

# Asset supply and liquidity transformation in HANK<sup>†</sup>

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## Abstract

We study how the financial sector affects fiscal and monetary policy in heterogeneous agent New Keynesian (HANK) economies. We show that, in a large class of models of financial intermediation, relevant features of the financial sector are summarized by the elasticities of a liquid asset supply function. The financial sector in these models affects aggregate responses only through its ability to perform liquidity transformation (i.e., issue liquid assets to finance illiquid capital). If liquid asset supply responds inelastically to returns on capital (low cross-price elasticities), disturbances in the liquid asset market generate large responses in aggregate demand through adjustments in capital prices. Assumptions about the financial sector are not innocuous quantitatively. In commonly used setups that imply different liquid asset supply elasticities, aggregate output responses to an unexpected deficit-financed government transfer can differ by a factor of three.

**Keywords:** financial frictions, liquidity, monetary and fiscal policy, HANK

**JEL code:** E2, E6, H3, H6

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# 1 Introduction

How does the financial sector interact with the real sector in shaping aggregate responses to macroeconomic policies, such as fiscal and monetary policies? One market that sits at the center of these issues is the liquid asset market. The liquid asset market is a broad term we use to describe markets for a set of closely linked assets such as deposits, treasury debt, and money market funds. This market is important because it is where monetary policy takes place, and fiscal policy is financed. Effectively, monetary policy works by affecting the prices of liquid assets, and the issuance of treasury debt directly increases the quantity of liquid assets. In almost all economic models, prices and quantities of liquid assets are crucial for households' consumption-saving responses to macro policies. A large share of these assets is created by financial intermediaries who perform "liquidity transformation". They issue deposits in the liquid asset market to finance productive but illiquid capital. The extent of their activities is limited by financial frictions. However, in models of frictional financial intermediation, financial constraints arise due to various micro-foundations, and their exact form varies greatly between models. As a result, what features of the financial sector (if any) are crucial for the transmission of macroeconomic policies remains inconclusive.

In this paper, we characterize key features of the financial sector that affect aggregate response to fiscal and monetary policies. We provide a general framework of the financial sector and embed it in a heterogeneous-agent New Keynesian (HANK) economy in which the real sector is rich enough to study fiscal and monetary policy. The financial sector in our framework is general in that it nests a large class of models of frictional financial intermediation. Our main contribution is to reformulate equilibrium condition in this class of models as a demand-and-supply system and show that relevant features of the financial sector can be "summarized" by a *liquid asset supply function*: as far as aggregate outcomes are concerned, details of micro-foundations of financial frictions matter only to the extent that they lead to a different liquid asset supply function.

Our formulation allows us to study the role of the financial sector through two sets of key (intertemporal) elasticities that can be measured in the data: the own-price and cross-price elasticities of liquid asset supply. The own-price elasticities measure how much the financial sector responds to the rate of return on liquid assets and determine the strength of monetary policy. The cross-price elasticities tell us how much the financial sector responds to returns on capital, which is critical to understanding the interaction between the real sector and the liquid asset market. We show, that in some of the most commonly used setups, these two sets of elasticities are effectively governed by three parameters. The first parameter governs the own-price elasticities, the second governs the cross-price elasticities, and the third governs the "forward-looking" component of financial constraints. This representation is useful because it allows us to do a systematic comparison between various models of financial frictions by doing comparative statics with respect to just these three parameters.

A demand-and-supply system of the goods and assets markets allows us to decompose equilibrium responses into separate forces working through these markets. We decompose aggregate output responses to fiscal and monetary policy into three channels: (1) a direct effect of macro policies on aggregate demand, such as consumption responding to transfers, (2) an indirect effect of macro policies on aggregate demand through the liquid asset market, and (3) a modified Keynesian cross channel, where aggregate income feeds back into aggregate demand in general equilibrium. Channels (2) and (3) depend on the liquid asset market and crucially on *the cross-price elasticities*. When cross-price elasticities are low, the financial sector absorbs disturbances in the liquid asset market only when there are large changes in capital prices and expected returns on capital. Large changes in capital prices lead to strong investment responses. Under common specifications, an excess liquid asset supply (say, due to government debt issuance) increases capital prices and aggregate demand through channel (2). On the contrary, the Keynesian feedback (channel (3)) between aggregate income and demand is dampened. An increase in aggregate income absorbs excess liquid assets, reduces capital prices, and weakens aggregate demand response.

We demonstrate the quantitative importance of cross-price elasticities by studying an increase in deficit-financed government transfers. If the cross-price elasticities are low, a disturbance in the liquid asset market, such as an increase in government debt, leads to strong responses in capital returns, prices, and output. We target key moments on the household and financial sector balance sheets and compare output responses under different assumptions about the financial sector. Our model comparison ranges from one extreme where private liquid asset supply is perfectly inelastic (as in the standard HANK models) to another with perfectly elastic liquid asset supply. For the same shock, the aggregate output response on impact is three times larger when cross-price elasticities are zero compared to the perfectly elastic case. We decompose aggregate output response into the three channels discussed above and show that responses differ between models because movements in the liquid asset market generate drastically different adjustments in capital price and investment (Channel (2)). This exercise shows that assumptions about the financial sector, as summarized by the key elasticities in our framework, are not innocuous quantitatively.

## Literature

Our work is related to an extensive literature that emphasizes the importance of household heterogeneity in understanding the effects of macroeconomic policies (e.g. [Gornemann et al. \(2012\)](#), [McKay et al. \(2016\)](#), [Guerrieri and Lorenzoni \(2017\)](#), [Kaplan et al. \(2018\)](#)). We contribute to this literature by providing a framework that allows us to understand how general equilibrium responses in these models depend on assumptions about the financial market, as summarized by properties of asset supply curves. Our formulation of equilibrium as a demand-supply system is similar to [Auclert et al. \(2018\)](#). They show that households' intertemporal marginal propensity to consume summarizes the general equilibrium responses to fiscal policy in a wide range of heterogeneous-agent models. They interpret feedback between output and consumption as an *intertemporal Keynesian cross*. While their analysis builds on specific assumptions about the asset supply side, we provide a general

framework to model asset supply as a result of financial intermediation. One can interpret our framework as a version of *intertemporal IS-LM* model, in which the liquid asset supply curve reflects key features of the financial sector.

Our framework nests a large class of models with frictional financial intermediation with various micro-foundations. Models nested include frictions originating from asset diversion in [Gertler and Karadi \(2011\)](#), costly-state verification in [Bernanke et al. \(1999\)](#), and reduced-form leverage cost in [Cúrdia and Woodford \(2016\)](#), among other numerous variations. We contribute to this literature by identifying the key elasticities that summarize the financial sector’s features that matter for aggregate responses. We show that all micro-foundations of financial frictions in the models cited above generate different aggregate responses only to the extent they result in different elasticities of the private liquid asset supply curve.

Our paper is also related to a recent body of work that incorporates frictional financial intermediation into heterogeneous-agent models (e.g., [Lee et al. \(2020\)](#), [Fernández-Villaverde et al. \(2020\)](#), [Lee \(2021\)](#), [Mendicino et al. \(2021\)](#), [Faria-e Castro \(2017\)](#), [Schroth \(2021\)](#), [Ferrante and Gornemann \(2022\)](#)). The advantage of our unifying approach is that it allows us to study the role of financial intermediation in a rich HA model without having to take a stand on particular micro-foundations of financial frictions.

## 2 Model

Time is discrete,  $t \in \{0, \dots, \infty\}$ . A continuum of households make consumption-savings decisions under idiosyncratic income shocks. Households can save in liquid and illiquid assets. Households need to incur a portfolio adjustment cost to access the illiquid asset, but returns from the two assets are potentially different. A banking sector performs liquidity transformation by issuing liquid assets to finance productive capital. The extent of liquidity transformation is limited and depends on net worth of banks and a financial constraint.

## 2.1 Household

Households are indexed by  $i \in [0, 1]$ . Household  $i$  derives utility from final good consumption  $c_{i,t}$  and disutility from labor  $h_{i,t}$ . Preferences are time separable, and the future is discounted with factor  $\beta \in (0, 1)$ :

$$\max_{a_{i,t}, b_{i,t}, c_{i,t}} \mathbb{E} \sum_{t \geq 0} \beta^t u(c_{i,t}, h_{i,t}),$$

where household  $i$  chooses consumption  $c_{i,t}$  and trades in two types of assets  $a_{i,t}$  and  $b_{i,t}$ , with real returns  $r_t^A$  and  $r_t^B$  respectively. We interpret  $b_{i,t}$  as a liquid asset and  $a_{i,t}$  as an illiquid asset. Portfolio adjustment costs are captured by a function  $\Phi_{i,t}(a_{i,t}, a_{i,t-1})$ , which is potentially time-household specific. The budget constraint faced by household  $i$  is:

$$\begin{aligned} & a_{i,t} + b_{i,t} + c_{i,t} + \Phi_{i,t}(a_{i,t}, a_{i,t-1}) \\ &= (1 + r_t^A) a_{i,t-1} + (1 + r_t^B) b_{i,t-1} + (1 - \tau_t) \left( \frac{W_t}{P_t} z_{i,t} h_{i,t} \right)^{1-\lambda} \end{aligned}$$

We assume that

$$a_{i,t} \geq \underline{a}, \quad b_{i,t} \geq \underline{b}, \quad \int \Phi_{i,t} di = 0.$$

Expectations are taken over realizations of idiosyncratic earnings shocks  $z_{i,t}$ . There is no aggregate uncertainty. In each period, household  $i$  receives real after-tax income that depends on the nominal wage per efficiency unit of labor,  $W_t$ , the price of a unit of final good,  $P_t$ , and on  $\tau_t$  and  $\lambda$ , which control the average and marginal tax rates. Labor  $h_{i,t}$  is taken as exogenous by each household and is determined by monopolistically competitive labor unions to be described shortly below.

## 2.2 The financial sector

The illiquid asset  $a_t = \int a_{i,t} di$  is held as a passive mutual fund. The fund consists of the net worth of a representative bank  $n_t$  and the value of capital  $q_t k_t^F$ . The balance

sheet of the fund is given by

$$a_t = q_t k_t^F + n_t.$$

Let  $\nu_{t+1}$  denote the return on bank net worth and  $r_{t+1}^K$  the return on capital. The rate of return on illiquid assets is

$$r_{t+1}^A = \frac{1}{a_t} (r_{t+1}^K q_t k_t^F + \nu_{t+1} n_t). \quad (1)$$

The representative bank has the technology to issue liquid assets and holds illiquid capital. At time  $t$ , given net worth  $n_t$ , the bank issues net liquid asset  $d_t$ <sup>1</sup> and holds capital  $k_t^B$  to maximize its flow return  $\nu_{t+1}$ :

$$\nu_{t+1} n_t = \max_{k_t^B, d_t} r_{t+1}^K q_t k_t^B - r_{t+1}^B d_t$$

subject to their balance sheet and a financial constraint:

$$q_t k_t^B = d_t + n_t, \quad q_t k_t^B \leq \Theta_t \left( \{r_{s+1}^K, r_{s+1}^B; \theta_s\}_{s \geq t} \right) n_t$$

The existence of the bank allows households to finance capital without incurring portfolio adjustment costs  $\Phi_{i,t}$  when households need to liquidate assets quickly. This captures how banks perform *liquidity transformation* in the economy. Banks' ability to fund firms by issuing deposits is limited, and we represent this limit by the financial constraint. The degree of financial friction in the banking sector potentially depends on the entire path of future returns  $r_s^B$  and  $r_s^K$ , which captures the future funding cost and investment opportunities in the economy. The severity of the financial frictions is parameterized by exogenous shifters  $\theta_t$ . This forward-looking specification of the financial constraint allows us to nest agency problems as in [Gertler and Kiyotaki \(2010\)](#) and [Gertler and Karadi \(2011\)](#) but also common regulatory requirements as

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<sup>1</sup>We define a bank's net liquid assets issuance as its total liquid asset issuance minus its liquid asset holding (e.g. its holding of government debt). In our model, government debt and bank deposits are perfect substitutes, and it does not matter whether government debt is held directly by households or indirectly through banks.

special cases of our framework. We discuss this nesting property in Section 3.1 and in Appendix B in detail.

We assume that the bank follows an exogenous rule that pays out a fraction  $f$  of the accumulated net worth as dividends and receives a constant equity injection  $m$  from the fund.<sup>2</sup> The net worth of the banking sector evolves according to

$$n_{t+1} = (1 - f)n_t(1 + \nu_{t+1}) + m. \quad (2)$$

## 2.3 Production

*Final good production*

A representative firm operates a Cobb-Douglas technology to produce final good  $y_t$  using capital  $k_{t-1}$  and differentiated types of labor  $h_{\ell,t}$ ,  $\ell \in [0, 1]$ :

$$y_t = k_{t-1}^\alpha h_t^{1-\alpha}, \quad \text{where } h_t = \left( \int_0^1 h_{\ell,t}^{\frac{\varepsilon_W-1}{\varepsilon_W}} d\ell \right)^{\frac{\varepsilon_W}{\varepsilon_W-1}},$$

where  $h_{\ell,t} = \int z_{i,t} h_{i,\ell,t} di$  is supplied by labor union  $\ell$ , and  $\varepsilon_W > 1$  is the elasticity of substitution between labor types. Given wages  $W_{\ell,t}$  set by union  $\ell$  and the rental rate of capital  $R_t$ , firms rent capital and hire labor to maximize profit:

$$\max_{k_{t-1}, \{h_{\ell,t}\}} P_t y_t - R_t k_{t-1} - \int W_{\ell,t} h_{\ell,t} d\ell.$$

Capital in the economy consists of that held by the mutual fund directly and that held by the bank:

$$k_t = k_t^F + k_t^B.$$

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<sup>2</sup>We relax the assumption that  $m_t = m$  in Appendix A.2.1 and allow for  $m_t = \xi(1 + r_t^K) q_{t-1} n_{t-1}$ , where  $\xi$  is a constant, as in Gertler and Kiyotaki (2010).



Over time, capital evolves according to

$$k_t = (1 - \delta + \Gamma(\iota_t)) k_{t-1}, \quad \iota_t := \frac{x_t}{k_{t-1}}$$

where  $x_t, \iota_t$  denote the investment level and investment rate,  $\delta$  is the depreciation rate, and  $\Gamma(\cdot)$  captures capital adjustment cost. Holding capital over periods earns a return on capital

$$1 + r_{t+1}^K = \max_{\hat{i}_{t+1}} \frac{R_{t+1} + q_{t+1} (1 + \Gamma(\hat{\iota}_{t+1}) - \delta) - \hat{\iota}_{t+1}}{q_t},$$

where  $q_t$  is the price of capital.

### *Labor supply*

We model labor supply following [Schmitt-Grohé and Uribe \(2005\)](#) and [Auclert et al. \(2018\)](#). There is a continuum of labor unions indexed by  $\ell \in [0, 1]$ . Every household  $i$  provides  $h_{i,\ell,t}$  hours of work to each of a continuum of unions, so its total labor supply is  $h_{i,t} = \int_0^1 h_{i,\ell,t} d\ell$ . Each union then aggregates labor provided by all households into union-specific labor services:

$$h_{\ell,t} = \int_0^1 z_{i,t} h_{i,\ell,t} di$$

Nominal wage  $W_{\ell,t}$  for each labor type  $\ell$  is set by a monopolistically competitive labor union  $\ell$ . Given labor demand  $h_{\ell,t}(W_{\ell,t})$  from the goods producer, each labor union  $\ell$  calls all households to provide the same hours of work. Households commit to supply any amount of hours  $h_{\ell,t}$  to union  $\ell$  to meet labor demand at the given wage. Labor demand from the final goods producer implies that the labor income of each household  $i$  is

$$W_t z_{i,t} h_{i,t} = \int_0^1 W_{\ell,t} z_{i,t} h_{i,\ell,t} d\ell,$$

where  $W_t$  is the ideal wage index.

