

A Behavioral New Keynesian Model of a Small Open Economy under Limited Foresight*

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Abstract

This paper investigates exchange rate dynamics in open economies by incorporating bounded rationality. We develop a small open-economy New Keynesian model with an incomplete asset market, wherein decision-makers possess limited foresight and can only plan for a finite distance into the future. The equilibrium dynamics depend on the degree of foresight and the decision-makers' belief updating behaviors that approximate continuation values at the end of their planning horizons. This limited foresight leads to persistent, non-monotonic forecast errors in the real exchange rate across time horizons and distinguishes between short- and long-term expectations. This framework hence provides a micro-foundation for understanding time-horizon variability in uncovered interest parity (UIP) puzzles.

Keywords: Finite planning horizon; value function learning; small open economy; incomplete market; UIP violations

JEL codes: E43; E70; F31; F41

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1 Introduction

A burgeoning body of research studies and survey evidence shows that economic decision-makers often exhibit strong biases in forming expectations, deviating from the standard assumption of rational expectations (RE). In particular, recent empirical evidence concerning open economies has suggested that subjective expectations, rather than alternative forces, could be the main drivers of many RE-based violations.¹ Whereas some existing works focusing on theoretical models in closed economies, prompted by the “forward guidance puzzle” (Del Negro et al., 2015), have found significant policy consequences by relaxing the RE assumption,² relatively little attention has been given to open-economy macro models. In this paper, we aim to bridge this gap between theory and empirical evidence by introducing bounded rationality within an open-economy context.

Our goals are twofold. First, we develop a small open-economy New Keynesian (SOE-NK) model, integrating the concept of bounded rationality. We consider a particular approach of modeling bounded rationality developed by Woodford (2019)—limited foresight. The model is broad enough for applications in typical open-economy studies, while also encapsulating RE analyses when decision-makers’ foresight extends infinitely into the future. We further investigate how limited foresight affects the equilibrium dynamics and the forecast error of the real exchange rate, in comparison to the standard RE case. Second, we demonstrate that our model provides an intrinsic micro-foundation for several well-known puzzles related to the uncovered interest parity (UIP) under RE, particularly those concerning discrepancies across different time horizons.

To facilitate comparison, we build upon the standard SOE-NK model, as outlined by Galí and Monacelli (2005) as our reference model, and introduce two departures from this standard model. First, we assume that decision-makers form expectations subject to limited foresight, that is, they consider only a finite planning horizon (henceforth, FH) into the future. To evaluate potential situations that may be reached at the end of their planning horizons, they employ a *coarse* continuation value function learned from past experiences. This signifies that decision-makers incorporate all relevant information and policy alterations within the FH. However, when approximating the more distant future beyond their planning horizons, their value function becomes less accurate compared to the RE scenario. Second, we assume an incomplete asset market. This assumption stems from the idea that provided

¹Kalemli-Özcan and Varela (2022) document that several uncovered interest parity (UIP) violations, such as the Fama (1984) puzzle and the predictability reversal puzzles of the real exchange rate (e.g., Engel, 2016; Valchev, 2020), disappear in advanced economies when one uses the actual expectations of exchange rates from survey data instead of ex post realized exchange rates.

²For closed-economy studies, see, among others, Angeletos and Lian (2018), Gabaix (2020), Woodford and Xie (2019, 2022), and Dupraz et al. (2022).

decision-makers cannot make state-contingent plans as they can in the RE benchmark, assuming Arrow-Debreu markets would be logically inconsistent. From a practical viewpoint, this incomplete market structure also allows us to examine the impact of external shocks originating from the international financial market on the domestic economy.

The value function in our FH model significantly differs from the RE benchmark in two key aspects. First, unlike the RE model, the value function in the FH model does not encapsulate all state variables of the economy. Specifically, we assume that the value function only includes individual state variables, excluding aggregate state variables such as aggregate stochastic shocks affecting the economy. In consequence, when envisioning the future beyond their planning horizons, decision-makers do not factor in the evolution of these aggregate state variables.

Second, the construction of the coarse value function is backward-looking, in contrast to the forward-looking nature of dynamic programming in the RE benchmark. Decision-makers with limited foresight extend the value function over time by updating it based on past experiences, reflecting the concept that obtaining a value function as precise as in the RE benchmark is computationally implausible. For the sake of simplicity, we model this value function update as a constant-gain learning process. Inherently, our FH model nests the RE reference model as a limiting case. As decision-makers' planning horizons extend towards infinity, or when they adopt the precise value function aligned with RE, their planning problem yields the same outcomes for variables as found in the unique RE equilibrium of the reference model.

Our economy is buffeted by one domestic shock and one external shock; a domestic productivity shock and a foreign interest rate shock. We compare the equilibrium dynamics between FH and RE models by examining impulse response functions. This comparison reveals that the FH model generates more persistent, hump-shaped movements of aggregate variables, together with significant forecast errors of the real exchange rate, a feature absent in the RE benchmark. The learning behavior of households in the FH model results in hump-shaped dynamics for their value functions over time, influencing the dynamic patterns of other macro variables. In the case of a productivity shock or foreign interest rate shock, the value functions exhibit U-shaped or inverse U-shaped dynamics, respectively.

The dynamics of real exchange rate forecast errors in the FH model exhibit a particularly interesting feature. The FH model inherently produces systematic forecast errors due to the gap between expectation and realization. Given the length of the planning horizon, the dynamics of these forecast errors significantly rely on value function learning behavior. The decomposition of the real exchange rate forecast errors reveals that value function extrapolation plays a dominant role due to its non-monotonic, hump-shaped behavior, causing

an overshoot of the forecast error and leading to a sign reversal over the time horizon.

Our FH model offers explanations for various puzzling violations of the RE-based UIP condition, especially those related to time-horizon variability. The RE-UIP condition, a renowned empirical rebuttal of theoretical predictions from the RE-based open-economy macro model, dictates that an increase in the interest rate differential should match the RE-based expected future domestic currency appreciation. This results in *unpredictable* excess returns on currency bonds for any time horizon and predicts the *horizon invariance* on the exchange rate reaction to the expected interest rate differentials.

However, comprehensive empirical studies have refuted these RE-based UIP conditions, particularly in advanced countries. Observations reveal predictable short-term excess returns for higher interest rate currency bonds (Fama, 1984; Eichenbaum and Evans, 1995), with excess return predictability reversing its sign across time horizons (e.g., Bacchetta and van Wincoop, 2010; Engel, 2016; Valchev, 2020). Additionally, the horizon-invariance of the real exchange rate is challenged, with a documented short-term overreaction and long-term underreaction of the exchange rate to future interest rate differentials (Galí, 2020). Whereas our model could also speak to other forms of UIP violations in the literature, we focus particularly on these two, as the FH models offer a compelling micro-founded explanation for the excess return predictability and the breakdown of the horizon invariance.

The forecast errors of the real exchange rate in our FH model reflect the dynamic patterns of predictable excess returns, in line with the empirical findings. We perform a regression analysis on the predictability of excess returns using simulated data from the calibrated FH model. The results demonstrate that the FH model produces an unconditional profile of excess return predictability and a reversal of predictability that are qualitatively consistent with the empirical literature. Furthermore, we find that while the FH model predicts opposite profiles of excess return predictability conditioned on the two types of shocks, the foreign interest rate shock predominantly drives the unconditional profile of excess return predictability.

In addition, our FH model suggests that the empirical evidence of the breakdown of the horizon invariance implies the rejection of the Law of Iterated Expectations (LIE), as expectations formed at any time are contingent on the value functions at that same time. When decision-makers learn and update their value functions, expectations of future endogenous variables differ from the expectations of expected future endogenous variables. This disrupts the horizon-invariance specification of the UIP condition.

Through model simulated data, our model generates empirically consistent short-term overreaction and long-term underreaction of the exchange rate to future interest rate differentials. What is the role of the value function in the empirical specification? The decision-

makers form expectations at time t for the future period $t + k$ based on the assumption that the counterfactual endogenous variables are constructed on aggregate conditions with a remaining planning horizon $h - k$. This implies that as k increases, the expectations become more dependent on the decision-makers' value functions. Consequently, the value functions significantly impact the counterfactual real interest rate differential in the long term.

Related Literature. This paper contributes to the recent development in the literature that scrutinizes the RE assumption by considering behavioral biases or alternatives to assess their implications on policy analyses and empirical relevance.³ In this context, we explore the consequences of limited foresight in an open-economy setting. Compared with other popular behavioral variants, the FH approach naturally differentiates between short-term forecasts (influenced by the planning horizons) and long-term forecasts (guided by the value function that approximates the continuation values). Thus, our model provides micro-founded explanations for the RE-UIP violations, particularly those over the time horizon.

Our paper builds on and expands the scope of FH models. In terms of modeling bounded rationality, [Gust et al. \(2022\)](#) show that [Woodford \(2019\)](#)'s approach is particularly relevant for aligning with aggregate data. [Xie \(2020\)](#) posits that this method facilitates the examination of equilibrium dynamics without having to tackle equilibrium selection issues. [Woodford and Xie \(2022\)](#) emphasize the importance of the bounded rationality approach for policy analysis and its welfare consequences, particularly under the zero lower bound conditions.⁴ Notably, these studies were conducted within a closed-economy setting. Our paper broadens this framework to an open economy, underlining its implications for foreign shock transmissions and the dynamics of exchange rate and interest rate differential. Moreover, from a methodological perspective, we augment the FH model by introducing a method to incorporate additional endogenous state variables — the real exchange rate and net foreign asset position — into our analysis.

The open-economy environment of this paper begins with the literature on the macro models of monetary transmission mechanisms with open capital account back to the 1960s, in which the equilibrium analyses are based on the UIP condition with no excess returns. Subsequent advancements in dynamic stochastic general equilibrium (DSGE) open-economy macro models, under the full-information RE hypothesis, continued to rely on the UIP condition.⁵ However, numerous empirical studies report frequent rejections of the RE-UIP

³The literature has developed several approaches to model bounded rationality that address the forward guidance puzzle, such as cognitive discounting ([Gabaix, 2020](#)), level- k thinking (e.g., [García-Schmidt and Woodford, 2019](#); [Farhi and Werning, 2019](#)), lack of common knowledge ([Angeletos and Lian, 2018](#)), and finite planning horizons ([Woodford, 2019](#)).

⁴See [Woodford and Xie \(2022\)](#) for empirical evidence that supports the assumption of FHs.

⁵For examples of the earliest contributions, see [Mundell \(1963\)](#), [Fleming \(1962\)](#), and [Dornbusch \(1976\)](#).

condition. A line of works aforementioned documents predictable excess returns of the real exchange rate and its reversal of the sign over the time horizon. It also reports the breakdown of the horizon-invariance as implied by the RE-UIP condition.

In particular, several studies utilizing expectation survey data indicate that in advanced economies, systematic forecast errors from the subjective expectations of economic agents are the primary source of RE-UIP violations (Froot and Frankel, 1989; Chinn and Frankel, 2019; Kalemlı-Özcan and Varela, 2022; Candian and De Leo, 2023), rather than alternative explanations such as risk premia or financial frictions (e.g., Gabaix and Maggiori, 2015). Inspired by these findings, our paper applies a subjective expectation formation—featuring agents with limited foresight to the standard open-economy New Keynesian model. Our work thus offers a new theoretical framework capable of explaining the time-horizon aspects of the UIP puzzles.⁶ Furthermore, our model presents a unique perspective on the source of UIP wedges compared to the recent work by Itskhoki and Mukhin (2021). They propose that a segmented financial market with noise traders and limits-to-arbitrage results in a wedge in the RE-UIP condition. In contrast, our model attributes endogenous deviations from the RE-UIP condition to decision-makers’ behavioral responses to aggregate shocks.

This paper aligns with the literature on open-economy macro models employing bounded-rational agents to address exchange rate puzzles. Gourinchas and Tornell (2004) model investors with distorted beliefs, resulting in misperceptions about the relative weight of persistent versus transitory interest rate shocks. Their model can predict positive excess returns for bonds bearing higher interest rates but fails to predict the reversal of excess return across different time horizons. Candian and De Leo (2023) extend this work by incorporating investors’ extrapolation of underlying shocks, showing that the model can predict the excess return reversal if investors’ perception of shocks over-extrapolates the actual shocks. Valente et al. (2021) also model decision-makers receiving noisy signals and extrapolating an exogenous interest rate process to address various exchange rate puzzles. These analyses focus on decision-makers’ misperception and extrapolation of underlying shocks. Kolasa et al. (2022) tackle UIP puzzles by developing an open-economy macro model with cognitive discounting, as in Gabaix (2020).⁷

Unlike the explanations above-mentioned, decision-makers do not misperceive an exoge-

Among many others, see Clarida et al. (2001) and Galí and Monacelli (2005) for DSGE open-economy macro models.

⁶The literature has also developed various approaches in the RE framework to address some parts of these UIP puzzles, such as by considering time-varying risk premia (Verdelhan, 2010), infrequent portfolio adjustments (Bacchetta and van Wincoop, 2010), or the convenience yield (Valchev, 2020).

⁷Crucini et al. (2020) argue that the high persistence in purchasing power parity (PPP) deviation and the law of one price (LOP) deviation cannot be explained solely by nominal rigidities, and they also provide a rationale through cognitive discounting.

nous process in our model. Instead, the behavioral biases stem from their limited foresight in planning, along with the coarse value functions they use to assess future possibilities at the end of their planning horizon. This approach also affects their expectations for endogenous variables. Our model underscores the importance of learning in decision-makers' value functions, addressing both the predictability reversal and the breakdown of horizon invariance in the UIP puzzles. It lays a micro-foundation for contrasting short- and long-run forecasts.

This paper proceeds as follows. Section 2 illustrates an SOE-NK model in which decision-makers are subject to limited foresight. Section 3 summarizes the full equilibrium conditions when decision-makers share a homogeneous planning horizon and discusses the solution method of the model. Section 4 analyzes equilibrium dynamics of the FH model. Section 5 applies the FH model to address the UIP puzzles. Section 6 discusses the robustness of the main results by considering alternative setups of the model, including extending to heterogeneous planning horizons. Section 7 concludes the paper.

2 A Small Open-Economy New Keynesian Model under Limited Foresight

We develop an SOE-NK model in which decision-makers have bounded rationality. The model setup is similar to the standard SOE-NK DSGE model developed in Galí and Monacelli (2005). The world consists of a continuum of small open economies that lie on the unit interval. The consumption basket of households includes domestically produced goods and imported foreign goods. Producers of domestic goods have market power and can set prices in the domestic currency (i.e., producer currency pricing). We assume nominal rigidity in goods prices whereby firms have a limited ability to reset their prices following Calvo pricing.

Our model differs from the benchmark SOE-NK model in the following aspects. First, we replace the assumption of infinite-horizon dynamic planning with planning under limited foresight à la Woodford (2019). Second, we assume incomplete asset markets such that only state noncontingent claims are available to trade. We consider two types of bonds. One is a domestic currency bond which is only traded domestically, and the other is an international bond (denominated in foreign currency) which is traded with the rest of the world. In addition, we also introduce random foreign interest rate shock from the rest of the world to study the effects of aggregate disruptions stemming from the international financial market. These disruptions include instances of country premium shocks and foreign monetary policy shocks. In this section, we show that the assumption of FH yields behavioral analogs of the structural equations of open-economy macro models, such as the open-economy Euler

equations for domestic and foreign bonds and the open-economy New Keynesian supply curve.

2.1 Households

Let us begin with a description of the households' forward planning problem. The small open economy consists of infinitely many identical households indexed in the unit interval $[0, 1]$. At any time t , household i seeks to maximize

$$\hat{\mathbb{E}}_t^i \sum_{\tau=t}^{\infty} \beta^{\tau-t} (u(C_\tau^i) - \varpi(N_\tau^i)), \quad (2.1)$$

where C_τ^i is the consumption composite at date τ and N_τ^i is the labor supply of household i . Function $u(\cdot)$ denotes a periodic utility, which is strictly increasing and concave, while function $\varpi(\cdot)$ denotes a periodic disutility of labor supply, which is strictly increasing and convex. Parameter $0 < \beta < 1$ is the subjective discount factor. Operator \mathbb{E}_t^i describes the subjective expectation operator of household i at time t , and we will specify this expectation operator later such that it features the assumption of limited foresight.

The consumption basket is a composite index of home goods, $C_{H,\tau}^i$, and imported foreign goods, $C_{F,\tau}^i$. The home good is a constant elasticity of substitution (CES) aggregation of different varieties of home-made goods, $C_{H,\tau}^i(j)$, where $j \in [0, 1]$. The imported goods are purchased from a variety of small open economies; thus, they are an aggregate of goods from each such country, $l \in [0, 1]$, denoted as $C_{l,\tau}^i$. Finally, a good imported from country l is an aggregator of different varieties of goods made in country l , $C_{l,\tau}(j)$. These aggregates are given by

$$C_\tau^i = \left[(1 - \alpha)^{\frac{1}{\eta}} (C_{H,\tau}^i)^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} (C_{F,\tau}^i)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}},$$

where $1 - \alpha$ governs the degree of home bias, and

$$C_{H,\tau}^i = \left(\int_0^1 C_{H,\tau}^i(j)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}}, \quad C_{F,\tau}^i = \left(\int_0^1 C_{l,\tau}^i \frac{\gamma-1}{\gamma} dl \right)^{\frac{\gamma}{\gamma-1}}, \quad C_{l,\tau}^i = \left(\int_0^1 C_{l,\tau}(j)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}},$$

where η , ϵ , and γ represent the elasticities of substitution between home and foreign goods, within-country varieties, and across-country varieties, respectively.

The household faces the following sequential budget constraint:

$$\int_0^1 P_{H,\tau}(j) C_{H,\tau}^i(j) dj + \int_0^1 \int_0^1 P_{l,\tau}(j) C_{l,\tau}^i(j) dj dl + \frac{B_{\tau+1}^i}{1 + i_\tau^i} + \frac{\mathcal{E}_\tau B_{\tau+1}^{*,i}}{1 + i_\tau^{*,i}} = B_\tau^i + \mathcal{E}_\tau B_\tau^{*,i} + W_\tau N_\tau^i + \Phi_\tau,$$

where $P_{H,\tau}$ is the nominal price index for domestically produced final goods, $P_{l,\tau}(j)$ is the nominal price for final good j produced in country l , $B_{\tau+1}^i$ is the nominal payoff in period $\tau + 1$ of the domestic bond portfolio that household i holds at the end of period τ , and $1 + i_\tau^i$ is the (one-period-ahead) riskless gross domestic nominal interest rate of the domestic bond. Variable \mathcal{E}_τ is the effective nominal exchange rate between the home country and the rest of the world in units of domestic currency. Variable $B_{\tau+1}^{*,i}$ is the nominal payoff of an international bond in international currency, and $1 + i_\tau^{*,i}$ is the nominal interest rate for the international bond. Variable W_τ is the nominal wage, and Φ_τ is the nominal profit of firms transferred to individual household i (households own the domestic firms, but the dividends transfer is beyond household i 's control).

The household's static cost minimization problem for consumption expenditure yields the following demand functions for consumption goods:

$$C_{H,\tau}^i(j) = \left(\frac{P_{H,\tau}(j)}{P_{H,\tau}} \right)^{-\epsilon} C_{H,\tau}^i, \quad C_{l,\tau}^i(j) = \left(\frac{P_{l,\tau}(j)}{P_{l,\tau}} \right)^{-\epsilon} C_{l,\tau}^i, \quad C_{l,\tau}^i = \left(\frac{P_{l,\tau}}{P_{F,\tau}} \right)^{-\gamma} C_{F,\tau}^i,$$

$$C_{H,\tau}^i = (1 - \alpha) \left(\frac{P_{H,\tau}}{P_\tau} \right)^{-\eta} C_\tau^i, \quad C_{F,\tau}^i = \alpha \left(\frac{P_{F,\tau}}{P_\tau} \right)^{-\eta} C_\tau^i,$$

where the price indices for domestically produced final goods and foreign-produced goods are given as

$$P_{H,\tau} = \left(\int_0^1 P_{H,\tau}(j)^{1-\epsilon} dj \right)^{\frac{1}{1-\epsilon}}, \quad P_{l,\tau} = \left(\int_0^1 P_{l,\tau}(j)^{1-\epsilon} dj \right)^{\frac{1}{1-\epsilon}}, \quad P_{F,\tau} = \left(\int_0^1 P_{l,\tau}^{1-\gamma} dl \right)^{\frac{1}{1-\gamma}}.$$

The aggregate consumer price index (CPI) is given as

$$P_\tau = [(1 - \alpha)P_{H,\tau}^{1-\eta} + \alpha P_{F,\tau}^{1-\eta}]^{\frac{1}{1-\eta}},$$

which also implies

$$P_\tau C_\tau^i = \int_0^1 P_{H,\tau}(j) C_{H,\tau}^i(j) dj + \int_0^1 \int_0^1 P_{l,\tau}(j) C_{l,\tau}^i(j) dj dl.$$

Then, the sequential budget constraint can be rewritten as

$$P_\tau C_\tau^i + \frac{B_{\tau+1}^i}{1 + i_\tau} + \frac{\mathcal{E}_\tau B_{\tau+1}^{*,i}}{1 + i_\tau^*} = B_\tau^i + \mathcal{E}_\tau B_\tau^{*,i} + W_\tau N_\tau^i + \Phi_\tau. \quad (2.2)$$

For the sake of parsimony, we follow the assumption on the labor market in [Woodford](#)

(2019). That is, the country has a labor organization in which each household is asked by firms to supply its share of the aggregate domestic labor demand N_τ . Thus, the expected path of $N_\tau^i = N_\tau$ is beyond household i 's control. This implies that the expected path of equilibrium income (in domestic currency), i.e.,

$$W_\tau N_\tau + \Phi_\tau = P_{H,\tau} Y_\tau, \quad (2.3)$$

is exogenous to individual household i .

Now, we characterize household i 's intertemporal consumption-saving decisions under limited foresight. Suppose that in each period, household i engages in explicit forward planning for finite h periods ahead; that is, household i has a planning horizon h . For simplicity, we assume that h is exogenous and time-invariant. At time t with the vector of aggregate state variables \mathbf{z}_t , the household chooses state-contingent plans $\{C_\tau^i(\mathbf{z}_\tau)\}$ only for the possible states \mathbf{z}_τ within periods $t \leq \tau \leq t+h$. The household chooses the finite-horizon plans to maximize the objective

$$\mathbb{E}_t^h \left[\sum_{\tau=t}^{t+h} \beta^{\tau-t} u(C_\tau^i) + \beta^{h+1} v(\mathcal{B}_{t+h+1}^i, \mathcal{B}_{t+h+1}^{*,i}; \mathbf{z}_{t+h}) \right], \quad (2.4)$$

where $v(\cdot; \mathbf{z}_{t+h})$ is the value function that the household uses to approximate continuation values for each possible state \mathbf{z}_{t+h} at the end of planning horizon. It is a *coarse* value function such that it is contingent only on individual states instead of the complete state-contingent structure as under the RE case. Here $\mathcal{B}_\tau \equiv B_\tau/P_{\tau-1}$ and $\mathcal{B}_\tau^* \equiv B_\tau^*/P_{\tau-1}^*$ denote the nominal value of bonds maturing in period τ deflated by last-period price indices, and thus \mathcal{B}_τ and \mathcal{B}_τ^* are real variables that are purely predetermined in period $\tau - 1$.

By combining (2.2) and (2.3), household i 's budget constraint can be expressed in real terms as follows:

$$C_\tau^i + \frac{\mathcal{B}_{\tau+1}^i}{1+i_\tau} + \frac{Q_\tau \mathcal{B}_{\tau+1}^{*,i}}{1+i_\tau^*} = \frac{\mathcal{B}_\tau^i}{\Pi_\tau} + \frac{Q_\tau \mathcal{B}_\tau^{*,i}}{\Pi_\tau^*} + \mathcal{S}_\tau Y_\tau, \quad (2.5)$$

where $\Pi_{\tau+1} \equiv P_{\tau+1}/P_\tau$ and $\Pi_{\tau+1}^* \equiv P_{\tau+1}^*/P_\tau^*$ are the domestic and foreign inflation rates, $Q_\tau \equiv P_\tau^* \mathcal{E}_\tau / P_\tau$ is the real exchange rate, and $\mathcal{S}_\tau Y_\tau$ is the household income from wages and dividends (beyond the household's control). Here, we have used the condition that firms' total income is equal to their wage and dividend payment to households, that is,

$$\frac{W_\tau N_\tau + \Phi_\tau}{P_\tau} = \frac{P_{H,\tau} Y_\tau}{P_\tau} = \mathcal{S}_\tau Y_\tau,$$

where $\mathcal{S}_\tau \equiv P_{H,\tau}/P_\tau$ is the ratio between the price index for domestically produced goods

and the price index for the aggregate consumption basket.

In the forward planning exercise with horizon $h \geq 1$ at any date $t \leq \tau \leq t + h - 1$, the first-order conditions to maximize lifetime utility with respect to C_τ , $\mathcal{B}_{\tau+1}$, and $\mathcal{B}_{\tau+1}^*$ yield

$$u'(C_\tau^i) = \beta \mathbb{E}_t^h [(1 + i_\tau) u'(C_{\tau+1}^i) / \Pi_{\tau+1} | \mathbf{z}_\tau], \quad (2.6)$$

$$u'(C_\tau^i) = \beta \mathbb{E}_t^h [(1 + i_\tau^*) u'(C_{\tau+1}^i) (Q_{\tau+1} / Q_\tau) (1 / \Pi_{\tau+1}^*) | \mathbf{z}_\tau], \quad (2.7)$$

for each possible state \mathbf{z}_τ , given state \mathbf{z}_t at the time of the planning exercise. In terminal period $\tau = t + h$ where the forward planning is truncated, or in the case of $h = 0$, the first-order conditions related to the value function are

$$u'(C_{t+h}^i) = \beta(1 + i_{t+h}) v_1(\mathcal{B}_{t+h+1}^i, \mathcal{B}_{t+h+1}^{*,i}; \mathbf{z}_{t+h}), \quad (2.8)$$

$$u'(C_{t+h}^i) = \beta(1 + i_{t+h}^*) v_2(\mathcal{B}_{t+h+1}^i, \mathcal{B}_{t+h+1}^{*,i}; \mathbf{z}_{t+h}) / Q_{t+h}, \quad (2.9)$$

for each possible state \mathbf{z}_{t+k} .

Note that in the household's finite-horizon forward planning problem, if the household's subjective expectation operator $\mathbb{E}_t^h[\cdot]$ is the model-consistent expectation and the value function $v(\cdot; \mathbf{z}_{t+h})$ is the accurate model-consistent value function with a complete state-contingent structure (as in standard dynamic programming under infinite planning horizons), the household's optimization problem replicates the conventional intertemporal optimization problem. That is, in such a case, the households make the optimal infinite-horizon contingent plans under RE.

