Z_t is a nonstationary trend component that grows at a constant rate γ, *i.e.*,

$$\frac{Z_t}{Z_{t-1}} = \gamma.$$

In a monopolistically competitive market, each firm sets the price of its products in the presence of a Rotemberg (1982)-type adjustment cost and indexation to a weighted average of the past inflation rate for the domestically produced goods $\pi_{H,t-1} := P_{H,t-1}/P_{H,t-2}$ and the steady-state inflation rate π to maximize the present discounted value of its profit:

$$\mathbb{E}_t \sum_{n=0}^{\infty} m_{t,t+n} \left[\frac{P_{H,t+n}(j)}{P_{H,t+n}} - \frac{\exp(\mu_t) W_{t+n}}{\exp(z_t) Z_t P_{H,t+n}} - \frac{\phi}{2} \left(\frac{P_{H,t+n}(j)}{\pi^{\omega}_{H,t+n-1} \pi^{1-\omega} P_{H,t+n-1}(j)} - 1 \right)^2 \right] Y_{t+n}(j),$$

subject to the firm-level resource constraint

$$Y_t(j) = C_{H,t}(j) + C^*_{H,t}(j) + \frac{\phi}{2} \left(\frac{P_{H,t}(j)}{\pi^{\omega}_{H,t-1} \pi^{1-\omega} P_{H,t-1}(j)} - 1 \right)^2 Y_t(j),$$

and the downward sloping demand curves, which are obtained from the household's optimization problem in each country,

$$C_{H,t}(j) = \left[\frac{P_{H,t}(j)}{P_{H,t}} \right]^{-\epsilon} C_{H,t},$$

$$C^*_{H,t}(j) = \left[\frac{P^*_{H,t}(j)}{P^*_{H,t}} \right]^{-\epsilon} C^*_{H,t},$$

where $m_{t,t+n}$ is the stochastic discount factor, $C^*_{H,t}$ is the demand for the domestically produced goods in the foreign country, $P^*_{H,t}$ is the export price of the domestically produced goods in terms of the foreign currency, ϕ is the adjustment cost parameter, and $\omega \in [0,1]$ is the weight of price indexation to past inflation relative to steady-state inflation. Note that a marginal cost shock μ_t is embedded to capture exogenous cost-push factors.

Assuming the law of one price and the symmetric equilibrium, we obtain

$$P_{H,t} = S_t P^*_{H,t}.$$

Let $p_{H,t} := P_{H,t}/P_t$. Then, $\pi_{H,t} := P_{H,t}/P_{H,t-1}$ can be expressed as

$$\pi_{H,t} = \frac{p_{H,t}\pi_t}{p_{H,t-1}},$$

where $\pi_t := P_t/P_{t-1}$. The real exchange rate e_t is defined as

$$e_t := \frac{S_t P^*_t}{P_t}.$$

Aggregating the firm-level resource constraint leads to

$$Y_t = C_{H,t} + C^*_{H,t} + \frac{\phi}{2}\left(\frac{\pi_t}{\pi^{\omega}_{t-1}\pi^{1-\omega}} - 1\right)^2 Y_t.$$

The balance of payments identity is given by

$$P_{H,t}Y^*_{H,t} - P_{F,t}Y_{F,t} = S_t\left(A^*_t - \Phi_{t-1}i^*_{t-1}A^*_{t-1}\right),$$

where $Y^*_{H,t}$ is exports of the domestically produced goods, $P_{F,t}$ is the import price of the foreign goods expressed in foreign currency, and $Y_{F,t}$ is imports of the foreign goods.

3.3 Monetary policy

The central bank in the home country adjusts the nominal interest rate in response to deviations of inflation, output growth, and nominal exchange rate depreciation from their steady state values with policy smoothing:[4]

$$i_t = i^{\rho}_{t-1}\left[i\left(\frac{\pi_t}{\pi}\right)^{\alpha_\pi}\left(\frac{Y_t}{\gamma Y_{t-1}}\right)^{\alpha_y}\left(\frac{S_t\pi^*}{S_{t-1}\pi}\right)^{\alpha_s}\right]^{1-\rho}\exp\left(u_t\right),$$

[4] Our model does not explicitly consider the zero lower bound on the nominal interest rate because the policy interest rate in Canada was constrained at its effective lower bound only for four quarters after the global financial crisis.

where ρ is the degree of interest rate smoothing, and α_π, α_y, and α_s are the degrees of monetary policy responses to their target variables. u_t is a monetary policy shock. We include exchange rate depreciation in the policy rule following Justiniano and Preston (2010), who estimated a small open-economy model similar to ours for the Canadian economy.

3.4 Equilibrium conditions and detrending

The equilibrium conditions of the model are presented in Appendix A. To ensure the stationarity of the system of equations, real variables are detrended by the nonstationary trend component Z_t as follows: $y_t := Y_t/Z_t$, $y_{H,t} := Y_{H,t}/Z_t$, $y_{F,t} := Y_{F,t}/Z_t$, $y_t^* := Y_t^*/Z_t$, and $c_t := C_t/Z_t$. The steady-state conditions in terms of detrended variables are presented in Appendix B, whereas the log-linearized version of the detrended system of equations is shown in Appendix C.

3.5 Exogenous shock processes

In addition to the fundamental shocks mentioned above, we treat the foreign output $\hat{y}_t^*$, inflation $\hat{\pi}_t^*$, and the nominal interest rate $\hat{i}_t^*$ as exogenous shocks.[5] We assume that all the shocks except for the monetary policy shock follow stationary AR(1) processes:

$$
\begin{aligned}
\hat{\psi}_t &= \rho_\psi \hat{\psi}_{t-1} + \varepsilon_{\psi,t}, \\
z_t &= \rho_z z_{t-1} + \varepsilon_{z,t}, \\
\tilde{\mu}_t &= \rho_\mu \tilde{\mu}_{t-1} + \varepsilon_{\mu,t}, \\
u_t &= \varepsilon_{u,t}, \\
\hat{y}_t^* &= \rho_{y^*} \hat{y}_{t-1}^* + \varepsilon_{y^*,t}, \\
\hat{\pi}_t^* &= \rho_{\pi^*} \hat{\pi}_{t-1}^* + \varepsilon_{\pi^*,t}, \\
\hat{i}_t^* &= \rho_{i^*} \hat{i}_{t-1}^* + \varepsilon_{i^*,t},
\end{aligned}
$$

[5] Justiniano and Preston (2010) endogenize these foreign variables by adding a small-scale New Keynesian model into the system of equations. We treat them as exogenous because our estimation sample includes the zero lower bound periods in the US, which cannot be well captured by linearized equations.

where $\tilde{\mu}_t = (\epsilon - 1)/\{\phi(1+\beta\omega)\}\mu_t$, ρ_x for $x \in \{z, \mu, \psi, y^*, \pi^*, i^*\}$ are the autoregressive parameters and $\varepsilon_{x,t} \sim$ i.i.d. $N(0, \sigma_x^2)$ for $x \in \{z, \mu, u, \psi, y^*, \pi^*, i^*\}$.

3.6 Modified UIP condition and equilibrium indeterminacy

3.6.1 Modified UIP condition

From the optimality conditions for home and foreign bond holdings, we can derive the modified UIP condition as follows:

$$i_t = \mathbb{E}_t \left\{ s_{t+1} \exp\left[-\phi_a (a_t^* - a^*) - \phi_s \left(s_{t+1} - \frac{\pi}{\pi^*} \right) + \psi_t \right] i_t^* \right\},$$

where $s_t := S_t/S_{t-1}$ represents the depreciation of the nominal exchange rate and $a_t^* := S_t A_t^*/P_t Z_t$ is detrended real net foreign assets in the home currency.

Assuming that $\pi = \pi^*$, log-linearizing this equation around the steady state gives

$$\mathbb{E}_t \hat{s}_{t+1} = \frac{1}{1-\phi_s}(\hat{i}_t - \hat{i}_t^*) + \frac{1}{1-\phi_s}(\phi_a a^* \hat{a}_t^* - \hat{\psi}_t), \tag{2}$$

where hatted variables denote percentage deviation from their corresponding steady-state values. Note that setting $\phi_s = \phi_a = 0$ leads to the standard UIP condition:

$$\mathbb{E}_t \hat{s}_{t+1} = \hat{i}_t - \hat{i}_t^* - \hat{\psi}_t. \tag{3}$$

As addressed in Section 2, the UIP relationship given by (3) has been frequently rejected for various international data by conducting the UIP regression:

$$\log S_{t+1} - \log S_t = \alpha_0 + \alpha_1(\log i_t - \log i_t^*) + v_t, \tag{4}$$

with the null hypothesis $H_0 : \alpha_0 = 0$ and $\alpha_1 = 1$. Indeed, the OLS estimator of α_1 is -0.012 with the standard error of 0.702 according to our preliminary estimation of (4) using a Canada–US data set, described in Section 4.3, ranging from 1984:Q1 to 2019:Q4. Assuming rational expectations and ignoring the endogeneity of $\hat{a}_t^*$,[6] the regression coefficient α_1 is

[6] We can ignore the endogeneity of $\hat{a}_t^*$ because this term is needed to ensure stationarity of the small-open economy model with incomplete asset markets and because ϕ_a is commonly set at very small values less than

consistent with $1/(1-\phi_s)$ in the modified UIP condition given by (2). A novel feature of our analysis is that we allow for values of ϕ_s greater than one so that the modified UIP condition given by (2) can be consistent with negative coefficients in the UIP regressions.

Because of the empirical failure of the standard UIP relationship, the literature that estimates open-economy DSGE models with a UIP equation such as (3) finds very large contributions of the UIP shocks $\hat{\psi}_t$ to exchange rate fluctuations.[7] This conventional wisdom in the literature might be overturned once we embed the modified UIP condition (2) into the model and allow for the observed negative relationship between expected exchange rate depreciation $\mathbb{E}_t\hat{s}_{t+1}$ and the interest rate differential $\hat{i}_t - \hat{i}_t^*$.

3.6.2 Equilibrium indeterminacy

In dynamic general equilibrium economies, equilibrium can be indeterminate, depending on model structures and parameters that characterize them, and in such a case, sunspot shocks, which are nonfundamental disturbances, can affect economic fluctuations. In what follows, we demonstrate that the equilibrium can be indeterminate, depending on the parameters in the modified UIP condition (2):

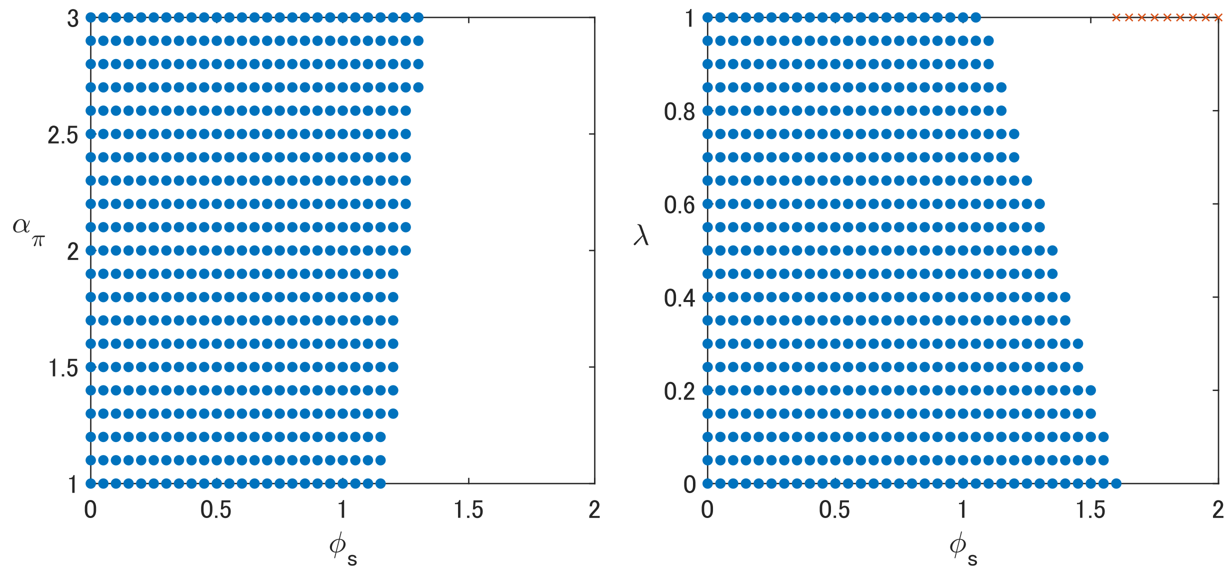
$$\mathbb{E}_t\hat{s}_{t+1} = \frac{1}{1-\phi_s}(\hat{i}_t - \hat{i}_t^*) + \frac{1}{1-\phi_s}(\phi_a a^* \hat{a}_t^* - \hat{\psi}_t). \tag{5}$$

Figure 1 presents the equilibrium determinacy regions in the parameter space for (ϕ_s, α_π) and (ϕ_s, λ), respectively, given the prior means of the other parameters reported in Table 1 (Section 4). The first panel illustrates that sufficiently large values of ϕ_s lead to equilibrium indeterminacy and that its threshold value for indeterminacy increases as α_π (monetary policy reaction to inflation) increases. Likewise, the second panel shows that large values of ϕ_s give rise to indeterminacy, although decreases in λ (relative weight on domestically produced goods in aggregate consumption) expand the determinacy region. A notable finding from these two panels is that, regardless of the values of α_π and λ, indeterminacy arises for such large values of ϕ_s that the modified UIP condition exhibits a negative relationship between expected exchange rate depreciation $\mathbb{E}_t\hat{s}_{t+1}$ and interest rate differential $\hat{i}_t - \hat{i}_t^*$ as observed

0.01.

[7]See Lubik and Schorfheide (2006), Hirose (2013), and Chen et al. (2021).

Figure 1: Determinacy regions in the parameter space



Notes: Each panel of this figure displays the equilibrium determinacy region in the parameter space for (ϕ_s, α_π) and (ϕ_s, λ), respectively, given the prior mean of the other parameters reported in Table 1. '$\times$' denotes the region for nonexistence of equilibrium.

in international data.