Each labor union  $\ell$  sets nominal wage growth rate  $\pi_{W,\ell,t} := \frac{W_{\ell,t}}{W_{\ell,t-1}} - 1$ , subject to

a quadratic adjustment cost to maximize utilitarian welfare of its members with uniform Pareto weights:

$$\sum_{t=0}^{\infty} \beta^t \left\{ \int_0^1 \{u(c_{i,t}, h_{i,t})\} di - \frac{\kappa_W}{2} \pi_{W,\ell,t}^2 d\ell \right\}.$$

Parameter  $\kappa_W > 0$  governs the wage adjustment cost, which introduces nominal rigidity in the economy. The wage adjustment cost is borne in units of utility by the labor union, and does not enter the resource constraint of the economy.

## 2.4 Government

The government sets a path for government purchases  $g_t$  and tax revenue (net of transfers)  $T_t$ .<sup>3</sup> Government liabilities  $b_t^G$  are liquid and evolve according to

$$b_t^G = (1 + r_t^B) b_{t-1}^G + g_t - T_t. \quad (3)$$

Tax revenue collected by the government is

$$T_t = \frac{W_t}{P_t} h_t - \int (1 - \tau_t) \left( \frac{W_t}{P_t} z_{i,t} h_{i,t} \right)^{1-\lambda} di.$$

The government adjusts  $\tau_t$  to collect its target net tax revenue  $T_t$ . As in [Woodford \(2011\)](#) and [Auclert et al. \(2018\)](#), we assume that the government sets the nominal interest rate  $i_t^B$  to keep the real interest rate equal to its desired level<sup>4</sup>

$$r_t^B = r_t.$$

This includes a special case with  $r_t = \bar{r}$ , where  $\bar{r}$  is the real return on liquid assets in the stationary equilibrium. Moreover, we assume that the real rate of return on liquid assets paid in period 0 is predetermined and cannot be affected by the

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<sup>3</sup>We focus on paths  $\{g_t, T_t\}$  that satisfy the intertemporal budget constraint of the government.

<sup>4</sup>This allows us to solve for equilibrium responses of real variables directly, and back out responses of nominal variables given the real variables from the wage Philips curve.

monetary authority.<sup>5</sup>

## 2.5 Equilibrium definition

Given government policy  $\{g_t, T_t, r_t\}$ , an equilibrium consists of prices  $\{q_t, P_t, R_t, \{W_{\ell,t}\}\}$  returns  $\{r_t^A, r_t^B, r_t^K\}$ , aggregate allocations  $\{y_t, k_t, h_t, n_t, d_t, a_t, k_t^F, k_t^B, b_t^G, x_t, \iota_t\}$ , individual allocations  $\{a_{i,t}, b_{i,t}, c_{i,t}, \{h_{i,\ell,t}\}\}$  such that: (1) households choose  $a_{i,t}, b_{i,t}, c_{i,t}$  to maximize utility subject to budget constraints; (2) firms choose  $y_t, \{h_{\ell,t}\}, k_{t-1}$  to maximize profit, (3)  $\{W_{\ell,t}\}$  maximize payoff of the labor unions; (4)  $\iota_t := x_t/k_t$  maximizes the return on capital (5) the bank chooses  $k_t^B, d_t$  to maximize return on net worth subject to its financial constraint and balance sheet, and aggregate dynamics of the banking sector are satisfied; (6) balance sheet of the fund holds and  $r^A$  is given by the weighted returns on capital and banking net worth; (7) the government budget constraint holds, and (8) markets clear:

$$\begin{aligned} \int c_{i,t} di + x_t + g_t &= y_t, \\ \int b_{i,t} di &= d_t + b_t^G, \\ \int a_{i,t} di &= q_t k_t^F + n_t, \\ k_t^F + k_t^B &= k_t, \end{aligned}$$

where (i) in the goods market, aggregate output equals the total of aggregate consumption, goods used for investment, and government purchases; (ii) the liquid asset market clears when the sum of public (government liabilities) and private (net liquid assets issued by banks) liquid asset equals the total households' holdings of liquid assets; and (iii) the illiquid asset market clearing condition is satisfied when the fund net worth equates the total households' holdings of illiquid assets; (iv) bank's and fund's holdings of capital are equal to the aggregate stock of capital. Finally, labor market clearing is embedded in the notation.

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<sup>5</sup>Liquid assets are real and there is no default.

### 3 A Supply and Demand Representation

We now represent the aggregate behavior of households, banks, and firms as a demand and supply system. For each type of agent in this economy, given aggregate variables, we can solve their problem along the transition path. For example, given the path of aggregate output, taxes, and returns on assets, we can obtain households' demand for consumption goods and both types of assets. Together with the initial asset distribution, this delivers aggregate demand for liquid and illiquid assets and aggregate demand for consumption goods. These functions contain all relevant information about household heterogeneity. We obtain similar supply and demand functions for firms and the bank. In particular, given the path of returns on assets, we solve the bank's problem and obtain its demand for capital and supply of liquid assets (i.e. its deposit issuance). With this representation, we show that, as far as aggregate dynamics are concerned, all differences between particular details of the financial sector matter only to the extent that they imply a different liquid asset supply function.

**Lemma 1** *The following supply and demand system characterizes an equilibrium of the economy:*

$$\begin{aligned} \mathcal{C}_t(\{y_s, r_s^A; r_{s+1}^B, T_s\}_{s=0}^\infty) + \mathcal{X}_t(\{y_s, r_{s+1}^K\}_{s=0}^\infty) + g_t &= y_t, \\ \mathcal{B}_t(\{y_s, r_s^A; r_{s+1}^B, T_s\}_{s=0}^\infty) &= \mathcal{D}_t(\{y_s, r_{s+1}^K; r_{s+1}^B\}_{s=0}^\infty) + b_t^G, \end{aligned}$$

where  $\{y_t, r_{t+1}^K\}_{t=0}^\infty$  are endogenous variables to be solved for,  $b_t^G$  satisfies Equation 3, and

$$r_{t+1}^A = \mathcal{R}_{t+1}^A(\{y_s, r_{s+1}^K; r_{s+1}^B\}_{s=0}^\infty, \mathcal{D}_t(\{y_s, r_{s+1}^K; r_{s+1}^B\}_{s=0}^\infty)).^6$$

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<sup>6</sup>Note that in Lemma 1 we suppress dependence on time 0 returns,  $r_0^K, r_0^B$ . We assume that  $r_0^B$  is predetermined (liquid assets are real).  $r_0^K$  is not predetermined, and any change in capital prices or output at time 0 will move them. However, unlike returns at  $t \geq 1$ , these returns do not directly affect agents' incentives but only change households' initial wealth and banks' initial net worth.  $r_0^K$  returns can be expressed as a function of  $\{y_t, r_{t+1}^K\}_{t=0}^\infty$ . Its dependence on  $\{y_t, r_{t+1}^K\}_{t=0}^\infty$  is embedded in functions  $\mathcal{X}_t(\cdot), \mathcal{D}_t(\cdot)$ . Similarly,  $r_0^A$  affects households only by its effect on their

*Proof.* See Appendix A.1. □

Lemma 1 characterizes an equilibrium with two market clearing conditions - the goods market and the liquid asset market. The *aggregate private liquid asset supply* function resulting from the bank’s decisions,  $\mathcal{D}_t(\cdot)$ , is the key object in our analysis.  $\mathcal{C}_t(\cdot)$ ,  $\mathcal{B}_t(\cdot)$   $\mathcal{X}_t(\cdot)$  are the *aggregate consumption function*, the *aggregate liquid asset demand* and the *aggregate investment function*. These functions result from the household and production blocks of the model, and  $\mathcal{R}_t^A(\cdot)$  corresponds to the accounting identity in Equation 1. Note that we can focus on the two markets — the goods market and the liquid asset market — because the illiquid asset market clearing condition is redundant by the Walras’ law. In principle, one can reformulate Lemma 1 with any two of the three markets. We choose to focus on the goods market because fiscal policy (government purchases and tax changes) operates directly in the goods market, and the role of marginal propensities to consume in determining its effects is well-understood (eg. Auclert et al. (2018)). We focus on the liquid asset market because it is the market where monetary policy directly influences the price,  $r_t^B$ , and where fiscal policy obtains debt financing.

Lemma 1 demonstrates a crucial step of our analysis: all relevant properties of the financial sector are summarized by  $\mathcal{D}_t(\{y_s, r_{s+1}^K; r_{s+1}^B\}_{s=0}^\infty)$ . This result is closely related to that from Auclert et al. (2018), who demonstrate that household heterogeneity matters only insofar it determines  $\mathcal{C}_t(\cdot)$  and its dependence on after-tax income. They focus on special cases in which goods market clearing is sufficient, and the liquid asset market is redundant.<sup>7</sup> By contrast, the liquid asset market plays a key role in understanding how the financial sector affects aggregate responses. The financial sector affects the transmission between the liquid asset and final goods markets. Various specifications of the financial sector matter only insofar as they determine  $\mathcal{D}_t(\cdot)$ . We now proceed to show how the liquid asset supply function  $\mathcal{D}_t(\{y_s, r_{s+1}^K; r_{s+1}^B\}_{s=0}^\infty)$

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wealth. It can be expressed as a function of  $r_0^K$  and  $r_0^B$  by using Equation 1. For the purpose of exposition we do not do it in Lemma 1, because it would introduce dependence of households’ functions on  $\mathcal{D}_t(\cdot)$ . See the proof of Lemma 1 in Appendix A.1 for details.

<sup>7</sup>In their baseline analysis, Auclert et al. (2018) assume that all assets pay the same real return, and the return is fully controlled by monetary policy.

depends on assumptions about the financial frictions  $\Theta_t(\cdot)$ .

### 3.1 Liquid Asset Supply: Nesting models of financial frictions

We characterize the liquid asset supply function  $\mathcal{D}_t(\cdot)$  when the underlying financial frictions result in  $\Theta_t(\cdot)$  that has a special structure. This structure is present in most common models of financial frictions that our framework nests.

**Lemma 2** *Suppose that  $\Theta_t(\cdot)$  satisfies*

$$\frac{\partial \Theta_t}{\partial r_{s+1}^K} = \gamma^{s-t} \bar{\Theta}_{r^K}, \quad \frac{\partial \Theta_t}{\partial r_{s+1}^B} = -\gamma^{s-t} \bar{\Theta}_{r^B}, \quad \forall s \geq t,$$

with  $\gamma \in [0, 1)$  and  $\bar{\Theta}_{r^K}, \bar{\Theta}_{r^B} \geq 0$ . Then

$$\begin{aligned} \mathbf{D}_{r^K} &= \bar{\Theta}_{r^K} \mathbf{N}(\gamma) + (\bar{\Theta} - 1) \bar{\Theta} [\mathbf{N}_0 + (1 - \delta) \mathbf{n}_0 \mathbf{q}_{r^K}], \\ \mathbf{D}_{r^B} &= -\bar{\Theta}_{r^B} \mathbf{N}(\gamma) - (\bar{\Theta} - 1)^2 \mathbf{N}_0, \\ \mathbf{D}_y &= (\bar{\Theta} - 1) \bar{\Theta} \mathbf{n}_0 \left[ \frac{\alpha}{\bar{k}} \mathbf{e}_1 + (1 - \delta) \mathbf{q}_y \right], \end{aligned}$$

where the  $(t, s)$  elements of  $\mathbf{D}_{r^K}, \mathbf{D}_{r^B}, \mathbf{D}_y$  are the first-order responses of  $\mathcal{D}_t$  to expected returns and aggregate output in period  $s$ .  $\bar{\Theta} \geq 1$  is the steady state leverage.  $\mathbf{N}_0, \mathbf{N}(\gamma)$  are nonnegative matrices with entries pinned down by  $\gamma$  and steady-state levels of variables.  $\mathbf{n}_0$  is a column vector such that its  $t^{\text{th}}$  element satisfies  $\mathbf{n}_{0(t)} = \mathbf{N}_{0(t+1,1)}$ .  $\mathbf{q}_{r^K}$  and  $\mathbf{q}_y$  are row vectors capturing how the initial capital price depends on returns and output. They depend only on the production side of the economy.  $\mathbf{e}_1$  is a row vector with 1 as its first entry, and zeros elsewhere and  $\bar{k}$  is the steady-state level of capital.

*Proof.* See Appendix A.2. □

Lemma 2 shows that the key elasticities of (intertemporal) liquid asset supply are

governed by a few key parameters given the special structure of  $\Theta_t(\cdot)$ . The structure of  $\Theta_t(\cdot)$  implies that the timing of changes in returns does not matter, but only how far they are in the future. Parameters  $\bar{\Theta}_{r^K}, \bar{\Theta}_{r^B}$  capture how much banks leverage up when there is a change in future returns. They are the key determinants of the cross-price and own-price elasticities of liquid asset supply  $\mathcal{D}_t$  with respect to future returns. Since  $\mathbf{N}(\gamma)$  is nonnegative, we can see that entries of  $\mathbf{D}_{r^K}$  are weakly increasing in  $\bar{\Theta}_{r^K}$  and those of  $\mathbf{D}_{r^B}$  weakly decreasing in  $\bar{\Theta}_{r^K}$ .<sup>8</sup> In this sense larger values of  $\bar{\Theta}_{r^K}, \bar{\Theta}_{r^B}$  correspond to larger elasticities. The parameter  $\gamma$  captures how much the impact of future returns decays with the distance in time  $s - t$ . Matrix  $\mathbf{N}_0$  characterizes the change in liquid asset supply due to net worth accumulation (holding leverage constant), and  $\mathbf{N}(\gamma)$  summarizes how the response of leverage to changes in future returns today propagates over time. Column vector  $\mathbf{n}_0$  is closely related to  $\mathbf{N}_0$  and reflects the effects of a change in net worth at  $t = 0$ . Finally, the structure of  $\mathbf{D}_y$  shows that holding returns constant, liquid asset supply  $\mathcal{D}_t$  depends on output only through its effect on the initial price of capital  $q_0$  and the rental rate at  $t = 0$ . Therefore,  $\mathbf{D}_y$  is pinned down by the production function and the capital adjustment cost function  $\Gamma(\cdot)$ , and does not depend on  $\Theta_t$ .

Lemma 2 also allows us to understand how different models of financial frictions in the literature lie within this framework through the structure of liquid asset supply elasticities  $\mathbf{D}_{r^K}, \mathbf{D}_{r^B}$ . Regulatory requirements that put an upper bound on the leverage ratio  $\Theta_t$  (e.g., [Van den Heuvel \(2008\)](#)) correspond to a case where  $\bar{\Theta}_{r^K} = \bar{\Theta}_{r^B} = 0$ , and net worth accumulation,  $\mathbf{N}_0$ , is all that matters for liquid asset supply. Models of costly leverage (such as [Cúrdia and Woodford \(2016\)](#)) impose a tight link between today's leverage ratio and the expected spread between  $r^K$  and  $r^B$  tomorrow. At the same time,  $\Theta_t$  does not depend on returns more than one period ahead. This corresponds to  $\bar{\Theta}_{r^K}, \bar{\Theta}_{r^B} \neq 0$ , but  $\gamma = 0$ . In this case, liquid asset supply elasticities contain no forward-looking component:  $\mathbf{N}(\gamma)$  is lower triangular if  $\gamma = 0$ . The same is true for the model of [Bernanke et al. \(1999\)](#), in which financial frictions arise due to costly state verification. Models in which agency problems create a link between

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<sup>8</sup>For  $\gamma > 0$   $\mathbf{D}_{r^K}$  is strictly increasing and  $\mathbf{D}_{r^B}$  strictly decreasing.

the leverage ratio and the continuation value of a bank, most notably [Gertler and Kiyotaki \(2010\)](#), [Gertler and Karadi \(2011\)](#), corresponds to the case where all three parameters are at work:  $\bar{\Theta}_{r^K}, \bar{\Theta}_{r^B}, \gamma \neq 0$ . Finally, when there are no frictions in the banking sector and banks can immediately exploit arbitrage opportunities, we have  $\bar{\Theta}_{r^K}, \bar{\Theta}_{r^B} \rightarrow \infty$  and  $\gamma = 0$ . We provide a detailed derivation of the nesting result for each model in [Appendix B](#).