However, the decision-making under limited foresight features optimal plans and expectation formations that deviate from the infinite-horizon RE benchmark. At date t , the household constructs a contingent plan for the subsequent h forward dates but implements the plan only for the current date t . When the following date $t + 1$ arrives, the household reconstructs the contingent plans for future h dates, which are not necessarily identical to those made at the "previous" date t . The household implements the new plans only for the current date $t + 1$. In terms of expectation formation, at each date t , the h -horizon household makes a contingent plan up to date $t + h$. At each date τ within the planning horizon $t \leq \tau \leq t + h$, the household is assumed to plan forward for the remaining $t + h - \tau$ dates. In addition, the household assumes that spending and pricing decisions made by other households and firms at any date τ within its planning horizon are made with the same remaining planning horizon $t + h - \tau$.

The expectation operator for the h -horizon household can be converted with the model-consistent expectation. For any endogenous variable X_τ determined at date τ ($t \leq \tau \leq t + h$),

the household's expectation conditional on state \mathbf{z}_t at date t is assumed to satisfy

$$\mathbb{E}_t^h[X_\tau|\mathbf{z}_t] = \mathbb{E}_t[X_\tau^{t+h-\tau}], \quad (2.10)$$

where operator $\mathbb{E}[\cdot]$ is the standard model-consistent expectation operator and $t + h - \tau$ represents the remaining planning horizon at date τ . The household's expectation for X_τ conditional on future state \mathbf{z}_τ in its period- t planning exercise is given by

$$\mathbb{E}_t^h[X_\tau|\mathbf{z}_\tau] = \mathbb{E}_\tau[X_\tau^{t+h-\tau}]. \quad (2.11)$$

Finally, the household's expectation for $X_{\tau+1}$ conditional on the same information structure above is given by

$$\mathbb{E}_t^h[X_{\tau+1}|\mathbf{z}_\tau] = \mathbb{E}_\tau[X_{\tau+1}^{t+h-\tau-1}]. \quad (2.12)$$

Under the expectation formation, the first-order conditions (2.6)-(2.9) can be rewritten with the model-consistent operator as follows:

$$u'(C_\tau^{t+h-\tau}) = \beta \mathbb{E}_\tau \left[(1 + i_\tau^{t+h-\tau}) \frac{u'(C_{\tau+1}^{t+h-\tau-1})}{\Pi_{\tau+1}^{t+h-\tau-1}} \right], \quad (2.13)$$

$$u'(C_\tau^{t+h-\tau}) = \beta \mathbb{E}_\tau \left[(1 + i_\tau^{*,t+h-\tau}) \frac{u'(C_{\tau+1}^{t+h-\tau-1})}{\Pi_{\tau+1}^{*,t+h-\tau-1}} \frac{Q_{\tau+1}^{t+h-\tau-1}}{Q_\tau^{t+h-\tau}} \right], \quad (2.14)$$

$$u'(C_{t+h}^0) = \beta(1 + i_{t+h}^0) v_1(\mathcal{B}_{t+h+1}^0, \mathcal{B}_{t+h+1}^{*,0}; \mathbf{z}_{t+h}), \quad (2.15)$$

$$u'(C_{t+h}^0) = \beta(1 + i_{t+h}^{*,0}) v_2(\mathcal{B}_{t+h+1}^0, \mathcal{B}_{t+h+1}^{*,0}; \mathbf{z}_{t+h}) / Q_{t+h}^0. \quad (2.16)$$

In the nonstochastic steady state, we have that $1 + \bar{i} = \beta^{-1} \bar{\Pi}$ and $1 + \bar{i}^* = \beta^{-1} \bar{\Pi}^*$, and the value function in the steady state is given by

$$v(\mathcal{B}, \mathcal{B}^*) = (1 - \beta)^{-1} u \left(\frac{(1 - \beta) \mathcal{B}}{\bar{\Pi}} + \frac{(1 - \beta) \bar{Q} \mathcal{B}^*}{\bar{\Pi}^*} + \bar{S} \bar{Y} \right).$$

Details of deriving the steady state value function can be found in Appendix A.

We define the domestic variables after log-linear approximation as

$$\begin{aligned} \hat{c}_t &\equiv \log \left(\frac{C_t}{\bar{C}} \right), & \hat{y}_t &\equiv \log \left(\frac{Y_t}{\bar{Y}} \right), & \hat{i}_t &\equiv \log \left(\frac{1 + i_t}{1 + \bar{i}} \right), \\ \hat{b}_t &\equiv \frac{\mathcal{B}_t - \bar{\mathcal{B}}}{\bar{\Pi} \bar{C}}, & \hat{q}_t &\equiv \log \left(\frac{Q_t}{\bar{Q}} \right), & \pi_t &\equiv \log \left(\frac{\Pi_t}{\bar{\Pi}} \right), \end{aligned}$$

and for the foreign variables,

$$\hat{i}_t^* \equiv \log \left(\frac{1 + i_t^*}{1 + \bar{i}^*} \right), \quad \hat{b}_t^* \equiv \frac{\bar{Q}(\mathcal{B}_t^* - \bar{\mathcal{B}}^*)}{\bar{\Pi}^* \bar{C}}, \quad \pi_t^* \equiv \log \left(\frac{\Pi_t^*}{\bar{\Pi}^*} \right),$$

where the definition of \hat{b}^* indicates the share of foreign bonds in *domestic currency* to domestic consumption. Throughout the paper, we use lowercase to denote variables after taking logs unless otherwise stated and further use hats to denote log-deviation from the steady state.

Log-linearizing equations (2.13) and (2.14) yield

$$\hat{c}_\tau^{t+h-\tau} = \mathbb{E}_\tau[\hat{c}_{\tau+1}^{t+h-\tau-1}] - \sigma^{-1} [\hat{i}_\tau^{t+h-\tau} - \mathbb{E}_\tau \pi_{\tau+1}^{t+h-\tau-1}], \quad (2.17)$$

$$\hat{c}_\tau^{t+h-\tau} = \mathbb{E}_\tau[\hat{c}_{\tau+1}^{t+h-\tau-1}] - \sigma^{-1} \left[\hat{i}_\tau^{*,t+h-\tau} + \mathbb{E}_\tau(\hat{q}_{\tau+1}^{t+h-\tau-1} - \hat{q}_\tau^{t+h-\tau} - \pi_{\tau+1}^{*,t+h-\tau-1}) \right], \quad (2.18)$$

for any date $t \leq \tau \leq t + h - 1$ with horizon $h \geq 1$, where $\sigma^{-1} \equiv -u'(\bar{C})/(u''(\bar{C})\bar{C})$ is the elasticity of intertemporal substitution of households.

Under the assumption that households adopt the steady-state value function in their forward planning exercise and that there is no learning in the value function, log-linearizing equations (2.15) and (2.16) yields

$$\hat{c}_\tau^0 = -\sigma^{-1} \hat{i}_\tau^0 + (1 - \beta) \hat{b}_{\tau+1}^0 + (1 - \beta) \hat{b}_{\tau+1}^{*,0}, \quad (2.19)$$

$$\hat{c}_\tau^0 = -\sigma^{-1} \hat{i}_\tau^{*,0} + (1 - \beta) \hat{b}_{\tau+1}^0 + (1 - \beta) \hat{b}_{\tau+1}^{*,0} + \sigma^{-1} \hat{q}_\tau^0. \quad (2.20)$$

Details of the derivation can be found in Appendix B. In Section 2.5, we will also introduce a learning process in the value function.

2.2 Firms

A set of continuum producers $f \in [0, 1]$ in the economy produce a variety of differentiated intermediate goods as inputs for the domestically produced final goods. The intermediate goods market is monopolistically competitive, and the producers of each intermediate good can be price-setters in domestic currency (that is, PCP) but face staggered pricing, as in the style of Calvo (1983) and Yun (1996). Specifically, we assume that at each time, fraction $1 - \theta$ of firms are randomly selected to be able to reoptimize their prices. Producer j that belongs to the remaining fraction θ cannot reset its price, and we assume that its price satisfies $P_{H,t}(f) = P_{H,t-1}(f) \bar{\Pi}_H$, where $\bar{\Pi}_H$ is the inflation rate of the domestic goods in the nonstochastic steady state. This implies that the prices are automatically revised by considering the long-run inflation rate for domestically produced goods. This assumption is

an open-economy variation of [Woodford \(2019\)](#), which implies that all equilibrium prices of domestic goods are the same as those under flexible prices in the steady state.

At time t , similar to the objective function of households, firm f with a k -period planning horizon that can reset its price chooses $P_{H,t}^f$ to maximize

$$\max_{P_{H,t}^f} \mathbb{E}_t^f \left[\sum_{\tau=t}^{t+k} (\beta\theta)^{\tau-t} \lambda_\tau H(r_\tau^f; \mathcal{S}_\tau, \mathbf{z}_\tau) + \tilde{v}(r_{t+k}^f; \mathcal{S}_{t+k}, \mathbf{s}_{t+k}) \right], \quad (2.21)$$

where $\lambda_\tau \equiv \int u_c(C_\tau^i) di$ is the average marginal utility of household consumption and $r_\tau^f \equiv \frac{P_{H,t}^f}{P_{H,\tau}} \bar{\Pi}_H^{\tau-t}$ denotes the relative price between firm f 's goods price and domestically produced final goods. $H(r_\tau^f; \mathcal{S}_\tau, \mathbf{z}_\tau)$ represents the real profits of the firm at date τ , where \mathbf{z}_τ is the vector of real state variables that are beyond firm f 's control. A detailed expression of the functional form $H(\cdot)$ can be found in [Appendix C](#).

The last term $\tilde{v}(r_{t+k}^f; \mathcal{S}_{t+k}, \mathbf{s}_{t+k})$ in [\(2.21\)](#) is the firm's value function at the end of the planning horizon that is used to approximate the value of discounted future real profits from date $t+k+1$ onward. Here \mathbf{s}_{t+k} is a coarse vector for the real state variables, which is a subset of \mathbf{z}_{t+k} .

We begin with the assumption that the firm's value function $\tilde{v}(\cdot)$ is the one learned from the nonstochastic steady-state equilibrium; that is, we now consider the following steady-state firm value function:

$$\tilde{v}(r^f) = (1 - \theta\beta)^{-1} \bar{\lambda} H(r^f; \bar{\mathcal{S}}, \bar{\mathbf{z}}), \quad (2.22)$$

where $\bar{\lambda} = u_c(\bar{C})$ is the constant value of λ_τ in the steady state. In [Section 2.5](#), we relax this assumption by incorporating a learning process into $\tilde{v}(\cdot)$.

The firm's expectation formation $\mathbb{E}_t^f[\cdot]$ is isomorphic to that of the household. That is, the firm with planning horizon k assumes that the endogenous variables determined at any date τ in $t \leq \tau \leq t+k$ are based on the decisions of all agents in the economy with the remaining planning horizon $t+k-\tau$. Therefore, the firm's subjective expectation operator for endogenous variables is represented by the model-consistent expectation in the same fashion as in [equations \(2.10\)-\(2.12\)](#), where h is now replaced with k .

Then, with the notation $p_{H,t}^f \equiv \log[P_{H,t}^f / (P_{H,t-1} \bar{\Pi}_H)]$, any firm f that reoptimizes its price at time t with a k -period planning horizon sets $p_{H,t}^f = p_{H,t}^k$, which is given by

$$p_{H,t}^k = \mathbb{E}_t \sum_{\tau=t}^{t+k} (\beta\theta)^{\tau-t} \left[\pi_{H,\tau}^{t+k-\tau} + (1 - \beta\theta) \widehat{m} \widehat{c}_\tau^{t+k-\tau} \right], \quad (2.23)$$

where $\widehat{mc}_\tau \equiv \log\left(\frac{MC_\tau}{\bar{MC}}\right)$ is the log-deviation of the real marginal cost, $MC_\tau \equiv \frac{MC_\tau}{P_{H,\tau}}$, around its steady state at date τ . In particular, it satisfies $\widehat{mc}_t = -H'(1; 1, \mathbf{z}_t)/H''(1; 1, \bar{\mathbf{z}})$. The details of deriving (2.23) and \widehat{mc}_t can be found in Appendix C.

The evolution of the aggregate domestic price index $P_{H,t}$ satisfies

$$P_{H,t}^{1-\varepsilon} = \theta(P_{H,t-1}\bar{\Pi}_H)^{1-\varepsilon} + (1-\theta)(P_{H,t}^f)^{1-\varepsilon},$$

and its log-linear approximation around the steady state with constant domestic inflation rate $\bar{\Pi}_H$ yields

$$\pi_{H,t}^k = (1-\theta)p_{H,t}^k.$$

Thus, equation (2.23) becomes

$$\pi_{H,t}^k = (1-\theta)\mathbb{E}_t \sum_{\tau=t}^{t+k} (\beta\theta)^{\tau-t} \left[\pi_{H,\tau}^{t+k-\tau} + (1-\beta\theta)\widehat{mc}_\tau^{t+k-\tau} \right]. \quad (2.24)$$

The isomorphic form of the equation holds if we replace k with any horizon $j \geq 0$. That is, $\{\pi_{H,t}^j\}$ for any horizon $j \geq 1$ satisfies the following recursive form:

$$\pi_{H,t}^j = \kappa\widehat{mc}_t^j + \beta\mathbb{E}_t\pi_{H,t+1}^{j-1}, \quad (2.25)$$

where $\kappa \equiv (1-\theta)(1-\beta\theta)/\theta$, and when $j = 0$, we have

$$\pi_{H,t}^0 = \kappa\widehat{mc}_t^0. \quad (2.26)$$

2.3 Labor Market and Real Marginal Cost

The wage is determined following the approach of Woodford (2019), which abstracts labor supply decision-making from any individual household while maintaining the aggregate labor-supply curve as in the canonical New Keynesian models. As mentioned in Section 2.1, the labor market organization has representatives who bargain for wages on behalf of households. Henceforth, we drop the superscripts on the planning horizon for the sake of parsimony when they are redundant for explicitly understanding the equilibrium relationships. We formally state the full equilibrium conditions with the FH in Section 3.

A representative determines the number of working hours provided by households for any given wage, and households must supply that number of hours and receive the same wage. There are many such representatives, and no representative has any market power. Then, the representatives choose the number of hours N_t to maximize the average utility of the

households in the economy, which yields the following labor supply:

$$\varpi_N(N_t) = \lambda_t \frac{W_t}{P_t}.$$

As in [Galí and Monacelli \(2005\)](#), we assume the standard disutility function of the labor supply in the form of

$$\varpi(N_t) = \frac{N_t^{1+\varphi}}{1+\varphi},$$

where φ is the inverse of the Frisch elasticity of labor supply. Then, the labor supply equation after taking log becomes

$$\varphi n_t = -\sigma c_t + w_t - p_t. \quad (2.27)$$

Each firm j has a linear technology represented by the production function

$$Y_t(j) = A_t N_t(j),$$

hence the marginal cost is common across domestic firms. The real marginal cost in terms of domestic prices is then given by

$$\mathcal{MC}_t = (1 - \tau) \frac{W_t}{P_{H,t} A_t},$$

where τ is an employment subsidy,⁸ and the log of the real marginal cost becomes:

$$mc_t = \log(1 - \tau) + w_t - p_{H,t} - a_t. \quad (2.28)$$

2.4 Closing the Economy

Exchange Rate and the Terms of Trade. From the definition of the real exchange rate, we have the following accounting relationship between the log of the nominal exchange rate, real exchange rate, and domestic and foreign price indices:

$$q_t = e_t + p_t^* - p_t, \quad (2.29)$$

⁸As discussed in [Galí and Monacelli \(2005\)](#), the constant employment subsidy τ is set to correct the distortion associated with firms' market power, which requires $1 - \tau = 1 - 1/\epsilon$. It is financed via contemporaneous lump-sum taxes on households. Thus, Φ_t in [\(2.3\)](#) is the net income of the dividend transfer less taxes.

which yields the following log-linearized equation

$$\hat{\varepsilon}_t = \hat{q}_t - \hat{q}_{t-1} + \pi_t - \pi_t^*. \quad (2.30)$$

Here, $\hat{\varepsilon}_t \equiv \hat{e}_t - \hat{e}_{t-1}$ is the log-linearized nominal devaluation rate.

Without loss of generality, the price of foreign composite goods in the foreign currency is normalized to one, i.e., $p_t^* = 1$. We further assume that the law of one price always holds, hence $e_t = p_{F,t}$. Then, taking into account the linearized CPI index

$$p_t = (1 - \alpha)p_{H,t} + \alpha p_{F,t}, \quad (2.31)$$

we have

$$\pi_t = (1 - \alpha)\pi_{H,t} + \alpha\hat{\varepsilon}_t. \quad (2.32)$$

Define the terms of trade between the domestic country and the rest of the world $S_t \equiv P_{F,t}/P_{H,t}$. It yields

$$s_t = p_{F,t} - p_{H,t}. \quad (2.33)$$

By further combining (2.29) and (2.31), the terms of trade in the first-order approximation satisfies

$$\hat{s}_t = \frac{\hat{q}_t}{(1 - \alpha)}. \quad (2.34)$$

International Goods-Market Clearing. The international market clearing condition for domestically produced goods j is

$$Y_t(j) = C_{H,t}(j) + \int_0^1 C_{H,t}^{*,i}(j) di \quad (2.35)$$

for all $j \in [0, 1]$ and any t , where $C_{H,t}^{*,i}(j)$ is the demand from foreign country i for good j produced in the domestic country.

With the assumption of identical preferences across countries and the law of one price for any goods j , aggregating (2.35) across j and utilizing (2.34) yield

$$\hat{y}_t = \vartheta_{yc}\hat{c}_t + \vartheta_{ys}\hat{s}_t, \quad (2.36)$$

where $\vartheta_{yc} \equiv (1 - \alpha)\bar{\mathcal{S}}^{-\eta}\bar{C}/\bar{Y}$ and $\vartheta_{ys} \equiv \alpha[\gamma + \eta(1 - \alpha)]\bar{\mathcal{S}}^{-\eta}\bar{C}/\bar{Y}$. Details of the derivations can be found in Appendix D.

Utilizing the production function $\hat{y}_t = \hat{a}_t + \hat{n}_t$, (2.27), together with (2.31), (2.33), and

(2.36), the log-linearized real marginal cost (2.28) becomes

$$\widehat{mc}_t = (\sigma + \varphi\vartheta_{yc})\hat{c}_t + (\alpha + \varphi\vartheta_{ys})\hat{s}_t - (1 + \varphi)\hat{a}_t. \quad (2.37)$$

Domestic Monetary Policy. For the domestic interest rate, we consider a monetary policy rule intended to stabilize the domestic CPI inflation rate following a standard Taylor-type form, that is,⁹

$$\hat{i}_t = \phi_\pi \pi_t, \quad (2.38)$$

where $\phi_\pi > 1$ is a constant parameter.

Evolution of Foreign Asset Position. Without loss of generality, we assume that the foreign inflation rate is always one, that is, $\Pi_t^* = \bar{\Pi}^* = 1$ and $\pi_t^* = 0$ for any t . Then, the foreign nominal interest rate \hat{i}_t^* is equal to \hat{r}_t^* and the steady-state relationship satisfies $1 + \bar{r}^* = \beta^{-1}$. Additionally, we assume that the net supply of domestic bonds in the small open economy is zero ($\mathcal{B}_t = 0$ for any t). This assumption is predicated on the notion that domestic agents possess homogeneous characteristics, thus enabling individual issues to be consolidated into a representative household model. Moreover, given that domestic bonds are non-tradable, the principle of zero net supply maintains its validity in equilibrium.

In equilibrium, the domestic resource constraint (2.5) now becomes:

$$\frac{Q_t \mathcal{B}_{t+1}^*}{1 + i_t^*} = \frac{Q_t \mathcal{B}_t^*}{\Pi_t^*} + \mathcal{S}_t Y_t - C_t.$$

After log-linearization around the steady state and by noticing $\hat{\mathcal{S}}_t = -\alpha\hat{s}_t$, we have

$$\hat{b}_{t+1}^* = \beta^{-1}(\hat{b}_t^* + \vartheta_1 \hat{q}_t - \vartheta_2 \alpha \hat{s}_t + \vartheta_2 \hat{y}_t - \hat{c}_t) - \vartheta_1 \hat{q}_t + \vartheta_1 \hat{r}_t^*, \quad (2.39)$$

where $\vartheta_1 \equiv \frac{\bar{B}^* \bar{Q}}{C}$ and $\vartheta_2 \equiv \frac{\bar{S} \bar{Y}}{C}$.

Foreign Interest Rate and External Bond. A small open-economy feature of our model is that the foreign real interest rate, denoted \hat{r}_t^* , is exogenously set by the rest of the world. [Schmitt-Grohé and Uribe \(2003\)](#) show that linearized small open-economy models under RE with an incomplete asset market yield equilibrium dynamics with random walk components for open-economy related variables, if the rate of return of the internationally traded assets is exogenously determined abroad. Thus, one needs stationarity-inducing devices to solve

⁹Our numerical results and conclusions are also robust to targeting domestic inflation measured by producer price index (PPI) $\pi_{H,t}$.

small open-economy models under RE with an incomplete asset market.

Following the standard practice in the literature as proposed in [Schmitt-Grohé and Uribe \(2003\)](#), we assume that the foreign interest rate faced by the domestic country endogenously responds to its level of foreign asset position:

$$\hat{r}_t^* = \phi_b \hat{b}_{t+1}^* + \mu_t, \quad (2.40)$$

where \hat{b}_{t+1}^* is pre-determined in period t , $\phi_b < 0$ is a constant, and μ_t is a random foreign interest rate shock. In particular, $\phi_b < 0$ indicates that as the domestic country borrows more from the rest of the world ($\hat{b}_{t+1}^* < 0$), it is charged with a higher interest rate by the international financial market.

Importantly, as discussed in [Woodford \(2019\)](#) and [Xie \(2020\)](#), a model with limited foresight by design always guarantees a unique equilibrium solution in the closed-economy setting, regardless of the restrictions on the monetary/fiscal policy reaction function. This feature of equilibrium determinacy also holds in our small open-economy model under limited foresight.¹⁰ However, we still impose the same assumption (2.40) in the model of limited foresight to facilitate its comparison with the counterpart model under RE, so that we can isolate the role of limited foresight on the different equilibrium performances.

2.5 Decision-makers' Learning in Value Functions

Thus far, we have assumed that the value functions of households $v(\mathcal{B}, \mathcal{B}^*)$ and of firms $\tilde{v}(r^f)$ are the fixed ones learned from the nonstochastic stationary environment. Starting from this section, we relax this assumption such that decision-makers update their value functions over time based on their past experiences. Similar to [Woodford \(2019\)](#), we assume that the learning behaviors of households and firms follow a constant-gain process:

$$v_{t+1}(\mathcal{B}, \mathcal{B}^*) = \gamma_v v_t^{est}(\mathcal{B}, \mathcal{B}^*) + (1 - \gamma_v) v_t(\mathcal{B}, \mathcal{B}^*),$$

$$\tilde{v}_{t+1}(r^f) = \gamma_{\tilde{v}} \tilde{v}_t^{est}(r^f) + (1 - \gamma_{\tilde{v}}) \tilde{v}_t(r^f),$$

where $\gamma_v, \gamma_{\tilde{v}} \in [0, 1]$ are learning gain parameters. That is, decision-makers *extrapolate* future value functions (which will be used in the planning exercise at time $t + 1$) using their beliefs about the value functions at time t and estimates of the value function obtained as a result of their planning exercises at time t . Therefore, the value functions that de-

¹⁰When there is no learning in agents' value function, the equilibrium under limited foresight is stationary even with an exogenous path of \hat{r}_t^* . However, when agents learn and update their value function, a device similar to (2.40) is necessary to guarantee the stationarity of equilibrium.

scribe decision-makers' perceptions of the future beyond their planning horizons reflect past and estimated value functions. This extrapolation can lead to slow-moving components of economic indicators.

We now consider a local approximation of the dynamics implied by the constant-gain learning rule through a perturbation of the steady-state solution. We parameterize a log-linear approximation of $v_1(\mathcal{B}, \mathcal{B}^*)$ in the household's optimal finite-horizon plan with respect to the domestic bond as

$$\log(v_{1,t}(\mathcal{B}, \mathcal{B}^*)/v_1^*(\bar{\mathcal{B}}, \bar{\mathcal{B}}^*)) = -\sigma \left[\nu_t + \chi_t \hat{b} + \zeta_t \hat{b}^* \right].$$

Here we use $v^*(\cdot)$ to denote the steady-state value function. Using this approximation, we can compute a log-linear approximation of the solution to the household's optimal finite-horizon plan in period t .

Let $C_t^i(\mathcal{B}, \mathcal{B}^*)$ denote the optimal expenditure plan of household i under the counterfactual assumption $\mathcal{B}^i = \mathcal{B}$. Then, the derivative of the estimated value function is equal to

$$v_{1,t}^{est}(\mathcal{B}, \mathcal{B}^*) = \mathbb{E}_t^i[u_C(C_t^i(\mathcal{B}, \mathcal{B}^*)/\Pi_t)].$$

By log-linearization, we then have

$$\log(v_{1,t}^{est}(\mathcal{B}, \mathcal{B}^*)/v_1^*(\bar{\mathcal{B}}, \bar{\mathcal{B}}^*)) = -\sigma \hat{c}_t^h(\hat{b}, \hat{b}^*) - \pi_t^h, \quad (2.41)$$

where h is the planning horizon of household i . Our log-linear approximation of the optimal household plan satisfies $\hat{c}_t^h(\hat{b}, \hat{b}^*) = \hat{c}_t^h(\bar{b}, \bar{b}^*) + \hat{c}_{1,t}^h \hat{b} + \hat{c}_{2,t}^h \hat{b}^*$. Approximating the left-hand side as $-\sigma \left[\nu_t^{est} + \chi_t^{est} \hat{b} + \zeta_t^{est} \hat{b}^* \right]$ of equation (2.41) directly implies

$$\nu_t^{est} = \hat{c}_t^h + \sigma^{-1} \pi_t^h - \zeta_t^{est} \hat{b}^*,$$

where we have used the condition $\hat{b}_t = 0$ for all t , and equating the coefficients on both sides by substituting the expression of $\hat{c}_t^h(\hat{b}, \hat{b}^*)$ yields

$$\chi_t^{est} = \hat{c}_{1,t}^h,$$

$$\zeta_t^{est} = \hat{c}_{2,t}^h.$$

The intercept term of the estimated marginal value of domestic currency bonds, ν_t^{est} , depends on current consumption, the CPI inflation, and the asset position of foreign bonds.

Together with the constant-gain learning rule, we have

$$\begin{aligned}\nu_{t+1} &= \gamma_v \nu_t^{est} + (1 - \gamma_v) \nu_t, \\ \chi_{t+1} &= \gamma_v \chi_t^{est} + (1 - \gamma_v) \chi_t, \\ \zeta_{t+1} &= \gamma_v \zeta_t^{est} + (1 - \gamma_v) \zeta_t.\end{aligned}$$

We can show that χ_t and ζ_t are univariately mean-reverting to a constant $1 - \beta$, and thus we assume that these two variables have converged. Details can be found in Appendix E.

Similarly, we parameterize a log-linear approximation of $v_2(\mathcal{B}, \mathcal{B}^*)$ in the households' optimal finite-horizon plan with respect to the foreign bond as

$$\log(v_{2,t}(\mathcal{B}, \mathcal{B}^*)/v_2^*(\bar{\mathcal{B}}, \bar{\mathcal{B}}^*)) = -\sigma \left[\nu_t^* + \chi_t' \hat{b} + \zeta_t' \hat{b}^* \right].$$

Using this approximation, we can compute a log-linear approximation of the solution to the household's optimal finite-horizon plan in period t .