To understand why large values of ϕ_s generate equilibrium indeterminacy, consider the case in which the monetary policy rule is of the simple form:

$$\hat{i}_t = \alpha_\pi \hat{\pi}_t. \tag{6}$$

In the model, the aggregate price is given by

$$P_t = P_{H,t}^{\lambda} \left(S_t P_t^* \right)^{1-\lambda},$$

which can be written in log-linearized form in terms of change rates and be solved for the exchange rate depreciation $\hat{s}_t$:

$$\hat{s}_t = \frac{1}{1-\lambda} \hat{\pi}_t - \frac{\lambda}{1-\lambda} \hat{\pi}_{H,t} - \hat{\pi}_t^*. \tag{7}$$

Substituting (6) and (7) into the modified UIP condition (2), we obtain

$$\frac{1}{1-\lambda}\mathbb{E}_t\hat{\pi}_{t+1} - \frac{\lambda}{1-\lambda}\mathbb{E}_t\hat{\pi}_{H,t+1} - \mathbb{E}_t\hat{\pi}^*_{t+1} = \frac{1}{1-\phi_s}(\alpha_\pi\hat{\pi}_t - \hat{i}^*_t) + \frac{1}{1-\phi_s}(\phi_a a^*\hat{a}^*_t - \hat{\psi}_t). \quad (8)$$

Isolating the relationship between $\mathbb{E}_t\hat{\pi}_{t+1}$ and $\hat{\pi}_t$ from this equation under the assumption that the stability of the other variables is guaranteed by other equations in the system, we have

$$\mathbb{E}_t\hat{\pi}_{t+1} = \varrho\hat{\pi}_t,$$

where $\varrho = \alpha_\pi(1-\lambda)/(1-\phi_s)$. In the analogy of Blanchard and Kahn (1980) conditions, expected inflation is uniquely pinned down (*i.e.*, determinate) if the composite parameter ϱ is outside the unit circle.[8] In a limiting case where $\alpha_\pi = 1$ and $\lambda = 0$, this condition is not satisfied for $\phi_s \geq 2$. Thus, sufficiently large values of ϕ_s lead to the equilibrium being indeterminate. As each of α_π and λ increases from the limiting case, the threshold value of ϕ_s for $|\varrho| \leq 1$ increases and decreases, respectively. This is consistent with the determinacy regions illustrated in Figure 1.

To gain intuition about why the negative relationship between $\mathbb{E}_t\hat{s}_{t+1}$ and $\hat{i}_t - \hat{i}^*_t$ in the modified UIP condition gives rise to equilibrium indeterminacy, suppose that agents believe a rise in future inflation without any changes in fundamentals. Then, taking one-period-ahead expectations for the both sides of (7), the future exchange rate is expected to depreciate (*i.e.*, $\mathbb{E}_t\hat{s}_{t+1}$ increases). If the coefficient on $\hat{i}_t - \hat{i}^*_t$ in the modified UIP condition is negative, expected future depreciation must coincide with a decrease in the domestic nominal interest rate, given the foreign one. The low interest rate stimulates the demand side of the economy and increases both output and inflation. Therefore, the non-fundamental belief on inflation can be self-fulfilling as an equilibrium. On the other hand, if the modified UIP equation exhibits a positive relationship between $\mathbb{E}_t\hat{s}_{t+1}$ and $\hat{i}_t - \hat{i}^*_t$, expected future depreciation is accompanied by an increase in the domestic nominal interest rate, which dampens output and inflation. Thus, the non-fundamental belief on inflation does not materialize and cannot be an equilibrium.

[8] Conditions for determinacy depend on eigenvalues in the autoregressive coefficient matrix of the whole system. In the present setting, however, we cannot characterize them analytically because of the high dimensionality of the system. Thus, we focus on the univariate relationship between $\mathbb{E}_t\hat{\pi}_{t+1}$ and $\hat{\pi}_t$ to illustrate the source of indeterminacy.

If we allow for indeterminacy, sunspot shocks, which are nonfundamental disturbances to the economy, can arise and affect equilibrium dynamics. As addressed above, the literature that estimates open-economy DSGE models has found that almost all fluctuations in exchange rates are driven by wedges to the UIP condition, *i.e.*, UIP shocks, reflecting the empirical failure of the standard UIP equation. Under indeterminacy, however, some portion of its contribution might be replaced with that from sunspot shocks. This point is also examined in the subsequent empirical analysis.

It should be noted that equilibrium indeterminacy does not occur for any $\phi_s > 0$ if we employ Adolfson et al. (2008)'s original specification for the endogenous risk premium on foreign bond holdings:

$$\Phi_t := \exp\left[-\phi_a\left(\frac{S_t A_t^*}{P_t Z_t} - a^*\right) - \phi_s\left(\frac{\mathbb{E}_t S_{t+1}}{S_t}\frac{S_t}{S_{t-1}} - \frac{\pi}{\pi^*}\right) + \psi_t\right],$$

where, differently from our specification (1), the risk premium depends on expected *consecutive* depreciation, or expected changes in the exchange rate from $t-1$ to $t+1$. Under this specification, the modified UIP condition is of the log-linearized form:

$$\mathbb{E}_t \hat{s}_{t+1} = \frac{1}{1-\phi_s}(\hat{i}_t - \hat{i}_t^*) + \frac{\phi_s}{1-\phi_s}\hat{s}_t + \frac{1}{1-\phi_s}(\phi_a a^* \hat{a}_t^* - \hat{\psi}_t).$$

Our preliminary estimation of the model with this modified UIP condition has confirmed that the model fits the data much worse than our baseline model. Moreover, we consider the hybrid specification between Adolfson et al. (2008)'s and ours:

$$\Phi_t := \exp\left[-\phi_a\left(\frac{S_t A_t^*}{P_t Z_t} - a^*\right) - \phi_s\left(\left(\frac{\mathbb{E}_t S_{t+1}}{S_t}\right)^{\phi_1}\left(\frac{S_t}{S_{t-1}}\right)^{1-\phi_1} - \frac{\pi}{\pi^*}\right) + \psi_t\right],$$

which gives another modified UIP condition:

$$\mathbb{E}_t \hat{s}_{t+1} = \frac{1}{1-\phi_s\phi_1}(\hat{i}_t - \hat{i}_t^*) + \frac{\phi_s(1-\phi_1)}{1-\phi_s\phi_1}\hat{s}_t + \frac{1}{1-\phi_s\phi_1}(\phi_a a^* \hat{a}_t^* - \hat{\psi}_t).$$

This specification coincides with our baseline specification when $\phi_1 = 1$. We estimated the model under this hybrid specification with a beta prior for ϕ_1 with mean 0.5 and standard deviation 0.2 and the other priors being the same as in the baseline estimation shown in

Section 4.3 (Table 1). According to the estimation results, the posterior mean estimate of ϕ_1 is very close to 1 (0.977), supporting our baseline specification. These preliminary estimation results are reported in Appendix D.

4 Estimation Strategy

As shown in the previous section, equilibrium indeterminacy can occur when the modified UIP condition exhibits a negative relationship between expected exchange rate depreciation $\mathbb{E}_t \hat{s}_{t+1}$ and the interest rate differential $\hat{i}_t - \hat{i}_t^*$, often found in the UIP regressions. If we restricted the model's parameter space to the determinacy region in estimation, it would be unlikely that the estimated model could replicate the observed pattern between the two.

To overcome this issue, we estimate the model allowing for indeterminacy, using full-information Bayesian methods based on Lubik and Schorfheide (2004). Specifically, the model's likelihood function is constructed not only for the determinacy region of its parameter space but also for the indeterminacy region. While Lubik and Schorfheide (2004) conduct model estimation separately for each region, we estimate the model for both the determinacy and indeterminacy regions in one step by adopting a sequential Monte Carlo (SMC) algorithm, as implemented by Hirose et al. (2020). This algorithm can deal with discontinuity in the likelihood function at the boundary of each region and help us find the entire posterior distribution of model parameters.

In this section, we begin by presenting solutions to linear rational expectations models, then explain how Bayesian inference over both the determinacy and indeterminacy regions of the model parameter space are made with the SMC algorithm, and lastly describe the data and prior distributions used in the model estimation.

4.1 Rational expectations solutions under indeterminacy

Lubik and Schorfheide (2003) derive a full set of solutions to the models under indeterminacy of the form

$$\mathbf{s}_t = \boldsymbol{\Phi}_1^I(\boldsymbol{\theta})\,\mathbf{s}_{t-1} + \boldsymbol{\Phi}_\varepsilon^I(\boldsymbol{\theta}, \tilde{\mathbf{M}})\,\boldsymbol{\varepsilon}_t + \boldsymbol{\Phi}_\zeta^I(\boldsymbol{\theta})\,\zeta_t, \tag{9}$$

where $\boldsymbol{\Phi}_1^I(\boldsymbol{\theta})$, $\boldsymbol{\Phi}_\varepsilon^I(\boldsymbol{\theta}, \tilde{\mathbf{M}})$, and $\boldsymbol{\Phi}_\zeta^I(\boldsymbol{\theta})$ are coefficient matrices that depend on the vector $\boldsymbol{\theta}$

of model parameters and an arbitrary matrix $\tilde{\mathbf{M}}$; $\mathbf{s}_t$ is a vector of endogenous variables; $\boldsymbol{\varepsilon}_t$ is a vector of fundamental shocks; and $\zeta_t \sim$ i.i.d. $N(0, \sigma_\zeta^2)$ is a reduced-form sunspot shock.[9] The indeterminacy solution (9) displays three characteristics. First, the equilibrium dynamics are driven not only by the fundamental shocks $\boldsymbol{\varepsilon}_t$ but also by the sunspot shock ζ_t. Second, the solution is not unique because of the presence of the arbitrary matrix $\tilde{\mathbf{M}}$. Third, the coefficient matrix $\boldsymbol{\Phi}_1^I(\boldsymbol{\theta})$ in the solution induces more persistent dynamics than its counterpart $\boldsymbol{\Phi}_1^D(\boldsymbol{\theta})$ in the determinacy solution presented below, because fewer autoregressive roots (*i.e.*, eigenvalues) in the matrix $\boldsymbol{\Phi}_1^I(\boldsymbol{\theta})$ are being suppressed to zero.[10]

In the case of determinacy, the solution form is reduced to

$$\mathbf{s}_t = \boldsymbol{\Phi}_1^D(\boldsymbol{\theta})\, \mathbf{s}_{t-1} + \boldsymbol{\Phi}_\varepsilon^D(\boldsymbol{\theta})\, \boldsymbol{\varepsilon}_t, \tag{10}$$

where the coefficient matrices $\boldsymbol{\Phi}_1^D(\boldsymbol{\theta})$ and $\boldsymbol{\Phi}_\varepsilon^D(\boldsymbol{\theta})$ depend only on the model parameters $\boldsymbol{\theta}$. Thus, the solution is uniquely determined and driven by only the fundamental shocks $\boldsymbol{\varepsilon}_t$.

Under indeterminacy, the matrix $\tilde{\mathbf{M}}$ must be determined to specify the law of motion of the endogenous variables $\mathbf{s}_t$. Following Lubik and Schorfheide (2004), we estimate the components of $\tilde{\mathbf{M}}$ along with the other parameters in the model. The prior distribution for $\tilde{\mathbf{M}}$ is set so that it is centered around the matrix $\mathbf{M}^*(\boldsymbol{\theta})$ given in a particular solution. That is, $\tilde{\mathbf{M}}$ is replaced with $\mathbf{M}^*(\boldsymbol{\theta}) + \mathbf{M}$, and the components of $\mathbf{M}$ are estimated with prior mean zero. As proposed by Lubik and Schorfheide (2004), the matrix $\mathbf{M}^*(\boldsymbol{\theta})$ is selected so that the contemporaneous impulse responses of endogenous variables to fundamental shocks (i.e., $\partial \mathbf{s}_t / \partial \boldsymbol{\varepsilon}_t$) are continuous at the boundary between the determinacy and indeterminacy regions of the model parameter space, which is called a "continuity solution." More specifically, for each set of $\boldsymbol{\theta}$, the procedure searches for a vector $\boldsymbol{\theta}^*$ that lies on the boundary of the determinacy region, and selects $\mathbf{M}^*(\boldsymbol{\theta})$ that minimizes the discrepancy between $\partial \mathbf{s}_t / \partial \boldsymbol{\varepsilon}_t(\boldsymbol{\theta}, \mathbf{M}^*(\boldsymbol{\theta}))$, and $\partial \mathbf{s}_t / \partial \boldsymbol{\varepsilon}_t(\boldsymbol{\theta}^*)$ using a least-squares criterion. In searching for $\boldsymbol{\theta}^*$, the procedure finds $\boldsymbol{\theta}^*$ numerically by perturbing the risk-premium parameter ϕ_s in the modified UIP condition (2),

[9] Instead of the term $\boldsymbol{\Phi}_\zeta^I(\boldsymbol{\theta})\, \zeta_t$ in the indeterminacy solution (9), Lubik and Schorfheide (2003) originally consider $\boldsymbol{\Phi}_\zeta^I(\boldsymbol{\theta}, \mathbf{M}_\zeta)\, \boldsymbol{\zeta}_t$, where $\mathbf{M}_\zeta$ is an arbitrary matrix and $\boldsymbol{\zeta}_t$ is a vector of sunspot shocks. For identification, however, Lubik and Schorfheide (2004) impose normalization on $\mathbf{M}_\zeta$ with the dimension of the sunspot shock vector being unity. Such a normalized shock is referred to as a "reduced-form sunspot shock" in that it contains beliefs associated with all the expectational variables.

[10] For details, see Lubik and Schorfheide (2003, 2004).

which is crucial for determinacy or indeterminacy, given the other parameters in $\boldsymbol{\theta}$.[11]

4.2 Bayesian inference

To conduct Bayesian inference over both the determinacy and indeterminacy regions of the model parameter space, we construct the likelihood function for a sample of observations $\mathbf{Y}^T = [\mathbf{Y}_1, ..., \mathbf{Y}_T]'$ as

$$p(\mathbf{Y}^T|\boldsymbol{\theta}, \mathbf{M}) = \mathbf{1}\{\boldsymbol{\theta} \in \boldsymbol{\Theta}^D\}\, p^D(\mathbf{Y}^T|\boldsymbol{\theta}) + \mathbf{1}\{\boldsymbol{\theta} \in \boldsymbol{\Theta}^I\}\, p^I(\mathbf{Y}^T|\boldsymbol{\theta}, \mathbf{M}),$$

where $\boldsymbol{\Theta}^D$ and $\boldsymbol{\Theta}^I$ are the determinacy and indeterminacy regions of the model parameter space; $\mathbf{1}\{\boldsymbol{\theta} \in \boldsymbol{\Theta}^i\}$, $i \in \{D, I\}$ is the indicator function that is equal to one if $\boldsymbol{\theta} \in \boldsymbol{\Theta}^i$ and zero otherwise; and $p^D(\mathbf{Y}^T|\boldsymbol{\theta})$ and $p^I(\mathbf{Y}^T|\boldsymbol{\theta}, \mathbf{M})$ are the likelihood functions of the state-space models that consist of observation equations and either the determinacy solution (10) or the indeterminacy solution (9). Then, by Bayes' theorem, updating a prior distribution $p(\boldsymbol{\theta}, \mathbf{M})$ with the sample observations $\mathbf{Y}^T$ leads to the posterior distribution

$$p(\boldsymbol{\theta}, \mathbf{M}|\mathbf{Y}^T) = \frac{p(\mathbf{Y}^T|\boldsymbol{\theta}, \mathbf{M})p(\boldsymbol{\theta}, \mathbf{M})}{p(\mathbf{Y}^T)} = \frac{p(\mathbf{Y}^T|\boldsymbol{\theta}, \mathbf{M})p(\boldsymbol{\theta}, \mathbf{M})}{\int p(\mathbf{Y}^T|\boldsymbol{\theta}, \mathbf{M})p(\boldsymbol{\theta}, \mathbf{M})d\boldsymbol{\theta}d\mathbf{M}}.$$

To approximate the posterior distribution, we adopt the generic SMC algorithm with likelihood tempering described in Herbst and Schorfheide (2014, 2015). The details of the algorithm are described in Appendix E. Based on particles from the final importance sampling in the algorithm, we make inferences on parameters and approximate the marginal data densities.