## 4 Aggregate Responses with Financial Frictions

We now study how the financial sector affects aggregate responses to government policies through the liquid asset market. Given government policies, we solve for the first-order responses of output and real return on capital — the two endogenous variables in [Lemma 1](#) determined in equilibrium. We focus on transitory policies such that  $\lim_{t \rightarrow \infty} dg_t = \lim_{t \rightarrow \infty} dT_t = \lim_{t \rightarrow \infty} dr_t^B = \lim_{t \rightarrow \infty} db_t^G = 0$  and the unique equilibrium for which first-order deviations of all variables go to zero as  $t \rightarrow \infty$ . To simplify notation, we use  $d\mathbf{r}^B$  to represent  $\{dr_{s+1}^B\}_{s=0}^{\infty}$ , the sequence of liquid rates paid in period  $s + 1$  for all  $s = 0, \dots, \infty$ , and similarly for  $d\mathbf{r}^K$ . We use  $d\mathbf{y}$  to represent  $\{dy_s\}_{s=0}^{\infty}$ , the sequence of output in period  $s, \forall s = 0, \dots, \infty$ , and similarly for  $d\mathbf{T}, d\mathbf{b}^G$  and all other variables. These sequences are represented as column vectors. By evaluating derivatives of aggregate functions  $\mathcal{X}_t(\cdot), \mathcal{B}_t(\cdot), \mathcal{C}_t(\cdot), \mathcal{D}_t(\cdot)$  and  $\mathcal{R}_t^A(\cdot)$  at the steady state, we obtain matrices such as  $\mathbf{C}_y$ , of which the  $(t, s)$  element is  $\partial \mathcal{C}_t / \partial y_s$ . In [Appendix A.3](#) we show the linearized version of equilibrium conditions stated in [Lemma 1](#) and details of these matrices.

We characterize the equilibrium in two steps. First, we take  $d\mathbf{y}$  as given and study how returns on capital  $d\mathbf{r}^K$  have to adjust to clear the liquid asset market given government policies. We then use the solution for  $d\mathbf{r}^K$  as a function of  $d\mathbf{y}$  (and government policies) to find the path of output that satisfies the goods market clearing condition.



## 4.1 Liquidity Transformation and Excess Liquid Asset Demand

An equilibrium in the liquid asset market is reached when the liquid asset demand from the households  $\mathcal{B}_t$  equals to liquid assets created by the financial sector, either through liquidity transformation,  $\mathcal{D}_t$ , or issued against government debt  $b_t^G$ . But note that the price of liquid assets,  $d\mathbf{r}^B$ , is determined exogenously by the monetary policy. Shifts in the liquid asset demand and supply due to government policies and changes in output will need to be offset by an adjustment in returns on capital  $d\mathbf{r}^K$ . We define *excess liquid asset demand* as

$$\mathcal{B}_t(\cdot) - b_t^G - \mathcal{D}_t(\cdot),$$

and we ask, for any shift in the excess liquid asset demand, by how much the path of returns on capital  $d\mathbf{r}^K$  will have to adjust to ensure market clearing.

We characterize responses in the liquid asset market using the following elasticities of excess liquid asset demand,  $\boldsymbol{\epsilon}$ 's, with respect to shifts in returns  $d\mathbf{r}^K$ ,  $d\mathbf{r}^B$ , output  $d\mathbf{y}$ , and tax  $d\mathbf{T}$ :

$$\begin{aligned} \boldsymbol{\epsilon}_{r^K} &:= \mathbf{B}_{r^A} \mathbf{R}_{r^K}^A + \mathbf{B}_{r_0^A, r^K} - \mathbf{D}_{r^K}, & \boldsymbol{\epsilon}_{r^B} &:= \mathbf{B}_{r^B} + \mathbf{B}_{r^A} \mathbf{R}_{r^B}^A - \mathbf{D}_{r^B}, \\ \boldsymbol{\epsilon}_y &:= \mathbf{B}_y + \mathbf{B}_{r^A} \mathbf{R}_y^A + \mathbf{B}_{r_0^A, y} - \mathbf{D}_y, & \boldsymbol{\epsilon}_T &:= \mathbf{B}_T. \end{aligned}$$

The first two matrices,  $\boldsymbol{\epsilon}_{r^K}$  and  $\boldsymbol{\epsilon}_{r^B}$ , are directly linked to key elasticities of the liquid asset supply.  $\boldsymbol{\epsilon}_{r^K}$  depends on the cross-price elasticity  $\mathbf{D}_{r^K}$  governed by  $\bar{\Theta}_{r^K}$ .  $\boldsymbol{\epsilon}_{r^B}$  depends on the own-price elasticity  $\mathbf{D}_{r^B}$ , which is controlled by  $\bar{\Theta}_{r^B}$ . Lemma 2 shows that entries of  $\mathbf{D}_{r^K}$  are nondecreasing in  $\bar{\Theta}_{r^K}$ . Similarly, entries of  $\mathbf{D}_{r^B}$  are nonincreasing in  $\bar{\Theta}_{r^B}$ . When these two parameters are large, excess liquid asset demand becomes more sensitive to changes in returns. Matrices  $\mathbf{R}_{r^K}^A$ ,  $\mathbf{R}_{r^B}^A$ , and  $\mathbf{R}_y^A$  reflect the accounting identity (1), in which the illiquid rate  $r^A$  depends on  $r^K$  and

$r^B$ .<sup>9</sup>

**Proposition 1** *In equilibrium, returns on capital satisfy*

$$d\mathbf{r}^K = (-\epsilon_{r^K})^{-1}[-d\mathbf{b}^G + \epsilon_T d\mathbf{T} + \epsilon_{r^B} d\mathbf{r}^B + \epsilon_y d\mathbf{y}]. \quad (4)$$

*Proof.* See Appendix A.4. □

In Proposition 1, we show how returns on capital respond to shifts in excess liquid asset demand due to exogenous policies and aggregate output. Intuitively, an increase in liquid asset supply (a decrease in excess liquid asset demand) pushes up the relative price of capital to liquid assets. At the original prices, households would be holding too much liquid assets, so they substitute towards illiquid assets (i.e., capital). In consequence, the relative price of capital now goes up. High capital price today means that the rate of return on capital will be low, diminishing banks' incentives to perform liquidity transformation. This force, the strength of which depends on the cross-price elasticity, reduces the endogenous supply of net liquid assets, helping to clear the liquid assets market.

The cross-price elasticity of excess liquid asset demand captures how shifts in excess liquid asset demand affect returns on capital consistent with liquid asset market clearing:

$$(-\epsilon_{r^K})^{-1} = (\mathbf{D}_{r^K} - \mathbf{B}_{r^A} \mathbf{R}_{r^K}^A - \mathbf{B}_{r_0^A, r^K})^{-1}.$$

Matrix  $\mathbf{B}_{r^A}$  captures the underlying features of the household side. Negative entries of this matrix correspond to substitutability between liquid and illiquid assets: households are willing to pay a relatively higher price for illiquid assets (accept lower expected illiquid return) if they have more liquid assets. If an entry of  $\mathbf{B}_{r^A} \mathbf{R}_{r^K}^A + \mathbf{B}_{r_0^A, r^K}$  is negative, an increase in the amount of liquid assets they hold makes households willing to accept a lower return on capital. If the elasticity is close to zero, a small

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<sup>9</sup>Matrices  $\mathbf{B}_{r_0^A, r^K}$  and  $\mathbf{B}_{r_0^A, y}$  capture the dependence of the household block on  $dr_0^A$ . We treat time 0 returns in a special way, because our notation for deviations of returns is such that  $d\mathbf{r}^A$  represents  $\{dr_{s+1}^A\}_{s=0}^\infty$ , the sequence of returns rates paid in period  $s+1$  for all  $s=0, \dots, \infty$ .  $dr_0^A$  can be expressed as a function of  $d\mathbf{r}^K$  and  $d\mathbf{y}$ .

increase in liquid assets makes households willing to hold capital at a much higher price with low expected returns.

On the other hand, banks finance capital by issuing liquid assets. The sensitivity of liquidity transformation to returns on capital is summarized by  $\mathbf{D}_{r^K}$ . When banks have low cross-price elasticities (when parameter  $\Theta_{r^K}$  is lower, and thus entries of  $\mathbf{D}_{r^K}$  are small), they can absorb changes in excess liquid asset demand only if returns on capital change a lot.

## 4.2 Aggregate Output Response

We now study how the financial sector affects the response of aggregate output to government policies. In Section 4.1, we showed that its characteristics matter because they result in different responses of  $d\mathbf{r}^K$ . We use the following semi-elasticities,  $\Psi$ 's, to understand how the aggregate demand  $\mathcal{C}_t(\cdot) + \mathcal{X}_t(\cdot) + g_t$  responds to the aggregate income  $d\mathbf{y}$ , returns on capital  $d\mathbf{r}^K$ , and government policies:

$$\begin{aligned}\Psi_{r^K} &:= \mathbf{C}_{r^A} \mathbf{R}_{r^K}^A + \mathbf{C}_{r_0^A, r^K} \mathbf{X}_{r^K}, & \Psi_{r^B} &:= \mathbf{C}_{r^B} + \mathbf{C}_{r^A} \mathbf{R}_{r^B}^A, \\ \Psi_y &:= \mathbf{C}_y + \mathbf{C}_{r^A} \mathbf{R}_y^A + \mathbf{C}_{r_0^A, y} + \mathbf{X}_y, & \Psi_T &:= \mathbf{C}_T.\end{aligned}$$

These matrices are predominantly determined by the household and production sector of the economy. For example,  $\Psi_y$  is a nonnegative matrix that contains marginal propensities to consume,  $\mathbf{C}_y$ , and to invest,  $\mathbf{X}_y$ . Similarly,  $\Psi_{r^K}$  describes how the aggregate demand responds to  $d\mathbf{r}^K$ . For a standard calibration, this matrix is mostly negative: households reduce consumption when returns on illiquid assets go up; higher expected returns on capital, holding output constant, are associated with lower capital price and lower investment.<sup>10</sup>

We totally differentiate the demand and supply functions in the goods market clearing condition and use the expression for returns on capital from Proposition 1 to characterize the aggregate output response as follows:

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<sup>10</sup>Holding  $y_{t+1}$  fixed, returns on capital,  $r_{t+1}^K$ , is high if  $k_t$  is low.  $r_{t+1}^K$  is also high, when  $q_t$  is low relative to  $q_{t+1}$ . Capital prices are positively related to investment rates.

**Theorem 1** Given  $\{dr^B, d\mathbf{T}, db^G, d\mathbf{g}\}$ , the aggregate output response is given by:

$$d\mathbf{y} = \underbrace{\left(I - \Psi_y - \Omega \epsilon_y\right)^{-1}}_{(3)} \times \left( \underbrace{d\mathbf{g} + \Psi_T d\mathbf{T} + \Psi_{r^B} dr^B}_{(1)} + \underbrace{\Omega(-db^G + \epsilon_T d\mathbf{T} + \epsilon_{r^B} dr^B)}_{(2)} \right),$$

where

$$\Omega := \Psi_{r^K}(-\epsilon_{r^K})^{-1}.$$

*Proof.* See Appendix A.5. □

Aggregate output responds to macroeconomic policies through three channels. The first channel (1) works directly in the goods market. It is a traditional channel through which investment and consumption respond to fiscal and monetary policy. The second (2) channel works through the liquid asset market. It captures how aggregate demand responds to shifts in excess liquid asset demand. The third channel (3) is a modified Keynesian cross shown in the formula as a multiplier. The modified Keynesian cross captures how aggregate demand responds to aggregate income through the goods and liquid asset markets. Channels (2) and (3) depend on the liquid asset market and crucially on the cross-price elasticities: the smaller they are, the larger the adjustment in capital prices and thus returns on capital, and the larger the aggregate demand responses.

The mechanism through which the liquid asset market affects aggregate output highlights the tight connection between monetary and fiscal policy. Consider, for example, the effect of an increase in government debt through channel (2). An increase in government debt (liquid asset supply) would have led to an increase in the real rate of return on liquid assets in a flexible price economy. However, with nominal rigidities, the liquid rate depends on the monetary policy. Suppose monetary policy targets a constant real rate (or adopts an even more accommodative policy). In that case, it will not increase the nominal rate enough to replicate the flexible price equilibrium, and there will be an increase in output due to higher inflation and lower real wage

— all these events triggered by an increase in government debt. As monetary policy stimulates the production of final goods to keep the liquid rate at the original level, it also increases the capital price as households and banks substitute for illiquid assets. High capital price generates more investment (capital supply from firms), and the unexpected gain in capital price allows the financial sector to hold more capital with less net liquid liability. The size of the indirect effect depends on the cross-price elasticities. The lower they are, the stronger the investment response to the same increase in government debt.

Matrix  $\Omega = \Psi_{r,K}(-\epsilon_{r,K})^{-1}$  captures how excess liquid asset demand affects aggregate output. Suppose an entry in  $(-\epsilon_{r,K})^{-1}$  is positive — meaning that an *fall* in excess liquid asset demand leads to higher demand for capital (a willingness to accept higher capital prices and lower expected returns on capital  $d\mathbf{r}^K$  at the original holding). And suppose the corresponding entry in  $\Psi_{r,K}$  is negative — meaning that an increase in capital price leads to more investment. Then through these entries, a fall in excess liquid asset demand leads to higher capital prices and increases aggregate demand.

The same mechanism works in channel (3), although it serves as a force that modifies the traditional Keynesian cross logic: when aggregate income increases, households increase their demand for liquid assets. If multiplied by a positive entry in  $\Omega$ , an increase in liquid asset demand tends to lower capital price (increase expected returns on capital) and decrease aggregate demand through investment. Therefore, a positive entry in  $\Omega$  is associated with a dampening force to the Keynesian cross logic, and the dampening force is more substantial with lower cross-price elasticities.

## 5 A Quantitative Study of Deficit-Financed Government Transfer

We demonstrate the importance of the key elasticities of liquid asset supply with a quantitative study of a deficit-financed government transfer shock. Our framework

is particularly suitable for studying such an event for two reasons. First, household heterogeneity is central to understanding the consumption-savings response to a transfer. Second, the financial sector is key to understanding the impact of increasing government debt and how it interacts with capital accumulation, given the monetary policy regime. The aggregate output response depends on the interaction between the real and financial sectors, as captured by the formula in Theorem 1, and especially on the  $\Omega$  matrix, which links the liquid asset market to aggregate demand. Because the key elasticities in the class of financial sector models we study hinge on a few parameters (Lemma 2), we can analyze this class of models by showing comparative static of aggregate responses with respect to these parameters. Moreover, we use the formula in Theorem 1 to decompose aggregate output response into different channels and show the strength of each channel depends crucially on features of the financial sector.

## 5.1 Policy Experiment and Comparative Static

We consider a transitory increase in government transfer financed by the issuance of government debt as follows:

$$d\tilde{T}_t = \rho_T d\tilde{T}_{t-1}, \quad db_t^G = -d\tilde{T}_t + \rho_{bG} db_{t-1}^G.$$

where gross transfer in each period is financed by the issuance of government debt of the same amount. Here  $d\tilde{T}_t$  is an exogenous sequence with negative values denoting there is a gross transfer. Parameter  $\rho_T$  specifies the persistence of transfer, and  $\rho_{bG}$  specifies how much the government roll over existing debt to the next period. Given the gross transfer and debt policy, we solve for the net tax/transfer  $dT$  that balances government budget constraints. For our policy experiment, we set  $\rho_T = .5$  and  $\rho_{bG} = .95$ . Under this policy, the government pays out 90% of the gross transfer in one year, and it takes around four years for government debt to return to half of its peak level. We assume that the monetary policy sets the real liquid rate constant at its steady-state level (therefore, the nominal rate adjusts one-to-one with expected inflation). This assumption allows us to focus on the role of the cross-price elasticity

of asset supply. Figure 1 shows the path of exogenous government policies we study in this section.