Let $C_t^i(\mathcal{B}, \mathcal{B}^*)$ denote the optimal expenditure plan of household i under the counterfactual assumption $\mathcal{B}^{*i} = \mathcal{B}^*$. Then, the derivative of the estimated value function will be equal to

$$v_{2,t}^{est}(\mathcal{B}, \mathcal{B}^*) = \hat{\mathbb{E}}_t^i [u_C(C_t^i(\mathcal{B}, \mathcal{B}^*)) Q_t / \Pi_t^*].$$

Note we have assumed that $\Pi_t^* = 1$ for any t . By log-linearization, we then have

$$\log(v_{2,t}^{est}(\mathcal{B}, \mathcal{B}^*)/v_2^*(\bar{\mathcal{B}}, \bar{\mathcal{B}}^*)) = -\sigma \hat{c}_t^h(\hat{b}, \hat{b}^*) + \hat{q}_t^h. \quad (2.42)$$

Our log-linear approximation of the optimal household plan satisfies $\hat{c}_t^h(\hat{b}, \hat{b}^*) = \hat{c}_t^h(\bar{b}, \bar{b}^*) + \hat{c}_{1,t}^h \hat{b} + \hat{c}_{2,t}^h \hat{b}^*$. Approximating the left-hand side as $-\sigma[\nu_t^{*,est} + \chi_t^{',est} \hat{b} + \zeta_t^{',est} \hat{b}^*]$ and equating coefficients yield

$$\nu_t^{*,est} = \hat{c}_t^h - \sigma^{-1} \hat{q}_t^h - (1 - \beta) \hat{b}_t^* = \nu_t^{est} - \sigma^{-1} (\hat{q}_t^h + \pi_t^h),$$

$$\chi_t^{',est} = \hat{c}_{1,t}^h = \chi_t^{est},$$

$$\zeta_t^{',est} = \hat{c}_{2,t}^h = \zeta_t^{est},$$

where we have utilized the fact that $\zeta_t^{est} = 1 - \beta$ as shown in Appendix E. Thus, the estimated marginal value of the foreign bond depends on the estimated marginal value of the domestic bond, the current real exchange rate, and CPI inflation.

Note further that the constant-gain learning rule yields

$$\nu_{t+1}^* = \gamma_v \nu_t^{*,est} + (1 - \gamma_v) \nu_t^*.$$

Therefore, we have characterized the learning process of the household value function.

For the firm (with planning horizon k), we similarly have

$$\tilde{\nu}_t^{est} = (1 - \theta)^{-1} \pi_{H,t}^k,$$

and

$$\tilde{\nu}_{t+1} = \gamma_{\tilde{\nu}} \tilde{\nu}_t^{est} + (1 - \gamma_{\tilde{\nu}}) \tilde{\nu}_t.$$

3 Equilibrium Characterization with Homogeneous Planning Horizons across Agents

In this section, we focus on the steps to pin down the equilibrium path with the assumption that all the agents share the same planning horizon h . Therefore, the equilibrium dynamics of aggregate variables satisfy $\hat{y}_t = \hat{y}_t^h$, $\pi_t = \pi_t^h$, etc. Focusing on the case of homogeneous agents allows us to abstract from the aggregation problem across the population, while we can still analyze how the equilibrium dynamics change with respect to the degree of foresight (i.e., the common planning horizon h). In Section 6, we extend the analyses to heterogeneous agents with different planning horizons and discuss the robustness of the main results.¹¹

First, given state variables $\{\hat{q}_{t-1}, \hat{b}_t^*, \nu_t, \nu_t^*, \tilde{\nu}_t\}$ and exogenous shocks $\{a_t, \mu_t\}$ that follow AR(1) processes, we solve the problem of the finite planning exercise in period t . Let $y_{\tau|t}^j$ be the value of \hat{y}_τ that is predicted at date τ as a result of aggregation of decisions made by agents with (counterfactual) planning horizon $j = h + t - \tau$, which is calculated at date t by agents with planning horizon h . It is a function of the state $\{\hat{q}_{t-1}, \hat{b}_t^*, \nu_t, \nu_t^*, \tilde{\nu}_t\}$ and $\{a_t, \mu_t\}$ in period t . Then, we have the actual aggregate output in period t given by $\hat{y}_t = \hat{y}_{t|t}^h$. Similarly, we can define other variables in the finite planning exercise with the same notation. The additional subscript $|t$ matters because different value functions are used in finite planning in different periods.

The equilibrium conditions for the finite planning exercise in period t are given in Appendix F. The system consists of a finite number of equations as a function of state variables $\{\hat{q}_{t-1}, \hat{b}_t^*, \nu_t, \nu_t^*, \tilde{\nu}_t\}$ and exogenous shocks $\{a_t, \mu_t\}$. Thus, we can solve for all endogenous vari-

¹¹Our conclusions are also robust when extending the set of exogenous shocks to include domestic demand shock and domestic interest rate shock, in addition to the productivity shock and foreign interest rate shock.

ables $\{\hat{c}_{\tau|t}^{h+t-\tau}, \hat{y}_{\tau|t}^{h+t-\tau}, \hat{i}_{\tau|t}^{h+t-\tau}, \hat{r}_{\tau|t}^{*,h+t-\tau}, \pi_{H,\tau|t}^{h+t-\tau}, \pi_{\tau|t}^{h+t-\tau}, \hat{q}_{\tau|t}^{h+t-\tau}, \hat{s}_{\tau|t}^{h+t-\tau}, \hat{\varepsilon}_{\tau|t}^{h+t-\tau}, \hat{b}_{\tau+1|t}^{*,h+t-\tau}\}_{\tau=t}^{t+h}$ with a unique solution. The actual aggregate variables in period t are then given by

$$\begin{aligned} \hat{c}_t &= \hat{c}_{t|t}^h, & \hat{y}_t &= \hat{y}_{t|t}^h, & \hat{i}_t &= \hat{i}_{t|t}^h, & \hat{r}_t^* &= \hat{r}_{t|t}^{*,h}, & \pi_{H,t} &= \pi_{H,t|t}^h, \\ \pi_t &= \pi_{t|t}^h, & \hat{q}_t &= \hat{q}_{t|t}^h, & \hat{s}_t &= \hat{s}_{t|t}^h, & \hat{\varepsilon}_t &= \hat{\varepsilon}_{t|t}^h, & \hat{b}_{t+1}^* &= \hat{b}_{t+1|t}^{*,h}. \end{aligned} \quad (3.1)$$

From period t to period $t+1$, the value functions evolve over time, that is,

$$\nu_{t+1} = \gamma_v \nu_t^{est} + (1 - \gamma_v) \nu_t, \quad (3.2)$$

$$\nu_{t+1}^* = \gamma_v \nu_t^{*,est} + (1 - \gamma_v) \nu_t^*, \quad (3.3)$$

$$\tilde{\nu}_{t+1} = \gamma_{\tilde{v}} \tilde{\nu}_t^{est} + (1 - \gamma_{\tilde{v}}) \tilde{\nu}_t, \quad (3.4)$$

where

$$\nu_t^{est} = \hat{c}_t + \sigma^{-1} \pi_t - (1 - \beta) \hat{b}_t^*, \quad (3.5)$$

$$\nu_t^{*,est} = \nu_t^{est} - \sigma^{-1} (\hat{q}_t + \pi_t), \quad (3.6)$$

$$\tilde{\nu}_t^{est} = (1 - \theta)^{-1} \pi_{H,t}. \quad (3.7)$$

Now, we describe how to solve the planned solution at date τ calculated in period t , which is characterized by the equilibrium conditions of the finite planning exercise as shown in Appendix F. We can write the solution to any endogenous variable $x_{\tau|t}^j$ except $\hat{b}_{\tau+1|t}^{*,j}$ in forward planning as a function of the state variables and exogenous shocks, that is,

$$x_{\tau|t}^j = \psi_{x,q}^j \hat{q}_{\tau-1|t}^{j+1} + \psi_{x,b}^j \hat{b}_{\tau|t}^{*,j+1} + \psi_{x,a}^j a_{\tau} + \psi_{x,\mu}^j \mu_{\tau} + \psi_{x,\nu}^j \nu_t + \psi_{x,\tilde{\nu}}^j \tilde{\nu}_t + \psi_{x,\nu^*}^j \nu_t^*, \quad (3.8)$$

for any (counterfactual) $j \geq 0$, and similarly,

$$\hat{b}_{\tau+1|t}^{*,j} = \psi_{b,q}^j \hat{q}_{\tau-1|t}^{j+1} + \psi_{b,b}^j \hat{b}_{\tau|t}^{*,j+1} + \psi_{b,a}^j a_{\tau} + \psi_{b,\mu}^j \mu_{\tau} + \psi_{b,\nu}^j \nu_t + \psi_{b,\tilde{\nu}}^j \tilde{\nu}_t + \psi_{b,\nu^*}^j \nu_t^*. \quad (3.9)$$

Then, one can solve the undetermined coefficients via the equilibrium conditions of the finite planning exercise for any j in the following steps: (i) utilizing the equilibrium conditions for $j = 0$ and equating the coefficients yields the coefficients for $j = 0$; (ii) the undetermined coefficients for any (counterfactual) j can be solved by forward induction; that is, given the coefficients for $j - 1$, the undetermined coefficients for j are uniquely given by the equilibrium conditions for j . See Appendix G for the detailed procedure.

Thus far, we have derived the solution of the entire forward planning calculated in period t . Then, one can easily solve for the equilibrium path (3.1) with the evolution of the state

Table 1: Calibrated Parameters

| Parameter | Value | Description |
|--------------|--------|--|
| h | 8 | Length of Planning Horizon (quarter) |
| β | 0.99 | Subjective Discount Factor |
| σ | 2 | Inverse of Intertemporal Elasticity of Substitution |
| α | 0.15 | 1 - Home Bias |
| φ | 1 | Inverse of the Frisch Elasticity of Labor Supply |
| θ | 0.75 | Calvo-Yun Sticky Price Parameter |
| γ | 1.5 | Elasticity of Substitution between Goods of Foreign Countries |
| η | 1.5 | Elasticity of Substitution between Goods of Home and Rest-of-the-World |
| ϕ_π | 2.15 | Monetary Policy Reaction Coefficient to the CPI Inflation Rate |
| ϕ_b | -0.01 | External Bond Sensitivity of the Foreign Interest Rate |
| ρ_x | 0.9 | Persistence of Shocks |
| γ_v | 0.165 | Household's Learning Gain |
| σ_a | 0.0114 | Std. Dev. of TFP Shock |
| σ_μ | 0.0050 | Std. Dev. of Foreign Interest Rate Shock |

variables (3.2)-(3.4) together with exogenous shocks.

4 Equilibrium Analyses

In this section, we investigate the equilibrium features of the FH model. We first calibrate the model parameters to match key moments of the aggregate dynamics of Canada and also set some structural parameters to be consistent with the literature. Next, we analyze the equilibrium dynamics by examining impulse response functions of the FH model, comparing it with its RE counterpart and the case of the FH model without learning in the value function. In particular, we also discuss how the models with limited foresight generate systematic forecast errors in the real exchange rate.

4.1 Calibrated Parameters

We calibrate the FH model to a quarterly frequency; see Table 1. We assume a symmetric steady state across the domestic country and the rest of the world, with $\bar{B}^* = 0$, $1 + \bar{r}^* = 1/\beta$, and $\bar{Q} = 1$, as is commonly assumed in open-economy macro literature. The steady-state assumptions establish $\bar{S} = \bar{S} = 1$ and $\bar{C} = \bar{Y}$ in the steady state. Following this, the coefficients ϑ_1 , ϑ_2 , ϑ_{yc} , and ϑ_{ys} are defined in relation to the corresponding structural parameters, which are calibrated below.

The following parameters are standard in the literature. We set the subjective discount factor $\beta = 0.99$, the inverse of intertemporal elasticity of substitution $\sigma = 2$, and the inverse

of the Frisch elasticity of labor supply $\varphi = 1$. The parameter of home bias parameter is set to $\alpha = 0.15$. The Calvo-Yun price stickiness parameter is $\theta = 0.75$, implying an average duration of four quarters between two consecutive price adjustments. We set the parameter of policy reaction in the standard Taylor rule $\phi_\pi = 2.15$, following [Clarida et al. \(1999\)](#). We also set the parameters of trade elasticity γ and η to 1.5, following the values used in [Backus et al. \(1994\)](#) and [Chari et al. \(2002\)](#). We fix the sensitivity of the foreign interest rate to external bond holdings ϕ_b to -0.01, following [Benigno \(2009\)](#) and [Justiniano and Preston \(2010\)](#).

Following [Woodford and Xie \(2022\)](#), we set the length of the planning horizon $h = 8$ (that is, eight quarters). This is a conservative value because empirical findings suggest an even shorter planning horizon; for instance, [Gust et al. \(2022\)](#) estimates an average planning horizon being one-quarter of the U.S. economy.¹² If the planning horizon is smaller, our conclusions are strengthened as they deviate more from the case under RE and the numerical results become quantitatively much stronger.

We assume that both exogenous shocks exhibit a persistence of 0.9, following the assumption of shock persistence as posited by [Candian and De Leo \(2023\)](#). We then calibrate the three remaining parameters: the household’s learning gain parameter γ_v and the standard deviations of the two exogenous shocks σ_a and σ_μ . The three parameters are jointly calibrated to match the following five moments, utilizing Canadian data from 1961:2Q to 2007:4Q:¹³ (i) the standard deviation of output growth, $\sigma(\Delta y)$; (ii) the relative standard deviation of consumption growth to output growth, $\sigma(\Delta c)/\sigma(\Delta y)$; (iii) the relative standard deviation of real exchange rate growth to output growth, $\sigma(\Delta q)/\sigma(\Delta y)$; (iv) the persistence of the real exchange rate, $\rho(q)$; and (v) the cross-correlation between output growth and consumption growth, $\rho(\Delta y, \Delta c)$. We obtain a learning gain of $\gamma_v = 0.165$. Additionally, we get $\sigma_a = 0.0114$ and $\sigma_\mu = 0.0050$. Consequently, the relative standard deviation of domestic productivity and foreign interest shocks stands at 2.28.

Table 2 reports a number of moments for the macroeconomic aggregates, comparing both the data and the model-generated moments with the same calibrated parameters. The FH model matches the overall moments reasonably well, including the five targeted moments and other non-targeted ones. In addition, the FH model fits the data uniformly better than the RE counterpart does, as the calibration is based on executing the FH model. Compared to the data, the calibrated FH model tends to overpredict the negative correlation between the consumption and real exchange rate growths, $\rho(\Delta c, \Delta q)$. However, this is due to the

¹²The survey evidence in [Coibion et al. \(2023\)](#) suggests that the households planning horizons in the US is no more than two years and they may well be capable of planning forward about three or four quarters.

¹³The data source and variable construction can be found in Appendix H.

Table 2: Data and Model-Implied Second-Order Moments

| | Data | Model-FH | Model-RE |
|--|-------|----------|----------|
| <i>Targeted Moments</i> | | | |
| $\sigma(\Delta y) \times 100$ | 0.89 | 0.89 | 0.84 |
| $\sigma(\Delta c)/\sigma(\Delta y)$ | 0.93 | 0.93 | 1.14 |
| $\sigma(\Delta q)/\sigma(\Delta y)$ | 2.19 | 2.03 | 2.30 |
| $\rho(q)$ | 0.98 | 0.96 | 0.90 |
| $\rho(\Delta y, \Delta c)$ | 0.49 | 0.41 | 0.35 |
| <i>Non-Targeted Moments</i> | | | |
| $\rho(\varepsilon)$ | 0.34 | 0.29 | 0.15 |
| $\sigma(\varepsilon)/\sigma(\Delta y)$ | 2.16 | 2.46 | 2.66 |
| $\rho(\varepsilon, \Delta q)$ | 0.94 | 0.99 | 0.99 |
| $\rho(\Delta c, \Delta q)$ | -0.07 | -0.39 | -0.55 |

Notes: To compute the model-implied second-order moments, we utilize the median value of the moments, derived from 10,000 simulations each spanning 188 quarters.

features of finite planning horizons, and in fact, the FH model performs significantly better than the RE model along this dimension.

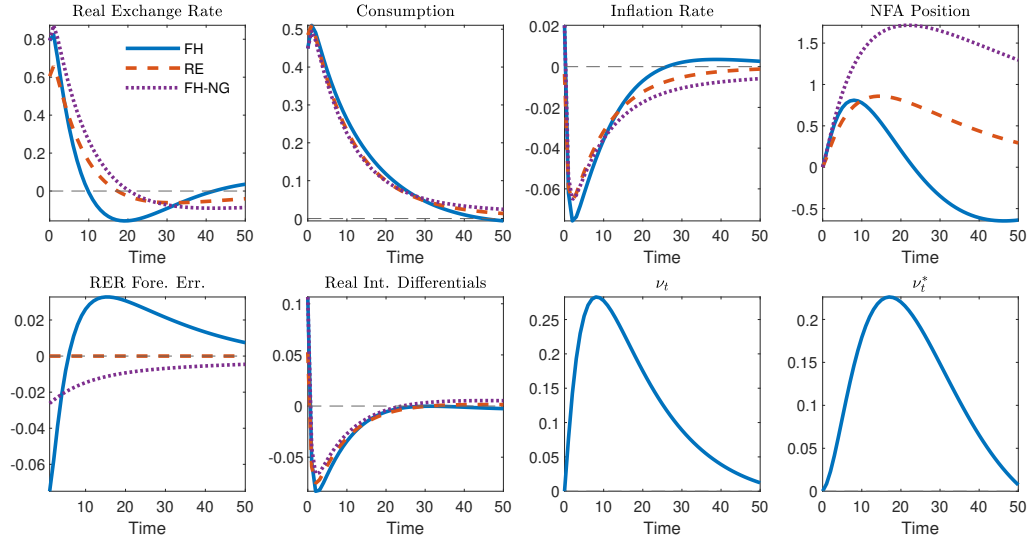
4.2 Impulse Responses

We now analyze the equilibrium dynamics by comparing the impulse response functions between the FH and RE models. In particular, to isolate the role of value function learning, we also compare with the FH model in which the learning gain in value function is zero (that is, $\gamma_v = 0$, labeled as FH-NG). We consider the actual transition dynamics of variables under perfect foresight shocks as impulse responses, where decision-makers are aware of the shock's complete future path upon its impact on the economy. The actual transition dynamics help study how value-function learning influences actual (not expected) aggregate dynamics, as decision-makers update their value function over time.

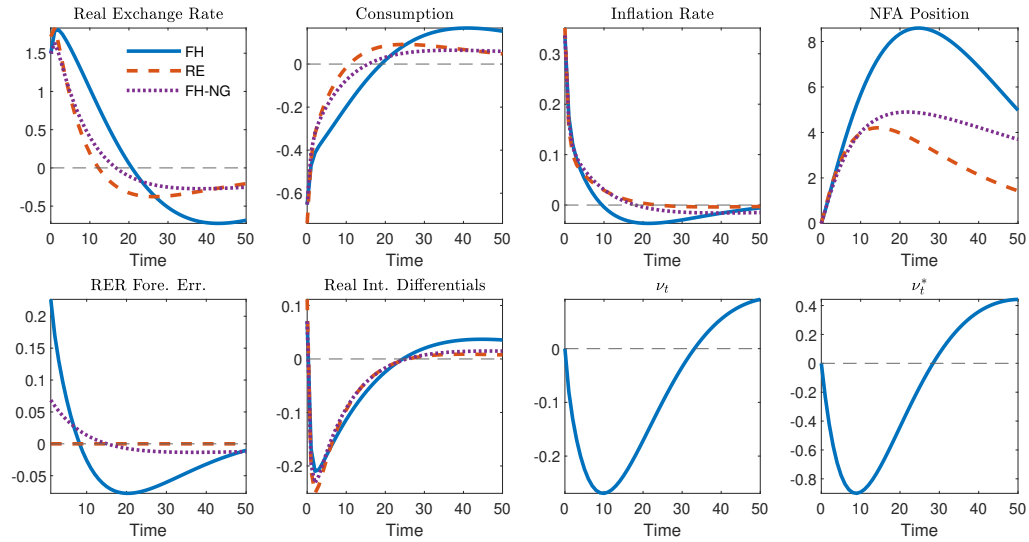
Figure 1 illustrates the impulse responses of the variables of interest to the two structural shocks, respectively, with a size of one standard deviation. In essence, across all three models, the domestic productivity and foreign interest rate shocks (shown in panels (a) and (b), respectively) conform to their roles in price and quantity determination. The former causes real exchange rate depreciation and consumption increase, triggering a temporary rise in CPI inflation due to real depreciation's pass-through effect, followed by a supply shock-induced inflation drop. The latter shock depreciates the real exchange rate, curbs consumption, and elevates CPI inflation via exchange rate pass-through. Both shocks prompt households to boost savings, resulting in a gradual rise in net foreign asset positions \hat{b}_t^* in

Figure 1: Impulse Response Functions of Selected Variables to Structural Shocks

(a) To Domestic Productivity Shock



(b) To Foreign Interest Rate Shock



Notes: This figure shows the impulse response functions of selected variables in three models, subject to one standard deviation domestic productivity shock (panel (a)) and to one standard deviation foreign interest rate shock (panel (b)). “FH” refers to the benchmark limited foresight model, “RE” refers to the rational expectation model, and “FH-NG” refers to the limited foresight model with no learning gain ($\gamma_v = 0$).

subsequent periods. Also, represented as $\hat{r}_t - \hat{r}_t^*$ (with \hat{r}_t being the nominal interest rate minus expected one-period ahead inflation), the real interest rate differential between domestic and

foreign bonds initially surges due to shocks but later adjusts downwards, indicating higher subsequent real returns from foreign bonds.¹⁴

Despite some common observations, notable differences persist across the models. Specifically, in comparison to the RE model, the FH model generally exhibits more persistent, hump-shaped movements of the aggregate variables. Additionally, the FH models generate non-trivial real exchange rate forecast errors, which is a feature absent in the RE model. We will examine these differences in detail.

The Effect of the Value Functions. To comprehend the responses of the aggregate dynamics in the FH model, we delve into the value functions ν_t and ν_t^* . In the FH model, when shocks occur at $t = 0$, households use the steady-state evaluated priors ν_0 and ν_0^* for planning, causing zero responses at this period. Post this period, they observe the realized macro outcomes and estimate their value function via (3.5) and (3.6). Households then update ν_1 and ν_1^* by averaging the estimated and prior value functions via (3.2) and (3.3), for planning in period 1. After observing the realized macroeconomic outcome at the end of period 1, they estimate ν_1^{est} and $\nu_1^{*,est}$, and use the updated ν_2 and ν_2^* for planning in period 2. This process is repeated thereafter.

Households' learning behavior results in hump-shaped dynamics for the value functions. With a learning gain of $\gamma_v = 0.165$, households replace 16.5% of the prior values with values estimated from previous planning exercises, which emphasizes the prior value functions and creates persistent dynamics. When exogenous shocks occur, households slowly update their value function using the estimated values, which then gradually revert to steady-state as the shocks fade, creating a hump-shaped dynamic. As depicted in in panel (a), ν_t and ν_t^* follow U-shaped dynamics with productivity shocks, while in panel (b), they display inverse U-shaped dynamics with foreign interest rate shocks. This hump-shaped behavior of the value functions impacts the dynamic pattern of macroeconomic variables.

To understand why the hump-shaped dynamics of value functions exhibit opposite directions based on two different shocks, we must examine the log-linear approximations of the estimated marginal values of domestic and foreign bonds, as detailed in Section 2.5. For the domestic bond, the approximation is $-\sigma \left[\nu_t^{est} + (1 - \beta) \hat{b}_t^* \right]$, positioning ν_t^{est} as the inverse of this value. As per equation (3.5), given a predetermined net foreign asset position \hat{b}_t^* , the bond's estimated marginal value increases with a decrease in both consumption and inflation. Intuitively, an increase in the marginal utility of consumption raises the marginal value of the real domestic bond via the standard wealth effect, while inflation reduces the

¹⁴The figure shows initial real interest rate differential surges in both the FH and RE models, indicating that the surge is not due to the FH-specific feature. Instead, the root cause of the initial surges is attributed to the substantial persistence of the shocks, which are set to 0.9.

bond's real value.

On the other hand, $\nu_t^{*,est}$ serves as the inverse of the estimated marginal value of the foreign bond. As per equation (3.6), given the current net foreign asset position, the value of $\nu_t^{*,est}$ rises as consumption decreases and the real exchange rate depreciates. This aligns with the notion that an increase in the marginal utility of consumption and a depreciation of the real exchange rate augment the real foreign bond's marginal value, as they respectively enable the standard wealth effect and higher real return on foreign currency bond investments.

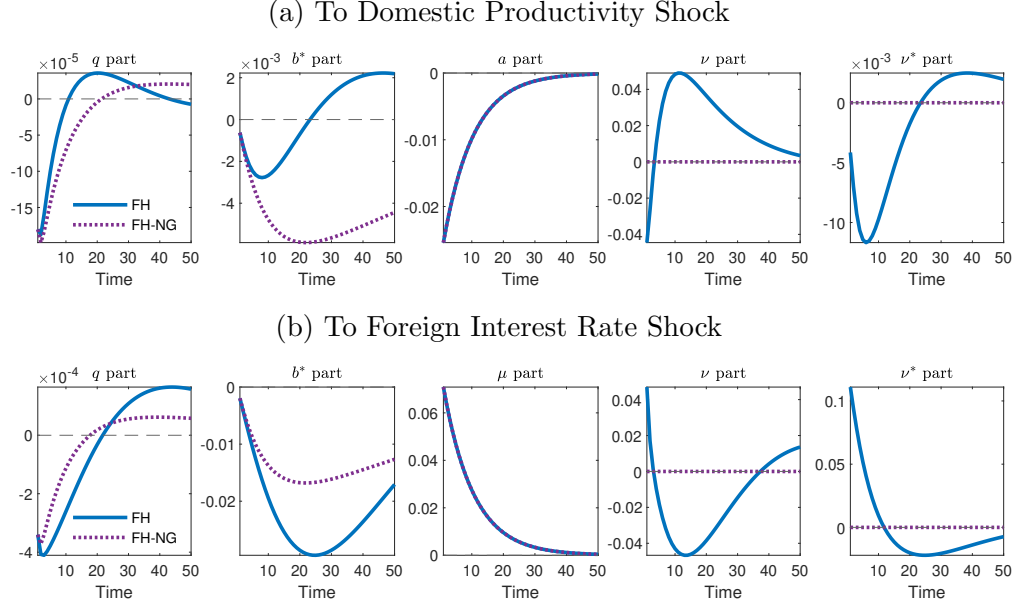
With σ and β calibrated as in Table 1, the equations for value function estimation (3.5) and (3.6) become $\nu_t^{est} = \hat{c}_t + 0.5\pi_t - 0.01\hat{b}_t^*$ and $\nu_t^{*,est} = \hat{c}_t - 0.5\hat{q}_t - 0.01\hat{b}_t^*$. As depicted in Figure 1, the two exogenous shocks impact the variables on the left-hand side of the equations differently, both in direction and magnitude. Concerning the coefficients on the variables, the responses of consumption, inflation, and the real exchange rate tend to be the primary influencers of ν_t^{est} and $\nu_t^{*,est}$. Consequently, combined with the households' learning behavior regarding value functions, ν_t and ν_t^* follow U-shaped dynamics in response to the productivity shock and inverse U-shaped dynamics in response to the foreign interest rate shock.