4.3 Data and prior distributions

The data used in the model estimation are seven quarterly time series on $\Delta \log Y_t$, $\log \pi_t$, $\log i_t$,$\Delta \log S_t$, $\hat{y}_t^*$, $\hat{\pi}_t^*$, and $\hat{i}_t^*$. The first four series are constructed from Canadian data: per

[11] In estimating a closed-economy New Keynesian model with nonzero trend inflation under both determinacy and indeterminacy, Hirose et al. (2020) employ a similar procedure, which finds $\boldsymbol{\theta}^*$ by perturbing a monetary policy reaction parameter on inflation. In this paper, the prior distribution for this parameter is truncated so that it does not cause indeterminacy (see Section 4.3), and hence the risk-premium parameter ϕ_s is the primary source of indeterminacy.

capita real GDP, GDP deflator, the Bank of Canada's policy rate (overnight rate), and the Canadian to US dollar exchange rate. The other series are proxied by the US data. In the model, foreign variables are treated as exogenous shocks that follow AR(1) processes. To identify these exogenous processes directly without considering the differences in trends and the steady states between Canada and the US, the data on $\hat{y}_t^*$, $\hat{\pi}_t^*$, and $\hat{i}_t^*$ are constructed by detrending per capita real GDP using the Hodrick-Prescott (HP) filter and demeaning the GDP deflator inflation rate and the federal funds rate. Thus, the observation equations that relate the data to the corresponding variables in the model are given by

$$\begin{bmatrix} 100\Delta\log Y_t \\ 100\log\pi_t \\ 100\log i_t \\ 100\Delta\log S_t \\ \hat{y}_t^*, \\ \hat{\pi}_t^* \\ \hat{i}_t^* \end{bmatrix} = \begin{bmatrix} \overline{\gamma} \\ \overline{\pi} \\ \overline{i} \\ \overline{\pi}-\overline{\pi}^* \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} \hat{y}_t-\hat{y}_{t-1} \\ \hat{\pi}_t \\ \hat{i}_t \\ \hat{s}_t \\ \hat{y}_t^* \\ \hat{\pi}_t^* \\ \hat{i}_t^* \end{bmatrix},$$

where $\overline{\gamma} := 100(\gamma-1)$, $\overline{\pi} := 100(\pi-1)$, $\overline{i} := 100(i-1)$, and $\overline{\pi}^* := 100(\pi^*-1)$. The sample period is from 1984:Q1 to 2019:Q4, so that we exclude the Great Inflation period and the COVID-19 pandemic period from the sample.

To avoid any identification issues, we fix two parameters in the model. The steady-state ratio of net foreign assets to GDP is fixed at $a^*/y = -1.2$, which is calculated from the steady-state relationship on the balance of payments identity and the sample average of net exports in Canada. The elasticity of substitution among differentiated goods is fixed at $\epsilon = 8$, following Justiniano and Preston (2010). We assume $\overline{\pi}^* = \overline{\pi}$ and estimate $\overline{\pi}$ as a parameter.[12]

All other parameters in the model are estimated.[13] Their prior distributions are presented in Table 1.

The novelty of this study is that we allow one of the risk premium parameters ϕ_s to

[12]This assumption is supported by the data: in our sample period, the quarterly inflation rates in Canada and the US are 0.557% and 0.543%, respectively.

[13]As for the subjective discount factor β, the steady-state condition $\beta = \pi\gamma/i$ is used in estimation.

Table 1: Prior distributions of parameters

Parameter	Distribution	Mean	S.D.
ϕ_s	Gamma	0.500	0.500
ϕ_a	Inverse gamma	0.010	2.000
b	Beta	0.500	0.100
η	Gamma	1.000	0.300
ϕ	Gamma	40.00	10.00
ω	Beta	0.500	0.100
λ	Beta	0.710	0.020
ρ	Beta	0.600	0.200
α_π	Gamma	1.800	0.300
α_y	Gamma	0.300	0.200
α_s	Gamma	0.300	0.200
$\overline{\gamma}$	Normal	0.290	0.050
$\overline{\pi}$	Normal	0.557	0.050
$\overline{i}$	Gamma	1.194	0.050
ρ_ψ	Beta	0.500	0.200
ρ_z	Beta	0.500	0.200
ρ_μ	Beta	0.500	0.200
ρ_{y^*}	Beta	0.500	0.200
ρ_{π^*}	Beta	0.500	0.200
ρ_{i^*}	Beta	0.500	0.200
σ_ψ	Inverse gamma	0.500	4.000
σ_z	Inverse gamma	0.500	4.000
σ_μ	Inverse gamma	0.500	4.000
σ_u	Inverse gamma	0.500	4.000
σ_{y^*}	Inverse gamma	0.500	4.000
σ_{π^*}	Inverse gamma	0.500	4.000
σ_{i^*}	Inverse gamma	0.500	4.000
σ_ζ	Inverse gamma	0.500	4.000
M_ψ	Normal	0.000	1.000
M_z	Normal	0.000	1.000
M_μ	Normal	0.000	1.000
M_u	Normal	0.000	1.000
M_{y^*}	Normal	0.000	1.000
M_{π^*}	Normal	0.000	1.000
M_{i^*}	Normal	0.000	1.000

Notes: The prior probability of equilibrium determinacy is 0.921. Inverse gamma distributions are of the form $p(\sigma|\nu, s) \propto \sigma^{-\nu-1}e^{-\nu s^2/2\sigma^2}$, where ν and s are respectively set at the values of Mean and S.D. in the table.

exceed one so that the modified UIP condition exhibits a negative relationship between the expected exchange rate depreciation and the interest rate differential. Thus, we impose a gamma distribution with mean 0.5 and standard deviation 0.5, while Adolfson et al. (2008) use a beta distribution with mean 0.5 and standard deviation 0.15. Note that our prior sets its mean and standard deviation at the same value and that the prior density takes the largest value at $\phi_s = 0$, which corresponds to the standard UIP equation as shown in (3). Hence, our prior strongly favors the conventional parameter values for ϕ_s, which is less than one, so that the equilibrium is determinate.[14] For another risk premium parameter ϕ_a, we use the same prior as Adolfson et al. (2008).

The prior mean for price adjustment cost is set at $\phi = 40$ so that the slope of the Phillips curve is 0.1 when $\epsilon = 8$ and $\omega = 0.5$. The priors for the other structural parameters on the household, firms, and central bank follow Justiniano and Preston (2010). We impose a gamma prior on the degree of monetary policy response to inflation α_π, but it is truncated at one so that this parameter itself cannot be the source of indeterminacy.

The prior means for the steady-state (quarterly) rates of GDP growth, inflation, and nominal interest are set at the respective averages of the data used in the estimation.

Regarding the shock parameters, the priors for AR(1) shock persistence parameters ρ_x, $x \in \{z, \mu, \psi, y^*, \pi^*, i^*\}$ are the same as those in Smets and Wouters (2007). For the shock standard deviations σ_x, $x \in \{z, \mu, u, \psi, y^*, \pi^*, i^*, \zeta\}$, we set $\nu = 0.5$ and $s = 4$ in the inverse gamma distribution of the form $p(\sigma|\nu, s) \propto \sigma^{-\nu-1}e^{-\nu s^2/2\sigma^2}$.

For the components $M_x, x \in \{z, \mu, u, \psi, y^*, \pi^*, i^*\}$ of the arbitrary matrix $\mathbf{M}$ in the indeterminacy solution, we use the same priors as those in Lubik and Schorfheide (2004), i.e., the normal distributions with mean zero and standard deviation one.

5 Estimation Results

In this section, we begin by presenting the parameter estimates in our baseline model and compare them with those obtained by estimating the same model only in the determinacy region of its parameter space. Next, we analyze how the dynamic properties of the model can differ between the two cases of determinacy and indeterminacy by comparing the impulse

[14] Indeed, the prior distributions presented here lead to the prior probability of determinacy of 0.921.

response functions. Furthermore, we conduct UIP regressions using simulated data and examine whether our model can replicate negative a relationship between expected exchange rate depreciation and interest rate differentials. Lastly, we conduct variance decompositions to investigate the source of macroeconomic fluctuations in the estimated model, in particular, exchange rate fluctuations.

5.1 Parameter estimates

Table 2 compares the posterior estimates of parameters in the baseline model with those in the model estimated only in the determinacy region of the parameter space. This table presents four key features.

First, in the baseline model, the posterior mean estimate of ϕ_s (risk-premium parameter associated with exchange rate depreciation in the modified UIP condition) is 3.678, which gives the negative slope coefficient $1/(1-\phi_s) = -0.373$ in the modified UIP equation (2). Thus, our modified UIP condition exhibits a negative relationship between the exchange rate depreciation and interest rate differentials. Because the estimate of ϕ_s is far above one, the equilibrium is indeterminate. Indeed, the posterior probability of determinacy $\mathbb{P}\{\boldsymbol{\theta} \in \boldsymbol{\Theta}^D|\mathbf{Y}^T\}$ shown in the last row of the table is zero, indicating that all the posterior draws of the model parameters lie in the indeterminacy region. In contrast, when the model is estimated only under determinacy, ϕ_s is estimated to be very small.

Second, the second last row shows that the log marginal data density $\log p(\mathbf{Y}^T)$ is much larger in the baseline estimation than the case in which we restrict the parameter space to its determinacy region. This huge difference in the marginal data density indicates that the data strongly favor indeterminacy over determinacy.

Third, the UIP shock's persistence parameter ρ_ψ indicates that the shock follows almost a unit-root process in both the baseline and determinacy cases. This result reflects the empirical properties of exchange rate dynamics such as its random-walk behavior and very persistent deviation from the one-to-one relationship between the exchange rate depreciation and interest rate differentials in the modified UIP condition. A crucial finding here is that such persistent deviation cannot be resolved even though we allow for a negative UIP relationship between the two as observed in the data.

Last, regarding the indeterminacy-related parameters, some of the components (M_ψ,

Table 2: Posterior estimates of parameters

	Baseline		Determinacy	
Parameter	Mean	90% interval	Mean	90% interval
ϕ_s	3.678	[3.183, 4.173]	0.093	[0.000, 0.211]
ϕ_a	0.006	[0.004, 0.008]	0.004	[0.003, 0.006]
b	0.360	[0.300, 0.430]	0.194	[0.111, 0.268]
η	1.223	[0.827, 1.617]	1.863	[1.151, 2.575]
ϕ	28.808	[18.361, 39.546]	32.278	[15.887, 49.508]
ω	0.171	[0.084, 0.252]	0.389	[0.244, 0.536]
λ	0.799	[0.778, 0.820]	0.863	[0.847, 0.882]
ρ	0.871	[0.844, 0.894]	0.808	[0.765, 0.851]
α_π	1.793	[1.514, 2.065]	2.196	[1.906, 2.506]
α_y	0.783	[0.462, 1.130]	0.106	[0.007, 0.193]
α_s	0.237	[0.145, 0.329]	0.266	[0.178, 0.363]
$\overline{\gamma}$	0.282	[0.239, 0.322]	0.187	[0.127, 0.251]
$\overline{\pi}$	0.540	[0.474, 0.610]	0.513	[0.447, 0.576]
$\overline{i}$	1.194	[1.118, 1.269]	1.200	[1.122, 1.278]
ρ_ψ	0.993	[0.988, 1.000]	0.991	[0.985, 0.998]
ρ_z	0.506	[0.212, 0.981]	0.993	[0.986, 0.999]
ρ_μ	0.944	[0.891, 0.998]	0.516	[0.409, 0.618]
ρ_{y^*}	0.840	[0.771, 0.904]	0.869	[0.806, 0.937]
ρ_{π^*}	0.621	[0.520, 0.729]	0.614	[0.495, 0.745]
ρ_{i^*}	0.951	[0.932, 0.971]	0.922	[0.903, 0.942]
σ_ψ	0.747	[0.510, 0.994]	0.171	[0.145, 0.196]
σ_z	0.539	[0.295, 0.774]	1.634	[1.365, 1.883]
σ_μ	0.710	[0.528, 0.864]	1.492	[1.110, 1.867]
σ_u	0.242	[0.212, 0.275]	0.332	[0.268, 0.394]
σ_{y^*}	0.489	[0.442, 0.535]	0.507	[0.454, 0.561]
σ_{π^*}	0.205	[0.185, 0.226]	0.210	[0.187, 0.232]
σ_{i^*}	0.151	[0.134, 0.166]	0.156	[0.140, 0.172]
σ_ζ	1.841	[1.038, 2.532]	-	-
M_ψ	-1.595	[-2.536, -0.713]	-	-
M_z	-1.238	[-2.600, 0.099]	-	-
M_μ	0.419	[-0.382, 1.277]	-	-
M_u	2.203	[1.019, 3.260]	-	-
M_{y^*}	0.601	[-0.012, 1.225]	-	-
M_{π^*}	0.258	[-0.800, 1.324]	-	-
M_{i^*}	0.447	[-0.848, 1.795]	-	-
$\log p(\mathbf{Y}^T)$	-781.985		-1032.170	
$\mathbb{P}\{\boldsymbol{\theta} \in \mathbf{\Theta}^D \mid \mathbf{Y}^T\}$	0.000		1.000	

Notes: This table reports the posterior mean and 90 percent highest posterior density intervals based on $10,000$ particles from the final importance sampling in the SMC algorithm. In the table, $\log p(\mathbf{Y}^T)$ represents the SMC-based approximation of log marginal data density and $\mathbb{P}\{\boldsymbol{\theta} \in \mathbf{\Theta}^D|\mathbf{Y}^T\}$ denotes the posterior probability of equilibrium determinacy.

M_z, and M_u) in the arbitrary matrix $\mathbf{M}$ are substantially different from zero. This finding suggests the importance of considering multiplicity of the equilibrium representation under indeterminacy. As shown in the following subsection, these estimates considerably alter the impulse response functions under indeterminacy.

5.2 Impulse response functions

Figure 2 depicts the impulse responses of output growth, inflation, interest rate, and exchange rate depreciation in terms of percentage deviations from steady-state values, to a one-standard-deviation shock to the modified UIP condition, technology, marginal cost, monetary policy, US output, US inflation, US interest rate, and sunspot, given the posterior mean estimates of the parameters in the baseline model allowing for indeterminacy and its counterpart estimated only in the determinacy region of the parameter space. To examine the effects of the estimated arbitrary matrix $\mathbf{M}$ on the propagation of shocks, the figure also presents the responses in the baseline model with $\mathbf{M} = 0$, given the other parameters fixed at the same values as in the baseline model.