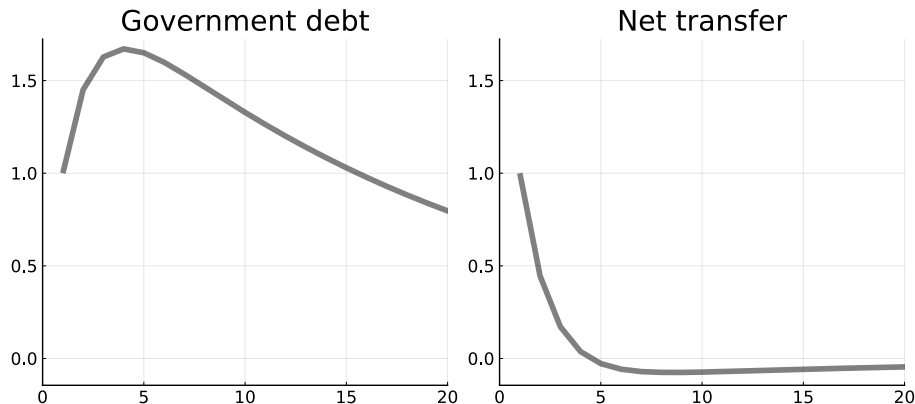


Figure 1: Exogenous government debt and net transfer policy; x-axis: quarters, y-axis: % of steady-state quarterly GDP.

For a comparative static concerning the key features of the financial sector, we start with  $\Theta_t(\cdot)$  implied by the financial sector in [Gertler and Karadi \(2011\)](#) as a baseline. We then depart from the baseline by increasing the cross-price elasticity of asset supply. Moreover, We compare it to a case in which the liquid asset supply is perfectly elastic and to a model in which the private liquid asset supply is perfectly inelastic. Steady-state equilibrium in all these models is identical.

Specifically, the Gertler-Karadi-Kiyotaki model imposes a tight link between steady-state bank balance sheets and the key elasticities of the financial sector. We use this link for our baseline calibration: we match the private liquid asset supply level and the banking sector leverage. These two steady-state moments imply values of the key parameters —  $\bar{\Theta}_{rK}, \bar{\Theta}_{rB}, \gamma$  — governing the liquid asset supply given the Gertler-Karadi-Kiyotaki specification. To compare between models, we consider the following comparative static exercise:

$$\hat{\Theta}_{rK} = \chi \bar{\Theta}_{rK}, \quad \forall \chi \in [1, 2].$$

When  $\chi = 1$ , we have a standard Gertler-Karadi model; when  $\chi = 2$ , the semi-elasticity of deposits to the liquid rate is close to the estimate from Drechsler et al. (2017).<sup>11</sup> We also consider  $\chi \rightarrow \infty$ . In this case, the model converges to an economy where the capital and liquid asset markets are linked by a financial sector that responds perfectly elastically to changes in capital returns and liquid rates. As a comparison, we also study a variant in which  $\mathbf{D}_{r^K}$ ,  $\mathbf{D}_{r^B}$ , and  $\mathbf{D}_y$  are all identically zero. In this case, both liquid asset supply and net worth of banks are constant. This specification is a modified version of Kaplan et al. (2018). The level of private liquidity asset supply reflects its empirical counterpart, but the elasticities are kept zero, as in most two-asset HANK models.

## 5.2 Calibration

*Returns:* We set the real rate of return on liquid assets to 1.6% per annum. The return on capital is 4.1%, and the rate of return on illiquid assets is 4.4%.

*Banks:* Banking sector in the quantitative model follows closely Gertler and Karadi (2011). Banks exit at an exogenous and idiosyncratic rate  $f$ . We assume they use the return on loans  $r_t^K$  to value their stream of cash flows. The fraction of assets they can divert is denoted by  $1/\theta$ . We set  $f = 0.05$ , which corresponds roughly to the average dividend rate of banks and is in the range of values in the literature (Gertler and Karadi (2011), Gertler and Kiyotaki (2010), Lee et al. (2020)). Given real returns and  $f$ , parameters  $m, \theta$  respectively determine banks' net worth and leverage. We calibrate them to match the aggregate bank balance sheet. We target a leverage ratio equal to 3 and banking net worth equal to 13.25% of annual GDP. The implied transfer of resources to the banks,  $m$ , is 1.5% of GDP, and  $1/\theta = 0.44$ , in the range of values in the literature. Details are discussed in Appendix C.

*Preferences:* We assume that households have a period utility function of the follow-

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<sup>11</sup>To nest the model of the banking sector in Drechsler et al. (2017), we need to depart from our framework by allowing for a wedge between the rate set by monetary policy and the rate on liquid liabilities of the financial sector.



ing form:

$$u(c, h) = \frac{c^{1-\sigma} - 1}{1 - \sigma} - \varsigma \frac{h^{1+\frac{1}{\varphi}}}{1 + \frac{1}{\varphi}}$$

with  $\sigma \geq 0$  and  $\varphi \geq 0$ . We set  $1/\sigma$ , the intertemporal elasticity of substitution (IES), to  $1/2$  and  $\varphi$ , the Frisch labor supply elasticity at the household level, to 1. The disutility parameter,  $\varsigma$ , is set so that average hours worked are equal to one-third in steady state.

*Income process:* We use a discrete-time version of the income process described in [Kaplan et al. \(2018\)](#), which fit the earnings process by targeting eight moments of the distribution of male earnings changes using Social Security Administration from [Guenen et al. \(2015\)](#). This process implies more idiosyncratic risk than a typical calibration of an AR process with normal innovations.

*Assets:* Households cannot have a negative asset position,  $\underline{a} = \underline{b} = 0$ . Adjustment of illiquid assets holdings is associated with a real adjustment cost similar to [Auclert et al. \(2021\)](#):

$$\Psi_{i,t}(a_{i,t}, a_{i,t-1}) = \frac{\chi_1}{\chi_2} \left| \frac{a_{i,t} - (1 + r_t^A)a_{i,t-1}}{(1 + r_t^A)a_{i,t-1} + \chi_0} \right|^{\chi_2} [(1 + r_t^A)a_{i,t-1} + \chi_0] - \bar{\Psi}_{i,t},$$

where  $\chi_0, \chi_1, \chi_2 > 0$  are parameters that characterize the adjustment cost, and  $\bar{\Psi}_{i,t}$  is a lump-sum rebate that guarantees

$$\int \Psi_{i,t}(a_{i,t}, a_{i,t-1}) di = 0.$$

$\chi_0$  is set to 0.1,  $\chi_2$  to 2. We calibrate  $\beta$  and  $\chi_1$  to match the steady-state ratio of liquid assets to annual GDP of 0.5 and 3.3 for illiquid assets. Given our calibration, 30% of the households are hand-to-mouth in the steady-state.

*Production:* The elasticity of output with respect to capital  $\alpha$  is set to 0.35. Depre-

ciation rate  $\delta$  is 6% per year. The capital production function is

$$\Gamma(\iota_t) = \bar{\iota}_1 \iota_t^{1-\kappa_I} + \bar{\iota}_2,$$

where  $\bar{\iota}_1, \bar{\iota}_2$  are set to ensure that the steady-state investment ratio is equal to  $\delta$ , and the price of capital is 1.  $\kappa_I$  is 0.5, which means that the elasticity of investment to capital price is 2. We set the elasticity of substitution between different varieties of labor,  $\varepsilon_W$ , to 6 and the constant  $\kappa_W$  that determines the degree of nominal wage rigidities to 200, so that the slope of the wage Phillips curve is 0.04.

*Government:* We set  $T$ , net tax revenue, to 15% of steady-state output. We set liquid assets provided by the government to 23% of the annual output. Government purchases are determined residually from the budget constraint and amount to 14.6% of GDP. We set  $\lambda$ , the parameter governing the tax system's progressivity, to 0.1.

### 5.3 Aggregate Response to Government Transfer

Figure 2 shows that output, consumption, investment, and capital price all respond positively to the government transfer shock. Red shades from light to dark represent models with increasing values for  $\hat{\Theta}_{r^K}$  for  $\chi \in [1, 2]$ . The yellow line indicates responses in the HANK model, in which liquid asset supply is perfectly inelastic. The black line indicates responses in a model in which cross-price elasticities approach infinity,  $\chi \rightarrow \infty$ . When the financial sector responds less elastically to returns on capital,  $d\mathbf{r}^K$ , the responses of output, consumption, investment, and asset prices are amplified. Moreover, we see that differences in output response are mostly driven by differences in investment. Increases in investment are due to firms' responses to capital price increases, associated with lower expected future return on capital  $r^K$ .

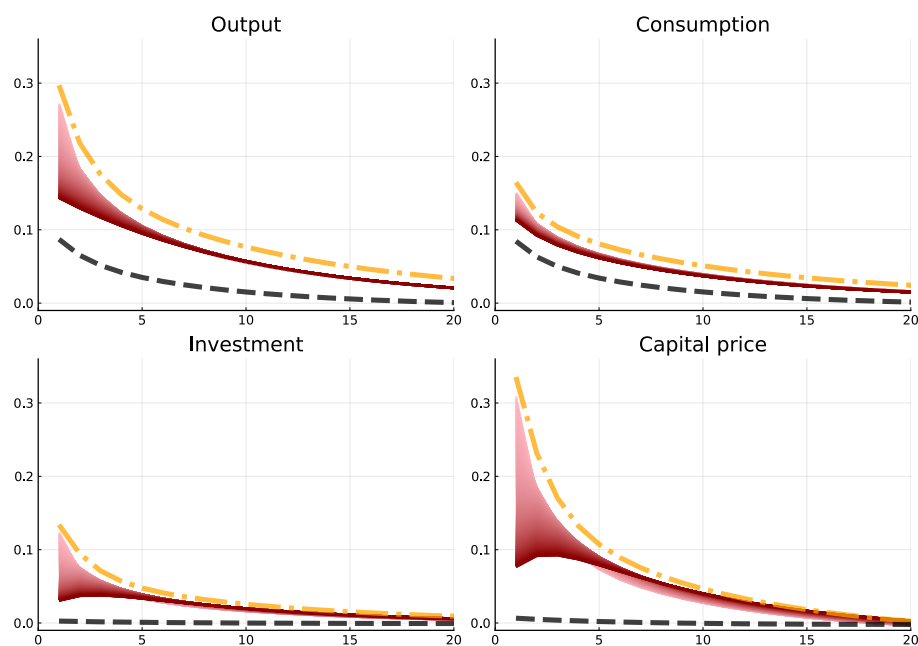


Figure 2: Impulse response functions to government spending shocks; x-axis: quarters, y-axis: % of GDP. Light red: low cross-price elasticities. Dark red: high cross-price elasticities. Yellow: inelastic supply. Black: perfectly elastic supply.

Figure 3 shows financial variables in response to the deficit-financed transfer shock. Issuance of government debt increases total liquid assets supply, but decreases net private liquid asset supply from liquidity transformation. This is because an increase in capital price lowers future expected returns on capital, and banks substitute away from capital towards government debt (a decrease in net liquid asset supply). Low cross-price elasticities imply larger increase in total liquid asset and smaller decrease in net liquid asset supply (less substitution away from capital by banks). Net worth of banks jumps up on impact and gradually declines. This reflects the initial realized return due to increases in capital price. Low cross-price elasticities are associated with a large increase in capital prices and net worth. When cross-price elasticities are low, increases in net worth allow banks to absorb additional capital resulting from increases in investment (except in our HANK specification, under which the value of capital holding is kept constant by assumption). These variables describe

the moving pieces in the liquid asset market, which accounts for the difference in real variables we show in Figure 2.

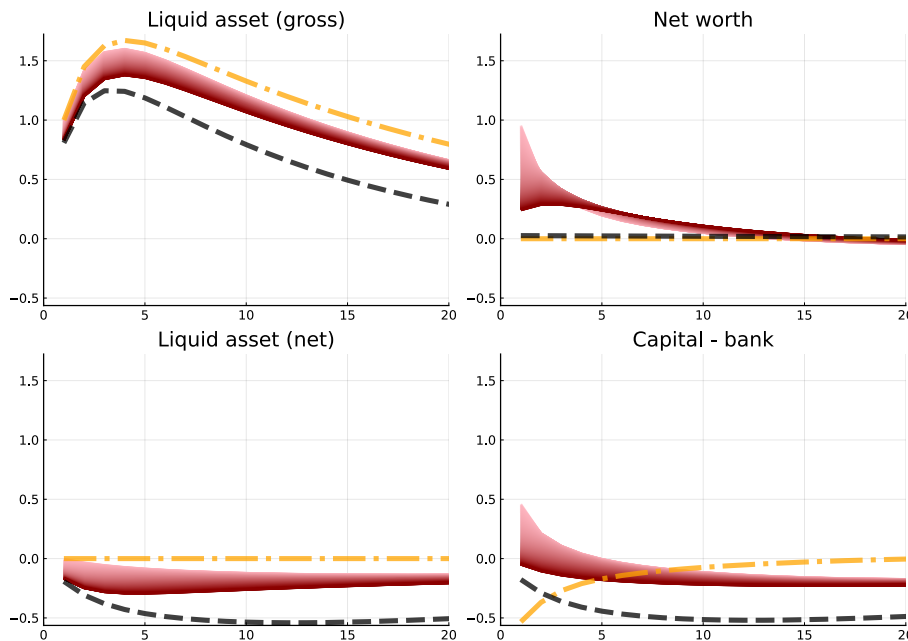


Figure 3: Impulse response functions to a transfer shock; x-axis: quarters, y-axis: % of GDP. Light red: low cross-price elasticities. Dark red: high cross-price elasticities. Yellow: inelastic supply. Black: perfectly elastic supply.

## 5.4 A Decomposition of Aggregate Output Response

To understand how the financial sector affects aggregate responses, we decompose the aggregate output response into the three channels discussed in Section 4.2, using the formula from Theorem 1:

$$d\mathbf{y} = \underbrace{\left( I - \Psi_y - \Omega \epsilon_y \right)^{-1}}_{(3)} \times \left( \underbrace{\Psi_T d\mathbf{T}}_{(1)} + \underbrace{\Omega \left( -d\mathbf{b}^G + \epsilon_T d\mathbf{T} \right)}_{(2)} \right),$$

The three panels in Figure 4 show the decomposition of total aggregate output response into the three channels: (1) a direct effect on aggregate demand through the

goods market,  $\Psi_T d\mathbf{T}$ , (2) an indirect effect on aggregate demand through the liquid asset market,  $\Omega(-db^G + \epsilon_T d\mathbf{T})$ , and (3) the general equilibrium effect resulting from the modified Keynesian cross, which we plot as the difference between  $dy$  and the sum of the first two effects.

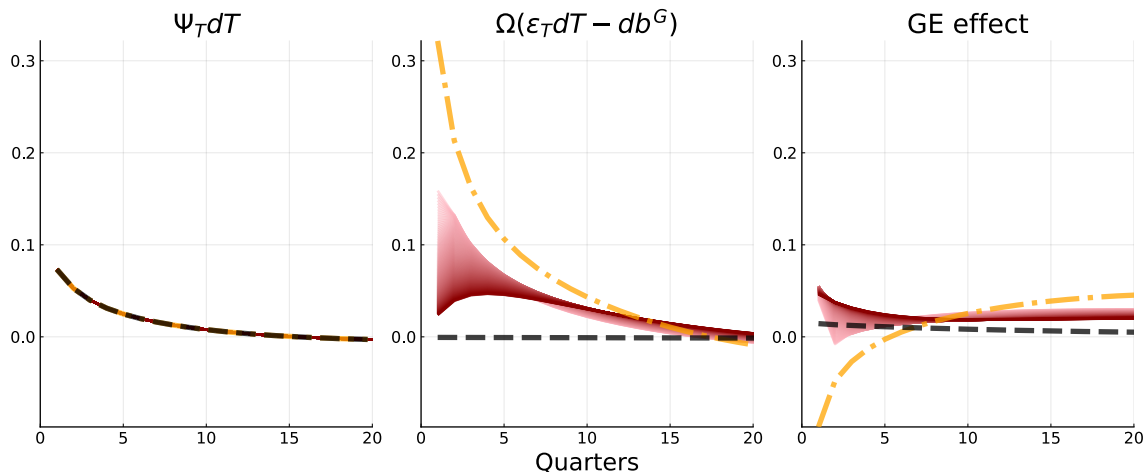


Figure 4: Decomposition of output response to a government spending shock; x-axis: quarters, y-axis: % of GDP. The decomposition uses formula from Theorem 1.