Dynamics of RER Forecast Errors. We now turn our attention to the dynamics of the forecast errors of the real exchange rate. As illustrated in Figure 1, the FH model inherently produce systematic forecast errors due to a conceptual disconnect between expectation and realization.

Additionally, the dynamics of the exchange rate forecast errors significantly depend on the learning behavior in the value function and display contrasting patterns based on the two shocks. When subjected to the domestic productivity shock, as shown in panel (a), the FH model initially presents a short-term negative forecast error that reverses to positive in subsequent periods. When responding to the foreign interest rate shock (panel (b)), the FH model initially shows a positive forecast error in the short term, which later reverses to negative in the ensuing periods. On the other hand, while the FH-NG model does display contrasting patterns of forecast errors after the two shocks, the magnitudes and dynamic reversals are significantly less pronounced.

Figure 2 displays the decomposition of the dynamics regarding the forecast errors of real exchange rate in Figure 1. In the FH model, the forecast error of the one-period ahead real

Figure 2: Decomposition of the Forecast Error Dynamics for the Real Exchange Rate



Notes: This figure shows the impulse response functions of the components of the one-period ahead forecast errors of the real exchange rate in the two models, subject to one standard deviation domestic productivity shock (panel (a)) and one standard deviation foreign interest rate shock (panel (b)). “FH” refers to the benchmark limited foresight model and “FH-NG” refers to the limited foresight model with $\gamma_v = 0$.

exchange rate becomes:

$$\begin{aligned}
 \hat{q}_{t+1} - \hat{\mathbb{E}}_t \hat{q}_{t+1} = & \underbrace{(\psi_{q,q}^8 - \psi_{q,q}^7) \hat{q}_t}_{q \text{ part}} + \underbrace{(\psi_{q,b}^8 - \psi_{q,b}^7) \hat{b}_{t+1}^*}_{b^* \text{ part}} + \underbrace{(\psi_{q,a}^8 - \psi_{q,a}^7) \rho_a a_t}_{a \text{ part}} + \underbrace{(\psi_{q,\mu}^8 - \psi_{q,\mu}^7) \rho_\mu \mu_t}_{\mu \text{ part}} \\
 & \underbrace{\hspace{10em}}_{\text{forecast errors from finite forward planning}} \\
 & + \underbrace{(\psi_{q,\nu}^8 \nu_{t+1} - \psi_{q,\nu}^7 \nu_t)}_{\nu \text{ part}} + \underbrace{(\psi_{q,\nu^*}^8 \nu_{t+1}^* - \psi_{q,\nu^*}^7 \nu_t^*)}_{\nu^* \text{ part}}. \tag{4.1} \\
 & \underbrace{\hspace{10em}}_{\text{forecast errors from value function extrapolation}}
 \end{aligned}$$

This suggests that the real exchange rate’s forecast error in the FH model can be decomposed into six components: $(q, b^*, a, \mu, \nu, \nu^*)$, categorized by forecast errors stemming from (i) finite forward planning and (ii) value function extrapolation (learning). In the FH-NG model, in contrast, the forecast error dynamics are influenced by the first four components, given that $\nu_t = \nu_t^* = 0$ for all t . In Figure 2, We observe that in the FH model, the forecast errors caused by value function extrapolation, absent in the FH-NG model, play a crucial role due to the value functions’ non-monotonic, hump-shaped behavior. This results in an overshoot of the forecast error, leading to a sign reversal over the horizon.

5 Addressing UIP Puzzles across Time Horizons

In this section, we show that the FH model can be used to address some puzzling features of the time-horizon aspects of the UIP condition under the RE framework. In this exercise, note that the calibrated parameters in Table 1 do not target any empirical moments related to UIP violations. Therefore, our approach demonstrates how the calibrated model is apt in explaining the qualitative features of expectation formations related to RE-UIP violations, while concurrently restricting the model's quantitative adaptability concerning the conventional aggregate moments in the data.

We begin by formally describing the RE-UIP condition and its implications.

5.1 RE-UIP Condition and Its Implications

The asset pricing equations of the real domestic currency bonds and the real foreign currency bonds in the RE framework (which correspond to equations (2.6)-(2.9) in the FH framework) are stated as follows:

$$u'(C_t) = \beta \mathbb{E}_t [(1 + i_t) u'(C_{t+1}) / \Pi_{t+1}], \quad (5.1)$$

$$u'(C_t) = \beta \mathbb{E}_t [(1 + i_t^*) u'(C_{t+1}) (Q_{t+1}/Q_t) (1/\Pi_{t+1}^*)]. \quad (5.2)$$

Combining (5.1) and (5.2) yields

$$\mathbb{E}_t \left[\frac{u'(C_{t+1})}{u'(C_t)} \left(\frac{1 + i_t}{\Pi_{t+1}} - \frac{1 + i_t^*}{\Pi_{t+1}^*} \frac{Q_{t+1}}{Q_t} \right) \right] = 0. \quad (5.3)$$

The log-linear approximation of equation (5.3) around the nonstochastic steady state gives:

$$\mathbb{E}_t \hat{q}_{t+1} - \hat{q}_t = \hat{r}_t - \hat{r}_t^*, \quad (5.4)$$

where $\hat{r}_t \equiv \hat{i} - \mathbb{E}_t \pi_{t+1}$ and $\hat{r}_t^* \equiv \hat{i}^* - \mathbb{E}_t \pi_{t+1}^*$. Equation (5.4) is the RE-UIP condition in real form. This implies that when the real interest rate differential between foreign and domestic currency bonds is positive (that is, $\hat{r}_t^* - \hat{r}_t > 0$), future real appreciation $\mathbb{E}_t \hat{q}_{t+1} - \hat{q}_t < 0$ should result of the same magnitude.¹⁵

We define the ex post real excess return on foreign currency bonds from period t to $t + 1$

¹⁵The literature also often use the nominal version of the RE-UIP condition. Choosing either nominal or real version is not consequential for our results. We use the real version just to facilitate the discussion with Section 4. Also, note that the RE-UIP condition (5.4) is a simple specification and it can be extended to more complex forms, for example, by considering a stationary trend in the real exchange rate dynamics. For the sake of parsimony, we use the simplest specification to emphasize the role of behavioral biases from the FH models in addressing the puzzles that we focus on in an essential modeling environment.

as follows:

$$\Delta_{t+1} \equiv \hat{q}_{t+1} - \hat{q}_t + \hat{r}_t^* - \hat{r}_t. \quad (5.5)$$

Then, the UIP condition (5.4) under RE implies

$$\mathbb{E}_t \Delta_{t+1} = 0, \quad (5.6)$$

indicating that the excess return should be unpredictable with the information set at time t .

In addition, iterating the expectation term of real exchange rate $\mathbb{E}_t \hat{q}_{t+1}$ forward in (5.4) by the law of iterated expectations (LIE) yields

$$\hat{q}_t = \sum_{k=0}^{\infty} \mathbb{E}_t [\hat{r}_{t+k}^* - \hat{r}_{t+k}] + \lim_{T \rightarrow \infty} \mathbb{E}_t \hat{q}_{t+T}.$$

Following Galí (2020), we decompose the sum of expectations into the short and long terms:

$$\hat{q}_t = D_t^S(M) + D_t^L(M) + \lim_{T \rightarrow \infty} \mathbb{E}_t \hat{q}_{t+T}, \quad (5.7)$$

where M is the threshold period for the short run and the long run, and

$$D_t^S(M) \equiv \sum_{k=0}^{M-1} \mathbb{E}_t [\hat{r}_{t+k}^* - \hat{r}_{t+k}], \quad D_t^L(M) \equiv \sum_{k=M}^{\infty} \mathbb{E}_t [\hat{r}_{t+k}^* - \hat{r}_{t+k}],$$

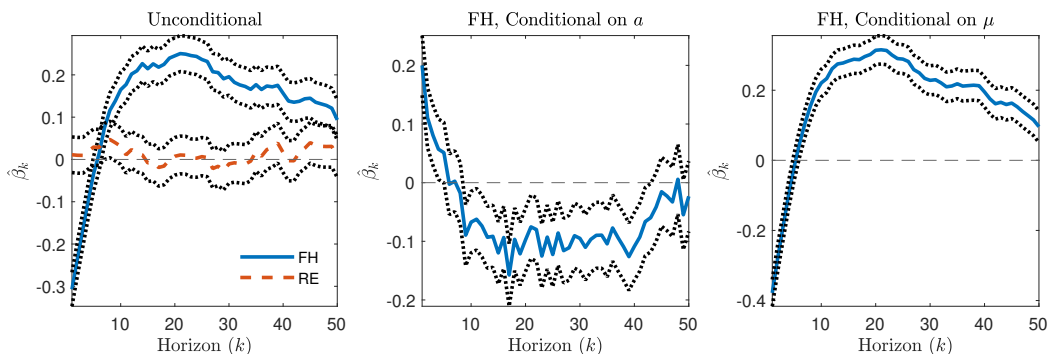
are defined as the sum of expectations on the short- and long-term real interest rate differentials, respectively.¹⁶ Since the real exchange rate is a stationary variable, one can assume that $\lim_{T \rightarrow \infty} \mathbb{E}_t \hat{q}_{t+T} = 0$. Then, equation (5.7) indicates that the RE-UIP condition predicts horizon-invariance for the impact of the real interest rate differential on the real exchange rate: the short-term interest rate differential $D_t^S(M)$ and long-term interest rate differential $D_t^L(M)$ have identical effects on the current real exchange rate with the same weight.

5.2 Excess Return Predictability and the Predictability Reversal

A challenge to the RE-UIP condition (5.4) is that a predictable excess return is observed in the data. Classic studies such as Fama (1984) and Eichenbaum and Evans (1995) show a short-term positive predictable excess returns of currency bonds that bear higher interest rates. Furthermore, recent studies (e.g., Bacchetta and van Wincoop, 2010; Engel, 2016;

¹⁶The definition of the real interest rate differential in Galí (2020) is represented as $\hat{r}^* - \hat{r}$, contradicts our earlier definition by being its negative. Nevertheless, in the discussion regarding horizon invariance, we follow Galí (2020)'s definition to maintain consistency.

Figure 3: Excess Return Predictability across Time Horizons: Regression Coefficients



Notes: This figure presents the estimates of excess return coefficients, $\hat{\beta}_k$, across the horizon (k), accompanied by 95% confidence intervals. These estimates were obtained using samples from 100 simulations, each spanning 188 quarters, of the models. First panel uses series generated by the two shocks, second panel uses series from the domestic productivity shock only, and the third panel uses series from the foreign interest rate shock only. In the first panel, “FH” refers to the benchmark limited foresight model and “RE” refers to the rational expectation model.

Valchev, 2020) document that the movements of the predictable excess return are more complicated over the time horizon: the excess return is positive in the short run, whereas it reverses to negative in the long run.

In the FH models, by construction, the excess return equals the forecast error of the real exchange rate:

$$\Delta_{t+1} \equiv \hat{q}_{t+1} - \hat{q}_t + \hat{r}_t^* - \hat{r}_t = \hat{q}_{t+1} - \hat{\mathbb{E}}_t q_{t+1}.$$

This formulation intentionally disregards other potential factors in excess return predictability driven by time-varying risk or liquidity premia. Thus, our model isolates the role of the forecast error from other factors and is also consistent with the empirical finding that the major sources of the UIP deviations in advanced countries are subjective forecast errors (e.g., Froot and Frankel, 1989; Chinn and Frankel, 2019; Kalemli-Özcan and Varela, 2022; Candian and De Leo, 2023).

By following the empirical setup in the literature, we run the following regression model using the simulated data from RE and FH models:

$$\Delta_{t+k} = \beta_0 + \beta_k(\hat{r}_t - \hat{r}_t^*) + \xi_t, \quad (5.8)$$

where Δ_{t+k} is the ex post one-period excess return between time $t+k-1$ and $t+k$. β_k captures the predictable excess return at time t for future horizon k . Since $\hat{r}_t - \hat{r}_t^*$ is an explanatory variable, a negative β_k suggests that the foreign currency bonds yield a positive real excess return when the foreign bonds carry a higher real interest rate ($\hat{r}_t^* > \hat{r}_t$). Conversely, a positive β_k implies the opposite.

Figure 3’s first panel illustrates the estimates of excess return coefficients, $\hat{\beta}_k$, across the horizon, complete with 95% confidence intervals, derived from the simulated series in both the FH and RE models. In the RE model, the confidence intervals for the estimates nearly always contain zero, highlighting the unpredictability of excess returns as implied by the RE-UIP condition. Conversely, the FH model demonstrates the predictability of excess returns. It initially exhibits a negative $\hat{\beta}_k$ for the first five horizons, which then reverses to positive, peaks at 21 horizon, and then diminishes towards zero. Thus, the FH model predicts that foreign currency bonds yield a short-term positive real excess return when foreign bonds bear a higher real interest rate. However, it predicts a negative excess return in the medium and long term, echoing empirical findings in the referenced literature.

It’s important to note that the first panel of Figure 3 displays *unconditional* profile of excess return predictability, which incorporates both shocks from the model. As depicted in Figure 1, the FH model’s exchange rate forecast errors respond in qualitatively opposite ways to the two shocks. This is mirrored in the second and third panels of Figure 3, which display the $\hat{\beta}_k$ estimates, each conditioned on a distinct shock. These dynamics inform the FH model’s prediction of opposite excess return predictability profiles, conditional on the two shocks. Notably, the foreign interest rate shock drives the unconditional profile of excess return predictability.

The sign reversal of forecast errors in the FH model, induced by the finite planning horizon and value function learning, introduces a fresh explanation for the reversal of predictability. This addition enriches existing theoretical explanations in literature that attribute this phenomenon to infrequent portfolio decisions (Bacchetta and van Wincoop, 2010), convenience yields (Valchev, 2020), and over-extrapolation on misperceived shocks (Candian and De Leo, 2023).

5.3 Breakdown of the Horizon-Invariance

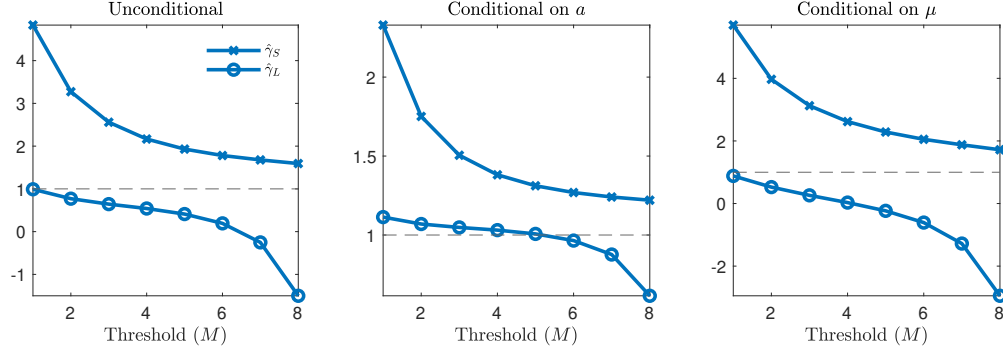
Another challenge to the RE-UIP condition is the empirical breakdown of the horizon-invariance in equation (5.7). Galí (2020) builds the following regression specification based on (5.7):

$$\hat{q}_t = \gamma_0 + \gamma_S D_t^S(M) + \gamma_L D_t^L(M) + \zeta_t, \quad (5.9)$$

where γ_0 , γ_S , and γ_L are regression coefficients and ζ_t is an orthogonal error term. If the horizon-invariance results hold, then the regression coefficients should be $\gamma_0 = 0$ and $\gamma_S = \gamma_L = 1$.

Galí (2020) tests specification (5.9) using the data of government zero-coupon bond yields from the U.S., the U.K., and Germany, and finds robust results on $\hat{\gamma}_S > 1$ and $\hat{\gamma}_L < 1$ for

Figure 4: Reaction of Real Exchange Rate to Short- and Long-Term Interest Rate Differentials: Regression Coefficients



Notes: This figure shows $\hat{\gamma}_S$ and $\hat{\gamma}_L$ estimates, representing the real exchange rate’s reaction to short and long-run interest rate differentials in the benchmark limited foresight model. These estimates were obtained using samples from 100 simulations, each spanning 188 quarters, of the models. First panel uses series generated by the two shocks, second panel uses series from the domestic productivity shock only, and the third panel uses series from the foreign interest rate shock only.

those countries. The empirical findings imply that the current real exchange rate *overreacts* to the short-term interest rate differential but *underreacts* to the long-term interest rate differential, rejecting the prediction of horizon-invariance.

We conduct the same exercise using the series generated by the FH model and show that the limited foresight model can address the short-term overreaction and the long-term underreaction of the real exchange rate. For a given the threshold horizon for the short-term and the long-term M and the given planning horizon h , we construct the expected real interest rate differentials under limited foresight as

$$D_t^S(M) = \sum_{k=0}^{M-1} \mathbb{E}_t^h[\hat{r}_{t+k}^* - \hat{r}_{t+k}], \quad D_t^L(M) = \sum_{k=M}^h \mathbb{E}_t^h[\hat{r}_{t+k}^* - \hat{r}_{t+k}], \quad (5.10)$$

where $0 < M \leq h$.¹⁷ We perform the regression exercise using the constructed variables from the FH model, as described in equation 5.9. Figure 4 displays the estimates of the regression coefficients $\hat{\gamma}_S$ and $\hat{\gamma}_L$ under the threshold $1 \leq M \leq 8$. The first panel demonstrates that, across all the threshold M , $\hat{\gamma}_S > 1$ and $\hat{\gamma}_L < 1$. This suggests that the real exchange rate overreacts to the short-term interest rate differential, but underreacts to the long-term differential. Furthermore, as the threshold M increases, the reaction coefficients decrease monotonically. This behavior indicates that the FH model qualitatively predicts the breakdown of horizon-invariance, aligning with the findings of Galí (2020). The foreign interest rate shock (as depicted in the third panel) primarily drives the breakdown of hori-

¹⁷The detailed procedure to construct the variables in 5.10 can be found in Appendix I.

zon invariance in this unconditional profile. The domestic productivity shock (shown in the second panel) plays a lesser role. This observation aligns with the influence of shocks on the excess return predictability, as presented in Figure 3.

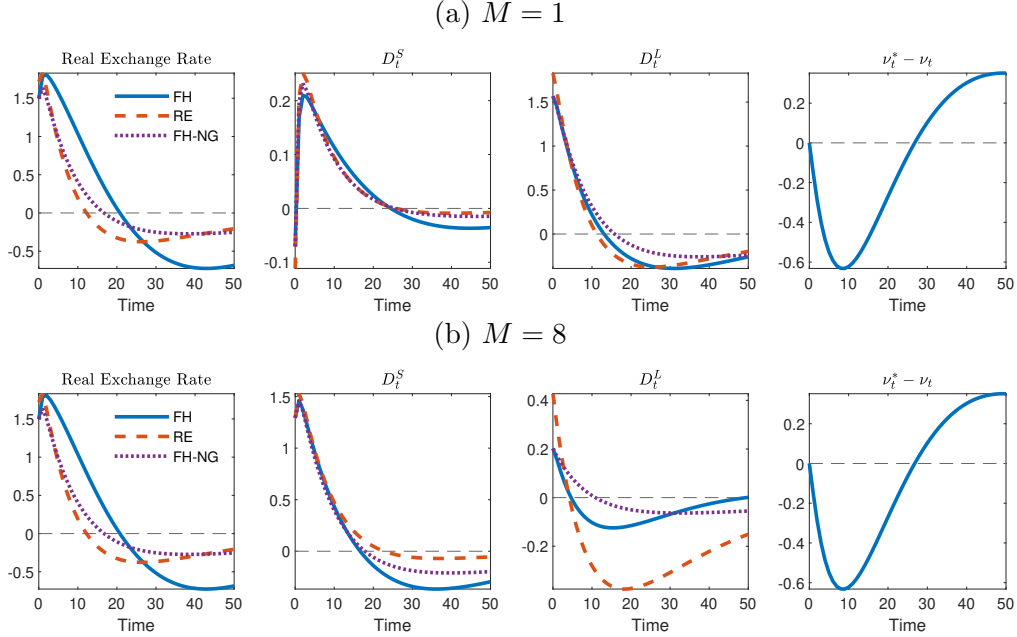
In both the RE and FH-NG models, the simulated series consistently yield $\hat{\gamma}_S = \hat{\gamma}_L = 1$. Hence, the dynamics of value function learning in the FH model play a pivotal role in the breakdown of horizon-invariance in Figure 4. But what are the mechanisms at play here? It's crucial to note that the law of iterated expectations doesn't apply in the FH models with value function learning. This is because expectations formed at any time t are contingent on the value functions at that same time t . When decision-makers learn and update their value functions, expectations of future endogenous variables differ from the *expectations of expected* future endogenous variables. For example, $\hat{\mathbb{E}}_t x_{t+2}$ is the subjective expectation for any endogenous variable x_{t+2} , which is based on time- t value functions, whereas $\hat{\mathbb{E}}_{t+1} x_{t+2}$ is the subjective expectation conditional on the value functions at time $t + 1$. As the value functions at times t and $t + 1$ are typically different (and their evolution follows a constant-gain learning process), in general $\hat{\mathbb{E}}_t x_{t+2} \neq \hat{\mathbb{E}}_t [\hat{\mathbb{E}}_{t+1} x_{t+2}]$. This variation disrupts the horizon-invariance specification (5.7), typically derived from forward iteration and the LIE. Thus, our FH model suggests that the empirical evidence of Galí (2020) also implies the rejection of LIE.

Next, we aim to understand the causes of the specific asymmetrical directions of the reaction coefficients $\hat{\gamma}_S$ and $\hat{\gamma}_L$ in Figure 4. It's important to acknowledge that decision-makers form expectations at time t for future periods $t + k$ based on the assumption that the counterfactual endogenous variables are constructed on aggregate conditions with a *remaining* planning horizon of $h - k$. Therefore, as k increases, the expectations become more dependent on the decision-makers' value functions. For example, with the current calibrated parameters and the planning horizon $h = 8$, the equilibrium condition (shown in equation (F.13) in Appendix F) implies that the decision-makers' perception of the counterfactual interest rate differential at 8 periods ahead is based on the following equilibrium condition:

$$\hat{r}_{t+8|t}^{*,0} - \hat{r}_{t+8|t}^0 = \hat{q}_{t+8|t}^0 + 2(\nu_t^* - \nu_t), \quad (5.11)$$

This implies that, given $\hat{q}_{t+8|t}$, the difference between the bond value functions ($\nu_t^* - \nu_t$) directly influences the equation for real interest rate differentials. Consequently, the term $\nu_t^* - \nu_t$ has a substantial impact on the counterfactual real interest rate differential at time $t + 8$. Equation (5.11) suggests that given $\hat{q}_{t+8|t}^0$, when decision-makers discern a higher relative marginal value of holding foreign over domestic bonds (signaled by a drop in $\nu_t^* - \nu_t$), they project these values onto their long-term counterfactual expectations. This leads to an

Figure 5: Impulse Response Functions of the Real Exchange Rate, D_t^S , D_t^L , and $\nu_t^* - \nu_t$ to Foreign Interest Rate Shock



Notes: This figure shows the impulse response functions of the real exchange rate, the sum of the short-term real interest rate differential (D_t^S), the sum of the long-term real interest rate differential (D_t^L), and the difference between variables regarding the marginal values of holding foreign and domestic bonds ($\nu_t^* - \nu_t$) in the four models, subject to foreign interest rate shock. Panel (a) presents the case where the threshold horizon for the short- and long-term (M) is set to 1, while panel (b) presents the case where M is set to 8. “FH” refers to the benchmark limited foresight model, “RE” refers to the rational expectation model, and “FH-NG” refers to the limited foresight model with no learning gain ($\gamma_v = 0$). The planning horizon (h) is set to 8.

increase in relative foreign bond prices and a corresponding decrease in the counterfactual interest rate ($\hat{r}_{t+8|t}^{*,0} - \hat{r}_{t+8|t}^0$) at time $t + 8$. Such an extrapolation is absent in both the RE and FH-NG models.

Thus, in the FH model, $D_t^L(M)$, which encompasses the expected counterfactual interest rate differentials for $t + 8$, is strongly linked with $\nu_t^* - \nu_t$. The equilibrium solutions (3.8) and (3.9) show that the value functions affect counterfactual variables across remaining horizons $h - k$ and current variables at time t . However, their impact diminishes as future horizons draw nearer, corresponding to an increase in planning horizon and a convergence of policy coefficients tied to the value functions towards zero, as seen in the RE case. Therefore, the impact of $\nu_t^* - \nu_t$ on $D_t^S(M)$ is weaker compared to $D_t^L(M)$ in the FH model.

Figure 5 illustrates these dynamics, showing the impulse responses to a foreign interest rate shock for varying threshold horizons $M = 1$ (panel (a)) and $M = 8$ (panel (b)). In the FH model, the real exchange rate, \hat{q}_t , deviates from the sum of short- and long-term interest rate differentials, $D_t^S(M)$ and $D_t^L(M)$. The FH model’s dynamics show a weaker

link between $D_t^S(M)$ and $\nu_t^* - \nu_t$, which is more pronounced when $M = 1$, whereas the relationship between $D_t^L(M)$ and $\nu_t^* - \nu_t$ strengthens for $M = 8$. In contrast, the RE and FH-NG models align \hat{q}_t with these sums, as per horizon-invariance. In addition, the formation of expectations across the three models diverges more for distant future expectations. It's also noteworthy that in the FH-NG model, the dynamics of \hat{q}_t bear a strong resemblance to $D_t^S(M)$, yet they diverge from $D_t^L(M)$. This divergence becomes more pronounced as M increases. Given that the accumulated short-term interest rate differential enlarges as M increases, Figure 5 offers an explanation for the regression analysis results illustrated in the first and third panels of Figure 4. It shows why the impact of $D_t^S(M)$ on \hat{q}_t exceeds 1, while $D_t^L(M)$ does not, and why these impacts decline as M increases.

6 Robustness

We evaluate the robustness of the main results in Section 5 by taking into account various planning horizons h , the firm's learning gain parameter γ_v , and by extending the model to incorporate agents with heterogeneous planning horizons. In these robustness checks, we set all structural parameters as in Section 4.2 except for the parameter of interest. The qualitative aspects of the main results remain broadly robust, with the principal takeaways unaltered by those considerations. To save space, we leave the details to Appendix J.