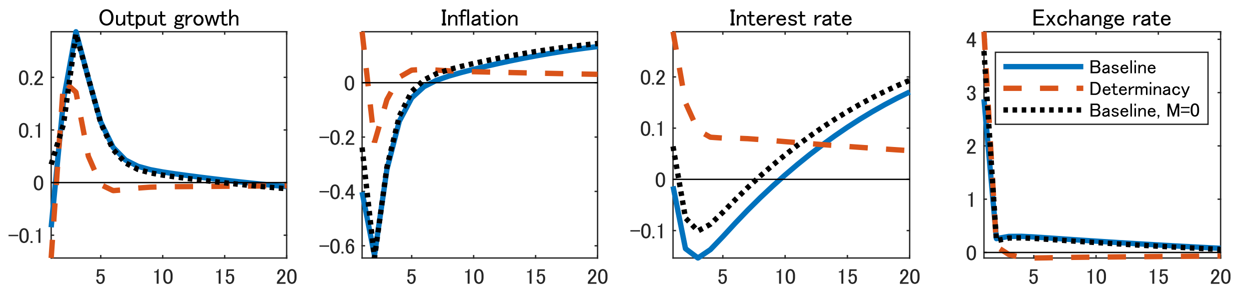
For all shocks, most of the impulse responses differ remarkably between the baseline (solid lines) and determinacy (dashed lines) cases, although our solution under indeterminacy is centered at the continuity solution as addressed in Section 4.1. In particular, the exchange rate responds in the opposite direction in response to the shocks in technology, marginal cost, and US output. These differences are attributed to the estimated components in the arbitrary matrix $\mathbf{M}$, which cause the indeterminacy solution to deviate from the continuity solution. If we do not allow for such deviation, responses in the opposite direction no longer occur, as shown by the dotted lines (Baseline, $\mathbf{M} = 0$).

In comparison with the empirical findings in the literature on the Canadian economy (Grilli and Roubini, 1996; Kim and Roubini, 2000), the panel (d) in Figure 2 shows that the exchange rate reacts excessively to the monetary policy shock in both the determinate model (dashed line) and the baseline model with $\mathbf{M} = 0$ (dotted line), approximately four to five times more strongly (in percentage points) than in the baseline model (solid line). In contrast, the baseline model exhibit a mild response, owing to the estimated components in the arbitrary matrix $\mathbf{M}$, which works as an equilibrium selection device under indeterminacy.

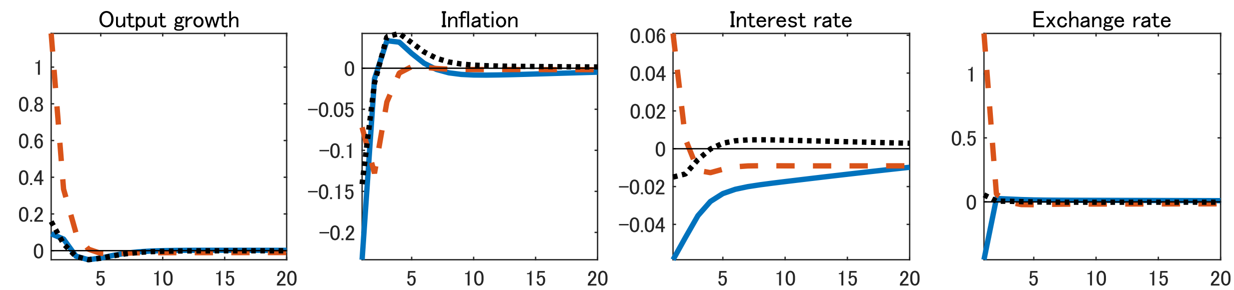
As shown in Figure 3, the sunspot shock affects equilibrium dynamics only in the base-

Figure 2: Impulse response functions

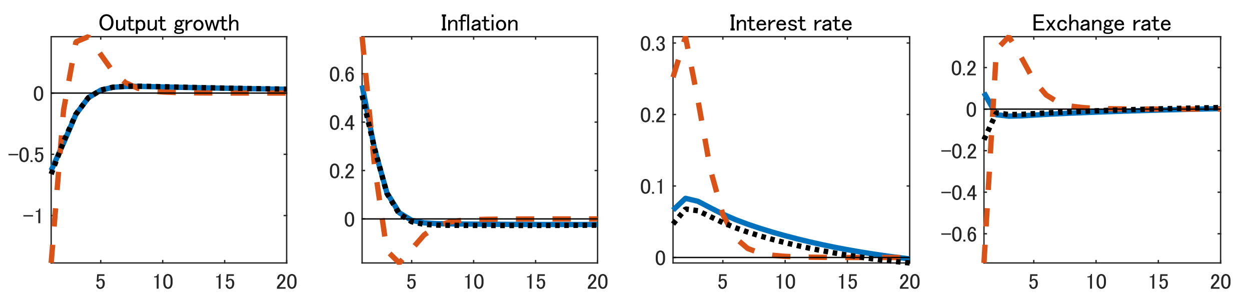
(a) UIP shock



(b) Technology shock



(c) Marginal cost shock



(d) Monetary policy shock

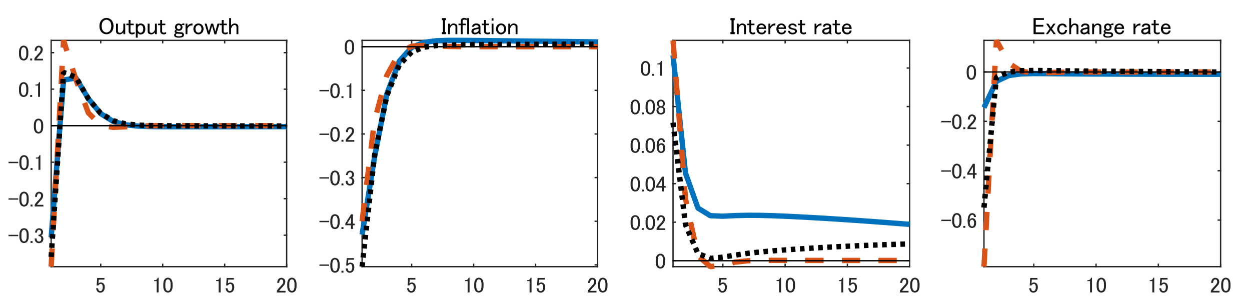
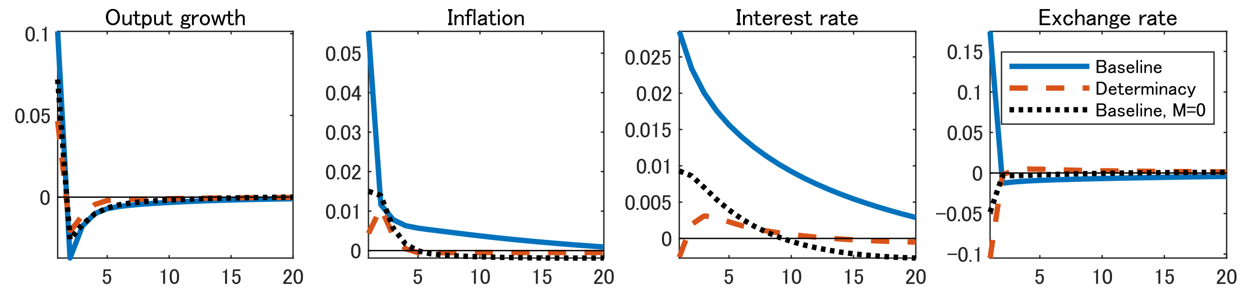
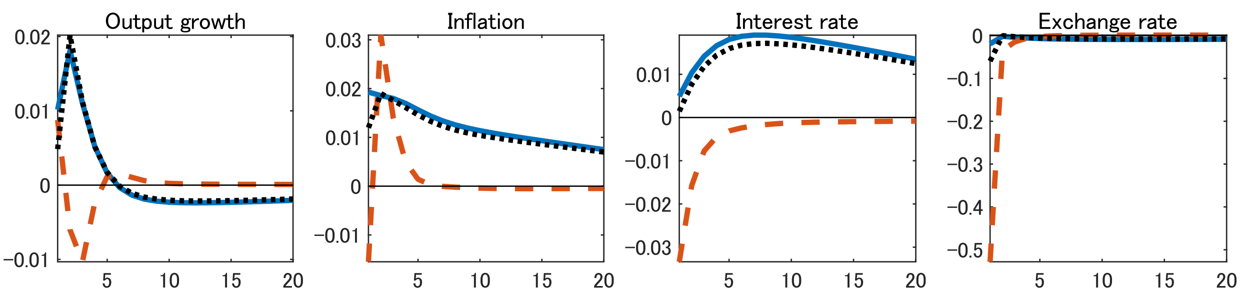


Figure 2: Impulse response functions (continued)

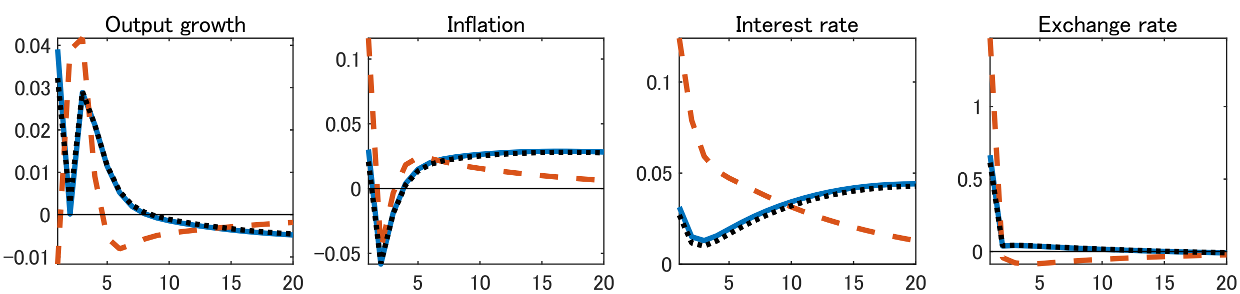
(e) US output shock



(f) US inflation shock

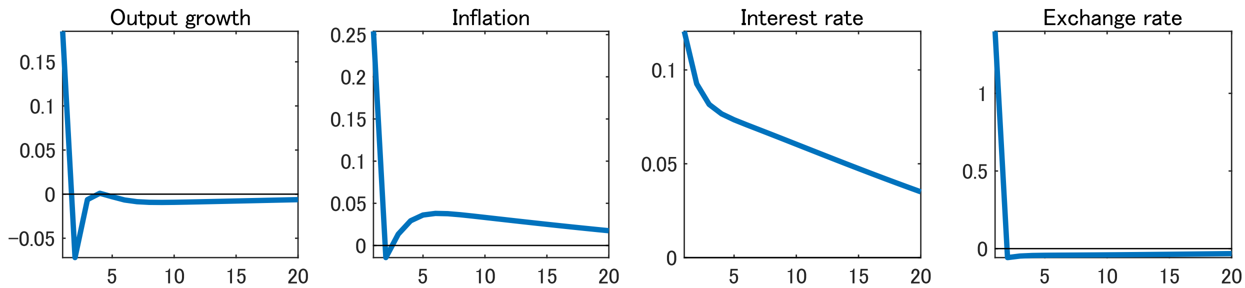


(g) US interest rate shock



Notes: This figure shows the impulse responses of output growth, inflation, interest rate, and exchange rate depreciation in terms of percentage deviations from steady-state values to a one-standard-deviation shock to the modified UIP condition, technology, marginal cost, monetary policy, US output, US inflation, and US interest rate, given the posterior mean estimates of parameters in the baseline model, in its counterpart estimated only under determinacy, and in the baseline model with $\mathbf{M} = 0$.

Figure 3: Impulse responses to sunspot shock



Notes: This figure shows the impulse responses of output growth, inflation, interest rate, and exchange rate depreciation in terms of percentage deviations from steady-state values to a one-standard-deviation sunspot shock, given the posterior mean estimates of parameters in the baseline model.

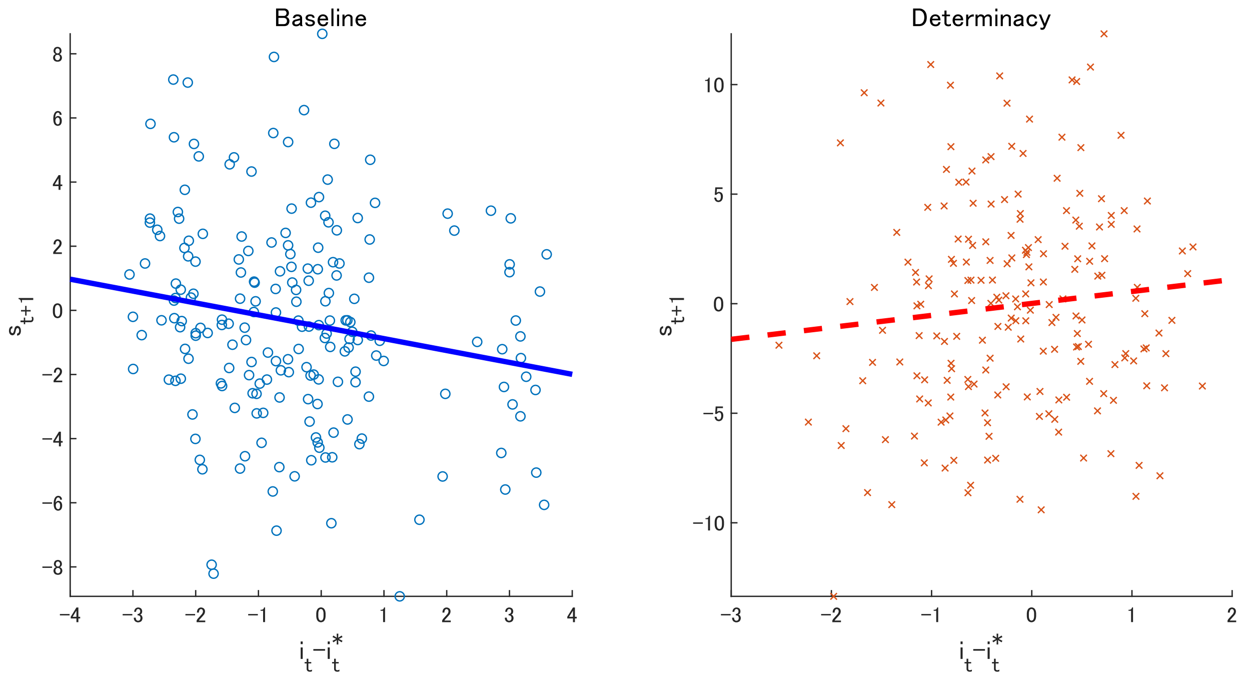
line model, which exhibits equilibrium indeterminacy. Upon impact, the identified sunspot shock has positive effects on all observables. The sunspot shock in the present model is constructed as a reduced-form sunspot shock following Lubik and Schorfheide (2004) and hence has positive effects on all expectational variables, irrelevant to fundamentals. Such nonfundamental beliefs are self-fulfilling under indeterminacy and have expansionary effects on their realizations. The rise in the interest rate, however, dampens these effects in the subsequent periods.

5.3 UIP regressions using simulated data

To examine whether our model can replicate negative a relationship between expected exchange rate depreciation and interest rate differentials, we conduct UIP regressions using simulated data. Figure 4 depicts the scatter plots and regression lines of exchange rate depreciation $\hat{s}_{t+1}$ and the interest rate differential $\hat{i}_t - \hat{i}_t^*$ simulated by the baseline model and its counterpart estimated only under determinacy.[15] This figure illustrates that the baseline model generates a negative correlation between the two *unconditionally* in the sense that the simulated data are driven by all shocks, whereas its determinacy counterpart does not. Thus, our baseline model can replicate a negative UIP coefficient as an unconditional phenomenon, as reported in numerous papers for a variety of international data.