The decomposition in Figure 4 shows how each channel is affected by the financial sector. First, the direct effect on aggregate demand (channel (1)) depends only on the household sector. It is not affected by any properties of the financial sector. On the other hand, the issuance of government debt and households' saving response shift excess liquid asset demand. The indirect effect (channel (2)) depends crucially on features of the financial sector through  $\Omega$  in response to a decrease in excess liquid asset demand. It is the key driving force of the differences in total output responses in Figure 2. The transfer shock initially leads to a decrease in excess liquid asset demand because there is a significant increase in government debt, and the marginal propensity to save out of transfer is relatively low. In response, return on capital  $r_{t+1}^K$  goes down, and capital price  $q_t$  jumps up. It induces banks to reduce liquidity transformation and supply less net liquid assets. At the same time, it makes

illiquid assets less attractive as a store of value, and households substitute towards demanding more liquid assets. An increase in capital price increases consumption and investment hence increasing aggregate demand. The less elastic the liquid asset supply, the larger the adjustment in return on capital, and the stronger output responses are.

Finally, the general equilibrium effect through the Keynesian cross generally amplifies the partial equilibrium effect: an increase in aggregate income leads to more consumption and investment. However, the Keynesian cross logic needs to be modified due to responses through the financial sector. When there is an increase in output, households demand more liquid assets, which leads to an increase in excess liquid asset demand, counteracting the first two channels. The dampening of the Keynesian cross logic is stronger when the financial sector responds inelastically because the capital price and returns need to respond strongly to balance the liquid asset market. In fact, when the financial sector is perfectly inelastic, the modified Keynesian cross causes an overall dampening effect on aggregate output in the initial periods, as shown by the yellow line. <sup>12</sup>

## 6 Conclusion

We provide a framework to understand how the effects of government policies depend on features of the financial sector. For a large class of financial friction models, relevant features of the financial sector are summarized by elasticities of a liquid asset supply function. The cross-price elasticities are crucial links between the liquid asset market and aggregate demand. When the liquid asset supply function responds inelastically to returns on capital (low cross-price elasticities), a disturbance in the liquid asset market generates large movements in capital prices and induces large

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<sup>12</sup>One should not interpret the ranking of lines in the right panel of Figure 4 as a direct comparison of the strength of the modified Keynesian cross. The plotted lines reflect both the size of the partial equilibrium response and the multiplier. For example, the black line in the right panel shows a smaller GE response than the red lines, but that is because the indirect response (channel (2)) is almost completely absent.

responses in aggregate demand. We show quantitatively that assumptions about the financial sector are not innocuous. In response to a deficit-financed government transfer shock, aggregate output responses with an inelastic liquid asset supply are three times larger than responses with a perfectly elastic liquid asset supply.

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## A Proofs and Derivations

### A.1 Proof of Lemma 1.

*Proof.* We first show how we obtain the aggregate demand and supply functions and then demonstrate that if the goods market and the liquid asset market clear, then by Walras’ law the illiquid asset market clears as well. We begin by showing that

$$(1 - \tau_t) \left( \frac{W_t}{P_t} z_{i,t} h_{i,t} \right)^{1-\lambda} = \frac{z_{i,t}^{1-\lambda}}{\int_0^1 z_{i,t}^{1-\lambda} di} [(1 - \alpha) y_t - T_t]$$

Recall that we have  $\frac{W_t}{P_t} h_t = (1 - \alpha) y_t$  and  $h_{i,t} = h_t$  so

$$(1 - \tau_t) \left( \frac{W_t}{P_t} z_{i,t} h_{i,t} \right)^{1-\lambda} = (1 - \tau_t) [(1 - \alpha) y_t z_{i,t}]^{1-\lambda}.$$

Now, since

$$T_t = \frac{W_t}{P_t} h_t - (1 - \tau_t) \int \left( \frac{W_t}{P_t} z_{i,t} h_{i,t} \right)^{1-\lambda} di$$

we have

$$1 - \tau_t = \frac{1}{\int_0^1 [(1 - \alpha) y_t z_{i,t}]^{1-\lambda} di} [(1 - \alpha) y_t - T_t]$$

and thus

$$(1 - \tau_t) [(1 - \alpha) y_t z_{i,t}]^{1-\lambda} = \frac{z_{i,t}^{1-\lambda}}{\int_0^1 z_{i,t}^{1-\lambda} di} [(1 - \alpha) y_t - T_t].$$

Using this in the household budget constraint, we see that optimal policy rules for consumption and savings in each type of assets depend on the aggregates only through the path of output  $\{y_t\}_{s=0}^{\infty}$ , taxes  $\{T_t\}_{s=0}^{\infty}$  and returns on both types of assets  $\{r_t^A, r_t^B\}_{s=0}^{\infty}$ . Therefore given the initial distribution of assets and productivity, we

obtain  $\tilde{\mathcal{A}}_t(\{y_s, r_s^A, r_s^B, T_s\}_{s=0}^\infty)$ ,  $\tilde{\mathcal{B}}_t(\{y_s, r_s^A, r_s^B, T_s\}_{s=0}^\infty)$  and  $\tilde{\mathcal{C}}_t(\{y_s, r_s^A, r_s^B, T_s\}_{s=0}^\infty)$ .

To obtain the investment function use the law of motion for capital to get the investment ratio

$$\frac{x_t}{k_{t-1}} = \Gamma^{-1} \left( \frac{k_t - (1 - \delta) k_{t-1}}{k_{t-1}} \right) =: \iota(k_t, k_{t-1})$$

and use this in the first order condition with respect to  $\iota_t$ :

$$q_t = \frac{1}{\Gamma'(\iota(k_t, k_{t-1}))} =: \hat{q}(k_t, k_{t-1})$$

All the above result in

$$1 + r_{t+1}^K = \frac{\alpha \frac{y_{t+1}}{k_t} + \hat{q}(k_{t+1}, k_t) \left( \frac{k_{t+1}}{k_t} \right) - \iota(k_{t+1}, k_t)}{\hat{q}(k_t, k_{t-1})},$$

which gives us capital in each period as a function of the path of output,  $r^K$  and  $k_{-1}$ :  $\mathcal{K}_t(\{y_s, r_{s+1}^K\}_{s=0}^\infty)$ . We then use the law of motion for capital again to back out the investment function  $\mathcal{X}_t(\{y_s, r_{s+1}^K\}_{s=0}^\infty)$ . Moreover  $q_t := \mathcal{Q}_t(\{y_s, r_{s+1}^K\}_{s=0}^\infty)$ . Similarly, given  $\{r_t^K\}_{t \geq 0}$  and  $\{r_t^B\}_{t \geq 0}$  we obtain the private liquid asset supply function  $\tilde{\mathcal{D}}_t(\{r_s^K, r_s^B\}_{s=0}^\infty)$ .

Functions  $\tilde{\mathcal{A}}_t(\cdot)$ ,  $\tilde{\mathcal{B}}_t(\cdot)$ , and  $\tilde{\mathcal{C}}_t(\cdot)$  differ from functions  $\mathcal{A}_t(\cdot)$ ,  $\mathcal{B}_t(\cdot)$ , and  $\mathcal{C}_t(\cdot)$  in Lemma 1. The former depend on returns on liquid assets in periods  $0, 1, \dots$ , the latter on returns on liquid assets in periods  $1, 2, \dots$ . Similarly,  $\tilde{\mathcal{D}}_t(\{r_s^K, r_s^B\}_{s=0}^\infty)$  depends on returns in periods  $0, 1, \dots$ , while  $\mathcal{D}_t(\{y_s, r_{s+1}^K, r_{s+1}^B\}_{s=0}^\infty)$  is a function of returns in periods  $1, 2, \dots$  but also on output in all periods.

$r_0^B$  is predetermined by assumption (real return on liquid assets is known one period in advance)<sup>13</sup>. This allows us to suppress dependence on it in our notation. Second,

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<sup>13</sup>This would not be true if liquid assets were nominal and time-0 inflation could erode their value. Extending the model to allow for that is straightforward.

we can express eliminate  $r_0^K$  by noting that

$$1 + r_0^K = \frac{\alpha \frac{y_0}{k_{-1}} + \hat{q}(k_0, k_{-1}) \left( \frac{k_0}{k_{-1}} \right) - \iota(k_0, k_{-1})}{\hat{q}(k_{-1}, k_{-2})},$$

where only  $y_0$  and  $k_0$  are not predetermined. By using  $\mathcal{K}_0(\{y_s, r_{s+1}^K\}_{s=0}^\infty)$  we obtain  $k_0$ . It allows us to write  $r_0^K$  as a function of  $\{y_s, r_{s+1}^K\}_{s=0}^\infty$ . We have

$$\begin{aligned} \mathcal{D}_t(\{y_s, r_{s+1}^K, r_{s+1}^B\}_{s=0}^\infty) &:= \tilde{\mathcal{D}}_t(r_0^K, \{r_{s+1}^K, r_{s+1}^B\}_{s=0}^\infty) \\ &= \tilde{\mathcal{D}}_t(\{y_s, r_{s+1}^K\}_{s=0}^\infty, \{r_{s+1}^K, r_{s+1}^B\}_{s=0}^\infty). \end{aligned}$$

Similarly, the dependence of the household functions on  $r_0^B$  is suppressed because of the assumption that  $r_0^B$  is a constant. We now derive the function  $\mathcal{R}_t^A(\cdot)$  using Equation 1 as follows:

$$\begin{aligned} 1 + r_t^A &= 1 + \frac{1}{a_{t-1}} (r_t^K q_{t-1} k_{t-1}^F + \nu_t n_{t-1}) \\ &= 1 + \frac{1}{a_{t-1}} (r_t^K q_{t-1} k_{t-1}^F + (r_t^K q_{t-1} k_{t-1}^B - r_t^B d_{t-1})) \\ &= \frac{1}{a_{t-1}} ((1 + r_t^K) q_{t-1} k_{t-1} - (1 + r_t^B) d_{t-1}) \\ &= \frac{1}{q_{t-1} k_{t-1} - d_{t-1}} ((1 + r_t^K) q_{t-1} k_{t-1} - (1 + r_t^B) d_{t-1}). \end{aligned}$$

Define

$$L_t := \frac{d_t}{q_t k_t}$$

This variable can be interpreted as a *liquidity transformation ratio*. As explained before, we have  $d_t = \mathcal{D}_t(\{y_s, r_{s+1}^K, r_{s+1}^B\}_{s=0}^\infty)$ ,  $q_t = \mathcal{Q}_t(\{y_s, r_{s+1}^K\}_{s=0}^\infty)$ , and  $k_t = \mathcal{K}_t(\{y_s, r_{s+1}^K\}_{s=0}^\infty)$  so we can write

$$L_t = \mathcal{L}_t(\{y_s, r_{s+1}^K, r_{s+1}^B\}_{s=0}^\infty)$$

and

$$1 + r_t^A = \frac{1}{1 - \mathcal{L}_{t-1}(\cdot)} (1 + r_t^K) - \frac{\mathcal{L}_{t-1}(\cdot)}{1 - \mathcal{L}_{t-1}(\cdot)} (1 + r_t^B).$$

The right hand side of the above depends on  $\{y_s, r_{s+1}^K, r_{s+1}^B\}_{s=0}^\infty$ . We can write it in a more compact way as

$$r_t^A := \mathcal{R}_t^A (\{y_s, r_{s+1}^K, r_{s+1}^B; \mathcal{D}\}_{s=0}^\infty).$$

Because  $b_t = \mathcal{B}_t (\{y_s, r_s^A, r_{s+1}^B, T_s\})$  and  $d_t = \mathcal{D}_t (\{y_s, r_{s+1}^K, r_{s+1}^B\})$

$$\mathcal{B}_t (\{y_s, r_s^A, r_{s+1}^B, T_s\}) = \mathcal{D}_t (\{y_s, r_{s+1}^K, r_{s+1}^B\}) + b_t^G$$

means that the liquid asset market clears. Since government debt satisfies Equation 3, the government budget constraint is satisfied. We can now obtain illiquid asset demand  $\int_0^1 a_{i,t} = \mathcal{A}_t (\{y_s, r_s^A, r_{s+1}^B; T_s\})$  for all  $t$ . By the Walras law, the illiquid asset market clears  $\mathcal{A}_t (\{y_s, r_s^A, r_{s+1}^B; T_s\}) = q_t k_t - d_t$ .

□

Notice that we can write

$$\begin{aligned} \mathcal{A}_t (\{y_s, r_s^A, r_{s+1}^B, T_s\}_{s=0}^\infty) &= \mathcal{A}_t (\{\mathcal{R}_s^A, y_s, r_s^B, T_s\}_{s=0}^\infty) \\ \mathcal{B}_t (\{y_s, r_s^A, r_{s+1}^B, T_s\}_{s=0}^\infty) &= \mathcal{B}_t (\{\mathcal{R}_s^A, y_s, r_s^B, T_s\}_{s=0}^\infty) \\ \mathcal{C}_t (\{y_s, r_s^A, r_{s+1}^B, T_s\}_{s=0}^\infty) &= \mathcal{C}_t (\{\mathcal{R}_s^A, y_s, r_s^B, T_s\}_{s=0}^\infty). \end{aligned}$$

and use the fact that  $r_0^A$  depends only on constants (including time 0 rate of return on liquid assets) and  $r_0^K$ . Recall that  $r_0^K$  is a function of  $\{y_s, r_{s+1}^K\}_{s=0}^\infty$ . This allows us to express demand for both types of assets and consumption as functions of  $\{y_s, r_{s+1}^K, r_s^B, T_s\}_{s=0}^\infty$

## A.2 Proof of Lemma 2.

*Proof.* We focus on a case with  $\theta_t = \bar{\theta}$ . To save on notation define  $\Theta_t := \Theta(\{r_{s+1}^B, r_{s+1}^K; \theta_s\}_{s \geq t})$ . To get the response of liquid asset supply recall that

$$d_t = (\Theta_t - 1) n_t,$$

so

$$d\mathcal{D}_t = d\Theta_t \bar{n} + (\bar{\Theta} - 1) dn_t.$$

Totally differentiating 2 and evaluating at the steady state results in

$$\begin{aligned} dn_t &= (1 - f) [(\bar{r}^K - \bar{r}^B) d\Theta_{t-1} + (dr_t^K - dr_t^B) \bar{\Theta} + dr_t^B] \bar{n} \\ &\quad + (1 - f) [(\bar{r}^K - \bar{r}^B) \bar{\Theta} + (1 + \bar{r}^B)] dn_{t-1} \end{aligned}$$

with

$$d\Theta_t = \sum_{s=0}^{\infty} \left( \frac{\partial \Theta_t}{\partial r_s^K} dr_s^K + \frac{\partial \Theta_t}{\partial r_s^B} dr_s^B \right).$$

Since

$$\frac{\partial \Theta_t}{\partial r_{s+1}^K} = \frac{\partial \Theta_t}{\partial r_{s+1}^B} = 0, \quad \forall s \leq t,$$

we have

$$d\Theta_t = \sum_{u=1}^{\infty} \left( \frac{\partial \Theta_t}{\partial r_{t+u}^K} dr_{t+u}^K + \frac{\partial \Theta_t}{\partial r_{t+u}^B} dr_{t+u}^B \right).$$

Define  $G := (1 - f) [(\bar{r}^K - \bar{r}^B) \bar{\Theta} + (1 + \bar{r}^B)] \geq 0$  to write

$$dn_t = (1 - f) \sum_{u=0}^t G^u [(\bar{r}^K - \bar{r}^B) d\Theta_{t-1-u} \bar{n} + (dr_{t-u}^K - dr_{t-u}^B) \bar{\Theta} \bar{n} + dr_{t-u}^B \bar{n}].$$

Now, consider a particular variation such that  $dr_s^K = 1$  and  $dr_u^K = 0$  for all  $u \neq s$ ,



and  $dr_u^B = 0$  for all  $u$ .<sup>14</sup> We have

$$dn_t = \begin{cases} \bar{n} (1-f) (\bar{r}^K - \bar{r}^B) \sum_{u=0}^{t-1} G^u \frac{\partial \Theta_{t-1-u}}{\partial r_s^K}, & s > t, \\ \bar{n} (1-f) (r^K - r^D) \sum_{u=t-s}^{t-1} G^u \frac{\partial \Theta_{t-1-u}}{\partial r_s^K} + \bar{n} (1-f) G^{t-s} \bar{\Theta}, & s \leq t. \end{cases}$$

The expression above shows that net worth of banks can move in response to a change in  $r^K$  for two reasons. First, if that change materialized in the past, it had a direct effect on net worth (and also for lending, holding the leverage ratio fixed). This is reflected by the term  $\bar{n} (1-f) G^{t-s} \bar{\Theta}$ . Second, if that change was expected, it affected the leverage ratio in the past through the dependence of  $\Theta_t$  on future returns.