7 Concluding Remarks

In this paper, we reconsider the conclusions from the RE assumption in a standard SOE-NK model by assuming that decision-makers are subject to limited foresight when making decisions. Our analysis indicates that the dynamics of the model's equilibrium are significantly influenced by both the degree of decision-makers' foresight and the manner in which they update their value functions. The FH model generates persistent and non-trivial forecast errors of the real exchange rate across time horizons, along with inherent differences in the formation of short-term and long-term expectations, none of which can be observed in the RE model. Our model provides a natural micro foundation for those renowned puzzles of the RE-UIP condition that feature a time-horizon variability. We show that our model can explain (i) the time-varying excess return predictability and its reversal of sign over the time horizon and (ii) the breakdown of horizon-invariance, marked by the real exchange rate's diverse responses to short-term and long-term expected real interest rate differentials.

The model environment presented in our paper is small-scale, which indicates the necessity for future research to enlarge the model with an increased emphasis on quantitative as-

pects. Natural extensions could involve the incorporation of additional frictions and wedges, along with endogenous variables such as capital. It may also be beneficial to estimate the quantitative model in line with dynamic moments from a more comprehensive data set, including financial and expectation-related elements. This approach could further enhance our understanding of the RE-UIP puzzles within a broader framework. We discuss these issues in a companion paper.

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Online Appendix: Not for Publication

“A Behavioral New Keynesian Model of a Small Open Economy under Limited Foresight”

Seunghoon Na and Yinxi Xie

A Steady-State Value Function of Households

The steady-state value function solves the following Bellman equation:

$$v(\mathcal{B}, \mathcal{B}^*) = \max_{C, \mathcal{B}', \mathcal{B}^{*'}} \{u(C) + v(\mathcal{B}', \mathcal{B}^{*'})\}$$

s.t.

$$\beta \mathcal{B}' + \beta \frac{\bar{Q} \mathcal{B}^{*'} \bar{\Pi}}{\bar{\Pi}^*} = \mathcal{B} + \frac{\bar{Q} \mathcal{B}^* \bar{\Pi}}{\bar{\Pi}^*} + (\bar{S} \bar{Y} - C) \bar{\Pi}.$$

The first-order conditions yield

$$\begin{aligned} v_1(\mathcal{B}', \mathcal{B}^{*'}) &= \frac{\beta u'(C)}{\bar{\Pi}}, & v_2(\mathcal{B}', \mathcal{B}^{*'}) &= \frac{\beta \bar{Q} u'(C)}{\bar{\Pi}^*}, \\ v_1(\mathcal{B}, \mathcal{B}^*) &= \frac{u'(C)}{\bar{\Pi}}, & v_2(\mathcal{B}, \mathcal{B}^*) &= \frac{\bar{Q} u'(C)}{\bar{\Pi}^*}, \end{aligned}$$

where the last two equations come from the envelope theorem.

It can be easily verified that the following solution satisfies the above system of first-order conditions, which is given by

$$v(\mathcal{B}, \mathcal{B}^*) = (1 - \beta)^{-1} u \left(\frac{(1 - \beta) \mathcal{B}}{\bar{\Pi}} + \frac{(1 - \beta) \bar{Q} \mathcal{B}^*}{\bar{\Pi}^*} + \bar{S} \bar{Y} \right).$$

B Log-Linearization of the F.O.C.s of Households in the Ending Period of Forward Planning

In this section, we show the steps of log-linearizing equations (2.15) and (2.16) under the assumption that households use a steady-state value function in their forward planning.

First, taking logs of (2.15) and conducting first-order Taylor expansion at the steady state with notation $\tau = t + h$ yields

$$\ln u'(C_\tau^0) = \ln \beta + \ln(1 + i_\tau^0) + \ln v_1(\mathcal{B}_{\tau+1}^0, \mathcal{B}_{\tau+1}^{*,0})$$

$$\begin{aligned}
\Rightarrow \frac{u''(\bar{C})}{u'(\bar{C})}(C_\tau^0 - \bar{C}) &= \frac{1 + i_\tau^0 - (1 + \bar{i})}{1 + \bar{i}} + \frac{v_{1,1}(\bar{\mathcal{B}}, \bar{\mathcal{B}}^*)}{v_1(\bar{\mathcal{B}}, \bar{\mathcal{B}}^*)}(\mathcal{B}_{\tau+1}^0 - \bar{\mathcal{B}}) + \frac{v_{1,2}(\bar{\mathcal{B}}, \bar{\mathcal{B}}^*)}{v_1(\bar{\mathcal{B}}, \bar{\mathcal{B}}^*)}(\mathcal{B}_{\tau+1}^{*,0} - \bar{\mathcal{B}}^*) \\
\Rightarrow \frac{u''(\bar{C})\bar{C}}{u'(\bar{C})}\hat{c}_\tau^0 &= \hat{i}_\tau^0 + \frac{u''(\bar{C})\bar{C}}{u'(\bar{C})} \frac{(1 - \beta)\mathcal{B}_{\tau+1}^0 - \bar{\mathcal{B}}}{\bar{\Pi}} + \frac{u''(\bar{C})\bar{C}}{u'(\bar{C})} \frac{(1 - \beta)\bar{Q}\mathcal{B}_{\tau+1}^{*,0} - \bar{\mathcal{B}}^*}{\bar{\Pi}^*}. \quad (\text{B.1})
\end{aligned}$$

Given the definition of $\sigma^{-1} \equiv -\frac{u'(\bar{C})}{u''(\bar{C})\bar{C}}$, (B.1) can be rewritten as

$$\hat{c}_\tau^0 = -\sigma^{-1}\hat{i}_\tau^0 + (1 - \beta)\hat{b}_{\tau+1}^0 + (1 - \beta)\hat{b}_{\tau+1}^{*,0},$$

which gives (2.19).

Similarly, we log-linearize equation (2.16), that is,

$$\begin{aligned}
\ln u'(C_\tau^0) &= \ln \beta + \ln(1 + i_\tau^{*,0}) + \ln v_2(\mathcal{B}_{\tau+1}^0, \mathcal{B}_{\tau+1}^{*,0}) - \ln Q_\tau^0 \\
\Rightarrow \frac{u''(\bar{C})}{u'(\bar{C})}(C_\tau^0 - \bar{C}) &= \frac{1 + i_\tau^{*,0} - (1 + \bar{i}^*)}{1 + \bar{i}^*} + \frac{v_{2,1}(\bar{\mathcal{B}}, \bar{\mathcal{B}}^*)}{v_2(\bar{\mathcal{B}}, \bar{\mathcal{B}}^*)}(\mathcal{B}_{\tau+1}^0 - \bar{\mathcal{B}}) + \frac{v_{2,2}(\bar{\mathcal{B}}, \bar{\mathcal{B}}^*)}{v_2(\bar{\mathcal{B}}, \bar{\mathcal{B}}^*)}(\mathcal{B}_{\tau+1}^{*,0} - \bar{\mathcal{B}}^*) - \frac{Q_\tau^0 - \bar{Q}}{\bar{Q}} \\
\Rightarrow \frac{u''(\bar{C})\bar{C}}{u'(\bar{C})}\hat{c}_\tau^0 &= \hat{i}_\tau^0 + \frac{u''(\bar{C})\bar{C}}{u'(\bar{C})} \frac{(1 - \beta)\mathcal{B}_{\tau+1}^0 - \bar{\mathcal{B}}}{\bar{\Pi}} + \frac{u''(\bar{C})\bar{C}}{u'(\bar{C})} \frac{(1 - \beta)\bar{Q}\mathcal{B}_{\tau+1}^{*,0} - \bar{\mathcal{B}}^*}{\bar{\Pi}^*} - \hat{q}_\tau^0.
\end{aligned}$$

After plugging the definition of σ , we have

$$\hat{c}_\tau^0 = -\sigma^{-1}\hat{i}_\tau^{*,0} + (1 - \beta)\hat{b}_{\tau+1}^0 + (1 - \beta)\hat{b}_{\tau+1}^{*,0} + \sigma^{-1}\hat{q}_\tau^0,$$

which gives (2.20).

C Firm Profit Function and Optimal Pricing Solution

In this section, we first show the explicit expression for profit function $H(\cdot)$ of the firms in each period and then derive the firms' optimal pricing decision given by (2.23). The period profit function of firms is the same regardless of whether they are infinitely forward-looking or have limited foresight. We therefore derive the profit function by considering the case of the standard RE framework with infinite planning horizons.

In the RE framework, a firm f that is able to reoptimize its goods price sets $P_{H,t}^f$ to maximize

$$\max_{P_{H,t}^f} \mathbb{E}_t \sum_{k=0}^{\infty} (\beta\theta)^k \left[\left(\frac{C_{t+k}}{C_t} \right)^{-\sigma} \left(\frac{P_t}{P_{t+k}} \right) Y_{t+k}(j) \left(P_{H,t}^f \bar{\Pi}_H^k - MC_{t+k} \right) \right], \quad (\text{C.1})$$

subject to the demand constraint

$$Y_{t+k}(j) \leq \left(\frac{P_{H,t}^f \bar{\Pi}_H^k}{P_{H,t+k}} \right)^{-\varepsilon} \underbrace{\left(C_{H,t+k} + \int_0^1 C_{H,t+k}^l dl \right)}_{\equiv Y_{t+k}}.$$

We rewrite the firm's problem (C.1) as follows:

$$\max_{P_{H,t}^f} \mathbb{E}_t \sum_{k=0}^{\infty} (\beta\theta)^k \left[C_{t+k}^{-\sigma} \left(\frac{P_{H,t}^f \bar{\Pi}_H^k}{P_{H,t+k}} \right)^{-\varepsilon} Y_{t+k} \frac{P_{H,t+k}}{P_{t+k}} \left(\frac{P_{H,t}^f \bar{\Pi}_H^k}{P_{H,t+k}} - \frac{MC_{t+k}}{P_{H,t+k}} \right) \right], \quad (\text{C.2})$$

where we have applied the demand constraint and dropped C_t and P_t (note that they are taken as given by firm f at time t , and dropping them does not change the solution to the optimality problem).

Now, let us define

$$\begin{aligned} \lambda_{t+k} &\equiv C_{t+k}^{-\sigma}, & r_{H,t+k}^f &\equiv \frac{P_{H,t}^f \bar{\Pi}_H^k}{P_{H,t+k}}, \\ \mathcal{S}_{t+k} &\equiv \frac{P_{H,t+k}}{P_{t+k}}, & \mathcal{MC}_{t+k} &\equiv \frac{MC_{t+k}}{P_{H,t+k}}. \end{aligned}$$

Then, the optimality problem (C.2) can be summarized as follows:

$$\max_{P_{H,t}^f} \mathbb{E}_t \sum_{k=0}^{\infty} (\beta\theta)^k \left[\lambda_{t+k} H(r_{H,t+k}^f; \mathcal{S}_{t+k}, \mathbf{z}_{t+k}) \right], \quad (\text{C.3})$$

where $H(r_{H,t+k}^f; \mathcal{S}_{t+k}, \mathbf{z}_{t+k})$ is the function of real profit in period $t+k$, i.e.,

$$H(r_{H,t+k}^f; \mathcal{S}_{t+k}, \mathbf{z}_{t+k}) = \left(r_{H,t+k}^f \right)^{-\varepsilon} Y_{t+k} \mathcal{S}_{t+k} \left(r_{H,t+k}^f - \mathcal{MC}_{t+k} \right), \quad (\text{C.4})$$

and \mathbf{z}_{t+k} is the vector of all real state variables at time $t+k$. In the steady state with $r_H^f = \mathcal{S} = 1$, the derivative of function $H(\cdot)$ becomes

$$H'(1; 1, \bar{\mathbf{z}}) = \bar{Y} (1 - \varepsilon + \varepsilon \cdot \overline{\mathcal{MC}}) = 0 \quad (\text{C.5})$$

by noting that the real marginal cost in the steady state is $\overline{\mathcal{MC}} = (\varepsilon - 1)/\varepsilon$.

Now, we show that the firms' optimal pricing decision is given by (2.23). We assume that the firms use a value function to approximate discounted future profits beyond its planning horizon that is learned from the nonstochastic steady state given by (2.22). Then,

the first-order condition of maximizing the firm's objective function (2.21) is

$$\mathbb{E}_t^f \left[\sum_{\tau=t}^{t+k} (\beta\theta)^{\tau-t} \lambda_{\tau} H_1 \left(\frac{P_{H,t}^f \bar{\Pi}_H^{\tau-t}}{P_{H,\tau}}; \mathcal{S}_{\tau}, \mathbf{z}_{\tau} \right) \frac{P_{H,t} \bar{\Pi}_H^{\tau-t}}{P_{H,\tau}} + \frac{(\beta\theta)^{k+1}}{1-\beta\theta} \bar{\lambda} H_1 \left(\frac{P_{H,t}^f \bar{\Pi}_H^k}{P_{H,t+k}}; \bar{\mathcal{S}}, \bar{\mathbf{z}} \right) \frac{P_{H,t} \bar{\Pi}_H^k}{P_{H,t+k}} \right] = 0. \quad (\text{C.6})$$

Log-linearizing (C.6) around the steady state yields

$$\mathbb{E}_t^f \left\{ \sum_{\tau=t}^{t+k} (\beta\theta)^{\tau-t} \left[p_{H,t}^f - \sum_{s=t}^{\tau} \pi_{H,s} - m_{\tau} \right] + \frac{(\beta\theta)^{k+1}}{1-\beta\theta} \left[p_{H,t}^f - \sum_{s=t}^{t+k} \pi_{H,s} \right] \right\} = 0, \quad (\text{C.7})$$

where

$$p_{H,t}^f \equiv \log \left(\frac{P_{H,t}(f)}{P_{H,t-1} \bar{\Pi}_H} \right), \quad \pi_{H,t} \equiv \log \left(\frac{\Pi_{H,t}}{\bar{\Pi}_H} \right),$$

and

$$m_t \equiv -\frac{H'(1; 1, \mathbf{z}_t)}{H''(1; 1, \bar{\mathbf{z}})} = \frac{Y_t}{\bar{Y}} \left(\frac{\varepsilon}{\varepsilon-1} m c_t - 1 \right). \quad (\text{C.8})$$

We define

$$\hat{m}_t \equiv m_t - \bar{m}, \quad \widehat{m}c_t \equiv \log \left(\frac{\mathcal{M}C_t}{\bar{\mathcal{M}C}} \right),$$

where \bar{m} is the value of m_t in the nonstochastic steady state.

By noting that $\bar{m} = \frac{\bar{Y}}{\varepsilon-1} (\frac{\varepsilon}{\varepsilon-1} \bar{\mathcal{M}C} - 1) = 0$, we have $\hat{m}_t = m_t$. Then, the log-linear approximation of (C.8) yields

$$m_t = \widehat{m}c_t.$$

Thus, by replacing m_t with $\widehat{m}c_t$ in (C.6) and reorganizing its expression, we have the firms' optimal pricing $p_{H,t}^f$ characterized by (2.23).

D International Goods-Market Clearing Condition

The international market clearing condition for domestically produced goods j is

$$Y_t(j) = C_{H,t}(j) + \int_0^1 C_{H,t}^{*,i}(j) di \quad (\text{D.1})$$

for all $j \in [0, 1]$ and all t , where $C_{H,t}^{*,i}(j)$ is the demand from foreign country i for good j produced in the domestic country. With the assumption of identical preferences across countries and the law of one price for all goods j , we can rewrite condition (D.1) as follows:

$$Y_t(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\varepsilon} \left[(1-\alpha) \left(\frac{P_{H,t}}{P_t} \right)^{-\eta} C_t + \alpha \int_0^1 \left(\frac{P_{H,t}}{\mathcal{E}_{i,t} P_{F,t}^{*,i}} \right)^{-\gamma} \left(\frac{P_{F,t}^{*,i}}{P_t^{*,i}} \right)^{-\eta} C_t^{*,i} di \right],$$

where $\mathcal{E}_{i,t}$ is the bilateral nominal exchange rate between the domestic country and the foreign country i (that is, the price of country i 's currency in units of domestic currency), $P_{F,t}^{*,i}$ and $P_t^{*,i}$ are price indices of imported goods and consumption goods in foreign country i in units of its own currency, respectively, and $C_t^{*,i}$ is foreign country i 's consumption.

By the definition of the aggregate output of the domestic economy, $Y_t \equiv \left[\int_0^1 Y_t(j)^{1-\frac{1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}}$, we can further rewrite the above equation as follows:

$$\begin{aligned} Y_t &= (1-\alpha) \left(\frac{P_{H,t}}{P_t} \right)^{-\eta} C_t + \alpha \int_0^1 \left(\frac{P_{H,t}}{\mathcal{E}_{i,t} P_{F,t}^{*,i}} \right)^{-\gamma} \left(\frac{P_{F,t}^{*,i}}{P_t^{*,i}} \right)^{-\eta} C_t^{*,i} di \\ &= \left(\frac{P_{H,t}}{P_t} \right)^{-\eta} \left[(1-\alpha) C_t + \alpha \int_0^1 \left(\frac{\mathcal{E}_{i,t} P_{F,t}^{*,i}}{P_{H,t}} \right)^{\gamma-\eta} Q_{i,t}^\eta C_t^{*,i} di \right], \\ &= \mathcal{S}_t^{-\eta} \left[(1-\alpha) C_t + \alpha \int_0^1 (S_t^i S_{i,t})^{\gamma-\eta} Q_{i,t}^\eta C_t^{*,i} di \right], \end{aligned} \quad (\text{D.2})$$

where $\mathcal{E}_{i,t} \equiv \frac{\mathcal{E}_{i,t} P_t^{*,i}}{P_t}$ is the bilateral real exchange rate between the domestic country and country i , $S_t^i \equiv \frac{P_{F,t}^{*,i}}{P_t^{*,i}}$ is the effective terms of trade for country i , and $S_{i,t} \equiv \frac{\mathcal{E}_{i,t} P_{H,t}^{*,i}}{P_{H,t}}$ is the bilateral terms of trade between the home country and country i .

To facilitate tractability, we further assume that in the steady state,

$$\bar{C} = (\bar{S}^i \bar{S}_i)^{\gamma-\eta} \bar{Q}_i^\eta \bar{C}^{*,i}$$

for any country i . Then, the log-linear approximation of equation (D.2) is given by

$$\begin{aligned} \bar{Y} \hat{y} &= (1-\alpha) \bar{\mathcal{S}}^{-\eta} \bar{C} (\hat{c}_t + \alpha \eta \hat{s}_t) + \alpha \bar{\mathcal{S}}^{-\eta} \bar{C} \left[\int_0^1 (\gamma - \eta) (\hat{s}_t^i + \hat{s}_{it}) + (\eta \hat{q}_{it} + \alpha \hat{s}_t + \hat{c}_t^i) di \right] \\ &= \bar{\mathcal{S}}^{-\eta} \bar{C} [(1-\alpha) \hat{c}_t + \alpha \gamma \hat{s}_t + \alpha \eta \hat{q}_t + \alpha \hat{c}_t^*], \end{aligned} \quad (\text{D.3})$$

where $\hat{c}_t^* \equiv \int_0^1 \hat{c}_t^i di$. Here we have utilized the conditions $\hat{S}_t = -\alpha \hat{s}_t$, $\int_0^1 \hat{s}_t^i di = 0$, and $\int_0^1 \hat{s}_{i,t} di = \hat{s}_t$.

The goods market clearing condition (D.3) nests the scenario analyzed in Galí and Monacelli (2005), in which all countries are assumed to be symmetric in the steady state and households feature perfect risk sharing. The symmetry assumption implies $\bar{Y} = \bar{C}$ and $\bar{\mathcal{S}} = 1$, while the perfect risk sharing condition implies $\hat{c}_t = \hat{c}_t^* + \frac{1}{\sigma} \hat{q}_t$. Plugging these two conditions into (D.3), we have

$$\hat{y} = \hat{c}_t + \alpha \gamma \hat{s}_t + \alpha \left(\eta - \frac{1}{\sigma} \right) \hat{q}_t,$$

which is exactly the same goods market clearing condition derived in Galí and Monacelli (2005) without the Cole-Obstfeld parameterization.

By assuming no foreign demand shocks, the world income is a constant, hence $\hat{c}_t^* = \hat{y}_t^* = 0$. Then, using the relation $\hat{s}_t = \frac{\hat{q}_t}{1-\alpha}$, we can rewrite equation (D.3) as

$$\hat{y}_t = \vartheta_{yc}\hat{c}_t + \vartheta_{ys}\hat{s}_t,$$

where $\vartheta_{yc} \equiv (1-\alpha)\bar{\mathcal{S}}^{-\eta}\bar{C}/\bar{Y}$ and $\vartheta_{ys} \equiv \alpha[\gamma + \eta(1-\alpha)]\bar{\mathcal{S}}^{-\eta}\bar{C}/\bar{Y}$. This gives equation (2.36).

E Proof of the Mean-Reverting Processes of χ_t and ζ_t

First, we log-linearize the resource constraint (2.5) as follows:

$$\hat{c}_t + \beta(\hat{b}_{t+1} + \hat{b}_{t+1}^*) - \beta\vartheta'_1\hat{i}_t + \beta\vartheta_1(\hat{q}_t - \hat{r}_t^*) = \hat{b}_t - \vartheta'_1\pi_t + \hat{b}_t^* + \vartheta_1\hat{q}_t + \vartheta_2(\hat{y}_t - \alpha\hat{s}_t),$$

where $\vartheta'_1 \equiv \frac{\bar{\mathcal{B}}}{\bar{\Pi}C}$. Here we have used the relation $\hat{\mathcal{S}}_t = -\alpha\hat{s}_t$, $\pi_t^* = 0$, hence $\hat{i}_t^* = \hat{r}_t^*$, together with the steady-state relationship $\bar{\Pi}^* = 1$ and $\beta^{-1} = (1 + \bar{i})/\bar{\Pi} = (1 + \bar{i}^*)/\bar{\Pi}^*$.

It can be rewritten as

$$\hat{b}_t + \hat{b}_t^* = \beta(\hat{b}_{t+1} + \hat{b}_{t+1}^*) + [\hat{c}_t - \beta\vartheta'_1\hat{i}_t + \beta\vartheta_1(\hat{q}_t - \hat{r}_t^*) + \vartheta'_1\pi_t - \vartheta_1\hat{q}_t - \vartheta_2(\hat{y}_t - \alpha\hat{s}_t)]. \quad (\text{E.1})$$

In the FH model, we can rewrite equation (E.1) at any date τ as the version of interest:

$$\hat{b}_\tau^{j+1} + \hat{b}_\tau^{*j+1} = \beta(\hat{b}_{\tau+1}^j + \hat{b}_{\tau+1}^{*j}) + [\hat{c}_\tau^j - \beta\vartheta'_1\hat{i}_\tau^j + \beta\vartheta_1(\hat{q}_\tau^j - \hat{r}_\tau^{*j}) + \vartheta'_1\pi_\tau^j - \vartheta_1\hat{q}_\tau^j - \vartheta_2(\hat{y}_\tau^j - \alpha\hat{s}_\tau^j)], \quad (\text{E.2})$$

where j is the (counterfactual) planning horizon at date τ .

Let t be the point at which forward planning occurs. Then, iterating (E.2) forward to the end of the planning horizon yields

$$\begin{aligned} & \hat{b}_t^{h+1} + \hat{b}_t^{*,h+1} \\ &= \mathbb{E}_t \sum_{j=0}^h \beta^j \left[\hat{c}_{t+j}^{h-j} - \beta\vartheta'_1\hat{i}_{t+j}^{h-j} + \beta\vartheta_1(\hat{q}_{t+j}^{h-j} - \hat{r}_{t+j}^{*,h-j}) + \vartheta'_1\pi_{t+j}^{h-j} - \vartheta_1\hat{q}_{t+j}^{h-j} - \vartheta_2(\hat{y}_{t+j}^{h-j} - \alpha\hat{s}_{t+j}^{h-j}) \right] \\ &+ \beta^{h+1}\mathbb{E}_t(\hat{b}_{t+h+1}^0 + \hat{b}_{t+h+1}^{*,0}), \end{aligned} \quad (\text{E.3})$$

where \hat{b}_t^{h+1} and $\hat{b}_t^{*,h+1}$ are the household's initial financial position in period t .

We parameterize the log-linear approximations of $v_1(\mathcal{B}, \mathcal{B}^*)$ and $v_2(\mathcal{B}, \mathcal{B}^*)$ by

$$\begin{aligned}\log(v_{1,t}(\mathcal{B}, \mathcal{B}^*)/v_1^*(\bar{\mathcal{B}}, \bar{\mathcal{B}}^*)) &= -\sigma(\nu_t + \chi_t \hat{b} + \xi_t \hat{b}^*), \\ \log(v_{2,t}(\mathcal{B}, \mathcal{B}^*)/v_2^*(\bar{\mathcal{B}}, \bar{\mathcal{B}}^*)) &= -\sigma(\nu_t^* + \chi_t' \hat{b} + \xi_t' \hat{b}^*).\end{aligned}$$

Then the first-order conditions of a household's optimality problem at the end of its planning horizon (2.15) and (2.16) can be log-linearized as

$$\begin{aligned}\hat{c}_{t+k}^0 &= -\sigma^{-1} \hat{i}_{t+k}^0 + \nu_t + \chi_t \hat{b}_{t+k+1}^0 + \xi_t \hat{b}_{t+k+1}^{*,0}, \\ \hat{c}_{t+k}^0 &= -\sigma^{-1} \hat{i}_{t+k}^{*,0} + \nu_t^* + \chi_t' \hat{b}_{t+k+1}^0 + \xi_t' \hat{b}_{t+k+1}^{*,0} + \sigma^{-1} \hat{q}_{t+k}^0,\end{aligned}$$

which implies $\chi_t = \chi_t'$ and $\xi_t = \xi_t'$.

In the standard dynamic programming problem under the RE assumption in the benchmark model, the household's holdings of domestic bonds \hat{b} and foreign bonds \hat{b}^* (in terms of the domestic currency) are perfect substitutes in their value functions. Since the bond holdings \hat{b} and \hat{b}^* are also perfect substitutes in the budget constraint of the household in the finite planning problem, we have $\chi_t = \xi_t$. Thus, the Euler equation at the end of the planning horizon reduces to

$$\tilde{b}_{t+h+1}^0 \equiv \hat{b}_{t+h+1}^0 + \hat{b}_{t+h+1}^{*,0} = \chi_t^{-1}(\hat{c}_{t+h}^0 - \nu_t + \sigma^{-1} \hat{i}_{t+h}^0), \quad (\text{E.4})$$

where \tilde{b} represents the total holdings of bond positions.

Similar to Woodford (2019), by the household's optimal expenditure conditions (2.17) and (2.19), together with (E.3), we have

$$\hat{c}_t^h = g_k(\chi_t) \tilde{b}_t^{h+1} + \text{rest},$$

where "rest" indicates the terms not including total asset position \tilde{b}_t^{h+1} and

$$g_k(\chi_t) \equiv \frac{\chi_t}{\beta^{k+1} + \left(\frac{1-\beta^{h+1}}{1-\beta}\right) \chi_t}.$$

Thus, we have $\chi_t^{est} = g_k(\chi_t)$. Since the evolution process of χ_t follows a constant-gain learning rule, that is,

$$\chi_{t+1} = \gamma g_k(\chi_t) + (1 - \gamma) \chi_t.$$

This implies the monotonic convergence of χ_t to fixed point $1 - \beta$. Similarly, since $\chi_t = \xi_t$ for any t , the same is true for the evolution process of ξ_t .