[15]Given the posterior mean estimates of parameters, each model is simulated for 250 periods, and the first 50 observations are discarded.

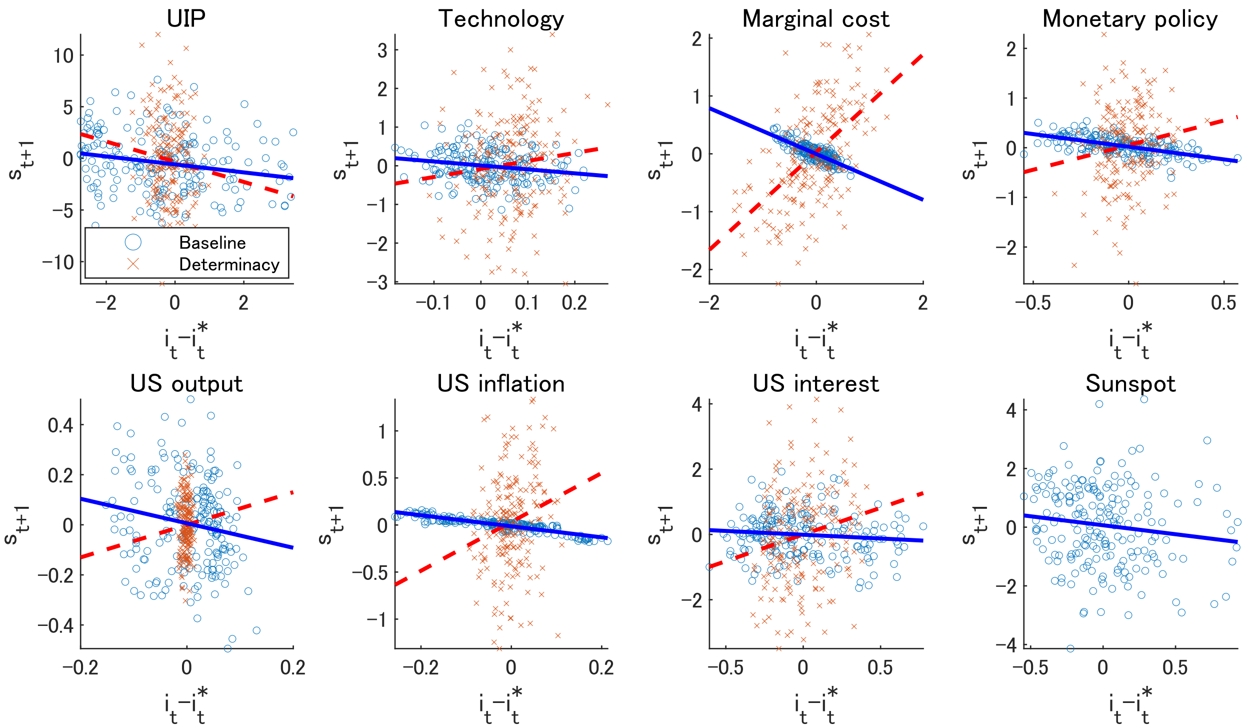
Figure 4: UIP regressions based on simulated data driven by all shocks



Notes: This figure shows the UIP regressions based on the simulated data of the exchange rate depreciation and the nominal interest rate differentials driven by all shocks in the baseline model and in its counterpart estimated only under determinacy.

In addition to the unconditional failure of the UIP relationship, the literature empirically demonstrates its failure in a conditional sense, *i.e.*, to a specific fundamental shock. In particular, Eichenbaum and Evans (1995) and Scholl and Uhlig (2008) show that the responses of the exchange and interest rates to a monetary policy shock are inconsistent with the standard UIP condition. To investigate this point, Figure 5 displays the UIP regressions using simulated data driven by each single shock in each model. The results based on the baseline model and its determinacy counterpart are shown in blue and red, respectively. The upper right panel in the figure demonstrates that the monetary policy shock generates a negative UIP coefficient in the baseline model as is consistent with the findings in Eichenbaum and Evans (1995) and Scholl and Uhlig (2008) but not in its determinacy counterpart. Moreover, all the other shocks replicate negative correlations in the baseline model. This is obviously owing to the negative slope coefficient in the modified UIP equation. In contrast, only the UIP shock can generate a negative correlation in the model estimated only under determinacy, where the slope coefficient is positive. Therefore, the negative slope coefficient

Figure 5: UIP regressions based on simulated data driven by each shock



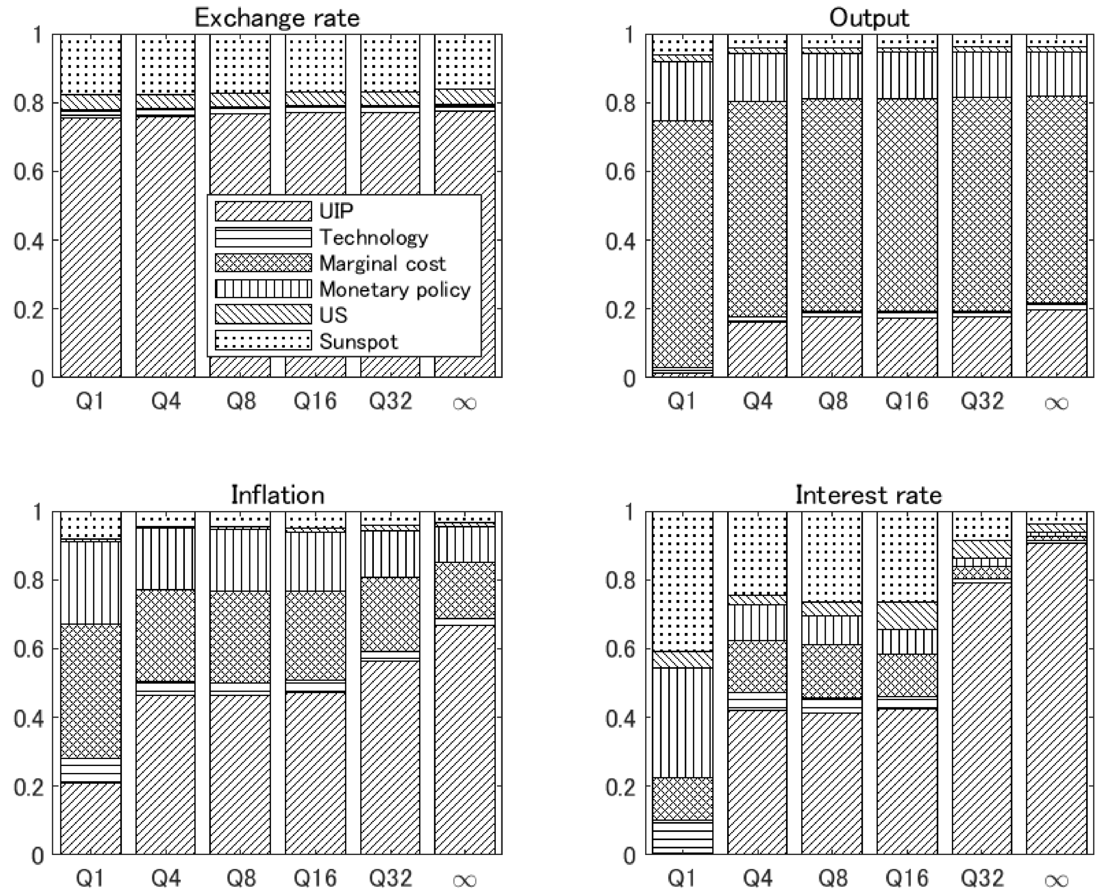
Notes: This figure shows the UIP regressions based on the simulated data of the exchange rate depreciation and the nominal interest rate differentials driven by each shock indicated at the top of each panel in the baseline model and in its counterpart estimated only under determinacy.

in the modified UIP condition is an essential feature for replicating the UIP puzzle both unconditionally and conditionally.

5.4 Variance decompositions

To assess the relative contribution of each shock to aggregate fluctuations, we conduct variance decompositions. In particular, we focus primarily on the sources of fluctuations in the nominal exchange rate. The literature has documented that estimations of open-economy DSGE models typically find that fluctuations in nominal exchange rates are little related to macroeconomic fundamentals and are mostly attributed to a direct shock to the exchange rate such as the UIP shock as specified in the standard UIP condition (3). Such an established view might be overturned if the model incorporates the modified UIP equation that replicates empirical regularities between exchange rate depreciation and interest rate differentials and allows for sunspot fluctuations, as considered in this paper.

Figure 6: Forecast error variance decompositions in the baseline model

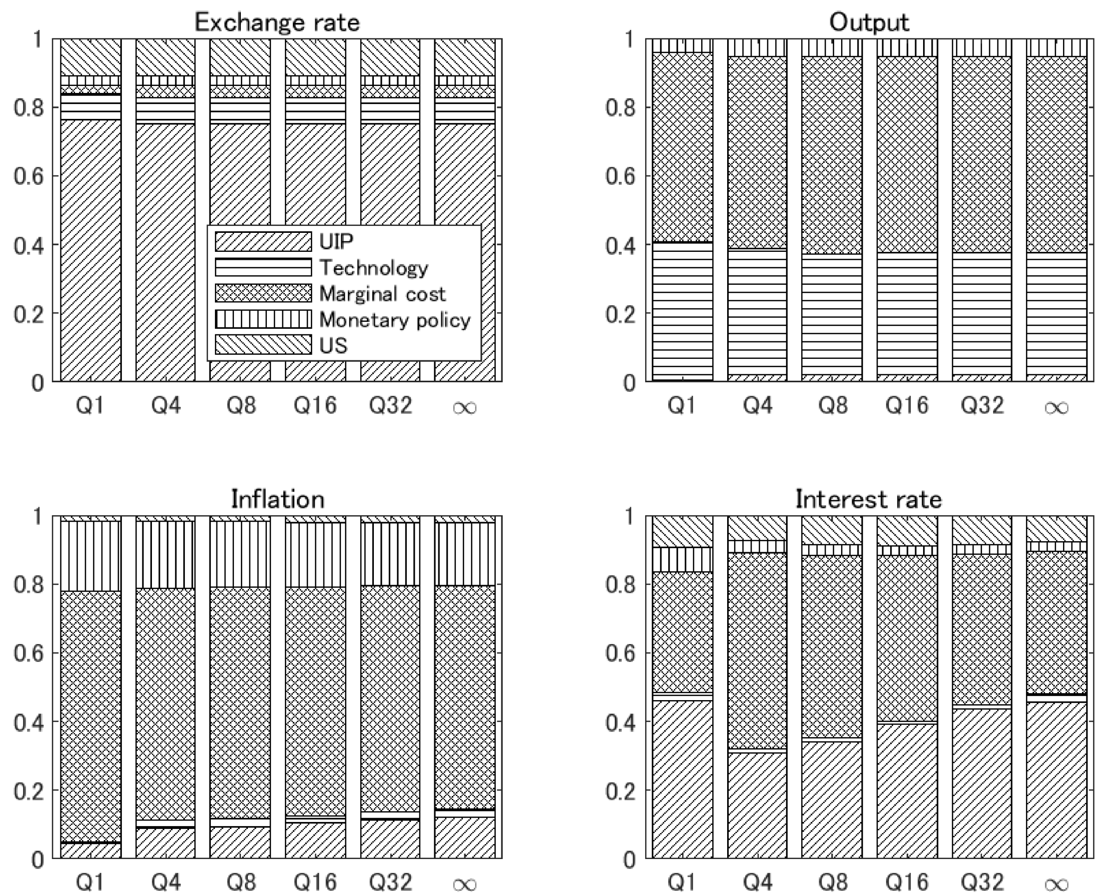


Notes: This figure shows the forecast error variance decompositions of exchange rate depreciation, output growth, inflation, and interest rate at various horizons, given the posterior mean estimates of the parameters in the baseline model. 'US' denotes the sum of the contributions of shocks to US output, inflation, and interest rate.

Figure 6 shows the forecast error variance decompositions of exchange rate depreciation, output growth, inflation, and the interest rate at various forecast horizons: 1, 4, 8, 16, and 32 quarters, and infinity, given the posterior mean estimates of the parameters in the baseline model. For comparison, Figure 7 presents the same decompositions based on the model estimated only in the determinacy region of the parameter space.

The upper-left panels in the two figures show that the UIP shock is the main driving force of exchange rate fluctuations in both the baseline model and its counterpart estimated only under determinacy. Thus, the conventional wisdom in the literature is not overturned. However, a novel finding here is that, in the baseline model, the sunspot shock plays a

Figure 7: Forecast error variance decompositions in the model under determinacy



Notes: This figure shows the forecast error variance decompositions of exchange rate depreciation, output growth, inflation, and interest rate at various horizons, given the posterior mean estimates of the parameters in the model estimated only under determinacy. 'US' denotes the sum of the contributions of shocks to US output, inflation, and interest rate.

secondary role in explaining exchange rate dynamics. The contributions of 16–18% for all the forecast horizons are considered to be substantial. Moreover, the sunspot shock plays nonnegligible roles for fluctuations in output growth, inflation, and nominal interest rate. In particular, its contribution to the interest rate is remarkable for short horizons of up to 16 quarters.

Why is the UIP shock still the primary source of exchange rate variability even though we allow for a negative slope coefficient in the modified UIP equation and sunspot fluctuations? This is because the UIP shock works as a direct shock to the exchange rate and can directly track exchange rate fluctuations. On the other hand, the sunspot shock can affect the exchange rate dynamics through non-fundamental revisions in expectations, so that its pass-through to the exchange rate is limited. Moreover, as addressed in Section 5.1 (Table 2), the estimated persistence parameter ρ_ψ in the UIP shock process is very close to one (0.993), indicating almost a unit-root process. In addition, the estimated standard deviation of the UIP shock σ_ψ is much larger (0.747) in the baseline model than in its counterpart estimated only under determinacy (0.171). Both of these estimates contribute to enhancing the effect of the UIP shock. However, the sunspot shock has no persistency, *i.e.*, i.i.d. by its construction. The contribution of such an i.i.d. shock is, *ceteris paribus*, smaller than that of a persistent shock.

The UIP shock accounts for not only exchange rate fluctuations but also the other aggregate fluctuations to a substantial extent under both determinacy and indeterminacy. Under determinacy (Figure 7), we find notable contributions of the UIP shock to inflation and the nominal interest rate. This finding is consistent with the argument in Itskhoki and Mukhin (2021), who offer microfoundations for a direct shock to the UIP condition and demonstrate the importance of the shock in explaining aggregate variables including exchange rates. Our results under indeterminacy enhance their argument further. Under indeterminacy (Figure 6), the contributions of the UIP shock to inflation and the nominal interest rate are much larger than under determinacy, and its substantial contribution to output growth emerges for horizons over four quarters.

6 Robustness Analysis

In this section, we investigate whether the results obtained from our baseline estimation are robust to alternative solution methods under indeterminacy and subsamples before and after the global financial crisis.