The assumption about the structure of  $\Theta_t$  implies

$$\frac{\partial \Theta_{t-1-u}}{\partial r_s^K} = \begin{cases} \gamma^{s-t+u} \bar{\Theta}_{r^K}, & s > t-1-u, \\ 0, & s \leq t-1-u. \end{cases}$$

which allows us to write

$$dn_t = \begin{cases} \bar{\Theta}_{r^K} \bar{n} (1-f) (\bar{r}^K - \bar{r}^B) \gamma^{s-t} \sum_{u=0}^{t-1} (\gamma G)^u dr_s^K, & s > t, \\ \bar{\Theta}_{r^K} \bar{n} (1-f) (r^K - r^D) G^{t-s} \sum_{l=0}^{s-1} (\gamma G)^l dr_s^K + \bar{n} (1-f) G^{t-s} \bar{\Theta} dr_s^K, & s \leq t. \end{cases}$$

Finally, define

$$P := (1-f) (\bar{\Theta} - 1) (\bar{r}^K - \bar{r}^B) \geq 0$$

to write

$$\frac{\partial \mathcal{D}_t}{\partial r_s^K} = \begin{cases} \bar{\Theta}_{r^K} \gamma^{s-t-1} \bar{n} + \bar{\Theta}_{r^K} \gamma^{s-t} P \sum_{u=0}^{t-1} (\gamma G)^u \bar{n}, & s > t, \\ \bar{\Theta}_{r^K} G^{t-s} P \sum_{l=0}^{s-1} (\gamma G)^l \bar{n} + G^{t-s} (\bar{\Theta} - 1) \bar{\Theta} (1-f) \bar{n}, & s \leq t, \end{cases} \quad (5)$$

which will correspond to the  $s$ -th column of  $\mathbf{D}_{r^K}$  for  $s \geq 1$ . Recall that we will treat  $s = 0$  in a special way.

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<sup>14</sup>Since derivation of  $\mathbf{D}_{r^B}$  follows the same logic, we will skip it in the proof.

We now reorganize  $\mathbf{D}_{r^k}$  as a sum of matrices shown in Lemma 2. Matrix  $\mathbf{N}_0$  consists of terms  $G^{t-s}(1-f)\bar{n}$ , present only for  $s \leq t$ . It captures the effect of net worth accumulation on liquid asset supply, holding the leverage ratio constant. Its  $(t+1, s)$ -th entry is  $G^{t-s}(1-f)\bar{n} \geq 0$ .

$$\mathbf{N}_0 = (1-f)\bar{n} \begin{bmatrix} 0 & 0 & 0 & 0 & \dots \\ 1 & 0 & 0 & 0 & \dots \\ G & 1 & 0 & 0 & \dots \\ G^2 & G & 1 & 0 & \dots \\ \dots & \dots & \dots & \dots & \dots \end{bmatrix}$$

Matrix  $\mathbf{N}(\gamma)$  consists of all other terms. Its  $(t+1, s)$ -th entry captures the effect of  $r_s^K$  on liquid asset supply in period  $t$  through changes in the leverage ratio (both in period  $t$  and in the past).

$$\mathbf{N}(\gamma) = \bar{n} \begin{bmatrix} 1 & \gamma & \gamma^2 & \dots \\ P & 1 + \gamma P & \gamma + \gamma^2 P & \dots \\ PG & P + \gamma PG & 1 + \gamma P + \gamma^2 PG & \dots \\ PG^2 & PG + \gamma PG^2 & P + \gamma PG + \gamma^2 PG^2 & \dots \\ \dots & \dots & \dots & \dots \end{bmatrix}$$

All entries of this matrix are non-negative. If  $\gamma = 0$ , then  $\mathbf{N}(\gamma)$  is a lower-triangular matrix with ones on the diagonal.

Let turn to the effect of changes in  $r_0^K$ . The sum

$$\bar{\Theta}_{r^k} \mathbf{N}(\gamma) + (\bar{\Theta} - 1) \bar{\Theta} \mathbf{N}_0$$

discussed above allows to capture the effects of changes in return on capital in periods  $s = 1, 2, \dots$ , but ignores the effect of  $r_0^K$ . The formula 5 can still be applied for  $s = 0$ .

Changes in liquid asset supply due to  $dr_0^K$  can be summarized as

$$\mathbf{n}_0 = (1 - f) \bar{n}^B \begin{bmatrix} 1 \\ G \\ G^2 \\ G^3 \\ \dots \end{bmatrix},$$

a vector such that its  $t$ -th element corresponds to the  $(t, 1)$ -th entry of  $\mathbf{N}_0$ . We also have

$$dr_0^K = \alpha \frac{1}{k} dy_0 + (1 - \delta) dq_0.$$

Capital price at  $t = 0$  can change only if the investment rate  $\iota_0$  changes. That depends on function  $\mathcal{X}_t(\cdot)$ . In a matrix form we can write

$$dr_0^K = \begin{bmatrix} 1 & 0 & 0 & \dots \end{bmatrix} d\mathbf{y} + (1 - \delta) (\mathbf{q}_y d\mathbf{y} + \mathbf{q}_{r^K} dr^K).$$

Where  $\mathbf{q}_y, \mathbf{q}_{r^K}$  are row vectors describing how the initial price of capital depends on output and return on capital. The total effect of  $dr^K$  on liquid asset supply is therefore

$$\mathbf{D}_{r^K} = \bar{\Theta}_{r^K} \mathbf{N}(\gamma) + (\bar{\Theta} - 1) \bar{\Theta} [\mathbf{N}_0 + (1 - \delta) \mathbf{n}_0 \mathbf{q}_{r^K}].$$

where the  $(1 - \delta) \mathbf{n}_0 \mathbf{q}_{r^K}$  term describes how returns on capital in the future move  $q_0$  and therefore  $r_0^K$ .

$$\mathbf{D}_y = (\bar{\Theta} - 1) \bar{\Theta} \left[ \frac{\alpha}{k} \mathbf{e}_1 + (1 - \delta) \mathbf{q}_y \right]$$

reflects the fact that  $q_0$  (and thus  $r_0^K$ ) depends also on the path of output. Note that  $d\mathbf{y}$  matters for liquid asset supply only because it affects  $r_0^K$ .

Derivation of  $\mathbf{D}_{r^B}$  follows the same steps. The main difference is that  $dr_t^B$  enters the law of motion for net worth with a coefficient  $1 - \bar{\Theta}$  instead of  $\bar{\Theta}$ .  $\square$

### A.2.1 Time-varying equity injection $m_t$ .

So far we assumed that equity injections are constant,  $m$ . We now relax this assumption and allow  $m_t$  as in [Gertler and Kiyotaki \(2010\)](#):

$$m_t = \xi (1 + r_t^K) q_{t-1} k_{t-1}^B.$$

Here  $\xi \geq 0$ . Totally differentiating [2](#) and evaluating at the steady state results in

$$\begin{aligned} dn_t = (1-f) [(\bar{r}^K - \bar{r}^B) d\Theta_{t-1} + (dr_t^K - dr_t^B) \bar{\Theta} + dr_t^B] \bar{n} \\ + (1-f) [(\bar{r}^K - \bar{r}^B) \bar{\Theta} + (1 + \bar{r}^B)] dn_{t-1} + dm_t \end{aligned}$$

where

$$dm_t = \xi [\bar{\Theta} \bar{n} dr_t^K + (1 + \bar{r}^K) (\bar{n} d\Theta_{t-1} + \bar{\Theta} dn_{t-1})].$$

Rewrite the linearized law of motion for  $n_t$  as

$$\begin{aligned} dn_t = (1-f) \left[ \left( \bar{r}^K - \bar{r}^B + \xi \frac{1 + \bar{r}^K}{1-f} \right) d\Theta_{t-1} + \left( \left( 1 + \xi \frac{1}{1-f} \right) dr_t^K - dr_t^B \right) \bar{\Theta} + dr_t^B \right] \bar{n} \\ + (1-f) \left[ (\bar{r}^K - \bar{r}^B) \bar{\Theta} + (1 + \bar{r}^B) + \xi \frac{1 + \bar{r}^K}{1-f} \right] dn_{t-1}. \end{aligned}$$

Define  $G := (1-f) \left[ (\bar{r}^K - \bar{r}^B) \bar{\Theta} + (1 + \bar{r}^B) + \xi \frac{1 + \bar{r}^K}{1-f} \right] \geq 0$  to write

$$\begin{aligned} dn_t = (1-f) \sum_{u=0}^t G^u \left[ \left( \bar{r}^K - \bar{r}^B - \xi \frac{1 + \bar{r}^K}{1-f} \right) d\Theta_{t-1-u} \bar{n} + \left( \left( 1 + \xi \frac{1}{1-f} \right) dr_{t-u}^K - dr_{t-u}^B \right) \bar{\Theta} \bar{n} \right] \\ + (1-f) \sum_{u=0}^t G^u dr_{t-u}^B \bar{n}. \end{aligned}$$

Observe that the form of the above expression is the same as with  $m_t = m$ . The only difference is in coefficients. Consider a particular variation such that  $dr_s^K = 1$

and  $dr_u^K = 0$  for all  $u \neq s$ , and  $dr_u^B = 0$  for all  $u$ . We have

$$dn_t = \begin{cases} \bar{n}(1-f) \left( \bar{r}^K - \bar{r}^B - \xi \frac{1+\bar{r}^K}{1-f} \right) \sum_{u=0}^{t-1} G^u \frac{\partial \Theta_{t-1-u}}{\partial r_s^K}, & s > t, \\ \bar{n}(1-f) \left( \bar{r}^K - \bar{r}^B - \xi \frac{1+\bar{r}^K}{1-f} \right) \sum_{u=t-s}^{t-1} G^u \frac{\partial \Theta_{t-1-u}}{\partial r_s^K} + \bar{n}(1-f) \left( 1 + \xi \frac{1}{1-f} \right) G^{t-s} \bar{\Theta}, & s \leq t. \end{cases}$$

We can define

$$P := (1-f) \left( \bar{r}^K - \bar{r}^B - \xi \frac{1+\bar{r}^K}{1-f} \right) \geq 0$$

to write

$$\frac{\partial \mathcal{D}_t}{\partial r_s^K} = \begin{cases} \bar{\Theta}_{r^K} \gamma^{s-t-1} \bar{n} + \bar{\Theta}_{r^K} \gamma^{s-t} P \sum_{u=0}^{t-1} (\gamma G)^u \bar{n}, & s > t, \\ \bar{\Theta}_{r^K} G^{t-s} P \sum_{l=0}^{s-1} (\gamma G)^l \bar{n} + G^{t-s} (\bar{\Theta} - 1) \bar{\Theta} (1-f) \left( 1 + \xi \frac{1}{1-f} \right) \bar{n}, & s \leq t, \end{cases}$$

By following the same steps as with constant  $m$  we obtain

$$\mathbf{D}_{r^K} = \bar{\Theta}_{r^K} \mathbf{N}(\gamma) + (\bar{\Theta} - 1) \bar{\Theta} \left( 1 + \xi \frac{1}{1-f} \right) [\mathbf{N}_0 + (1-\delta) \mathbf{n}_0 \mathbf{q}_{r^K}].$$

All matrices in the above formula have the same structure as before, but now  $P, G$  are different. Derivation of  $\mathbf{D}_y$  and  $\mathbf{D}_{r^B}$  follow similar steps and we get

$$\mathbf{D}_y = (\bar{\Theta} - 1) \bar{\Theta} \left( 1 + \xi \frac{1}{1-f} \right) \left[ \frac{\alpha}{\bar{k}} \mathbf{e}_1 + (1-\delta) \mathbf{q}_y \right]$$

and

$$\mathbf{D}_{r^B} = -\bar{\Theta}_{r^B} \mathbf{N}(\gamma) - (\bar{\Theta} - 1)^2 \mathbf{N}_0.$$

### A.3 Linearized equilibrium conditions

We use the following notation:  $d\mathbf{r}^B$  represents  $\{dr_{s+1}^B\}_{s=0}^\infty$ . The same convention applies to other rates of return. We use  $d\mathbf{y}$  to represent  $\{dy_s\}_{s=0}^\infty$ . Our notation is the same for other variables that are not rates of return. These are column vectors. By evaluating derivatives of aggregate functions  $\mathcal{X}_t(\cdot), \mathcal{B}_t(\cdot), \mathcal{C}_t(\cdot), \mathcal{D}_t(\cdot), \mathcal{R}_t^A(\cdot)$  and

so on at the steady state, we obtain matrices such as  $\mathbf{C}_y$ , of which the  $(t, s)$  element is  $\partial \mathcal{C}_t / \partial y_s$ . The  $(t, s)$  element of  $\mathbf{D}_{r^B}$  is  $\partial \mathcal{D}_t / \partial r_{s+1}^B$ .