F Equilibrium Conditions of the Forward Planning

This section summarizes the equilibrium conditions in the finite planning exercise calculated in period t . Let $y_{\tau|t}^j$ be the value of \hat{y}_{τ} that is predicted at date τ as a result of aggregation of decisions made by agents with (counterfactual) planning horizon $j = h + t - \tau$, which is calculated at date t by agents with planning horizon h . It is a function of the state $\{\hat{q}_{t-1}, \hat{b}_t^*, \nu_t, \nu_t^*, \tilde{\nu}_t\}$ and exogenous shocks $\{a_t, \mu_t\}$ in period t . Then, the actual aggregate output in period t is given by $\hat{y}_t = \hat{y}_{t|t}^h$. Similarly, we define other variables in the finite planning exercise with the same notation. The additional subscript $|t$ matters because different value functions are used in finite planning in different periods. All the exogenous shocks are assumed to follow an AR(1) process.

In the forward planning exercise by the agents in period t with planning horizon h , at any date $t \leq \tau < t + h - 1$, we have

$$\hat{c}_{\tau|t}^{h+t-\tau} = \mathbb{E}_{\tau} \hat{c}_{\tau+1|t}^{h+t-\tau-1} - \frac{1}{\sigma} (\hat{i}_{\tau|t}^{h+t-\tau} - \mathbb{E}_{\tau} \pi_{\tau+1|t}^{h+t-\tau-1}), \quad (\text{F.1})$$

$$\hat{q}_{\tau|t}^{h+t-\tau} = \mathbb{E}_{\tau} \hat{q}_{\tau+1|t}^{h+t-\tau-1} + \hat{r}_{\tau|t}^{*,h+t-\tau} - (\hat{i}_{\tau|t}^{h+t-\tau} - \mathbb{E}_{\tau} \pi_{\tau+1|t}^{h+t-\tau-1}), \quad (\text{F.2})$$

$$\hat{\varepsilon}_{\tau|t}^{h+t-\tau} = \hat{q}_{\tau|t}^{h+t-\tau} - \hat{q}_{\tau-1|t}^{h+t-\tau+1} + \pi_{\tau|t}^{h+t-\tau}, \quad (\text{F.3})$$

$$\hat{s}_{\tau|t}^{h+t-\tau} = \frac{\hat{q}_{\tau|t}^{h+t-\tau}}{1 - \alpha}, \quad (\text{F.4})$$

$$\pi_{H,\tau|t}^{h+t-\tau} = \kappa \widehat{mc}_{\tau|t}^{h+t-\tau} + \beta \mathbb{E}_{\tau} \pi_{H,\tau+1|t}^{h+t-\tau-1}, \quad (\text{F.5})$$

$$\widehat{mc}_{\tau|t}^{h+t-\tau} = (\sigma + \varphi \vartheta_{yc}) \hat{c}_{\tau|t}^{h+t-\tau} + (\alpha + \varphi \vartheta_{ys}) \hat{s}_{\tau|t}^{h+t-\tau} - (1 + \varphi) a_{\tau}, \quad (\text{F.6})$$

$$\hat{i}_{\tau|t}^{h+t-\tau} = \phi_{\pi} \pi_{\tau|t}^{h+t-\tau}, \quad (\text{F.7})$$

$$\hat{y}_{\tau|t}^{h+t-\tau} = \vartheta_{ys} \hat{s}_{\tau|t}^{h+t-\tau} + \vartheta_{yc} \hat{c}_{\tau|t}^{h+t-\tau}, \quad (\text{F.8})$$

$$\pi_{\tau|t}^{h+t-\tau} = (1 - \alpha) \pi_{H,\tau|t}^{h+t-\tau} + \alpha \hat{\varepsilon}_{\tau|t}^{h+t-\tau}, \quad (\text{F.9})$$

$$\hat{r}_{\tau|t}^{*,h+t-\tau} = \phi_b \hat{b}_{\tau+1|t}^{h+t-\tau} + \mu_{\tau}, \quad (\text{F.10})$$

$$\begin{aligned} \hat{b}_{\tau+1|t}^{*,h+t-\tau} &= \beta^{-1} (\hat{b}_{\tau|t}^{*,h+t+1-\tau} + \vartheta_1 \hat{q}_{\tau|t}^{h+t-\tau} - \vartheta_2 \alpha \hat{s}_{\tau|t}^{h+t-\tau} + \vartheta_2 \hat{y}_{\tau|t}^{h+t-\tau} - \hat{c}_{\tau|t}^{h+t-\tau}) \\ &\quad - \vartheta_1 \hat{q}_{\tau|t}^{h+t-\tau} + \vartheta_1 \hat{r}_{\tau|t}^{*,h+t-\tau}, \end{aligned} \quad (\text{F.11})$$

where $\hat{q}_{t-1|t}^{h+1}$ is simply a notational simplification defined by $\hat{q}_{t-1|t}^{h+1} \equiv \hat{q}_{t-1}$ and similarly $\hat{b}_{t|t}^{h+1} \equiv \hat{b}_t^*$. Here $\vartheta_{yc} = (1 - \alpha) \frac{\bar{S}^{-\eta} \bar{C}}{Y}$, $\vartheta_{ys} = \alpha [\gamma + \eta(1 - \alpha)] \frac{\bar{S}^{-\eta} \bar{C}}{Y}$, $\vartheta_1 = \frac{\bar{B}^* \bar{Q}}{C}$, and $\vartheta_2 = \frac{\bar{S} \bar{Y}}{C}$.

At the end of finite planning date $\tau = t + h$, we have

$$\hat{c}_{\tau|t}^0 = -\frac{1}{\sigma}\hat{i}_{\tau|t}^0 + (1 - \beta)\hat{b}_{\tau+1|t}^{*,0} + \nu_t, \quad (\text{F.12})$$

$$\hat{q}_{\tau|t}^0 = \hat{r}_{\tau|t}^{*,0} - \hat{i}_{\tau|t}^0 + \sigma(\nu_t - \nu_t^*), \quad (\text{F.13})$$

$$\hat{\varepsilon}_{\tau|t}^0 = \hat{q}_{\tau|t}^0 - \hat{q}_{\tau-1|t}^1 + \pi_{\tau|t}^0, \quad (\text{F.14})$$

$$\hat{s}_{\tau|t}^0 = \frac{\hat{q}_{\tau|t}^0}{1 - \alpha}, \quad (\text{F.15})$$

$$\pi_{H,\tau|t}^0 = \kappa\widehat{m}\hat{c}_{\tau|t}^0 + (1 - \theta)\beta\tilde{\nu}_t, \quad (\text{F.16})$$

$$\widehat{m}\hat{c}_{\tau|t}^0 = (\sigma + \varphi\vartheta_{yc})\hat{c}_{\tau|t}^0 + (\alpha + \varphi\vartheta_{ys})\hat{s}_{\tau|t}^0 - (1 + \varphi)a_\tau, \quad (\text{F.17})$$

$$\hat{i}_{\tau|t}^0 = \phi_\pi\pi_{\tau|t}^0, \quad (\text{F.18})$$

$$\hat{y}_{\tau|t}^0 = \vartheta_{ys}\hat{s}_{\tau|t}^0 + \vartheta_{yc}\hat{c}_{\tau|t}^0, \quad (\text{F.19})$$

$$\pi_{\tau|t}^0 = (1 - \alpha)\pi_{H,\tau|t}^0 + \alpha\hat{\varepsilon}_{\tau|t}^0, \quad (\text{F.20})$$

$$\hat{r}_{\tau|t}^{*,0} = \phi_b\hat{b}_{\tau+1|t}^0 + \mu_\tau, \quad (\text{F.21})$$

$$\hat{b}_{\tau+1|t}^{*,0} = \beta^{-1}(\hat{b}_{\tau|t}^{*,1} + \vartheta_1\hat{q}_{\tau|t}^0 - \vartheta_2\alpha\hat{s}_{\tau|t}^0 + \vartheta_2\hat{y}_{\tau|t}^0 - \hat{c}_{\tau|t}^0) - \vartheta_1\hat{q}_{\tau|t}^0 + \vartheta_1\hat{r}_{\tau|t}^{*,0}. \quad (\text{F.22})$$

The above system of equations consists of a finite number of equations as a function of state variables $\{\hat{q}_{t-1}, \hat{b}_t^*, \nu_t, \nu_t^*, \tilde{\nu}_t\}$ and exogenous shocks $\{a_t, \mu_t\}$. Thus, we can solve for all endogenous variables $\{\hat{c}_{\tau|t}^{h+t-\tau}, \hat{y}_{\tau|t}^{h+t-\tau}, \hat{i}_{\tau|t}^{h+t-\tau}, \hat{r}_{\tau|t}^{*,h+t-\tau}, \pi_{H,\tau|t}^{h+t-\tau}, \pi_{\tau|t}^{h+t-\tau}, \hat{q}_{\tau|t}^{h+t-\tau}, \hat{s}_{\tau|t}^{h+t-\tau}, \hat{\varepsilon}_{\tau|t}^{h+t-\tau}, \hat{b}_{\tau+1|t}^{*,h+t-\tau}\}_{\tau=t}^{t+h}$ with a unique solution. See Appendix G for the detailed solution method.

G Solution to Policy Functions

We now show the solution to the policy functions for the equilibrium characterized in Section 3 and Appendix F. Similar to expressions (3.8)-(3.9), we can write the solution to any endogenous variable $x_{\tau|t}^j$ except for $\hat{b}_{\tau+1|t}^{*,j}$ in forward planning as a function of the state variables with the extended set of shocks, that is,

$$x_{\tau|t}^j = \psi_{x,q}^j\hat{q}_{\tau-1|t}^{j+1} + \psi_{x,b}^j\hat{b}_{\tau|t}^{*,j+1} + \psi_{x,a}^j a_\tau + \psi_{x,\mu}^j \mu_\tau + \psi_{x,\nu}^j \nu_t + \psi_{x,\tilde{\nu}}^j \tilde{\nu}_t + \psi_{x,\nu^*}^j \nu_t^*, \quad (\text{G.1})$$

for any (counterfactual) $j \geq 0$, and

$$\hat{b}_{\tau+1|t}^{*,j} = \psi_{b,q}^j\hat{q}_{\tau-1|t}^{j+1} + \psi_{b,b}^j\hat{b}_{\tau|t}^{*,j+1} + \psi_{b,a}^j a_\tau + \psi_{b,\mu}^j \mu_\tau + \psi_{b,\nu}^j \nu_t + \psi_{b,\tilde{\nu}}^j \tilde{\nu}_t + \psi_{b,\nu^*}^j \nu_t^*. \quad (\text{G.2})$$

First, we aim to pin down the coefficients for $j = 0$. From (F.12)-(F.22), one can easily eliminate $\{\hat{\varepsilon}_{\tau|t}^0, \hat{s}_{\tau|t}^0, \widehat{m}\hat{c}_{\tau|t}^0, \hat{i}_{\tau|t}^0, \hat{r}_{\tau|t}^{*,0}\}$. Additionally, note that since $y_{\tau|t}^0$ only enters (F.19), we

only need to solve $\{\hat{c}_{\tau|t}^0, \hat{q}_{\tau|t}^0, \pi_{\tau|t}^0, \pi_{H,\tau|t}^0, \hat{b}_{\tau+1|t}^{*,0}\}$, and then $\hat{y}_{\tau|t}^0$ is uniquely pinned down by (F.19).

We solve $\{\hat{c}_{\tau|t}^0, \hat{q}_{\tau|t}^0, \pi_{\tau|t}^0, \pi_{H,\tau|t}^0\}$ by equating coefficients. Note that from (F.12),

$$\hat{c}_{\tau|t}^0 = -\frac{\phi_\pi}{\sigma}\pi_{\tau|t}^0 + (1-\beta)\hat{b}_{\tau+1|t}^{*,0} + \nu_t,$$

and equating the coefficients yields

$$\begin{aligned} \psi_{c,q}^0 &= -\frac{\phi_\pi}{\sigma}\psi_{\pi,q}^0 + (1-\beta)\psi_{b,q}^0, & \psi_{c,b}^0 &= -\frac{\phi_\pi}{\sigma}\psi_{\pi,b}^0 + (1-\beta)\psi_{b,b}^0, \\ \psi_{c,a}^0 &= -\frac{\phi_\pi}{\sigma}\psi_{\pi,a}^0 + (1-\beta)\psi_{b,a}^0, & \psi_{c,\mu}^0 &= -\frac{\phi_\pi}{\sigma}\psi_{\pi,\mu}^0 + (1-\beta)\psi_{b,\mu}^0, \\ \psi_{c,\nu}^0 &= -\frac{\phi_\pi}{\sigma}\psi_{\pi,\nu}^0 + (1-\beta)\psi_{b,\nu}^0 + 1, & \psi_{c,\tilde{\nu}}^0 &= -\frac{\phi_\pi}{\sigma}\psi_{\pi,\tilde{\nu}}^0 + (1-\beta)\psi_{b,\tilde{\nu}}^0, \\ \psi_{c,\nu^*}^0 &= -\frac{\phi_\pi}{\sigma}\psi_{\pi,\nu^*}^0 + (1-\beta)\psi_{b,\nu^*}^0. \end{aligned}$$

Similarly, from (F.13),

$$\hat{q}_{\tau|t}^0 = \phi_b\hat{b}_{\tau+1|t}^0 + \mu_\tau - \phi_\pi\pi_{\tau|t}^0 + \sigma(\nu_t - \nu_t^*),$$

which yields

$$\begin{aligned} \psi_{q,q}^0 &= \phi_b\psi_{b,q}^0 - \phi_\pi\psi_{\pi,q}^0, & \psi_{q,b}^0 &= \phi_b\psi_{b,b}^0 - \phi_\pi\psi_{\pi,b}^0, \\ \psi_{q,a}^0 &= \phi_b\psi_{b,a}^0 - \phi_\pi\psi_{\pi,a}^0, & \psi_{q,\mu}^0 &= \phi_b\psi_{b,\mu}^0 - \phi_\pi\psi_{\pi,\mu}^0 + 1, \\ \psi_{q,\nu}^0 &= \phi_b\psi_{b,\nu}^0 - \phi_\pi\psi_{\pi,\nu}^0 + \sigma, & \psi_{q,\tilde{\nu}}^0 &= \phi_b\psi_{b,\tilde{\nu}}^0 - \phi_\pi\psi_{\pi,\tilde{\nu}}^0, \\ \psi_{q,\nu^*}^0 &= \phi_b\psi_{b,\nu^*}^0 - \phi_\pi\psi_{\pi,\nu^*}^0 - \sigma. \end{aligned}$$

Similarly, from (F.16)-(F.17),

$$\pi_{H,\tau|t}^0 = \kappa(\sigma + \varphi\vartheta_{yc})\hat{c}_{\tau|t}^0 + \frac{\kappa(\alpha + \varphi\vartheta_{ys})}{1-\alpha}\hat{q}_{\tau|t}^0 - \kappa(1+\varphi)a_\tau + (1-\theta)\beta\tilde{\nu}_t,$$

which yields

$$\begin{aligned} \psi_{\pi_H,q}^0 &= \kappa(\sigma + \varphi\vartheta_{yc})\psi_{c,q}^0 + \frac{\kappa(\alpha + \varphi\vartheta_{ys})}{1-\alpha}\psi_{q,q}^0, & \psi_{\pi_H,b}^0 &= \kappa(\sigma + \varphi\vartheta_{yc})\psi_{c,b}^0 + \frac{\kappa(\alpha + \varphi\vartheta_{ys})}{1-\alpha}\psi_{q,b}^0, \\ \psi_{\pi_H,a}^0 &= \kappa(\sigma + \varphi\vartheta_{yc})\psi_{c,a}^0 + \frac{\kappa(\alpha + \varphi\vartheta_{ys})}{1-\alpha}\psi_{q,a}^0 - \kappa(1+\varphi), & \psi_{\pi_H,\mu}^0 &= \kappa(\sigma + \varphi\vartheta_{yc})\psi_{c,\mu}^0 + \frac{\kappa(\alpha + \varphi\vartheta_{ys})}{1-\alpha}\psi_{q,\mu}^0, \\ \psi_{\pi_H,\nu}^0 &= \kappa(\sigma + \varphi\vartheta_{yc})\psi_{c,\nu}^0 + \frac{\kappa(\alpha + \varphi\vartheta_{ys})}{1-\alpha}\psi_{q,\nu}^0, & \psi_{\pi_H,\tilde{\nu}}^0 &= \kappa(\sigma + \varphi\vartheta_{yc})\psi_{c,\tilde{\nu}}^0 + \frac{\kappa(\alpha + \varphi\vartheta_{ys})}{1-\alpha}\psi_{q,\tilde{\nu}}^0 + (1-\theta)\beta, \\ \psi_{\pi_H,\nu^*}^0 &= \kappa(\sigma + \varphi\vartheta_{yc})\psi_{c,\nu^*}^0 + \frac{\kappa(\alpha + \varphi\vartheta_{ys})}{1-\alpha}\psi_{q,\nu^*}^0. \end{aligned}$$

Similarly, from (F.14) and (F.20),

$$\pi_{\tau|t}^0 = (1 - \alpha)\pi_{H,\tau|t}^0 + \alpha(\hat{q}_{\tau|t}^0 - \hat{q}_{\tau-1|t}^1 + \pi_{\tau|t}^0),$$

which yields

$$\begin{aligned} (1 - \alpha)\psi_{\pi,q}^0 &= (1 - \alpha)\psi_{\pi_H,q}^0 + \alpha\psi_{q,q}^0 - \alpha, & (1 - \alpha)\psi_{\pi,b}^0 &= (1 - \alpha)\psi_{\pi_H,b}^0 + \alpha\psi_{q,b}^0, \\ (1 - \alpha)\psi_{\pi,a}^0 &= (1 - \alpha)\psi_{\pi_H,a}^0 + \alpha\psi_{q,a}^0, & (1 - \alpha)\psi_{\pi,\mu}^0 &= (1 - \alpha)\psi_{\pi_H,\mu}^0 + \alpha\psi_{q,\mu}^0, \\ (1 - \alpha)\psi_{\pi,\nu}^0 &= (1 - \alpha)\psi_{\pi_H,\nu}^0 + \alpha\psi_{q,\nu}^0, & (1 - \alpha)\psi_{\pi,\tilde{\nu}}^0 &= (1 - \alpha)\psi_{\pi_H,\tilde{\nu}}^0 + \alpha\psi_{q,\tilde{\nu}}^0, \\ (1 - \alpha)\psi_{\pi,\nu^*}^0 &= (1 - \alpha)\psi_{\pi_H,\nu^*}^0 + \alpha\psi_{q,\nu^*}^0. \end{aligned}$$

Similarly, from (F.19) and (F.22),

$$\begin{aligned} \hat{b}_{\tau+1|t}^{*,0} &= \beta^{-1}(\hat{b}_{\tau|t}^{*,1} + \vartheta_1\hat{q}_{\tau|t}^0 - \vartheta_2\alpha\hat{s}_{\tau|t}^0 + \vartheta_2\hat{y}_{\tau|t}^0 - \hat{c}_{\tau|t}^0) - \vartheta_1\hat{q}_{\tau|t}^0 + \vartheta_1\hat{r}_{\tau|t}^{*,0} \\ &= \beta^{-1} \left\{ \hat{b}_{\tau|t}^{*,1} + \left[\vartheta_1 - \vartheta_2 \frac{\alpha - \vartheta_{ys}}{(1 - \alpha)} \right] \hat{q}_{\tau|t}^0 - (1 - \vartheta_2\vartheta_{yc})\hat{c}_{\tau|t}^0 \right\} - \vartheta_1\hat{q}_{\tau|t}^0 + \vartheta_1\hat{r}_{\tau|t}^{*,0}, \end{aligned}$$

and it implies

$$(1 - \vartheta_1\phi_b)\hat{b}_{\tau+1|t}^{*,0} = \beta^{-1} \left\{ \hat{b}_{\tau|t}^{*,1} + \left[\vartheta_1 - \vartheta_2 \frac{\alpha - \vartheta_{ys}}{(1 - \alpha)} \right] \hat{q}_{\tau|t}^0 - (1 - \vartheta_2\vartheta_{yc})\hat{c}_{\tau|t}^0 \right\} - \vartheta_1\hat{q}_{\tau|t}^0 + \vartheta_1\mu_{\tau},$$

which yields

$$\begin{aligned} (1 - \vartheta_1\phi_b)\psi_{b,q}^0 &= \beta^{-1} \left\{ \left[\vartheta_1 - \vartheta_2 \frac{\alpha - \vartheta_{ys}}{(1 - \alpha)} \right] \psi_{q,q}^0 - (1 - \vartheta_2\vartheta_{yc})\psi_{c,q}^0 \right\} - \vartheta_1\psi_{q,q}^0, \\ (1 - \vartheta_1\phi_b)\psi_{b,b}^0 &= \beta^{-1} \left\{ 1 + \left[\vartheta_1 - \vartheta_2 \frac{\alpha - \vartheta_{ys}}{(1 - \alpha)} \right] \psi_{q,b}^0 - (1 - \vartheta_2\vartheta_{yc})\psi_{c,b}^0 \right\} - \vartheta_1\psi_{q,b}^0, \\ (1 - \vartheta_1\phi_b)\psi_{b,a}^0 &= \beta^{-1} \left\{ \left[\vartheta_1 - \vartheta_2 \frac{\alpha - \vartheta_{ys}}{(1 - \alpha)} \right] \psi_{q,a}^0 - (1 - \vartheta_2\vartheta_{yc})\psi_{c,a}^0 \right\} - \vartheta_1\psi_{q,a}^0, \\ (1 - \vartheta_1\phi_b)\psi_{b,\mu}^0 &= \beta^{-1} \left\{ \left[\vartheta_1 - \vartheta_2 \frac{\alpha - \vartheta_{ys}}{(1 - \alpha)} \right] \psi_{q,\mu}^0 - (1 - \vartheta_2\vartheta_{yc})\psi_{c,\mu}^0 \right\} - \vartheta_1\psi_{q,\mu}^0 + \vartheta_1, \\ (1 - \vartheta_1\phi_b)\psi_{b,\nu}^0 &= \beta^{-1} \left\{ \left[\vartheta_1 - \vartheta_2 \frac{\alpha - \vartheta_{ys}}{(1 - \alpha)} \right] \psi_{q,\nu}^0 - (1 - \vartheta_2\vartheta_{yc})\psi_{c,\nu}^0 \right\} - \vartheta_1\psi_{q,\nu}^0, \\ (1 - \vartheta_1\phi_b)\psi_{b,\tilde{\nu}}^0 &= \beta^{-1} \left\{ \left[\vartheta_1 - \vartheta_2 \frac{\alpha - \vartheta_{ys}}{(1 - \alpha)} \right] \psi_{q,\tilde{\nu}}^0 - (1 - \vartheta_2\vartheta_{yc})\psi_{c,\tilde{\nu}}^0 \right\} - \vartheta_1\psi_{q,\tilde{\nu}}^0, \\ (1 - \vartheta_1\phi_b)\psi_{b,\nu^*}^0 &= \beta^{-1} \left\{ \left[\vartheta_1 - \vartheta_2 \frac{\alpha - \vartheta_{ys}}{(1 - \alpha)} \right] \psi_{q,\nu^*}^0 - (1 - \vartheta_2\vartheta_{yc})\psi_{c,\nu^*}^0 \right\} - \vartheta_1\psi_{q,\nu^*}^0. \end{aligned}$$

Thus, in the case of $j = 0$, we have 45 simple linear equations for 35 undetermined coefficients. By solving these linear equations, we obtain the solution of $\{\hat{c}_{\tau|t}^0, \hat{q}_{\tau|t}^0, \pi_{\tau|t}^0, \pi_{H,\tau|t}^0, \hat{b}_{\tau+1|t}^{*,0}\}$ at date τ calculated in period t . Then one can easily obtain all other endogenous variables

at date τ calculated in period t .

Next, we solve the undermined coefficients for any (counterfactual) j as a function of the coefficients for $j-1$ through (F.1)-(F.11). Similarly, we can eliminate $\{\hat{\varepsilon}_{\tau|t}^j, \hat{s}_{\tau|t}^j, \widehat{m}c_{\tau|t}^j, \hat{v}_{\tau|t}^j, \hat{i}_{\tau|t}^{*,j}\}$ in (F.1)-(F.11). Additionally, since $\hat{y}_{\tau|t}^j$ only enters (F.8), we only need to solve $\{\hat{c}_{\tau|t}^j, \hat{q}_{\tau|t}^j, \pi_{\tau|t}^j, \pi_{H,\tau|t}^j, \hat{b}_{\tau+1|t}^{*,j}\}$, and $\hat{y}_{\tau|t}^j$ is pinned down by (F.8).