6.1 Alternative solutions under indeterminacy

6.1.1 Belief-shock specification

In the baseline estimation, we follow the approach of Lubik and Schorfheide (2004) to derive the full set of solutions for the linear rational expectations system under indeterminacy, in which we construct a reduced-form sunspot shock in that it contains beliefs associated with all the expectational variables. Lubik and Schorfheide (2003) propose another approach to constructing a sunspot shock called "a belief shock." In this approach, sunspots trigger a belief shock ζ_t^b that leads to a revision of the forecast of a specific expectational variable, say, $\mathbb{E}_t x_{t+1}$. Then, the definition of the rational expectations forecast error gives

$$x_t = (\mathbb{E}_{t-1} x_t + \zeta_t^b) + \tilde{\eta}_t^x,$$

where $\mathbb{E}_{t-1} x_t + \zeta_t^b$ is the revised forecast and $\tilde{\eta}_t^x$ is the error of this revised forecast. Lubik and Schorfheide (2003) show that such a belief shock affects equilibrium dynamics under indeterminacy and works like a sunspot shock. In what follows, we replace the reduced-form sunspot shock in the baseline estimation with a belief shock to the forecast of exchange rate depreciation:

$$\hat{s}_t = (\mathbb{E}_{t-1} \hat{s}_t + \zeta_t^b) + \tilde{\eta}_t^s,$$

and estimate the model with this belief-shock specification. We assume $\zeta_t^b \sim$ i.i.d. $N(0, \sigma_\zeta^2)$ and set the same inverse gamma prior for its standard deviation σ_ζ as in the baseline estimation.

The left part of Table 3 presents the estimation results of the model with the belief-shock specification. No notable differences are found in the parameter estimates, including the estimate of ϕ_s (risk-premium parameter associated with exchange rate depreciation in the modified UIP condition), compared with the baseline estimation results. Thus, the estimated model can account for the UIP puzzle. In terms of the model fit, the marginal data density (-786.4) slightly deteriorates, compared with the baseline estimation (-782.0, see Table 2). Therefore, the belief-shock specification does not improve the empirical performance of the model.

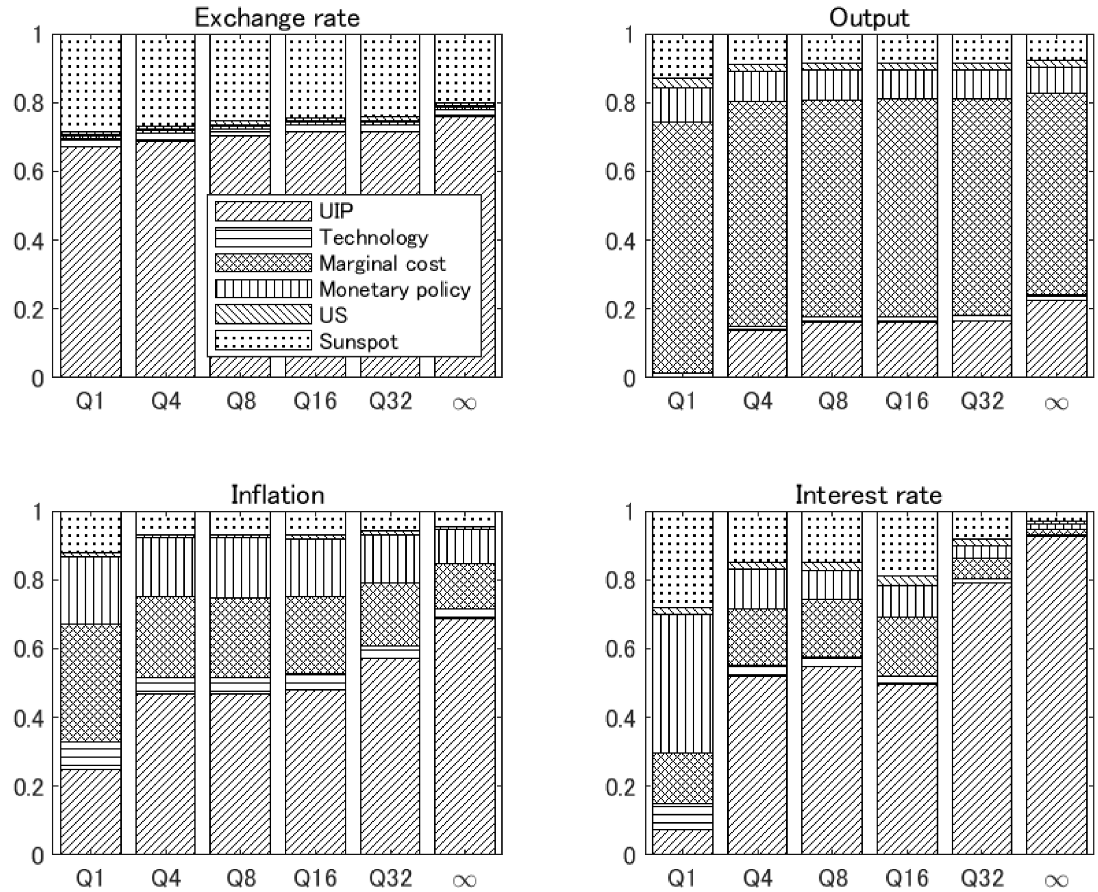
Figure 8 shows the variance decompositions based on the posterior mean estimates of pa-

Table 3: Posterior estimates of parameters with alternative solutions under indeterminacy

	Belief shock		$\mathbf{M} = 0$	
Parameter	Mean	90% interval	Mean	90% interval
ϕ_s	3.770	[3.095, 4.366]	4.878	[4.094, 5.693]
ϕ_a	0.009	[0.005, 0.014]	0.006	[0.004, 0.008]
b	0.427	[0.364, 0.491]	0.356	[0.299, 0.415]
η	1.412	[0.908, 1.936]	1.588	[1.004, 2.091]
ϕ	30.438	[18.915, 41.073]	30.242	[19.136, 40.221]
ω	0.159	[0.084, 0.232]	0.150	[0.077, 0.219]
λ	0.795	[0.772, 0.819]	0.791	[0.768, 0.813]
ρ	0.908	[0.889, 0.926]	0.865	[0.844, 0.888]
α_π	2.367	[1.968, 2.775]	1.714	[1.478, 1.925]
α_y	0.662	[0.197, 1.075]	0.852	[0.522, 1.177]
α_s	0.177	[0.060, 0.290]	0.266	[0.167, 0.359]
$\overline{\gamma}$	0.264	[0.217, 0.310]	0.281	[0.233, 0.324]
$\overline{\pi}$	0.570	[0.504, 0.635]	0.544	[0.475, 0.613]
$\overline{i}$	1.178	[1.100, 1.255]	1.179	[1.105, 1.257]
ρ_ψ	0.990	[0.982, 0.998]	0.995	[0.991, 0.999]
ρ_z	0.353	[0.053, 0.607]	0.516	[0.179, 0.829]
ρ_μ	0.953	[0.913, 0.995]	0.942	[0.902, 0.984]
ρ_{y^*}	0.839	[0.768, 0.912]	0.867	[0.814, 0.927]
ρ_{π^*}	0.575	[0.461, 0.691]	0.596	[0.488, 0.696]
ρ_{i^*}	0.957	[0.935, 0.977]	0.952	[0.930, 0.972]
σ_ψ	1.147	[0.897, 1.430]	0.867	[0.601, 1.158]
σ_z	0.618	[0.294, 0.942]	0.641	[0.282, 1.009]
σ_μ	0.711	[0.521, 0.912]	0.740	[0.536, 0.949]
σ_u	0.224	[0.194, 0.252]	0.248	[0.215, 0.278]
σ_{y^*}	0.492	[0.447, 0.542]	0.488	[0.442, 0.533]
σ_{π^*}	0.208	[0.187, 0.229]	0.206	[0.184, 0.226]
σ_{i^*}	0.147	[0.131, 0.160]	0.149	[0.133, 0.163]
σ_ζ	1.814	[1.470, 2.219]	2.000	[1.554, 2.401]
M_ψ	2.552	[1.961, 3.193]	-	-
M_z	-0.937	[-2.194, 0.457]	-	-
M_μ	0.754	[0.149, 1.354]	-	-
M_u	1.394	[0.239, 2.677]	-	-
M_{y^*}	0.478	[-0.253, 1.106]	-	-
M_{π^*}	0.889	[-0.277, 2.033]	-	-
M_{i^*}	0.724	[-0.616, 2.097]	-	-
$\log p(\mathbf{Y}^T)$	-786.411		-785.832	
$\mathbb{P}\{\boldsymbol{\theta} \in \mathbf{\Theta}^D \vert \mathbf{Y}^T\}$	0.000		0.000	

Notes: This table reports the posterior mean and 90 percent highest posterior density intervals based on 10,000 particles from the final importance sampling in the SMC algorithm. In the table, $\log p(\mathbf{Y}^T)$ represents the SMC-based approximation of log marginal data density and $\mathbb{P}\{\boldsymbol{\theta} \in \mathbf{\Theta}^D|\mathbf{Y}^T\}$ denotes the posterior probability of equilibrium determinacy.

Figure 8: Forecast error variance decompositions in the model with a belief shock



Notes: This figure shows the forecast error variance decompositions of exchange rate depreciation, output growth, inflation, and interest rate at various horizons, given the posterior mean estimates of the parameters in the model with a belief shock. 'US' denotes the sum of the contributions of shocks to US output, inflation, and interest rate.

rameters in the model under the belief-shock specification. The contribution of the sunspot shock to exchange rate fluctuations increases because we assume that the belief shock directly affects the revision of the forecast of the exchange rate. However, the UIP shock still plays a dominant role in explaining exchange rate volatility, and its contributions are also substantial for the other observables, because the estimated persistence of the UIP shock ρ_ψ and its standard deviation σ_ψ remain very close to one (0.990) and large (1.147), respectively. Therefore, the results are very similar to those in the baseline estimation.

6.1.2 $\mathbf{M} = 0$

An intrinsic feature of the full set of linear rational expectations solution given by (9) is that the arbitrary matrix $\mathbf{M}$ appears in the solution under indeterminacy, and $\mathbf{M}$ consists of purely free parameters. One might argue that the introduction of such free parameters improved the fit of the model dramatically and that the remarkable increase in the marginal data density in the baseline model, reported in Section 5.1, was attributed to these free parameters. To investigate this point, we estimate the baseline model with all the components of $\mathbf{M}$ fixed at zero.

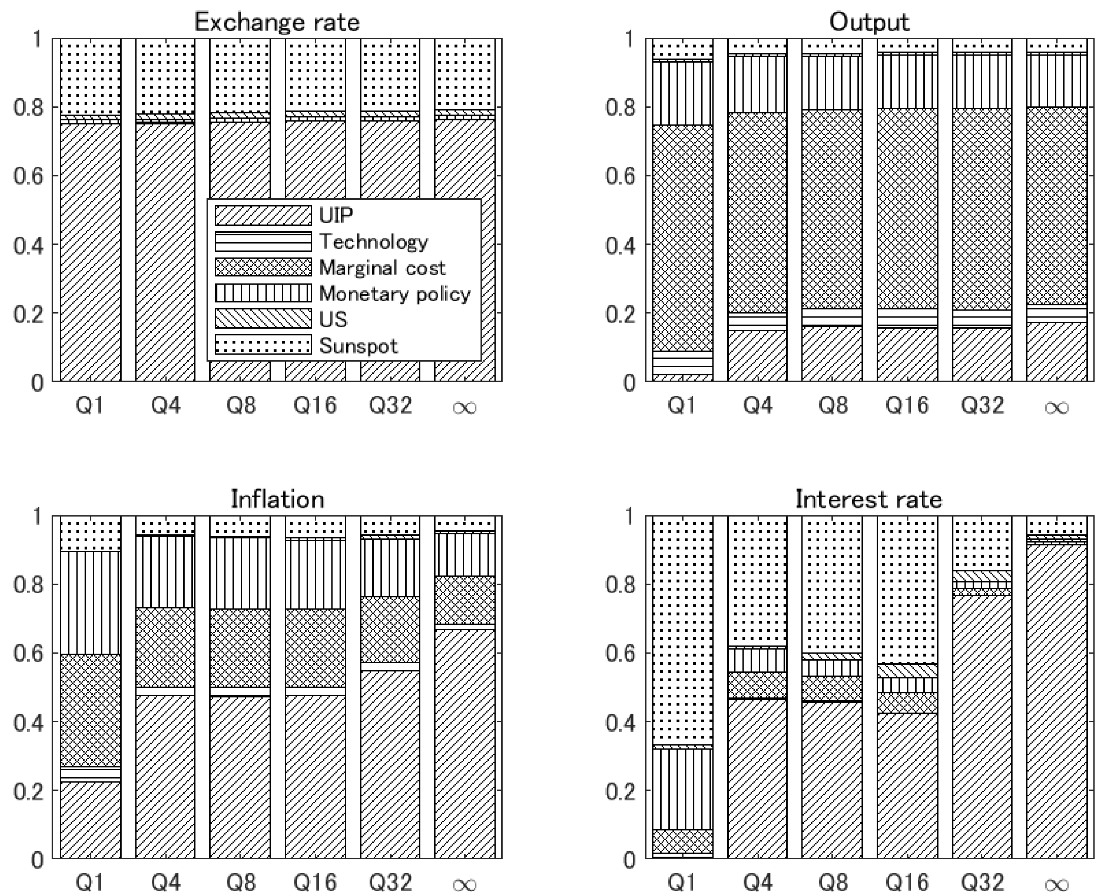
The last two columns of Table 3 show the estimation results when $\mathbf{M} = 0$. The marginal data density (-785.8) is lower than that in the baseline estimation (-782.0, see Table 2), but the difference is very marginal. Thus, the existence of the free parameters in $\mathbf{M}$ plays a minor role in the improved fit of the model. Because the parameter estimates are in line with those in the baseline estimation, the modified UIP equation (2) still exhibits a negative relationship between the expected exchange rate depreciation and interest rate differentials.

Figure 9 shows the variance decompositions based on the posterior mean estimates of parameters in the model with $\mathbf{M} = 0$. This figure is very similar to the one obtained from the baseline estimation (Figure 6). Although estimating the arbitrary matrix $\mathbf{M}$ alters the initial impact of the fundamental shocks as shown in Section 5.2 (Figure 2), the result here indicates that this effect on the variance decompositions is quite limited.