We start with obtaining some auxilliary results. Define

$$\mathbf{S}_{+1} := \begin{bmatrix} 0 & 1 & 0 & \cdots \\ 0 & 0 & 1 & \cdots \\ 0 & 0 & 0 & \cdots \\ \cdots & \cdots & \cdots & \cdots \end{bmatrix}, \mathbf{S}_{-1} := \begin{bmatrix} 0 & 0 & 0 & \cdots \\ 1 & 0 & 0 & \cdots \\ 0 & 1 & 0 & \cdots \\ \cdots & \cdots & \cdots & \cdots \end{bmatrix}.$$

Linearization of the formula for return on capital results in

$$d\mathbf{r}^K + \frac{(1+r^K)\bar{q}'}{k}(I - \mathbf{S}_{-1})d\mathbf{k} = \frac{\alpha}{k}\mathbf{S}_{+1}d\mathbf{y} - \frac{\alpha y}{k^2}d\mathbf{k} + \frac{\bar{q}' + \bar{q} - \bar{v}'}{k}(\mathbf{S}_{+1} - I)d\mathbf{k}$$

which allows us to express  $d\mathbf{k}$  as

$$d\mathbf{k} = \Xi^{-1} \left[ \frac{\alpha}{k}\mathbf{S}_{+1}d\mathbf{y} - d\mathbf{r}^K \right]$$

with

$$\Xi = \frac{\alpha y}{k^2}I + \frac{(1+r^K)\bar{q}'}{k}(I - \mathbf{S}_{-1}) - \frac{\bar{q}' + \bar{q} - \bar{v}'}{k}(\mathbf{S}_{+1} - I).$$

We can write it as

$$d\mathbf{k} = \mathbf{K}_y d\mathbf{y} + \mathbf{K}_{r^K} d\mathbf{r}^K$$

Therefore

$$d\mathbf{q} = \frac{\bar{q}'}{k}(I - \mathbf{S}_{-1})d\mathbf{k}$$

so

$$d\mathbf{q} = \mathbf{Q}_y d\mathbf{y} + \mathbf{Q}_{r^K} dr^K$$

and

$$d\mathbf{x} = \bar{v}'(I - \mathbf{S}_{-1})d\mathbf{k} + \bar{v}d\mathbf{k} = \mathcal{I}d\mathbf{k}$$

with

$$\mathcal{I} = \bar{v}'(I - \mathbf{S}_{-1}) + \bar{v}$$

so

$$d\mathbf{x} = \mathbf{X}_y d\mathbf{y} + \mathbf{X}_{r^K} dr^K$$

We now turn to the household sector of the economy. We want to eliminate  $\{dr_0^A, d\mathbf{r}^A\}$ . We deal with  $dr_0^A$  first. Define the following matrices

$$\mathbf{B}_{r_0^A} := \begin{bmatrix} \frac{\partial \mathcal{B}_0}{\partial r_0^A} & 0 & 0 & \dots \\ \frac{\partial \mathcal{B}_1}{\partial r_0^A} & 0 & 0 & \dots \\ \frac{\partial \mathcal{B}_2}{\partial r_0^A} & 0 & 0 & \dots \\ \dots & \dots & \dots & \dots \end{bmatrix}, \mathbf{C}_{r_0^A} := \begin{bmatrix} \frac{\partial \mathcal{C}_0}{\partial r_0^A} & 0 & 0 & \dots \\ \frac{\partial \mathcal{C}_1}{\partial r_0^A} & 0 & 0 & \dots \\ \frac{\partial \mathcal{C}_2}{\partial r_0^A} & 0 & 0 & \dots \\ \dots & \dots & \dots & \dots \end{bmatrix}.$$

We have

$$dr_0^A = \frac{1}{1-L} dr_0^K$$

where  $L$  is the steady state ratio  $d/qk$ . Moreover,

$$dr_0^K = \begin{bmatrix} 1 & 0 & 0 & \dots \end{bmatrix} d\mathbf{y} + (1-\delta)(\mathbf{q}_y d\mathbf{y} + \mathbf{q}_{r^K} dr^K)$$

where  $\mathbf{q}_y, \mathbf{q}_{r^K}$  are row vectors describing how the initial price of capital depends on

output and return on capital. They are obtained from matrices  $\mathbf{Q}_y, \mathbf{Q}_{r^K}$ . All this allows us to define.

$$\begin{aligned}\mathbf{B}_{r_0^A, y} &:= \frac{1}{1-L} \mathbf{B}_{r_0^A} \times \left[ \begin{bmatrix} 1 & 0 & 0 & \cdots \end{bmatrix} + (1-\delta) \mathbf{q}_y \right], \\ \mathbf{B}_{r_0^A, r^K} &:= \frac{1}{1-L} \mathbf{B}_{r_0^A} \times (1-\delta) \mathbf{q}_{r^K}, \\ \mathbf{C}_{r_0^A, y} &:= \frac{1}{1-L} \mathbf{C}_{r_0^A} \times \left[ \begin{bmatrix} 1 & 0 & 0 & \cdots \end{bmatrix} + (1-\delta) \mathbf{q}_y \right], \\ \mathbf{C}_{r_0^A, r^K} &:= \frac{1}{1-L} \mathbf{C}_{r_0^A} \times (1-\delta) \mathbf{q}_{r^K}.\end{aligned}$$

These matrices fully capture the effect of  $d\mathbf{y}$  and  $d\mathbf{r}^K$  on consumption and asset demand through  $dr_0^A$ .

We now proceed to eliminate  $dr_1^A, dr_2^A, \dots$  by using the condition that links returns on illiquid assets and on capital, Equation 1. We have

$$dr_t^A = \sum_{s=0}^{\infty} \frac{\partial \mathcal{R}_t^A}{\partial y_s} dy_s + \sum_{s=0}^{\infty} \frac{\partial \mathcal{R}_t^A}{\partial r_{s+1}^K} dr_{s+1}^K + \sum_{s=0}^{\infty} \frac{\partial \mathcal{R}_t^A}{\partial r_{s+1}^B} dr_{s+1}^B$$

They capture the effect of changes in rates of return and changes in output on the liquidity transformation ratio. As shown in Appendix A.1,  $\mathcal{R}_t^A$  depends on the liquidity transformation ratio  $L_t$ . Since  $L_t = \frac{qt_k t}{dt}$  we have

$$dL_t = -\frac{L}{q} dq_t - \frac{L}{k} dk_t + \frac{L}{d} dd_t$$

Define the following matrices

$$\mathbf{L}_{r^K} = \begin{bmatrix} \frac{\partial L_0}{\partial r_1^K} & \frac{\partial L_0}{\partial r_2^K} & \cdots \\ \frac{\partial L_1}{\partial r_1^K} & \frac{\partial L_1}{\partial r_2^K} & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix}, \mathbf{L}_{r^B} = \begin{bmatrix} \frac{\partial L_0}{\partial r_1^B} & \frac{\partial L_0}{\partial r_2^B} & \cdots \\ \frac{\partial L_1}{\partial r_1^B} & \frac{\partial L_1}{\partial r_2^B} & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix}, \mathbf{L}_y = \begin{bmatrix} \frac{\partial L_0}{\partial y_0} & \frac{\partial L_0}{\partial y_1} & \cdots \\ \frac{\partial L_1}{\partial y_0} & \frac{\partial L_1}{\partial y_1} & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix}.$$



They satisfy

$$\begin{aligned}\mathbf{L}_{r^K} &= -\frac{L}{q}\mathbf{Q}_{r^K} - \frac{L}{k}\mathbf{K}_{r^K} + \frac{L}{d}\mathbf{D}_{r^K} \\ \mathbf{L}_{r^B} &= \frac{L}{d}\mathbf{D}_{r^K} \\ \mathbf{L}_y &= -\frac{L}{q}\mathbf{Q}_y - \frac{L}{k}\mathbf{K}_y + \frac{L}{d}\mathbf{D}_y.\end{aligned}$$

Therefore,

$$\begin{aligned}\mathbf{R}_{r^K}^A &= \frac{1}{1-L}I + \frac{r^K - r^B}{(1-L)^2}\mathbf{L}_{r^K}, \\ \mathbf{R}_{r^B}^A &= -\frac{1}{1-L}I + \frac{r^K - r^B}{(1-L)^2}\mathbf{L}_{r^B} + I, \\ \mathbf{R}_y^A &= \frac{r^K - r^B}{(1-L)^2}\mathbf{L}_y.\end{aligned}$$

The goods market clearing condition in period  $t$  is:

$$\mathcal{C}_t(\{y_s, r_s^A, r_{s+1}^B, T_s\}_{s=0}^\infty) + \mathcal{X}_t(\{y_s, r_{s+1}^K\}_{s=0}^\infty) + g_t = y_t.$$

In first-order deviations

$$\begin{aligned}\sum_{s=0}^{\infty} \frac{\partial \mathcal{C}_t}{\partial y_s} dy_s + \sum_{s=0}^{\infty} \frac{\partial \mathcal{C}_t}{\partial r_s^A} dr_s^A + \sum_{s=0}^{\infty} \frac{\partial \mathcal{C}_t}{\partial r_{s+1}^B} dr_{s+1}^B + \sum_{s=0}^{\infty} \frac{\partial \mathcal{C}_t}{\partial T_s} dT_s \\ + \sum_{s=0}^{\infty} \frac{\partial \mathcal{X}_t}{\partial y_s} dy_s + \sum_{s=0}^{\infty} \frac{\partial \mathcal{X}_t}{\partial r_{s+1}^K} dr_{s+1}^K + dg_t = dy_t\end{aligned}$$

We can stack it for all periods and represent as

$$\begin{aligned}\mathbf{C}_y d\mathbf{y} + \mathbf{C}_{r^A} dr^A + \mathbf{C}_{r_0^A, y} dy + \mathbf{C}_{r_0^A, r^K} dr^K + \mathbf{C}_{r^B} dr^B + \mathbf{C}_T d\mathbf{T} \\ + \mathbf{X}_y d\mathbf{y} + \mathbf{X}_{r^K} dr^K + d\mathbf{g} = d\mathbf{y}\end{aligned}$$

Finally, use the fact that  $dr^A = \mathbf{R}_{r^K}^A dr^K + \mathbf{R}_{r^B}^A dr^B + \mathbf{R}_y^A dy$  to write

$$\begin{aligned} dy - \mathbf{C}_y dy - \mathbf{X}_y dy &= \mathbf{C}_{r^B} dr^B + \mathbf{C}_{r^A} (\mathbf{R}_{r^K}^A dr^K + \mathbf{R}_{r^B}^A dr^B + \mathbf{R}_y^A dy) \\ &\quad + \mathbf{C}_{r_0^A, y} dy + \mathbf{C}_{r_0^A, r^K} dr^K + \mathbf{C}_T dT + \mathbf{X}_{r^K} dr^K + dg. \end{aligned}$$

By following the same steps we can write the liquid asset market clearing condition as

$$\begin{aligned} (\mathbf{D}_{r^K} - \mathbf{B}_{r^A} \mathbf{R}_{r^K}^A - \mathbf{B}_{r_0^A, r^K}) dr^K &= (\mathbf{B}_y + \mathbf{B}_{r^A} \mathbf{R}_y^A + \mathbf{B}_{r_0^A, y} - \mathbf{D}_y) dy + \mathbf{B}_T dT - db^G \\ &\quad + (\mathbf{B}_{r^B} + \mathbf{B}_{r^A} \mathbf{R}_{r^B}^A - \mathbf{D}_{r^B}) dr^B. \end{aligned}$$

#### A.4 Proof of Proposition 1.

*Proof.*

$$\begin{aligned} (\mathbf{D}_{r^K} - \mathbf{B}_{r^A} \mathbf{R}_{r^K}^A - \mathbf{B}_{r_0^A, r^K}) dr^K &= (\mathbf{B}_y + \mathbf{B}_{r^A} \mathbf{R}_y^A + \mathbf{B}_{r_0^A, y} - \mathbf{D}_y) dy + \mathbf{B}_T dT - db^G \\ &\quad + (\mathbf{B}_{r^B} + \mathbf{B}_{r^A} \mathbf{R}_{r^B}^A - \mathbf{D}_{r^B}) dr^B. \end{aligned}$$

Use definitions

$$\begin{aligned} \epsilon_{r^K} &:= \mathbf{B}_{r^A} \mathbf{R}_{r^K}^A + \mathbf{B}_{r_0^A, r^K} - \mathbf{D}_{r^K}, & \epsilon_{r^B} &:= \mathbf{B}_{r^B} + \mathbf{B}_{r^A} \mathbf{R}_{r^B}^A - \mathbf{D}_{r^B}, \\ \epsilon_y &:= \mathbf{B}_y + \mathbf{B}_{r^A} \mathbf{R}_y^A + \mathbf{B}_{r_0^A, y} - \mathbf{D}_y, & \epsilon_T &:= \mathbf{B}_T \end{aligned}$$

to rewrite the linearized liquid asset market clearing condition as

$$\epsilon_{r^K} dr^K = \epsilon_y dy + \epsilon_T dT - db^G + \epsilon_{r^B} dr^B.$$

Left-multiply by the inverse of  $\epsilon_{r^K}$  to obtain 4. □

## A.5 Proof of Theorem 1.

*Proof.*

$$\begin{aligned} d\mathbf{y} - \mathbf{C}_y d\mathbf{y} - \mathbf{X}_y d\mathbf{y} &= \mathbf{C}_{r^B} d\mathbf{r}^B + \mathbf{C}_{r^A} (\mathbf{R}_{r^K}^A d\mathbf{r}^K + \mathbf{R}_{r^B}^A d\mathbf{r}^B + \mathbf{R}_y^A d\mathbf{y}) \\ &\quad + \mathbf{C}_{r_0^A, y} d\mathbf{y} + \mathbf{C}_{r_0^A, r^K} d\mathbf{r}^K + \mathbf{C}_T d\mathbf{T} + \mathbf{X}_{r^K} d\mathbf{r}^K + dg. \end{aligned}$$

Use definitions

$$\begin{aligned} \Psi_{r^K} &:= \mathbf{C}_{r^A} \mathbf{R}_{r^K}^A + \mathbf{C}_{r_0^A, r^K} + \mathbf{X}_{r^K}, & \Psi_{r^B} &:= \mathbf{C}_{r^B} + \mathbf{C}_{r^A} \mathbf{R}_{r^B}^A, \\ \Psi_y &:= \mathbf{C}_y + \mathbf{C}_{r^A} \mathbf{R}_y^A + \mathbf{C}_{r_0^A, y} + \mathbf{X}_y, & \Psi_T &:= \mathbf{C}_T, \end{aligned}$$

define

$$\Omega := \Psi_{r^K} (-\epsilon_{r^K})^{-1},$$

and use Proposition 1 to solve for

$$d\mathbf{y} = \left( I - \Psi_y - \Omega \epsilon_y \right)^{-1} \times \left( g + \Psi_T d\mathbf{T} + \Psi_{r^B} d\mathbf{r}^B + \Omega (-db^G + \epsilon_T d\mathbf{T} + \epsilon_{r^B} d\mathbf{r}^B) \right).$$

□

## B Nested Models of Financial Frictions

In this Appendix we show how our framework nests some commonly used models of financial frictions. We can do it by appropriately choosing the financial constraint  $\Theta_t \left( \{r_{s+1}^B, r_{s+1}^K; \theta_s\}_{s \geq t} \right)$ . We also demonstrate that in all these models financial frictions result in  $\Theta_t(\cdot)$  that has a special structure we use in Lemma 2.

## B.1 Gertler and Kiyotaki (2010) and Gertler and Karadi (2011)

In Gertler and Kiyotaki (2010) and Gertler and Karadi (2011) there is a continuum of banks indexed by  $j \in [0, 1]$ . Bank activity is subject to an agency problem. Every period, after receiving returns on assets and paying depositors, bank  $j$  exits with probability  $f$  and transfers its retained earnings as dividends to its owners. At the same time, a new bank enters and receives some initial net worth to operate with. Conditional on surviving, bank  $j$  chooses how much loans  $l_{j,t}^B$  and deposits  $d_{j,t}$  to issue. Banks cannot issue equity. Moreover, an agency problem constrains the amount of deposits they can issue. After obtaining funding from depositors and investing in assets (loans), bank  $j$  can divert fraction  $1/\theta_t$  of assets and run away. If this happens, depositors force it into bankruptcy and bank  $j$  has to close. The largest amount of funding a bank can receive from depositors depends on the franchise value  $v_{j,t}(n_{j,t})$ , where  $n_{j,t}$  is net worth — bank  $j$  must be better off continuing instead of running away. These microfoundations of financial frictions have been used in the recent literature studying interactions between the financial sector and household heterogeneity, for example in Lee et al. (2020) and Lee (2021). The optimization problem is:

$$v_{j,t}(n_{j,t}) = \max_{\{l_{j,t+s}^B, d_{j,t+s}, n_{j,t+s+1}\}_{s=0}^{\infty}} \sum_{s=1}^{\infty} \Lambda_{t,t+s}^{\text{bank}} (1-f)^{s-1} f n_{j,t+s}$$

subject to

$$\begin{aligned} l_{j,t}^B &\leq \theta_t v_{j,t}(n_{j,t}) \\ n_{j,t} + d_{j,t} &= l_{j,t}^B \\ n_{j,t+1} &= (1 + r_{t+1}^K) l_{j,t}^B - (1 + r_{t+1}^B) d_{j,t}. \end{aligned}$$

The first constraint is the incentive compatibility constraint resulting from the agency problem.  $\Lambda_{t,t+s}^{\text{bank}}$  is the discount factor used by banks. The recursive formulation of

the problem is:

$$v_{j,t}(n_{j,t}) = \max_{l_{j,t}^B, d_{j,t}, n_{j,t+1}} \Lambda_{t,t+1}^{\text{bank}} (f n_{j,t+1} + (1-f) v_{j,t+1}(n_{j,t+1}))$$

subject to

$$\begin{aligned} \frac{1}{\theta_t} l_{j,t}^B &\leq v_{j,t}(n_{j,t}) \\ n_{j,t} + d_{j,t} &= l_{j,t}^B \\ n_{j,t+1} &= (1 + r_{t+1}^K) l_{j,t}^B - (1 + r_{t+1}^B) d_{j,t} \end{aligned}$$

Guess linearity:  $v_{j,t}(n_{j,t}) = \eta_{j,t} n_{j,t}$ . We can write Bellman equation as

$$\begin{aligned} \eta_{j,t} n_{j,t} &= \max_{l_{j,t}^B, d_{j,t}} \Lambda_{t,t+1}^{\text{bank}} (f + (1-f) \eta_{j,t+1}) [(1 + r_{t+1}^K) l_{j,t}^B - (1 + r_{t+1}^B) d_{j,t}] \\ &\quad + \lambda_{j,t} \left[ \eta_{j,t} n_{j,t} - \frac{1}{\theta_t} l_{j,t}^B \right] + \mu_{j,t} [l_{j,t}^B - n_{j,t} - d_{j,t}]. \end{aligned}$$

Define

$$\psi_{j,t} := \frac{l_{j,t}^B}{n_{j,t}}$$

and write

$$\begin{aligned} \eta_{j,t} n_{j,t} &= \max_{\psi_{j,t}} \Lambda_{t,t+1}^{\text{bank}} (f + (1-f) \eta_{j,t+1}) [(r_{t+1}^K - r_{t+1}^B) \psi_{j,t} + (1 + r_{t+1}^B)] n_{j,t} \\ &\quad + \lambda_{j,t} \left[ \eta_{j,t} - \frac{1}{\theta_t} \psi_{j,t} \right] n_{j,t} \end{aligned}$$

First order condition with respect to  $\psi_{j,t}$  is

$$\Lambda_{t,t+1}^{\text{bank}} (f + (1-f) \eta_{j,t+1}) (r_{t+1}^L - r_{t+1}^D) = \frac{1}{\theta_t} \lambda_{j,t}$$

so

$$\eta_{j,t}n_{j,t} = \Lambda_{t,t+1}^{\text{bank}} (f + (1-f)\eta_{j,t+1}) (1 + r_{t+1}^D) n_{j,t} + \lambda_{j,t}\eta_{j,t}n_{j,t}$$

i.e.

$$\eta_{j,t} = \frac{1}{1 - \lambda_{j,t}} \Lambda_{t,t+1}^{\text{bank}} (f + (1-f)\eta_{j,t+1}) (1 + r_{t+1}^D)$$

The guess that  $v_{j,t}(n_{j,t}) = \eta_{j,t}n_{j,t}$  is verified if  $\lambda_{j,t} < 1$ .