To solve $\{\hat{c}_{\tau|t}^j, \hat{q}_{\tau|t}^j, \pi_{\tau|t}^j, \pi_{H,\tau|t}^j, \hat{b}_{\tau+1|t}^{*,j}\}$, from (F.1), we have

$$\hat{c}_{\tau|t}^j = \mathbb{E}_{\tau} \hat{c}_{\tau+1|t}^{j-1} - \frac{1}{\sigma} (\phi_{\pi} \pi_{\tau|t}^j - \mathbb{E}_{\tau} \pi_{\tau+1|t}^{j-1}),$$

and substituting with the policy functions yields

$$\begin{aligned} & \psi_{c,q}^j \hat{q}_{\tau-1|t}^{j+1} + \psi_{c,b}^j \hat{b}_{\tau|t}^{*,j+1} + \psi_{c,a}^j a_{\tau} + \psi_{c,\mu}^j \mu_{\tau} + \psi_{c,\nu}^j \nu_{\tau} + \psi_{c,\tilde{\nu}}^j \tilde{\nu}_{\tau} + \psi_{c,\nu^*}^j \nu_{\tau}^* \\ &= \mathbb{E}_{\tau} \{ \psi_{c,q}^{j-1} [\psi_{q,q}^j \hat{q}_{\tau-1|t}^{j+1} + \psi_{q,b}^j \hat{b}_{\tau|t}^{*,j+1} + \psi_{q,a}^j a_{\tau} + \psi_{q,\mu}^j \mu_{\tau} + \psi_{q,\nu}^j \nu_{\tau} + \psi_{q,\tilde{\nu}}^j \tilde{\nu}_{\tau} + \psi_{q,\nu^*}^j \nu_{\tau}^*] \\ &+ \psi_{c,b}^{j-1} [\psi_{b,q}^j \hat{q}_{\tau-1|t}^{j+1} + \psi_{b,b}^j \hat{b}_{\tau|t}^{*,j+1} + \psi_{b,a}^j a_{\tau} + \psi_{b,\mu}^j \mu_{\tau} + \psi_{b,\nu}^j \nu_{\tau} + \psi_{b,\tilde{\nu}}^j \tilde{\nu}_{\tau} + \psi_{b,\nu^*}^j \nu_{\tau}^*] \\ &+ \psi_{c,a}^{j-1} a_{\tau+1} + \psi_{c,\mu}^{j-1} \mu_{\tau+1} + \psi_{c,\nu}^{j-1} \nu_{\tau} + \psi_{c,\tilde{\nu}}^{j-1} \tilde{\nu}_{\tau} + \psi_{c,\nu^*}^{j-1} \nu_{\tau}^* \} \\ &- \frac{\phi_{\pi}}{\sigma} [\psi_{\pi,q}^j \hat{q}_{\tau-1|t}^{j+1} + \psi_{\pi,b}^j \hat{b}_{\tau|t}^{*,j+1} + \psi_{\pi,a}^j a_{\tau} + \psi_{\pi,\mu}^j \mu_{\tau} + \psi_{\pi,\nu}^j \nu_{\tau} + \psi_{\pi,\tilde{\nu}}^j \tilde{\nu}_{\tau} + \psi_{\pi,\nu^*}^j \nu_{\tau}^*] \\ &+ \frac{1}{\sigma} \mathbb{E}_{\tau} \{ \psi_{\pi,q}^{j-1} [\psi_{q,q}^j \hat{q}_{\tau-1|t}^{j+1} + \psi_{q,b}^j \hat{b}_{\tau|t}^{*,j+1} + \psi_{q,a}^j a_{\tau} + \psi_{q,\mu}^j \mu_{\tau} + \psi_{q,\nu}^j \nu_{\tau} + \psi_{q,\tilde{\nu}}^j \tilde{\nu}_{\tau} + \psi_{q,\nu^*}^j \nu_{\tau}^*] \\ &+ \psi_{\pi,b}^{j-1} [\psi_{b,q}^j \hat{q}_{\tau-1|t}^{j+1} + \psi_{b,b}^j \hat{b}_{\tau|t}^{*,j+1} + \psi_{b,a}^j a_{\tau} + \psi_{b,\mu}^j \mu_{\tau} + \psi_{b,\nu}^j \nu_{\tau} + \psi_{b,\tilde{\nu}}^j \tilde{\nu}_{\tau} + \psi_{b,\nu^*}^j \nu_{\tau}^*] \\ &+ \psi_{\pi,a}^{j-1} a_{\tau+1} + \psi_{\pi,\mu}^{j-1} \mu_{\tau+1} + \psi_{\pi,\nu}^{j-1} \nu_{\tau} + \psi_{\pi,\tilde{\nu}}^{j-1} \tilde{\nu}_{\tau} + \psi_{\pi,\nu^*}^{j-1} \nu_{\tau}^* \}, \end{aligned}$$

where we have substituted $\hat{q}_{\tau|t}^j = \psi_{q,q}^j \hat{q}_{\tau-1|t}^{j+1} + \psi_{q,b}^j \hat{b}_{\tau|t}^{*,j+1} + \psi_{q,a}^j a_{\tau} + \psi_{q,\mu}^j \mu_{\tau} + \psi_{q,\nu}^j \nu_{\tau} + \psi_{q,\tilde{\nu}}^j \tilde{\nu}_{\tau} + \psi_{q,\nu^*}^j \nu_{\tau}^*$ and the similar expression of $\hat{b}_{\tau+1|t}^{*,j}$.

By equating the coefficients, we obtain

$$\begin{aligned} \psi_{c,q}^j &= \psi_{c,q}^{j-1} \psi_{q,q}^j + \psi_{c,b}^{j-1} \psi_{b,q}^j - \frac{\phi_{\pi}}{\sigma} \psi_{\pi,q}^j + \frac{1}{\sigma} [\psi_{\pi,q}^{j-1} \psi_{q,q}^j + \psi_{\pi,b}^{j-1} \psi_{b,q}^j], \\ \psi_{c,b}^j &= \psi_{c,q}^{j-1} \psi_{q,b}^j + \psi_{c,b}^{j-1} \psi_{b,b}^j - \frac{\phi_{\pi}}{\sigma} \psi_{\pi,b}^j + \frac{1}{\sigma} [\psi_{\pi,q}^{j-1} \psi_{q,b}^j + \psi_{\pi,b}^{j-1} \psi_{b,b}^j], \\ \psi_{c,a}^j &= \psi_{c,q}^{j-1} \psi_{q,a}^j + \psi_{c,b}^{j-1} \psi_{b,a}^j + \rho_a \psi_{c,a}^{j-1} - \frac{\phi_{\pi}}{\sigma} \psi_{\pi,a}^j + \frac{1}{\sigma} [\psi_{\pi,q}^{j-1} \psi_{q,a}^j + \psi_{\pi,b}^{j-1} \psi_{b,a}^j + \rho_a \psi_{\pi,a}^{j-1}], \\ \psi_{c,\mu}^j &= \psi_{c,q}^{j-1} \psi_{q,\mu}^j + \psi_{c,b}^{j-1} \psi_{b,\mu}^j + \rho_{\mu} \psi_{c,\mu}^{j-1} - \frac{\phi_{\pi}}{\sigma} \psi_{\pi,\mu}^j + \frac{1}{\sigma} [\psi_{\pi,q}^{j-1} \psi_{q,\mu}^j + \psi_{\pi,b}^{j-1} \psi_{b,\mu}^j + \rho_{\mu} \psi_{\pi,\mu}^{j-1}], \\ \psi_{c,\nu}^j &= \psi_{c,q}^{j-1} \psi_{q,\nu}^j + \psi_{c,b}^{j-1} \psi_{b,\nu}^j + \psi_{c,\nu}^{j-1} - \frac{\phi_{\pi}}{\sigma} \psi_{\pi,\nu}^j + \frac{1}{\sigma} [\psi_{\pi,q}^{j-1} \psi_{q,\nu}^j + \psi_{\pi,b}^{j-1} \psi_{b,\nu}^j + \psi_{\pi,\nu}^{j-1}], \\ \psi_{c,\tilde{\nu}}^j &= \psi_{c,q}^{j-1} \psi_{q,\tilde{\nu}}^j + \psi_{c,b}^{j-1} \psi_{b,\tilde{\nu}}^j + \psi_{c,\tilde{\nu}}^{j-1} - \frac{\phi_{\pi}}{\sigma} \psi_{\pi,\tilde{\nu}}^j + \frac{1}{\sigma} [\psi_{\pi,q}^{j-1} \psi_{q,\tilde{\nu}}^j + \psi_{\pi,b}^{j-1} \psi_{b,\tilde{\nu}}^j + \psi_{\pi,\tilde{\nu}}^{j-1}], \\ \psi_{c,\nu^*}^j &= \psi_{c,q}^{j-1} \psi_{q,\nu^*}^j + \psi_{c,b}^{j-1} \psi_{b,\nu^*}^j + \psi_{c,\nu^*}^{j-1} - \frac{\phi_{\pi}}{\sigma} \psi_{\pi,\nu^*}^j + \frac{1}{\sigma} [\psi_{\pi,q}^{j-1} \psi_{q,\nu^*}^j + \psi_{\pi,b}^{j-1} \psi_{b,\nu^*}^j + \psi_{\pi,\nu^*}^{j-1}]. \end{aligned}$$

Similarly, from (F.2), we have

$$\hat{q}_{\tau|t}^j = \mathbb{E}_{\tau} \hat{q}_{\tau+1|t}^{j-1} + \phi_b \hat{b}_{\tau+1|t}^j + \mu_{\tau} - (\phi_{\pi} \pi_{\tau|t}^j - \mathbb{E}_{\tau} \pi_{\tau+1|t}^{j-1}),$$

and substituting with the policy functions yields

$$\begin{aligned} & \psi_{q,q}^j \hat{q}_{\tau-1|t}^{j+1} + \psi_{q,b}^j \hat{b}_{\tau|t}^{*,j+1} + \psi_{q,a}^j a_{\tau} + \psi_{q,\mu}^j \mu_{\tau} + \psi_{q,\nu}^j \nu_t + \psi_{q,\bar{\nu}}^j \tilde{\nu}_t + \psi_{q,\nu^*}^j \nu_t^* \\ &= \mathbb{E}_{\tau} \{ \psi_{q,q}^{j-1} [\psi_{q,q}^j \hat{q}_{\tau-1|t}^{j+1} + \psi_{q,b}^j \hat{b}_{\tau|t}^{*,j+1} + \psi_{q,a}^j a_{\tau} + \psi_{q,\mu}^j \mu_{\tau} + \psi_{q,\nu}^j \nu_t + \psi_{q,\bar{\nu}}^j \tilde{\nu}_t + \psi_{q,\nu^*}^j \nu_t^*] \\ &+ \psi_{q,b}^{j-1} [\psi_{b,q}^j \hat{q}_{\tau-1|t}^{j+1} + \psi_{b,b}^j \hat{b}_{\tau|t}^{*,j+1} + \psi_{b,a}^j a_{\tau} + \psi_{b,\mu}^j \mu_{\tau} + \psi_{b,\nu}^j \nu_t + \psi_{b,\bar{\nu}}^j \tilde{\nu}_t + \psi_{b,\nu^*}^j \nu_t^*] \\ &+ \psi_{q,a}^{j-1} a_{\tau+1} + \psi_{q,\mu}^{j-1} \mu_{\tau+1} + \psi_{q,\nu}^{j-1} \nu_t + \psi_{q,\bar{\nu}}^{j-1} \tilde{\nu}_t + \psi_{q,\nu^*}^{j-1} \nu_t^* \} \\ &+ \phi_b [\psi_{b,q}^j \hat{q}_{\tau-1|t}^{j+1} + \psi_{b,b}^j \hat{b}_{\tau|t}^{*,j+1} + \psi_{b,a}^j a_{\tau} + \psi_{b,\mu}^j \mu_{\tau} + \psi_{b,\nu}^j \nu_t + \psi_{b,\bar{\nu}}^j \tilde{\nu}_t + \psi_{b,\nu^*}^j \nu_t^*] \\ &+ \mu_{\tau} - \phi_{\pi} [\psi_{\pi,q}^j \hat{q}_{\tau-1|t}^{j+1} + \psi_{\pi,b}^j \hat{b}_{\tau|t}^{*,j+1} + \psi_{\pi,a}^j a_{\tau} + \psi_{\pi,\mu}^j \mu_{\tau} + \psi_{\pi,\nu}^j \nu_t + \psi_{\pi,\bar{\nu}}^j \tilde{\nu}_t + \psi_{\pi,\nu^*}^j \nu_t^*] \\ &+ \mathbb{E}_{\tau} \{ \psi_{\pi,q}^{j-1} [\psi_{q,q}^j \hat{q}_{\tau-1|t}^{j+1} + \psi_{q,b}^j \hat{b}_{\tau|t}^{*,j+1} + \psi_{q,a}^j a_{\tau} + \psi_{q,\mu}^j \mu_{\tau} + \psi_{q,\nu}^j \nu_t + \psi_{q,\bar{\nu}}^j \tilde{\nu}_t + \psi_{q,\nu^*}^j \nu_t^*] \\ &+ \psi_{\pi,b}^{j-1} [\psi_{b,q}^j \hat{q}_{\tau-1|t}^{j+1} + \psi_{b,b}^j \hat{b}_{\tau|t}^{*,j+1} + \psi_{b,a}^j a_{\tau} + \psi_{b,\mu}^j \mu_{\tau} + \psi_{b,\nu}^j \nu_t + \psi_{b,\bar{\nu}}^j \tilde{\nu}_t + \psi_{b,\nu^*}^j \nu_t^*] \\ &+ \psi_{\pi,a}^{j-1} a_{\tau+1} + \psi_{\pi,\mu}^{j-1} \mu_{\tau+1} + \psi_{\pi,\nu}^{j-1} \nu_t + \psi_{\pi,\bar{\nu}}^{j-1} \tilde{\nu}_t + \psi_{\pi,\nu^*}^{j-1} \nu_t^* \}. \end{aligned}$$

By equating the coefficients, we obtain

$$\begin{aligned} \psi_{q,q}^j &= \psi_{q,q}^{j-1} \psi_{q,q}^j + \psi_{q,b}^{j-1} \psi_{b,q}^j + \phi_b \psi_{b,q}^j - \phi_{\pi} \psi_{\pi,q}^j + \psi_{\pi,q}^{j-1} \psi_{q,q}^j + \psi_{\pi,b}^{j-1} \psi_{b,q}^j, \\ \psi_{q,b}^j &= \psi_{q,q}^{j-1} \psi_{q,b}^j + \psi_{q,b}^{j-1} \psi_{b,b}^j + \phi_b \psi_{b,b}^j - \phi_{\pi} \psi_{\pi,b}^j + \psi_{\pi,q}^{j-1} \psi_{q,b}^j + \psi_{\pi,b}^{j-1} \psi_{b,b}^j, \\ \psi_{q,a}^j &= \psi_{q,q}^{j-1} \psi_{q,a}^j + \psi_{q,b}^{j-1} \psi_{b,a}^j + \rho_a \psi_{q,a}^{j-1} + \phi_b \psi_{b,a}^j - \phi_{\pi} \psi_{\pi,a}^j + \psi_{\pi,q}^{j-1} \psi_{q,a}^j + \psi_{\pi,b}^{j-1} \psi_{b,a}^j + \rho_a \psi_{\pi,a}^{j-1}, \\ \psi_{q,\mu}^j &= \psi_{q,q}^{j-1} \psi_{q,\mu}^j + \psi_{q,b}^{j-1} \psi_{b,\mu}^j + \rho_{\mu} \psi_{q,\mu}^{j-1} + \phi_b \psi_{b,\mu}^j + 1 - \phi_{\pi} \psi_{\pi,\mu}^j + \psi_{\pi,q}^{j-1} \psi_{q,\mu}^j + \psi_{\pi,b}^{j-1} \psi_{b,\mu}^j + \rho_{\mu} \psi_{\pi,\mu}^{j-1}, \\ \psi_{q,\nu}^j &= \psi_{q,q}^{j-1} \psi_{q,\nu}^j + \psi_{q,b}^{j-1} \psi_{b,\nu}^j + \psi_{q,\nu}^{j-1} + \phi_b \psi_{b,\nu}^j - \phi_{\pi} \psi_{\pi,\nu}^j + \psi_{\pi,q}^{j-1} \psi_{q,\nu}^j + \psi_{\pi,b}^{j-1} \psi_{b,\nu}^j, \\ \psi_{q,\bar{\nu}}^j &= \psi_{q,q}^{j-1} \psi_{q,\bar{\nu}}^j + \psi_{q,b}^{j-1} \psi_{b,\bar{\nu}}^j + \psi_{q,\bar{\nu}}^{j-1} + \phi_b \psi_{b,\bar{\nu}}^j - \phi_{\pi} \psi_{\pi,\bar{\nu}}^j + \psi_{\pi,q}^{j-1} \psi_{q,\bar{\nu}}^j + \psi_{\pi,b}^{j-1} \psi_{b,\bar{\nu}}^j, \\ \psi_{q,\nu^*}^j &= \psi_{q,q}^{j-1} \psi_{q,\nu^*}^j + \psi_{q,b}^{j-1} \psi_{b,\nu^*}^j + \psi_{q,\nu^*}^{j-1} + \phi_b \psi_{b,\nu^*}^j - \phi_{\pi} \psi_{\pi,\nu^*}^j + \psi_{\pi,q}^{j-1} \psi_{q,\nu^*}^j + \psi_{\pi,b}^{j-1} \psi_{b,\nu^*}^j. \end{aligned}$$

Similarly, from (F.5)-(F.6), we have

$$\pi_{H,\tau|t}^j = \kappa(\sigma + \varphi \vartheta_{yc}) \hat{c}_{\tau|t}^j + \frac{\kappa(\alpha + \varphi \vartheta_{ys})}{1 - \alpha} \hat{q}_{\tau|t}^j - \kappa(1 + \varphi) a_{\tau} + \beta \mathbb{E}_{\tau} \pi_{H,\tau+1|t}^{j-1},$$

and substituting with the policy functions yields

$$\begin{aligned}
& \psi_{\pi_H,q}^j \hat{q}_{\tau-1|t}^{j+1} + \psi_{\pi_H,b}^j \hat{b}_{\tau|t}^{j+1} + \psi_{\pi_H,a}^j a_\tau + \psi_{\pi_H,\mu}^j \mu_\tau + \psi_{\pi_H,\nu}^j \nu_t + \psi_{\pi_H,\tilde{\nu}}^j \tilde{\nu}_t + \psi_{\pi_H,\nu^*}^j \nu_t^* \\
&= \kappa(\sigma + \varphi \vartheta_{yc}) [\psi_{c,q}^j \hat{q}_{\tau-1|t}^{j+1} + \psi_{c,b}^j \hat{b}_{\tau|t}^{j+1} + \psi_{c,a}^j a_\tau + \psi_{c,\mu}^j \mu_\tau + \psi_{c,\nu}^j \nu_t + \psi_{c,\tilde{\nu}}^j \tilde{\nu}_t + \psi_{c,\nu^*}^j \nu_t^*] \\
&+ \frac{\kappa(\alpha + \varphi \vartheta_{ys})}{1 - \alpha} [\psi_{q,q}^j \hat{q}_{\tau-1|t}^{j+1} + \psi_{q,b}^j \hat{b}_{\tau|t}^{j+1} + \psi_{q,a}^j a_\tau + \psi_{q,\mu}^j \mu_\tau + \psi_{q,\nu}^j \nu_t + \psi_{q,\tilde{\nu}}^j \tilde{\nu}_t + \psi_{q,\nu^*}^j \nu_t^*] \\
&- \kappa(1 + \varphi) a_\tau \\
&+ \beta \mathbb{E}_\tau \{ \psi_{\pi_H,q}^{j-1} [\psi_{q,q}^j \hat{q}_{\tau-1|t}^{j+1} + \psi_{q,b}^j \hat{b}_{\tau|t}^{j+1} + \psi_{q,a}^j a_\tau + \psi_{q,\mu}^j \mu_\tau + \psi_{q,\nu}^j \nu_t + \psi_{q,\tilde{\nu}}^j \tilde{\nu}_t + \psi_{q,\nu^*}^j \nu_t^*] \\
&+ \psi_{\pi_H,b}^{j-1} [\psi_{b,q}^j \hat{q}_{\tau-1|t}^{j+1} + \psi_{b,b}^j \hat{b}_{\tau|t}^{j+1} + \psi_{b,a}^j a_\tau + \psi_{b,\mu}^j \mu_\tau + \psi_{b,\nu}^j \nu_t + \psi_{b,\tilde{\nu}}^j \tilde{\nu}_t + \psi_{b,\nu^*}^j \nu_t^*] \\
&+ \psi_{\pi_H,a}^{j-1} a_{\tau+1} + \psi_{\pi_H,\mu}^{j-1} \mu_{\tau+1} + \psi_{\pi_H,\nu}^{j-1} \nu_t + \psi_{\pi_H,\tilde{\nu}}^{j-1} \tilde{\nu}_t + \psi_{\pi_H,\nu^*}^{j-1} \nu_t^* \}.
\end{aligned}$$

By equating the coefficients, we obtain

$$\begin{aligned}
\psi_{\pi_H,q}^j &= \kappa(\sigma + \varphi \vartheta_{yc}) \psi_{c,q}^j + \frac{\kappa(\alpha + \varphi \vartheta_{ys})}{1 - \alpha} \psi_{q,q}^j + \beta [\psi_{\pi_H,q}^{j-1} \psi_{q,q}^j + \psi_{\pi_H,b}^{j-1} \psi_{b,q}^j], \\
\psi_{\pi_H,b}^j &= \kappa(\sigma + \varphi \vartheta_{yc}) \psi_{c,b}^j + \frac{\kappa(\alpha + \varphi \vartheta_{ys})}{1 - \alpha} \psi_{q,b}^j + \beta [\psi_{\pi_H,q}^{j-1} \psi_{q,b}^j + \psi_{\pi_H,b}^{j-1} \psi_{b,b}^j], \\
\psi_{\pi_H,a}^j &= \kappa(\sigma + \varphi \vartheta_{yc}) \psi_{c,a}^j + \frac{\kappa(\alpha + \varphi \vartheta_{ys})}{1 - \alpha} \psi_{q,a}^j - \kappa(1 + \varphi) + \beta [\psi_{\pi_H,q}^{j-1} \psi_{q,a}^j + \psi_{\pi_H,b}^{j-1} \psi_{b,a}^j + \rho_a \psi_{\pi_H,a}^{j-1}], \\
\psi_{\pi_H,\mu}^j &= \kappa(\sigma + \varphi \vartheta_{yc}) \psi_{c,\mu}^j + \frac{\kappa(\alpha + \varphi \vartheta_{ys})}{1 - \alpha} \psi_{q,\mu}^j + \beta [\psi_{\pi_H,q}^{j-1} \psi_{q,\mu}^j + \psi_{\pi_H,b}^{j-1} \psi_{b,\mu}^j + \rho_\mu \psi_{\pi_H,\mu}^{j-1}], \\
\psi_{\pi_H,\nu}^j &= \kappa(\sigma + \varphi \vartheta_{yc}) \psi_{c,\nu}^j + \frac{\kappa(\alpha + \varphi \vartheta_{ys})}{1 - \alpha} \psi_{q,\nu}^j + \beta [\psi_{\pi_H,q}^{j-1} \psi_{q,\nu}^j + \psi_{\pi_H,b}^{j-1} \psi_{b,\nu}^j + \psi_{\pi_H,\nu}^{j-1}], \\
\psi_{\pi_H,\tilde{\nu}}^j &= \kappa(\sigma + \varphi \vartheta_{yc}) \psi_{c,\tilde{\nu}}^j + \frac{\kappa(\alpha + \varphi \vartheta_{ys})}{1 - \alpha} \psi_{q,\tilde{\nu}}^j + \beta [\psi_{\pi_H,q}^{j-1} \psi_{q,\tilde{\nu}}^j + \psi_{\pi_H,b}^{j-1} \psi_{b,\tilde{\nu}}^j + \psi_{\pi_H,\tilde{\nu}}^{j-1}], \\
\psi_{\pi_H,\nu^*}^j &= \kappa(\sigma + \varphi \vartheta_{yc}) \psi_{c,\nu^*}^j + \frac{\kappa(\alpha + \varphi \vartheta_{ys})}{1 - \alpha} \psi_{q,\nu^*}^j + \beta [\psi_{\pi_H,q}^{j-1} \psi_{q,\nu^*}^j + \psi_{\pi_H,b}^{j-1} \psi_{b,\nu^*}^j + \psi_{\pi_H,\nu^*}^{j-1}].
\end{aligned}$$

Similarly, from (F.3) and (F.9), we have

$$\pi_{\tau|t}^j = (1 - \alpha) \pi_{H,\tau|t}^j + \alpha (\hat{q}_{\tau|t}^j - \hat{q}_{\tau-1|t}^{j+1} + \pi_{\tau|t}^j),$$

and substituting with the policy functions yields

$$\begin{aligned}
& (1 - \alpha) [\psi_{\pi,q}^j \hat{q}_{\tau-1|t}^{j+1} + \psi_{\pi,b}^j \hat{b}_{\tau|t}^{j+1} + \psi_{\pi,a}^j a_\tau + \psi_{\pi,\mu}^j \mu_\tau + \psi_{\pi,\nu}^j \nu_t + \psi_{\pi,\tilde{\nu}}^j \tilde{\nu}_t + \psi_{\pi,\nu^*}^j \nu_t^*] \\
&= (1 - \alpha) [\psi_{\pi_H,q}^j \hat{q}_{\tau-1|t}^{j+1} + \psi_{\pi_H,b}^j \hat{b}_{\tau|t}^{j+1} + \psi_{\pi_H,a}^j a_\tau + \psi_{\pi_H,\mu}^j \mu_\tau + \psi_{\pi_H,\nu}^j \nu_t + \psi_{\pi_H,\tilde{\nu}}^j \tilde{\nu}_t + \psi_{\pi_H,\nu^*}^j \nu_t^*] \\
&+ \alpha [\psi_{q,q}^j \hat{q}_{\tau-1|t}^{j+1} + \psi_{q,b}^j \hat{b}_{\tau|t}^{j+1} + \psi_{q,a}^j a_\tau + \psi_{q,\mu}^j \mu_\tau + \psi_{q,\nu}^j \nu_t + \psi_{q,\tilde{\nu}}^j \tilde{\nu}_t + \psi_{q,\nu^*}^j \nu_t^*] - \alpha \hat{q}_{\tau-1|t}^{j+1}.
\end{aligned}$$

By equating the coefficients, we obtain

$$\begin{aligned}
(1 - \alpha)\psi_{\pi,q}^j &= (1 - \alpha)\psi_{\pi_H,q}^j + \alpha\psi_{q,q}^j - \alpha, & (1 - \alpha)\psi_{\pi,b}^j &= (1 - \alpha)\psi_{\pi_H,b}^j + \alpha\psi_{q,b}^j, \\
(1 - \alpha)\psi_{\pi,a}^j &= (1 - \alpha)\psi_{\pi_H,a}^j + \alpha\psi_{q,a}^j, & (1 - \alpha)\psi_{\pi,\mu}^j &= (1 - \alpha)\psi_{\pi_H,\mu}^j + \alpha\psi_{q,\mu}^j, \\
(1 - \alpha)\psi_{\pi,\nu}^j &= (1 - \alpha)\psi_{\pi_H,\nu}^j + \alpha\psi_{q,\nu}^j, & (1 - \alpha)\psi_{\pi,\tilde{\nu}}^j &= (1 - \alpha)\psi_{\pi_H,\tilde{\nu}}^j + \alpha\psi_{q,\tilde{\nu}}^j, \\
(1 - \alpha)\psi_{\pi,\nu^*}^j &= (1 - \alpha)\psi_{\pi_H,\nu^*}^j + \alpha\psi_{q,\nu^*}^j.
\end{aligned}$$

Finally, from (F.8) and (F.11), we have

$$(1 - \vartheta_1\phi_b)\hat{b}_{\tau+1|t}^{*,j} = \beta^{-1} \left\{ \hat{b}_{\tau|t}^{*,j+1} + \left[\vartheta_1 - \vartheta_2 \frac{\alpha - \vartheta_{ys}}{(1 - \alpha)} \right] \hat{q}_{\tau|t}^j - (1 - \vartheta_2\vartheta_{yc})\hat{c}_{\tau|t}^j \right\} - \vartheta_1\hat{q}_{\tau|t}^j + \vartheta_1\mu_\tau,$$

and substituting with the policy functions yields

$$\begin{aligned}
&(1 - \vartheta_1\phi_b)[\psi_{b,q}^j\hat{q}_{\tau-1|t}^{j+1} + \psi_{b,b}^j\hat{b}_{\tau|t}^{*,j+1} + \psi_{b,a}^j a_\tau + \psi_{q,\mu}^j\mu_\tau + \psi_{b,\nu}^j\nu_t + \psi_{b,\tilde{\nu}}^j\tilde{\nu}_t + \psi_{b,\nu^*}^j\nu_t^*] \\
&= \beta^{-1}\hat{b}_{\tau|t}^{*,j+1} + \vartheta_1\mu_\tau \\
&+ \left[(\beta^{-1} - 1)\vartheta_1 - \beta^{-1}\vartheta_2 \frac{\alpha - \vartheta_{ys}}{(1 - \alpha)} \right] [\psi_{q,q}^j\hat{q}_{\tau-1|t}^{j+1} + \psi_{q,b}^j\hat{b}_{\tau|t}^{j+1} + \psi_{q,a}^j a_\tau + \psi_{q,\mu}^j\mu_\tau + \psi_{q,\nu}^j\nu_t + \psi_{q,\tilde{\nu}}^j\tilde{\nu}_t + \psi_{q,\nu^*}^j\nu_t^*] \\
&- \beta^{-1}(1 - \vartheta_2\vartheta_{yc})[\psi_{c,q}^j\hat{q}_{\tau-1|t}^{j+1} + \psi_{c,b}^j\hat{b}_{\tau|t}^{j+1} + \psi_{c,a}^j a_\tau + \psi_{c,\mu}^j\mu_\tau + \psi_{c,\nu}^j\nu_t + \psi_{c,\tilde{\nu}}^j\tilde{\nu}_t + \psi_{c,\nu^*}^j\nu_t^*]
\end{aligned}$$