6.2 Subsample analysis

Bussière et al. (2022) argue that, in contrast to earlier findings, the coefficient on the interest rate differential in the UIP regression has become large and positive for several currencies including the Canadian dollar during and in the decade after the global financial crisis, which they term “the New Fama Puzzle.” To investigate whether their argument based on a single-equation estimation approach using monthly data carries over to our system estimation approach allowing for indeterminacy using quarterly data, we estimate the model for two subsamples: before and after the global financial crisis. More specifically, we split the full sample used in the baseline estimation into 1984:Q1–2007:Q2 and 2007:Q3–2019:Q4

Figure 9: Forecast error variance decompositions in the model with $\mathbf{M} = 0$



Notes: This figure shows the forecast error variance decompositions of exchange rate depreciation, output growth, inflation, and interest rate at various horizons, given the posterior mean estimates of the parameters in the model with $\mathbf{M} = 0$. 'US' denotes the sum of the contributions of shocks to US output, inflation, and interest rate.

samples.[16]

Table 4 presents the parameter estimates for the two subsamples. The posterior estimates of ϕ_s (risk-premium parameter associated with exchange rate depreciation in the modified UIP condition) are very close to each other between the two subsamples and are both much larger than one. Thus, the slope coefficient in the modified UIP equation (2) remains negative for both subsamples, leading the model to exhibit equilibrium indeterminacy. Consequently,

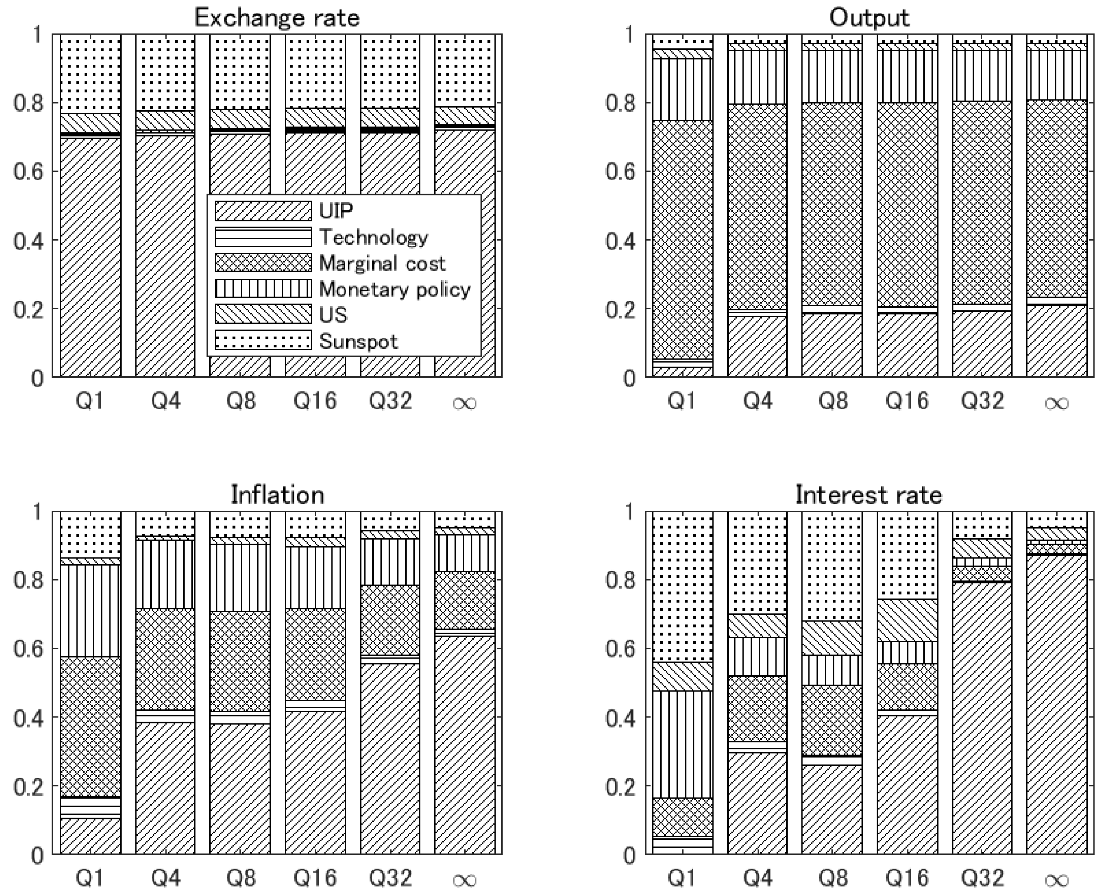
[16] Bussière et al. (2022) separate the monthly data into three subsamples: 1999:M01–2006:M04, 2006:M05–2018:M04, and 2018:M05–2021:M09. If we considered the same subsamples for our quarterly data, the first and third subsamples would be too short to estimate the model.

Table 4: Posterior estimates of parameters in two subsamples

	Pre-crisis period		Post-crisis period	
Parameter	Mean	90% interval	Mean	90% interval
ϕ_s	4.048	[3.357, 4.727]	4.281	[3.457, 5.048]
ϕ_a	0.007	[0.004, 0.010]	0.005	[0.003, 0.007]
b	0.317	[0.249, 0.382]	0.332	[0.239, 0.432]
η	1.324	[0.856, 1.808]	1.240	[0.757, 1.696]
ϕ	32.288	[21.598, 42.014]	31.481	[16.869, 44.027]
ω	0.246	[0.138, 0.349]	0.280	[0.158, 0.401]
λ	0.765	[0.742, 0.787]	0.781	[0.758, 0.804]
ρ	0.830	[0.797, 0.862]	0.914	[0.894, 0.937]
α_π	1.938	[1.668, 2.230]	1.732	[1.487, 1.983]
α_y	0.713	[0.301, 1.054]	0.592	[0.249, 0.940]
α_s	0.245	[0.129, 0.361]	0.280	[0.152, 0.408]
$\overline{\gamma}$	0.345	[0.296, 0.404]	0.089	[0.036, 0.142]
$\overline{\pi}$	0.612	[0.548, 0.676]	0.380	[0.313, 0.447]
$\overline{i}$	1.631	[1.558, 1.704]	0.353	[0.293, 0.420]
ρ_ψ	0.990	[0.981, 0.999]	0.992	[0.985, 1.000]
ρ_z	0.514	[0.151, 0.869]	0.543	[0.242, 0.847]
ρ_μ	0.948	[0.901, 0.996]	0.689	[0.520, 0.885]
ρ_{y^*}	0.847	[0.777, 0.919]	0.773	[0.663, 0.875]
ρ_{π^*}	0.659	[0.541, 0.785]	0.326	[0.144, 0.519]
ρ_{i^*}	0.900	[0.861, 0.937]	0.895	[0.827, 0.973]
σ_ψ	0.702	[0.437, 0.953]	1.147	[0.637, 1.747]
σ_z	0.489	[0.298, 0.706]	0.993	[0.344, 1.595]
σ_μ	0.563	[0.439, 0.681]	0.852	[0.645, 1.033]
σ_u	0.286	[0.239, 0.334]	0.197	[0.159, 0.234]
σ_{y^*}	0.471	[0.415, 0.524]	0.533	[0.449, 0.615]
σ_{π^*}	0.186	[0.165, 0.209]	0.261	[0.216, 0.301]
σ_{i^*}	0.176	[0.153, 0.195]	0.177	[0.144, 0.209]
σ_ζ	1.790	[0.992, 2.603]	1.180	[0.484, 1.762]
M_ψ	-0.744	[-1.750, 0.221]	-1.946	[-2.893, -1.041]
M_z	-0.823	[-2.586, 0.891]	-0.641	[-1.771, 0.575]
M_μ	0.522	[-0.416, 1.395]	-0.129	[-1.029, 0.692]
M_u	1.377	[0.213, 2.501]	0.095	[-1.254, 1.375]
M_{y^*}	0.732	[0.066, 1.415]	0.430	[-0.351, 1.207]
M_{π^*}	1.418	[0.215, 2.671]	0.144	[-0.964, 1.365]
M_{i^*}	0.899	[-0.527, 2.419]	-0.231	[-1.568, 1.328]
$\log p(\mathbf{Y}^T)$	-472.811		-322.782	
$\mathbb{P}\{\boldsymbol{\theta} \in \boldsymbol{\Theta}^D \mid \mathbf{Y}^T\}$	0.000		0.000	

Notes: This table reports the posterior mean and 90 percent highest posterior density intervals based on 10,000 particles from the final importance sampling in the SMC algorithm. In the table, $\log p(\mathbf{Y}^T)$ represents the SMC-based approximation of log marginal data density and $\mathbb{P}\{\boldsymbol{\theta} \in \boldsymbol{\Theta}^D|\mathbf{Y}^T\}$ denotes the posterior probability of equilibrium determinacy.

Figure 10: Forecast error variance decompositions in the pre-crisis period.

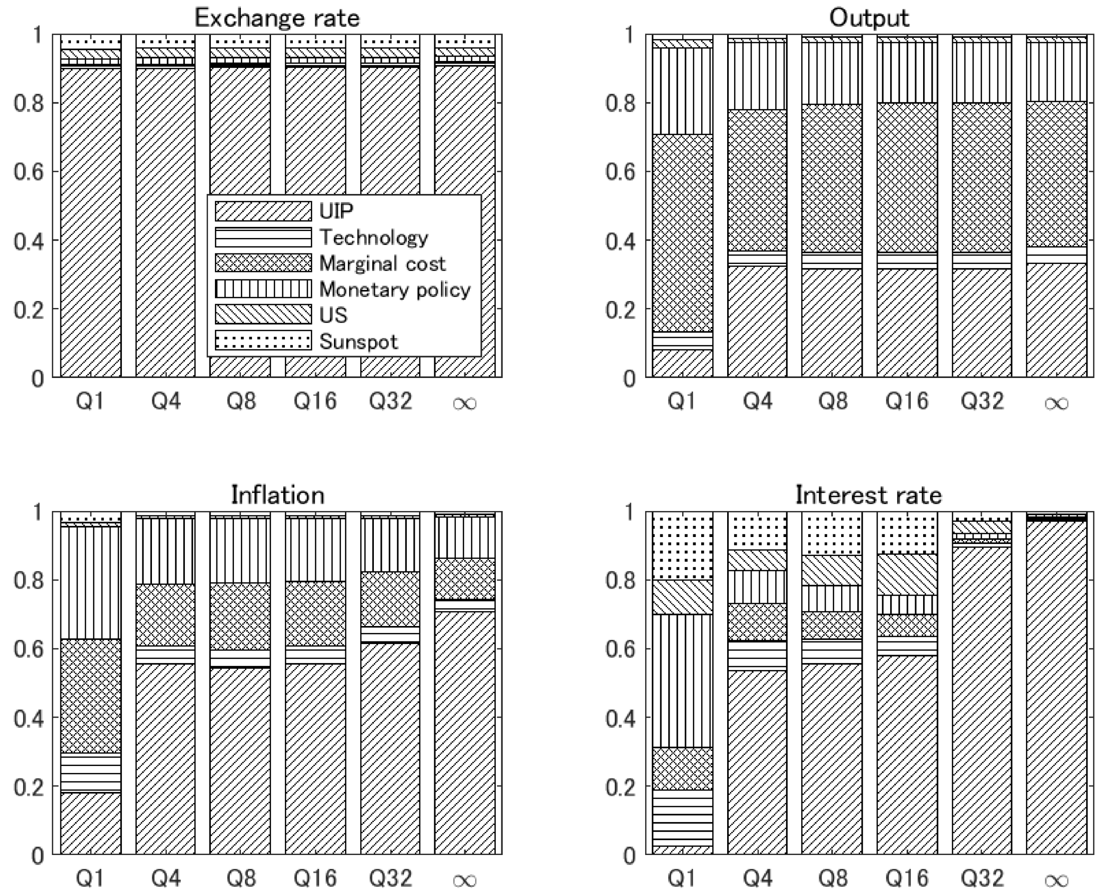


Notes: This figure shows the forecast error variance decompositions of exchange rate depreciation, output growth, inflation, and interest rate at various horizons, given the posterior mean estimates of the parameters in the model for the pre-crisis period. 'US' denotes the sum of the contributions of shocks to US output, inflation, and interest rate.

the posterior probabilities of determinacy $\mathbb{P}\{\boldsymbol{\theta} \in \boldsymbol{\Theta}^D | \mathbf{Y}^T\}$ shown in the last row are both zero. Therefore, there is no evidence for the New Fama Puzzle from our system estimation of the fully specified structural model for the Canadian economy.

While the other structural parameters on the household, firms, and central bank are not substantially different, several shock-related parameters vary across the two subsamples. The differences in the shock parameters produce different results in the variance decompositions of the exchange rate, as shown in Figures 10 and 11. The contribution of the sunspot shock to exchange rate fluctuations decreases from more than 20 percent to a few percent after the

Figure 11: Forecast error variance decompositions in the post-crisis period.



Notes: This figure shows the forecast error variance decompositions of exchange rate depreciation, output growth, inflation, and interest rate at various horizons, given the posterior mean estimates of parameters in the model for the post-crisis period. 'US' denotes the sum of the contributions of shocks to US output, inflation, and interest rate.

global financial crisis. However, the UIP shock is still the major source of the exchange rate volatility, and the variance decompositions of the other observables are not much different from those in the baseline estimation. Thus, our baseline results are robust to the subsample estimations.

7 Concluding Remarks

Using data for Canada and the US, we estimated a small open-economy model with an endogenous risk premium on the foreign bond holdings so that the UIP relationship can be characterized by the same sign as suggested by the UIP puzzle. Because the negative UIP coefficient can lead to equilibrium indeterminacy, we estimate the model using Bayesian methods allowing for both determinacy and indeterminacy of equilibrium.

According to the estimation results, the data strongly favor indeterminacy over determinacy, and hence the modified UIP condition exhibits the observed negative correlation between expected exchange rate depreciation and interest rate differentials. The propagation of shocks can be remarkably different between determinacy and indeterminacy, as a specific equilibrium is selected from multiple equilibria under indeterminacy. Forecast error variance decompositions based on the estimated model show that the UIP shock is the main driving force of the exchange rate dynamics, whereas the sunspot shock plays a secondary role.