By complementarity slackness  $\lambda_{j,t} \left[ \eta_{j,t} - \frac{1}{\theta_t} \psi_{j,t} \right] = 0$  and we can write

$$\eta_{j,t}n_{j,t} = \max_{\psi_{j,t}} \Lambda_{t,t+1}^{\text{bank}} (f + (1-f)\eta_{j,t+1}) \left[ (r_{t+1}^K - r_{t+1}^B) \psi_{j,t} + (1 + r_{t+1}^B) \right] n_{j,t}.$$

If the incentive compatibility constraint is binding, we have

$$\eta_{j,t} = \Lambda_{t,t+1}^{\text{bank}} (f + (1-f)\eta_{j,t+1}) \left[ (r_{t+1}^K - r_{t+1}^B) \eta_{j,t} \theta_t + (1 + r_{t+1}^B) \right]$$

which can be rearranged as

$$\eta_{j,t} = \frac{\Lambda_{t,t+1}^{\text{bank}} (f + (1-f)\eta_{j,t+1}) (1 + r_{t+1}^B)}{1 - \Lambda_{t,t+1}^{\text{bank}} (f + (1-f)\eta_{j,t+1}) (r_{t+1}^K - r_{t+1}^B) \theta_t}.$$

As all banks face the same rates of return, the marginal value of net worth  $\eta_{j,t}$  is the same for them,  $\eta_t$ . It follows that, if the incentive compatibility constraint is binding,

$$l_{j,t}^B = \theta_t \eta_t n_{j,t}$$

and so if  $\Lambda_{s-1,s}^{\text{bank}} = 1/(1 + r_s^B)$  or  $\Lambda_{s-1,s}^{\text{bank}} = 1/(1 + r_s^K)$  we can write

$$l_{j,t}^B = \Theta_t \left( \{r_{s+1}^B, r_{s+1}^K; \theta_s\}_{s \geq t} \right) n_{j,t}.$$

Aggregating individual banks  $\int_0^1 l_{j,t}^B dj = q_t k_t^B$  and  $\int_0^1 n_{j,t} dj = n_t^B$  we obtain

$$q_t k_t^B = \Theta_t \left( \{r_{s+1}^B, r_{s+1}^K; \theta_s\}_{s \geq t} \right) n_t$$

which coincides with the solution to the bank's problem described in Section 2.2. In this model, if  $\Lambda_{s-1,s}^{\text{bank}} = 1 / (1 + r_s^K)$ ,

$$\begin{aligned} \bar{\Theta}_{r^K} &= \frac{\Theta(\Theta - 1)}{1 + r^K} \\ \bar{\Theta}_{r^B} &= -\frac{\Theta(\Theta - 1)}{1 + r^B} \\ \gamma &= \frac{(1 - f)(1 + r^B + (r^K - r^B) \Theta)^2}{(1 + r^K)(1 + r^B)}, \end{aligned}$$

and if  $\Lambda_{s-1,s}^{\text{bank}} = 1 / (1 + r_s^B)$ ,

$$\begin{aligned} \bar{\Theta}_{r^K} &= \frac{1}{1 + r^B} \Theta^2 \\ \bar{\Theta}_{r^B} &= -\frac{1 + r^K}{1 + r^B} \Theta^2 \\ \gamma &= \frac{(1 - f)(1 + r^B + (r^K - r^B) \Theta)^2}{(1 + r^K)(1 + r^B)}. \end{aligned}$$

Here  $\Theta = \theta\eta$ , the steady state leverage ratio.

## B.2 Bernanke, Gertler, Gilchrist (1999)

In [Bernanke et al. \(1999\)](#) financial frictions arise because of "costly state verification" ([Townsend \(1979\)](#)). In their model there is a continuum of entrepreneurs that need to finance capital purchases. Their realized returns are idiosyncratic and cannot be observed by the lenders, unless they incur a monitoring cost. This creates a link between entrepreneurs' capital expenditures, their net worth and the spread between the expected return on capital and the safe rate. Entrepreneurs face a constant probability of exit  $f$  and consume their retained earnings upon exiting. We

can map this model to our framework by reinterpreting entrepreneurs as banks. The key condition in [Bernanke et al. \(1999\)](#) is Equation 3.8 (p. 1353)

$$q_t k_t^B = \psi \left( \frac{1 + r_{t+1}^K}{1 + r_{t+1}^B} \right) n_t$$

with  $\psi'(\cdot) > 0$  and  $\psi(1) = 1$ .<sup>15</sup> If we define

$$\Theta_t \left( \{r_{s+1}^K, r_{s+1}^B; \theta_s\}_{s \geq t} \right) := \psi \left( \frac{1 + r_{t+1}^K}{1 + r_{t+1}^B} \right)$$

the solution to the bank's problem described in Section 2.2 and dynamics of bank net worth will coincide with the one in [Bernanke et al. \(1999\)](#). Notice that here the financial friction at time  $t$  depends only on  $r_{t+1}^K$  and  $r_{t+1}^B$  and not on returns more than one period ahead. In this model

$$\begin{aligned} \bar{\Theta}_{r^K} &= \psi' \left( \frac{1 + r^K}{1 + r^B} \frac{1}{1 + r^B} \right) \\ \bar{\Theta}_{r^B} &= -\psi' \left( \frac{1 + r^K}{1 + r^B} \right) \frac{1 + r^K}{(1 + r^B)^2} \\ \gamma &= 0. \end{aligned}$$

### B.3 Costly leverage

[Uribe and Yue \(2006\)](#), [Eggertsson et al. \(2019\)](#), [Chi et al. \(2021\)](#) and [Cúrdia and Woodford \(2011\)](#) consider reduced form financial frictions. They assume that banks need to incur a resource cost that depends on the level of financial intermediation. Since the marginal cost of intermediation is increasing in the scale of intermediation, there will be a link between the leverage ratio and the spread between returns on assets held by banks and deposits. Our framework allows us to nest these models without any modification to the framework if we assume that this cost is borne in

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<sup>15</sup>There is no aggregate uncertainty in our framework and this explain why there is no expectation operator in front of  $r_{t+1}^K$ .



units of utility or that it is rebated back lump-sum to the bank. We need to make this change to ensure that the law of motion for  $n_t$ , Equation 2, remains the same. More specifically, assume that the bank maximizes

$$\nu_{t+1}n_t = \max_{k_t^B, d_t} r_{t+1}^K q_t k_t^B - r_{t+1}^B d_t - \Upsilon_t \left( \frac{q_t k_t^B}{n_t} \right) n_t + \tilde{\Upsilon}_t$$

subject to balance sheet

$$q_t k_t^B = d_t + n_t.$$

Here  $\Upsilon_t \left( \frac{q_t k_t^B}{n_t} \right) n_t$  captures costs related to financial intermediation.  $\tilde{\Upsilon}_t$  is the lump-sum rebate, equal to intermediation costs in equilibrium. Assume it is strictly increasing in the leverage ratio  $\psi_t := q_t k_t^B / n_t$ . First order condition is

$$r_{t+1}^K - r_{t+1}^B = \Upsilon_t' \left( \frac{q_t k_t^B}{n_t} \right)$$

which can be rewritten as

$$q_t k_t^B = \Upsilon_t'^{-1} (r_{t+1}^K - r_{t+1}^B) n_t.$$

It is enough to define

$$\Theta_t \left( \{r_{s+1}^K, r_{s+1}^B; \theta_s\}_{s \geq t} \right) := \Upsilon_t'^{-1} (r_{t+1}^K - r_{t+1}^B)$$

to ensure that the solution to the bank's problem described in Section 2.2 will be the same as the one to the problem stated above. Note that  $\Theta_t$  does not depend on returns more than one period in the future. Moreover, since  $\Upsilon_t \left( \frac{q_t k_t^B}{n_t} \right) n_t = \tilde{\Upsilon}_t$ ,

$\nu_{t+1}n_t$  is the same as in section. In this model

$$\begin{aligned}\bar{\Theta}_{r^K} &= \frac{1}{\Upsilon''\left(\frac{qk^B}{n}\right)} \\ \bar{\Theta}_{r^B} &= -\frac{1}{\Upsilon''\left(\frac{qk^B}{n}\right)} \\ \gamma &= 0.\end{aligned}$$

## B.4 Collateral constraints and regulator constraints

Kiyotaki and Moore (1997) consider the following collateral constraint

$$(1 + r_{t+1}^B) d_t \leq q_t k_t^B.$$

By using the balance sheet we can rewrite it as

$$q_t k_t^B \leq \left(1 + \frac{1}{r_{t+1}^B}\right) n_t$$

and we can map it to our framework by defining

$$\Theta_t \left( \{r_{s+1}^K, r_{s+1}^B; \theta_s\}_{s \geq t} \right) := 1 + \frac{1}{r_{t+1}^B}$$

In a more general case in which only a fraction  $1/\theta_t$  of  $q_t k_t^B$  can be pledged as collateral we have

$$\Theta_t \left( \{r_{s+1}^K, r_{s+1}^B; \theta_s\}_{s \geq t} \right) = \frac{1 + r_{t+1}^B}{1 + r_{t+1}^B - \frac{1}{\theta_t}}.$$

Here we have

$$\begin{aligned}\bar{\Theta}_{r^K} &= 0 \\ \bar{\Theta}_{r^B} &= -\frac{1}{\theta} \frac{1}{\left(1 + r^B - \frac{1}{\theta}\right)^2} \\ \gamma &= 0.\end{aligned}$$

Regulatory constraints that limit the leverage ratio as in, for example, [Van den Heuvel \(2008\)](#), can be analyzed in our framework by setting

$$\Theta_t \left( \left\{ r_{s+1}^K, r_{s+1}^B; \theta_s \right\}_{s \geq t} \right) = \theta_t.$$

Here  $\theta_t$  is simply the maximum allowed leverage ratio. In this case

$$\begin{aligned}\bar{\Theta}_{r^K} &= 0 \\ \bar{\Theta}_{r^B} &= 0 \\ \gamma &= 0.\end{aligned}$$

## C Bring Model to Data: Balance sheets

We take household and banks balance sheet data from the Financial Accounts of the United States, 2019 Q3: <https://www.federalreserve.gov/releases/z1/20210923/z1.pdf>

**Household balance sheet:** Table B.101h, 2021

- liquid asset: checkable deposit, time deposit, money market fund share, and government debt (treasury plus municipal debt)
- liquid liability: credit card loans in Table L.222 (included under consumer credit in B.101h)
- illiquid liability: total liability - liquid liability

Households			
assets		liability and equity	
- liquid asset	16944	equity	109998
(checkable deposit,	( 1003,		
time deposit,	9602,		
mmf share,	2097,		
gov debt)	4242.)	liquid liability	1092
- illiquid asset (others)	108651	illiquid liability	14505

- Consolidate household illiquid asset with illiquid liability: 108651 - 14505. Note: holders of illiquid liability now directly hold the illiquid assets that are netted out.
- Subtract underfunded government DB amount (L.119.b and L.120, Claims of pension fund on sponsor): 1673 (federal) + 4157 (state)
- Consolidate government debt held by government DB to get claims of net government debt outstanding: 1893 (federal) + (367 + 4) (state)
- Subtract government debt held in household's illiquid asset outside public pensions (4456)
  - Treasury Securities (L.210) : treasury debt held by nonfinancial business (63 + 74), insurance companies (153 + 215), private pension (440), mutual fund(1310), the rest of the domestic financial sector (2+231+175+33+213+55)
  - Municipal Securities (L.212): muni debt held by insurance companies (285 + 215), mutual fund (831), the rest of the domestic financial sector (93+49+3+16)
- Net illiquid asset:  $108651 - 14505 - (1673 + 4157) - (1893 + 367 + 4) - 4456 - (63 + 74 + 153 + 215 + 440 + 1310 + 2 + 231 + 175 + 33 + 213 + 55) - (285 + 215 + 831 + 93 + 49 + 3 + 16) = 77140$

**Bank balance sheet:** Table L.110

Banks (Private Depository Institution)			
assets		liability and equity	
- liquid assets	3726	liquid liability	15532
(cash,	(83,	(checkable deposit,	(2648,
reserves,	1548,	time deposit)	12884)
fed fund,	738,		
treasury,	879,		
muni,	478)		
- credit card loan	1100		
- asset (others)	15230	other liability	4790
- goodwill	2752	equity	2486

- Bank equity is calculated from:  $(11486 + 83)$  (corporate and noncorporate equity in domestic financial sector, L.223)  $\times (3726 + 16330) / (107423 - 11042 - 3038)$  (bank asset share in the domestic financial sector, excluding pension funds, mutual funds, monetary market fund, L.108) = 2486. This assumes the same leverage ratio as in other financial institutions in banks. Goodwill is calculated as a residual:  $15532 + 4790 + 2486 - 3726 - 1100 - 15230$
- non-household deposit (from the HH and Bank tables above):  $15532 - 1042 - 9061 = 5429$
- net capital held by banks:  $15230 + 2752$  (goodwill)  $- 5429$  (non-household deposit)  $- 4790$  (other liability) = 7763 (consolidate non-household borrowing and lending, as well as other liability)

Consolidating liquid assets and liability, we have the following tables corresponding to the model:

Households			
assets		liability and equity	
- liquid asset	15844	equity	92992
- illiquid asset	77140		

Banks (Private Depository Institution)			
assets		liability and equity	
- liquid asset	3726	liquid liability	9003
- capital	7763	equity	2486

As a fraction of GDP, we have

Households			
assets		liability and equity	
- liquid asset	0.74	equity	4.34
- illiquid asset	3.6		

Banks (Private Depository Institution)			
assets		liability and equity	
- liquid asset	0.18	liquid liability	0.43
- capital	0.37	equity	0.12