By equating the coefficients, we obtain

$$\begin{aligned}
(1 - \vartheta_1\phi_b)\psi_{b,q}^j &= \left[(\beta^{-1} - 1)\vartheta_1 - \beta^{-1}\vartheta_2 \frac{\alpha - \vartheta_{ys}}{(1 - \alpha)} \right] \psi_{q,q}^j - \beta^{-1}(1 - \vartheta_2\vartheta_{yc})\psi_{c,q}^j \\
(1 - \vartheta_1\phi_b)\psi_{b,b}^j &= 1 + (\beta^{-1} - 1) + \left[(\beta^{-1} - 1)\vartheta_1 - \beta^{-1}\vartheta_2 \frac{\alpha - \vartheta_{ys}}{(1 - \alpha)} \right] \psi_{q,b}^j - \beta^{-1}(1 - \vartheta_2\vartheta_{yc})\psi_{c,b}^j \\
(1 - \vartheta_1\phi_b)\psi_{b,a}^j &= \left[(\beta^{-1} - 1)\vartheta_1 - \beta^{-1}\vartheta_2 \frac{\alpha - \vartheta_{ys}}{(1 - \alpha)} \right] \psi_{q,a}^j - \beta^{-1}(1 - \vartheta_2\vartheta_{yc})\psi_{c,a}^j \\
(1 - \vartheta_1\phi_b)\psi_{b,\mu}^j &= \left[(\beta^{-1} - 1)\vartheta_1 - \beta^{-1}\vartheta_2 \frac{\alpha - \vartheta_{ys}}{(1 - \alpha)} \right] \psi_{q,\mu}^j - \beta^{-1}(1 - \vartheta_2\vartheta_{yc})\psi_{c,\mu}^j + \vartheta_1 \\
(1 - \vartheta_1\phi_b)\psi_{b,\nu}^j &= \left[(\beta^{-1} - 1)\vartheta_1 - \beta^{-1}\vartheta_2 \frac{\alpha - \vartheta_{ys}}{(1 - \alpha)} \right] \psi_{q,\nu}^j - \beta^{-1}(1 - \vartheta_2\vartheta_{yc})\psi_{c,\nu}^j \\
(1 - \vartheta_1\phi_b)\psi_{b,\tilde{\nu}}^j &= \left[(\beta^{-1} - 1)\vartheta_1 - \beta^{-1}\vartheta_2 \frac{\alpha - \vartheta_{ys}}{(1 - \alpha)} \right] \psi_{q,\tilde{\nu}}^j - \beta^{-1}(1 - \vartheta_2\vartheta_{yc})\psi_{c,\tilde{\nu}}^j \\
(1 - \vartheta_1\phi_b)\psi_{b,\nu^*}^j &= \left[(\beta^{-1} - 1)\vartheta_1 - \beta^{-1}\vartheta_2 \frac{\alpha - \vartheta_{ys}}{(1 - \alpha)} \right] \psi_{q,\nu^*}^j - \beta^{-1}(1 - \vartheta_2\vartheta_{yc})\psi_{c,\nu^*}^j.
\end{aligned}$$

Thus, given the undetermined coefficients for $j - 1$, we have 35 simple linear equations for the undetermined coefficients for j , which yields a unique solution. Since we have derived the undetermined coefficients for the case of $j = 0$, we can solve the expressions for $\{\hat{c}_{\tau|t}^j, \hat{q}_{\tau|t}^j, \pi_{\tau|t}^j, \pi_{H,\tau|t}^j, b_{\tau|t}^{*,j+1}\}$ by forward induction. All the other endogenous variables can be

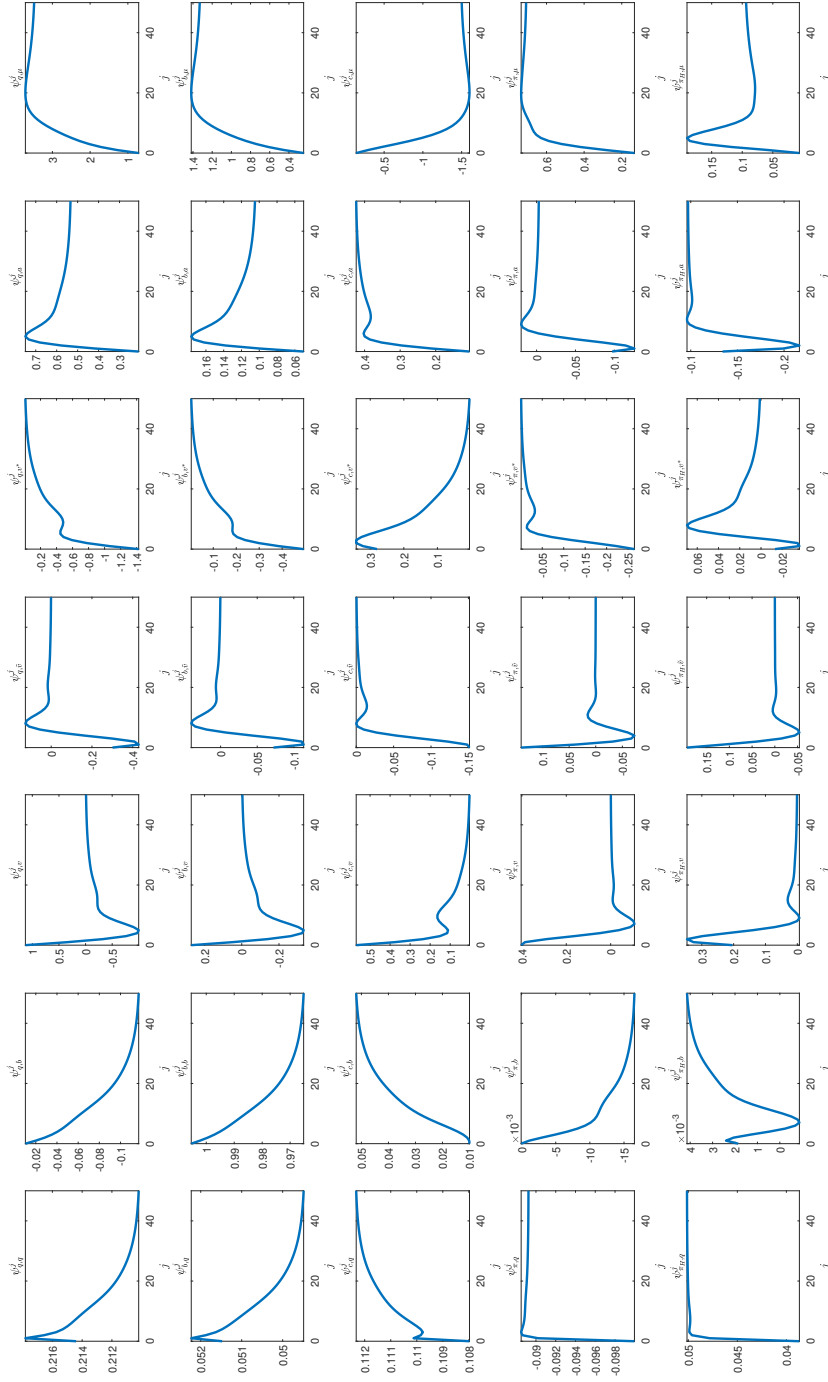
easily derived with the solutions of these four endogenous variables.

Thus far, we have derived the solution of the entire forward planning calculated in period t . Then, one can easily solve the equilibrium path solution (3.1) with the evolution of the state variables (3.2)-(3.4).

Figure G.6 reports the policy coefficients of the five variables $x \in \{q, b, c, \pi, \pi_H\}$ for varying planning horizons $j \in [0, 50]$ quarters, with the calibrated structural parameters reported in Table 1. As j increases, all coefficients converge to the unique RE equilibrium values, with household value functions $\psi_{x,\nu}^j, \psi_{x,\tilde{\nu}}^j, \psi_{x,\nu^*}^j$ becoming zero. This reflects redundancy in coarse value functions under RE equilibrium due to time-consistent, infinitely future-aware decision-making. As planning horizon j decreases, policy coefficients deviate from RE values, exhibiting non-monotonic, bumpy movements. Some coefficients, such as $\psi_{q,\nu}^j$, may change sign with shorter horizons.

These policy coefficients entail several behavioral implications. Shorter planning horizons give rise to deviations from the RE equilibrium, attributable to limited foresight. Moreover, these biases do not consistently follow the same direction, as indicated by the non-monotonic policy coefficients. Consequently, when decision-makers operate within a relatively short planning horizon, the resulting equilibrium can diverge in various ways from the RE equilibrium. Conversely, even with an extended planning horizon, expectations regarding distant future variables—assessed using policy coefficients for a truncated remaining planning horizon—may substantially deviate from RE.

Figure G.6: Policy Coefficients of the Five Variables with Planning Horizon $j \in [0, 50]$



H Data Sources and Variable Construction

The data used in the calibration of Section 4.1 are constructed as follows, all at a quarterly frequency:

- Canadian per capita real output growth and consumption growth.** We obtain the Canadian data of real GDP [V6E06896], household real final consumption expenditure [V6A89012], and population [V1] from Statistics Canada (StatsCan). The code in brackets represents StatsCan mnemonic. We divide the two data series by population to calculate the per capita real output and consumption and take the log difference over time to get the corresponding growth rates.
- US and Canada price level and inflation.** We obtain the headline consumer price index (CPI) of the US and Canada from Federal Reserve Economic Data [FRED: CPIAUCSL and CPALCY01CAQ661N]. The US CPI data is seasonally adjusted, whereas the later is not. Thus, we follow the US census bureau model X-13ARIMA-SEATS to seasonally adjust the Canadian CPI data series. We further normalize the data series such that the average price level for both the US and Canada in 2015 is 100. The relevant inflation rate is then calculated by taking log-difference over time for each series.
- US-Canada real exchange rate.** We obtain the average of the daily nominal exchange rate between US and Canadian dollars from Federal Reserve Economic Data [FRED: CCUSMA02CAM618N]. The data is in units of Canadian dollars. We calculate the real exchange rate by multiplying the nominal exchange rate with the US price level and dividing by the Canadian price level.

I Construction of Short-Term and Long-Term Interest Rate Differentials

We construct the corresponding model-generated series via the following steps. First, to construct the simulated expected real interest rate differentials under limited foresight, for each time t and for any horizon k , we need to pin down $\{\hat{q}_{t+k|t}^{h-k}, \hat{b}_{t+k+1|t}^{h-k}\}_{k=0}^h$ to back up $\{\hat{\pi}_{t+k|t}^{h-k}, \hat{v}_{t+k|t}^{h-k}, \hat{r}_{t+k|t}^{*,h-k}\}_{k=0}^h$. For each k , the solution to the forward planning exercise (3.8) yields

$$\mathbb{E}_t \hat{q}_{t+k|t}^{h-k} = \psi_{q,q}^{h-k} \mathbb{E}_t \hat{q}_{t+k-1|t}^{h-k+1} + \psi_{q,b}^{h-k} \mathbb{E}_t \hat{b}_{t+k|t}^{h-k+1} + \psi_{q,a}^{h-k} \rho_a^k a_t + \psi_{q,\mu}^{h-k} \rho_\mu^k \mu_t + \psi_{q,\nu}^{h-k} \nu_t + \psi_{q,\nu^*}^{h-k} \nu_t^*,$$

where $\mathbb{E}_t \hat{q}_{t-1|t}^{h+1} = \hat{q}_{t-1}$ and $\mathbb{E}_t \hat{b}_{t|t}^{h+1} = \hat{b}_t$ (that is, the equilibrium pre-determined real exchange rate and the net foreign asset position at time t). Then, the expected inflation under limited foresight satisfies

$$\mathbb{E}_t^h \pi_{t+k} \equiv \mathbb{E}_t \pi_{t+k|t}^{h-k} = \psi_{\pi,q}^{h-k} \mathbb{E}_t \hat{q}_{t+k-1|t}^{h-k+1} + \psi_{\pi,b}^{h-k} \mathbb{E}_t \hat{b}_{t+k|t}^{h-k+1} + \psi_{\pi,a}^{h-k} \rho_a^k a_t + \psi_{\pi,\mu}^{h-k} \rho_\mu^k \mu_t + \psi_{\pi,\nu}^{h-k} \nu_t + \psi_{\pi,\nu^*}^{h-k} \nu_t^*.$$

Since $\hat{i}_{\tau|t}^{h+t-\tau} = \phi_\pi \hat{\pi}_{\tau|t}^{h+t-\tau}$, we have the expected nominal interest rate given by

$$\mathbb{E}_t^h \hat{i}_{t+k} \equiv \mathbb{E}_t \hat{i}_{t+k|t}^{h-k} = \phi_\pi \mathbb{E}_t \pi_{t+k|t}^{h-k},$$

and then we construct the expected real interest rate under limited foresight as

$$\mathbb{E}_t^h \hat{r}_{t+k} \equiv \mathbb{E}_t \hat{i}_{t+k|t}^{h-k} - \mathbb{E}_t \pi_{t+k+1|t}^{h-k-1}, \quad (\text{I.1})$$

for any $0 \leq k < h$, and for the case of $k = h$, we construct $\mathbb{E}_t^h \hat{r}_{t+h} \equiv \mathbb{E}_t \hat{i}_{t+h|t}^0$.

Second, for the expected foreign real interest rate under limited foresight, since (2.40) yields $\hat{r}_{\tau|t}^{*,h+t-\tau} = \phi_b \hat{b}_{\tau+1|t}^{h+t-\tau} + \mu_\tau$ for any $t \leq \tau \leq t+h$, we construct

$$\mathbb{E}_t^h \hat{r}_{t+k}^* \equiv \mathbb{E}_t \hat{r}_{t+k|t}^{h-k} = \phi_b \mathbb{E}_t \hat{b}_{t+k+1|t}^{h-k} + \rho_\mu^k \mu_t. \quad (\text{I.2})$$

Therefore, for a given the threshold horizon for the short-term and the long-term M and the given planning horizon h , we can construct the expected real interest rate differentials under limited foresight as

$$D_t^S(M) = \sum_{k=0}^{M-1} \mathbb{E}_t^h [\hat{r}_{t+k}^* - \hat{r}_{t+k}], \quad D_t^L(M) = \sum_{k=M}^h \mathbb{E}_t^h [\hat{r}_{t+k}^* - \hat{r}_{t+k}], \quad (\text{I.3})$$

where $0 < M \leq h$.

J Robustness Checks

Planning Horizon h . First, we consider planning horizon $h \in \{2, 4, 40\}$ in the benchmark model with homogeneous agents. In the model for each planning horizon h , which is subject to two exogenous shocks, Figure J.7 illustrates the regression coefficients for both excess return predictability and the real exchange rate's response to short- and long-term real interest rate differentials. In summary, the attributes of the primary findings in Section 5 become stronger when planning horizons shorten and weaker as they lengthen, in comparison to the benchmark planning horizon 8. This is a natural outcome inherent in the FH model.

Firm's Learning Gain $\gamma_{\bar{v}}$. Next, we consider firm's learning behavior on its value function with various learning gain $\gamma_{\bar{v}} \in \{0.1, 0.5, 0.99\}$. Figure J.8 illustrates the related regression coefficients based on each learning gain $\gamma_{\bar{v}}$. Clearly, the outcomes for excess return predictability are not significantly impacted by variations in $\gamma_{\bar{v}}$. Moreover, the results of the breakdown of horizon invariance are found to be unaffected by $\gamma_{\bar{v}}$.

Heterogeneous Planning Horizons across Agents. Lastly, we extend the model to heterogeneous planning horizons across agents and examine its consequences for the aggregate variables. We assume that fraction $\omega_h \in (0, 1)$ of households and fraction $\tilde{\omega}_h \in (0, 1)$ of firms have planning horizon h . Following Woodford (2019), we further assume that those agents with horizon h make their decisions by assuming that all other agents have the same planning horizon of h . After the planning exercises in each period, the estimated value functions across households are aggregated by

$$\nu_t^{est} = \sum_h \omega_h \left[\hat{c}_t^h + \sigma^{-1} \pi_t^h - (1 - \beta) \hat{b}_t^{*,h} \right], \quad (\text{J.1})$$

$$\nu_t^{*,est} = \sum_h \omega_h \left[\nu_t^{est} - \sigma^{-1} (\hat{q}_t^h + \pi_t^h) \right], \quad (\text{J.2})$$

and the estimated value functions across firms are aggregated by

$$\tilde{\nu}_t^{est} = \sum_h \tilde{\omega}_h (1 - \theta)^{-1} \pi_{H,t}^h. \quad (\text{J.3})$$

We consider a special case in which fractions ω_h and $\tilde{\omega}_h$ follow the same geometric distributions:

$$\omega_h = \tilde{\omega}_h = \rho^h (1 - \rho)$$

for any $h \geq 0$, where $\rho \in (0, 1)$. Then, the aggregate variables become the weighted average among the population. The current endogenous variables are aggregated as follows:

$$\begin{aligned} \hat{c}_t &= \sum_h \omega_h \hat{c}_t^h, & \hat{y}_t &= \sum_h \omega_h \hat{y}_t^h, & \hat{i}_t &= \sum_h \omega_h \hat{i}_t^h, & \hat{r}_t^* &= \sum_h \omega_h \hat{r}_t^{*,h}, & \pi_{H,t} &= \sum_h \omega_h \pi_t^h, \\ \pi_t &= \sum_h \omega_h \pi_t^h, & \hat{q}_t &= \sum_h \omega_h \hat{q}_t^h, & \hat{s}_t &= \sum_h \omega_h \hat{s}_t^h, & \hat{\varepsilon}_t &= \sum_h \omega_h \hat{\varepsilon}_t^h, & \hat{b}_{t+1}^* &= \sum_h \omega_h \hat{b}_{t+1}^h. \end{aligned} \quad (\text{J.4})$$

The k -period ahead expectations for CPI inflation are aggregated as follows:

$$\begin{aligned}\mathbb{E}_t^{agg}\pi_{t+k} &\equiv \sum_h w_h \mathbb{E}_t^h \pi_{t+k} = \sum_h w_h \mathbb{E}_t \pi_{t+k|t}^{h-k} \\ &= \mathbb{E}_t \left[\rho^k (1-\rho) \pi_{t+k|t}^0 + \rho^{k+1} (1-\rho) \pi_{t+k|t}^1 + \rho^{k+2} (1-\rho) \pi_{t+k|t}^2 + \dots \right] \\ &= \rho^k \mathbb{E}_t \left[\pi_{t+k|t} \right].\end{aligned}$$

Similarly, we obtain

$$\begin{aligned}\mathbb{E}_t^{agg}\hat{q}_{t+k} &= \rho^k \mathbb{E}_t \left[\hat{q}_{t+k|t} \right], & \mathbb{E}_t^{agg}\hat{\theta}_{t+k+1}^* &= \rho^k \mathbb{E}_t \left[\hat{\theta}_{t+k+1|t}^* \right], \\ \mathbb{E}_t^{agg}\hat{i}_{t+k} &= \rho^k \mathbb{E}_t \left[\hat{i}_{t+k|t} \right], & \mathbb{E}_t^{agg}r_{t+k}^* &= \rho^k \mathbb{E}_t \left[r_{t+k|t}^* \right].\end{aligned}$$

Thus, we have

$$\begin{aligned}\mathbb{E}_t \left[\pi_{t+k|t} \right] &= \rho^{-k} \mathbb{E}_t^{agg} \pi_{t+k}, & \mathbb{E}_t \left[\hat{q}_{t+k|t} \right] &= \rho^{-k} \mathbb{E}_t^{agg} \hat{q}_{t+k}, \\ \mathbb{E}_t \left[\hat{i}_{t+k|t} \right] &= \rho^{-k} \mathbb{E}_t^{agg} \hat{i}_{t+k}, & \mathbb{E}_t \left[\hat{r}_{t+k|t}^* \right] &= \rho^{-k} \mathbb{E}_t^{agg} \hat{r}_{t+k}^*,\end{aligned}$$

and the k -step ahead domestic real interest rate becomes

$$\mathbb{E}_t[\hat{r}_{t+k|t}] = \mathbb{E}_t[\hat{i}_{t+k|t}] - \mathbb{E}_t[\pi_{t+k+1|t}]. \quad (\text{J.5})$$

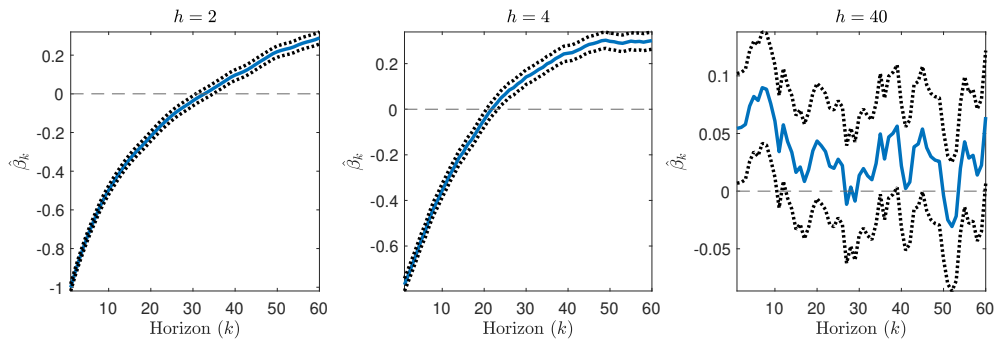
We can then construct $D_t^S(M) = \sum_{k=0}^{M-1} \mathbb{E}_t[\hat{r}_{t+k|t}^* - \hat{r}_{t+k|t}]$ and $D_t^L(M) = \sum_{k=M}^{\infty} \mathbb{E}_t[\hat{r}_{t+k|t}^* - \hat{r}_{t+k|t}]$.

For the numerical exercise, we set $\rho = 8/9$ so that the average planning horizon in the economy is $\rho/(1-\rho) = 8$ quarters, the same as the planning horizon of the homogeneous agents in Section 4.¹⁸ The Figure J.9 illustrates the related regression coefficients. Clearly, the coefficients for excess return predictability across various time horizons display a pattern similar to our benchmark results, as shown in Section 5. Additionally, there is a breakdown of horizon invariance, as depicted in the second and third panels. One observation is that the long-run reaction coefficient of the real exchange rate, $\hat{\gamma}_S$, exhibits quantitatively minor variations across different thresholds. This is because the long-term expected real interest rate differential $D_t^L(M)$ also shows relatively minor differences across these thresholds, due to the aggregation effect of the value functions across heterogeneous planning horizons.

¹⁸For numerical aggregation, we consider planning horizons ranging from 0 to 200. Additionally, we persist in nullifying the firms' learning in the value function.

Figure J.7: Planning Horizons $h \in \{2, 4, 40\}$

(a) Excess Return Predictability across Time Horizons



(b) Reaction of Real Exchange Rate to Short- and Long-Term Interest Rate Differentials

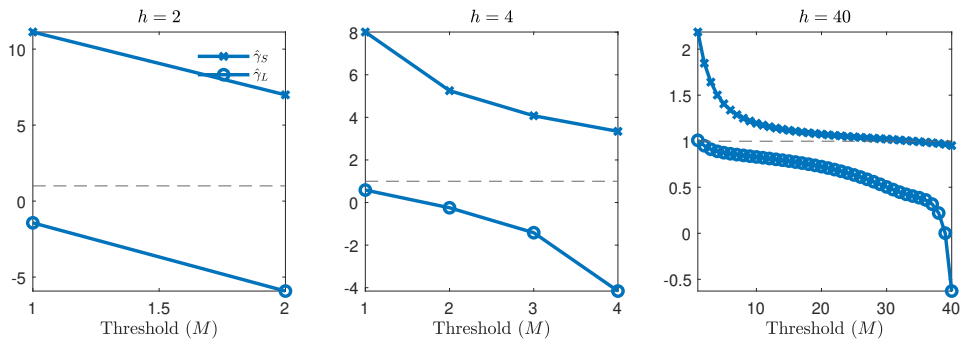
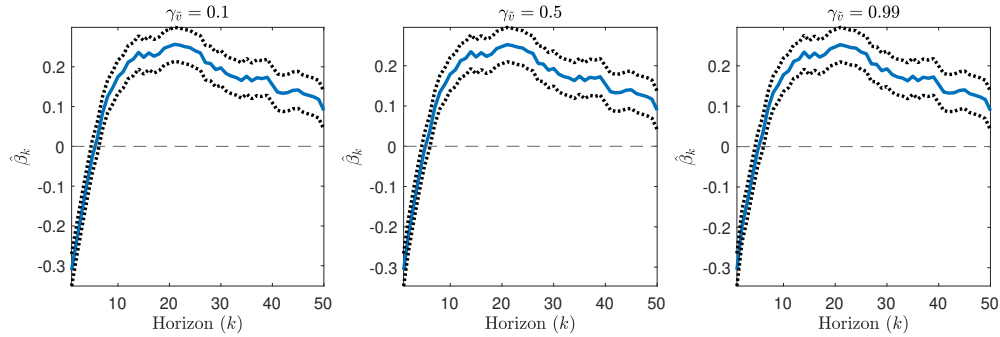


Figure J.8: Firm's Learning Gains $\gamma_{\tilde{v}} \in \{0.1, 0.5, 0.99\}$

(a) Excess Return Predictability across Time Horizons



(b) Reaction of Real Exchange Rate to Short- and Long-Term Interest Rate Differentials

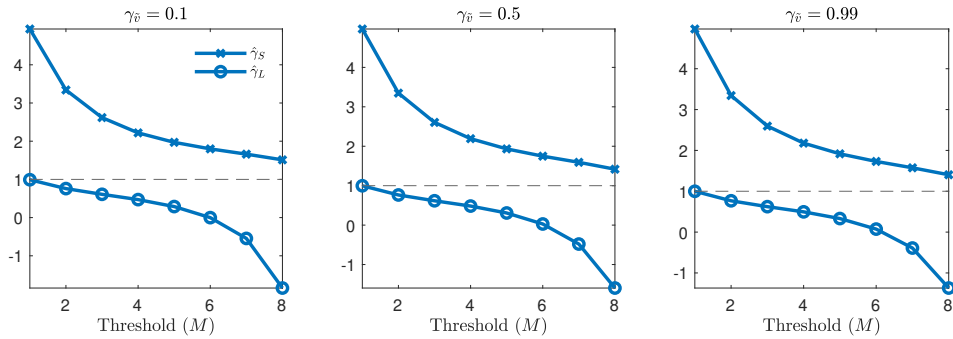


Figure J.9: Heterogeneous Planning Horizon across Agents with $\rho = 8/9$: Excess Return Predictability and Reactions of Real Exchange Rate