Appendix

A Equilibrium Conditions

$$(C_t - bC_{t-1})^{-\sigma} = \beta \mathbb{E}_t \left[i_t \frac{(C_{t+1} - bC_t)^{-\sigma}}{\pi_{t+1}} \right],$$

$$Z_t^{1-\sigma} h_t^{\eta} = (C_t - bC_{t-1})^{-\sigma} \frac{W_t}{P_t},$$

$$S_t (C_t - bC_{t-1})^{-\sigma} = \beta \mathbb{E}_t \left\{ S_{t+1} \exp\left[-\phi_a \left(\frac{S_t A_t^*}{P_t Z_t} - \bar{a}^* \right) - \phi_s \left(\frac{S_{t+1}}{S_t} - \frac{\bar{\pi}}{\pi^*} \right) + \psi_t \right] i_t^* \frac{(C_{t+1} - bC_t)^{-\sigma}}{\pi_{t+1}} \right\},$$

$$\begin{aligned} &(1-\epsilon) + \epsilon \frac{W_t}{\exp(z_t) Z_t P_{H,t}} - \phi \left(\frac{\pi_{H,t}}{\pi_{t-1}^{\omega} \bar{\pi}^{1-\omega}} - 1 \right) \frac{\pi_{H,t}}{\pi_{t-1}^{\omega} \bar{\pi}^{1-\omega}} \\ &= -\beta \mathbb{E}_t \left[\phi \frac{(C_{t+1} - bC_t)^{-\sigma}}{(C_t - bC_{t-1})^{-\sigma} \pi_{t+1}} \left(\frac{\pi_{H,t+1}}{\pi_t^{\omega} \bar{\pi}^{1-\omega}} - 1 \right) \frac{Y_{t+1}}{Y_t} \frac{\pi_{H,t+1}^2}{(\pi_t^{\omega} \bar{\pi}^{1-\omega})^2} \right], \end{aligned}$$

$$Y_{H,t} = \lambda \left(\frac{P_{H,t}}{P_t} \right)^{-1} C_t,$$

$$Y_{F,t} = (1-\lambda) \left(\frac{S_t P_{F,t}^*}{P_t} \right)^{-1} C_t,$$

$$P_t = P_{H,t}^{\lambda} \left(S_t P_{F,t}^* \right)^{1-\lambda},$$

$$i_t = i_{t-1}^{\rho} \left[i \left(\frac{\pi_t}{\bar{\pi}} \right)^{\alpha_\pi} \left(\frac{Y_t}{\gamma Y_{t-1}} \right)^{\alpha_y} \left(\frac{S_t \pi^*}{S_{t-1} \pi} \right)^{\alpha_s} \right]^{1-\rho} \exp(u_t),$$

$$Y_{H,t} + Y_{H,t}^* + \frac{\phi}{2} \left(\frac{\pi_t}{\pi_{t-1}^{\omega} \bar{\pi}^{1-\omega}} - 1 \right)^2 Y_t = Y_t,$$

$$P_{H,t} Y_{H,t}^* - P_{F,t} Y_{F,t} = S_t \left\{ A_t^* - \exp\left[-\phi_a \left(\frac{S_{t-1} A_{t-1}^*}{P_{t-1} Z_{t-1}} - \bar{a}^* \right) - \phi_s \left(\frac{S_t}{S_{t-1}} - \frac{\bar{\pi}}{\pi^*} \right) + \psi_{t-1} \right] i_{t-1}^* A_{t-1}^* \right\},$$

$$Y_t = \exp(z_t) Z_t h_t,$$

$$Y_{H,t}^* = \lambda^* \left(\frac{P_{H,t}^*}{P_t^*} \right)^{-1} Y_t^*,$$

$$P_{H,t} = S_t P_{H,t}^*,$$

$$P_{F,t} = S_t P_t^*,$$

$$\pi_{H,t} = \frac{P_{H,t}}{P_{H,t-1}}.$$

B Steady-State Conditions

Let χ denote the ratio of net foreign assets over GDP in the steady state, *i.e.*, $\chi := a^*/y$. Then, we can derive the following steady-state conditions analytically:

$$s = \frac{\pi}{\pi^*},$$

$$\pi_H = \pi,$$

$$i = \frac{\gamma\pi}{\beta},$$

$$p_H = 1,$$

$$w = \frac{\epsilon - 1}{\epsilon},$$

$$h = \left(\frac{\frac{\gamma}{\gamma-b}\frac{\epsilon-1}{\epsilon}}{1 + \frac{1-\beta}{\beta}\chi} \right)^{\frac{1}{1+\eta}},$$

$$c = \left(1 + \frac{1-\beta}{\beta}\chi\right)^{\frac{\eta}{1+\eta}} \left(\frac{\gamma}{\gamma - b}\frac{\epsilon - 1}{\epsilon}\right)^{\frac{1}{1+\eta}},$$

$$y_H = \lambda \left(1 + \frac{1-\beta}{\beta}\chi\right)^{\frac{\eta}{1+\eta}} \left(\frac{\gamma}{\gamma - b}\frac{\epsilon - 1}{\epsilon}\right)^{\frac{1}{1+\eta}},$$

$$y_F = (1-\lambda) \left(1 + \frac{1-\beta}{\beta}\chi\right)^{\frac{\eta}{1+\eta}} \left(\frac{\gamma}{\gamma - b}\frac{\epsilon - 1}{\epsilon}\right)^{\frac{1}{1+\eta}},$$

$$y = \left(\frac{\frac{\gamma}{\gamma-b}\frac{\epsilon-1}{\epsilon}}{1 + \frac{1-\beta}{\beta}\chi} \right)^{\frac{1}{1+\eta}},$$

$$a^* = \chi \left(\frac{\frac{\gamma}{\gamma-b}\frac{\epsilon-1}{\epsilon}}{1 + \frac{1-\beta}{\beta}\chi} \right)^{\frac{1}{1+\eta}},$$

$$\lambda^* y^* = \left(\frac{\gamma}{\gamma - b}\frac{\epsilon - 1}{\epsilon}\right)^{\frac{1}{1+\eta}} \left(1 + \frac{1-\beta}{\beta}\chi\right)^{\frac{\eta}{1+\eta}} \left(\frac{1}{1 + \frac{1-\beta}{\beta}\chi} - \lambda\right).$$

The last equation implies that the parameters satisfy the following condition:

$$\lambda < \frac{1}{1 + \frac{1-\beta}{\beta}\chi}.$$

As $0 < \lambda < 1$ and $\frac{1-\beta}{\beta}\chi \approx 0$, it must be satisfied unless λ is very close to unity.

C Log-Linearized Equilibrium Conditions

$$\hat{c}_t = -\frac{\gamma - b}{\gamma + b}\left(\hat{i}_t - \mathbb{E}_t \hat{\pi}_{t+1}\right) + \frac{\gamma}{\gamma + b}\mathbb{E}_t \hat{c}_{t+1} + \frac{b}{\gamma + b}\hat{c}_{t-1},$$

$$\eta \hat{h}_t = \hat{w}_t - \frac{\gamma}{\gamma - b}\hat{c}_t + \frac{b}{\gamma - b}\hat{c}_{t-1},$$

$$\hat{i}_t - \hat{i}_t^* = (1 - \phi_s s)\,\mathbb{E}_t s_{t+1} - \phi_a \bar{a}^* \hat{a}_t^* + \hat{\psi}_t,$$

$$\hat{\pi}_{H,t} = \frac{\omega}{1 + \beta\omega}\hat{\pi}_{H,t-1} + \frac{\beta}{1 + \beta\omega}\mathbb{E}_t \hat{\pi}_{H,t+1} + \frac{\epsilon - 1}{\phi\,(1 + \beta\omega)}\left(\hat{w}_t - \hat{p}_{H,t} - z_t\right) + \tilde{\mu}_t,$$

$$\hat{y}_{H,t} = -\hat{p}_{H,t} + \hat{c}_t,$$

$$\hat{y}_{F,t} = -\hat{e}_t + \hat{c}_t,$$

$$0 = \lambda \hat{p}_{H,t} + (1 - \lambda)\,\hat{e}_t,$$

$$\hat{i}_t = \rho \hat{i}_{t-1} + (1 - \rho)\left[\alpha_\pi \hat{\pi}_t + \alpha_y\left(\hat{y}_t - \hat{y}_{t-1}\right) + \alpha_S \hat{s}_t\right] + u_t,$$

$$\frac{y}{\lambda^* y^*}\hat{y}_t = \frac{y_H}{\lambda^* y^*}\hat{y}_{H,t} + \hat{y}_t^* - \hat{p}_{H,t} + \hat{e}_t,$$

$$\hat{a}_t^* = \frac{\lambda^* y^*}{a^*}\hat{y}_t^* - \frac{y_F}{a^*}\hat{y}_{F,t} + \frac{\lambda^* y^* - y_F}{a^*}\hat{e}_t + \frac{1}{\beta}\left[(1 - \phi_a a^*)\hat{a}_{t-1}^* + (1 - \phi_s s)\hat{s}_t + \hat{i}_{t-1}^* - \hat{\pi}_t + \hat{\psi}_{t-1}\right],$$

$$\hat{y}_t = z_t + \hat{h}_t,$$

$$\hat{s}_t = \hat{e}_t - \hat{e}_{t-1} + \hat{\pi}_t - \hat{\pi}_t^*,$$

$$\hat{\pi}_{H,t} = \hat{p}_{H,t} - \hat{p}_{H,t-1} + \hat{\pi}_t,$$

where $\tilde{\mu}_t = \frac{\epsilon - 1}{\phi(1+\beta\omega)}\mu_t$ is a (reduced-form) marginal cost shock.

D Preliminary Estimation Results

Table 5 presents the posterior estimates of parameters in the model with Adolfson et al. (2008)'s specification of the endogenous risk premium on the foreign bond holdings and that with the hybrid specification between Adolfson et al. (2008)'s and ours.

Table 5: Posterior estimates of parameters under alternative specifications for risk premium

	Adolfson et al. (2008)		Hybrid	
Parameter	Mean	90% interval	Mean	90% interval
ϕ_1	-	-	0.977	[0.959, 0.996]
ϕ_s	0.848	[0.759, 0.935]	3.782	[3.223, 4.393]
ϕ_a	0.005	[0.003, 0.007]	0.006	[0.004, 0.008]
b	0.673	[0.627, 0.718]	0.424	[0.367, 0.484]
η	0.328	[0.180, 0.478]	1.767	[1.235, 2.276]
ϕ	58.787	[44.097, 74.098]	38.967	[27.889, 50.046]
ω	0.458	[0.336, 0.589]	0.140	[0.066, 0.206]
λ	0.850	[0.833, 0.869]	0.800	[0.776, 0.823]
ρ	0.902	[0.884, 0.925]	0.886	[0.867, 0.908]
α_π	2.608	[2.200, 3.003]	1.807	[1.534, 2.067]
α_y	0.084	[0.014, 0.147]	0.440	[0.146, 0.713]
α_s	0.088	[0.021, 0.146]	0.187	[0.088, 0.280]
$\overline{\gamma}$	0.318	[0.246, 0.389]	0.262	[0.224, 0.299]
$\overline{\pi}$	0.630	[0.564, 0.694]	0.554	[0.483, 0.615]
$\overline{i}$	1.170	[1.092, 1.250]	1.212	[1.137, 1.294]
ρ_ψ	0.857	[0.800, 0.921]	0.992	[0.985, 0.999]
ρ_z	0.998	[0.996, 1.000]	0.220	[0.050, 0.387]
ρ_μ	0.655	[0.573, 0.743]	0.939	[0.890, 0.987]
ρ_{y^*}	0.873	[0.812, 0.932]	0.836	[0.774, 0.900]
ρ_{π^*}	0.595	[0.484, 0.704]	0.628	[0.526, 0.744]
ρ_{i^*}	0.951	[0.929, 0.974]	0.954	[0.933, 0.975]
σ_ψ	2.230	[1.742, 2.768]	1.017	[0.664, 1.329]
σ_z	3.214	[2.613, 3.834]	0.863	[0.462, 1.204]
σ_μ	0.615	[0.514, 0.710]	0.625	[0.513, 0.733]
σ_u	0.241	[0.209, 0.270]	0.223	[0.193, 0.251]
σ_{y^*}	0.490	[0.441, 0.539]	0.482	[0.436, 0.527]
σ_{π^*}	0.203	[0.183, 0.223]	0.207	[0.187, 0.227]
σ_{i^*}	0.151	[0.133, 0.168]	0.148	[0.131, 0.164]
σ_ζ	0.599	[0.247, 0.905]	1.011	[0.365, 1.673]
M_ψ	-0.345	[-2.053, 1.431]	-1.190	[-2.160, -0.250]
M_z	-0.450	[-1.839, 1.066]	1.752	[1.156, 2.338]
M_μ	-0.379	[-1.858, 0.918]	1.227	[0.511, 1.948]
M_u	-0.370	[-1.824, 1.195]	1.473	[0.200, 2.725]
M_{y^*}	-0.215	[-1.842, 1.418]	0.572	[-0.052, 1.202]
M_{π^*}	0.627	[-0.843, 2.053]	-0.403	[-1.480, 0.700]
M_{i^*}	-0.340	[-1.857, 1.273]	-0.798	[-2.066, 0.608]
$\log p(\mathbf{Y}^T)$	-893.375		-872.922	
$\mathbb{P}\{\boldsymbol{\theta} \in \boldsymbol{\Theta}^D \mid \mathbf{Y}^T\}$	1.000		0.000	

Notes: This table reports the posterior mean and 90 percent highest posterior density intervals based on $10,000$ particles from the final importance sampling in the SMC algorithm. In the table, $\log p(\mathbf{Y}^T)$ represents the SMC-based approximation of log marginal data density and $\mathbb{P}\{\boldsymbol{\theta} \in \boldsymbol{\Theta}^D|\mathbf{Y}^T\}$ denotes the posterior probability of equilibrium determinacy.

E Sequential Monte Carlo Algorithm

To approximate the posterior distribution of model parameters, we employ the generic SMC algorithm with likelihood tempering described in Herbst and Schorfheide (2014, 2015). In the algorithm, a sequence of tempered posteriors is defined as

$$\varpi_n(\boldsymbol{\theta}) = \frac{[p(\mathbf{Y}^T|\boldsymbol{\theta}, \mathbf{M})]^{\tau_n} p(\boldsymbol{\theta}, \mathbf{M})}{\int [p(\mathbf{Y}^T|\boldsymbol{\theta}, \mathbf{M})]^{\tau_n} p(\boldsymbol{\theta}, \mathbf{M}) d\boldsymbol{\theta} d\mathbf{M}}, \quad n = 0, ..., N_\tau,$$

where N_τ denotes the number of stages and is set at $N_\tau = 200$. The tempering schedule $\{\tau_n\}_{n=0}^{N_\tau}$ is determined by $\tau_n = (n/N_\tau)^\mu$, where μ is a parameter that controls the shape of the tempering schedule and is set at $\mu = 2$, following Herbst and Schorfheide (2014, 2015). The SMC algorithm generates parameter draws $\{\boldsymbol{\theta}_n^{(i)}, \mathbf{M}_n^{(i)}\}$ and associated importance weights $w_n^{(i)}$, called particles, from the sequence of posteriors $\{\varpi_n\}_{n=1}^{N_\tau}$; that is, at each stage, $\varpi_n(\boldsymbol{\theta})$ is represented by a swarm of particles $\{\boldsymbol{\theta}_n^{(i)}, \mathbf{M}_n^{(i)}, w_n^{(i)}\}_{i=1}^N$, where N denotes the number of particles. In the subsequent estimation, the algorithm uses $N = 10,000$ particles. For $n = 0, ..., N_\tau$, the algorithm sequentially updates the swarm of particles $\{\boldsymbol{\theta}_n^{(i)}, \mathbf{M}_n^{(i)}, w_n^{(i)}\}_{i=1}^N$ through importance sampling.[17]

Posterior inferences on model parameters are made based on the particles $\{\boldsymbol{\theta}_{N_\tau}^{(i)}, \mathbf{M}_{N_\tau}^{(i)}, w_{N_\tau}^{(i)}\}_{i=1}^N$ from the final importance sampling. The SMC-based approximation of the marginal data density is given by

$$p(\mathbf{Y}^T) = \prod_{n=1}^{N_\tau} \left(\frac{1}{N} \sum_{i=1}^{N} \tilde{w}_n^{(i)} w_{n-1}^{(i)} \right),$$

where $\tilde{w}_n^{(i)}$ is the incremental weight defined as $\tilde{w}_n^{(i)} = [p(\mathbf{Y}^T|\boldsymbol{\theta}_{n-1}^{(i)}, \mathbf{M}_{n-1}^{(i)})]^{\tau_n - \tau_{n-1}}$. The posterior probability of equilibrium determinacy can be calculated as

$$\mathbb{P}\{\boldsymbol{\theta} \in \boldsymbol{\Theta}^D|\mathbf{Y}^T\} = \frac{1}{N} \sum_{i=1}^{N} \mathbf{1}\{\boldsymbol{\theta}_{N_\tau}^{(i)} \in \boldsymbol{\Theta}^D\}.$$

Likewise, the prior probability of equilibrium determinacy can be computed using prior draws.

[17] This process includes one step of a single-block random-walk Metropolis–Hastings algorithm.

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