

Firm Size and Business Cycles with Credit Shocks ^{*}

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Abstract

I study the macroeconomic implications of firm heterogeneity in the presence of financial frictions. I build a business cycle model in which firm size is jointly determined by idiosyncratic productivity and collateral constraints. I estimate idiosyncratic shocks and align the model with the empirical evidence on firm size, leverage, and investment moments. The extent of resource misallocation is driven by a small number of highly productive firms that are prevented from becoming very large. A credit shock severely affects such firms, further constraining their ability to borrow. This generates a large and persistent economic downturn that is comparable to the Great Recession.

Keywords: business cycles, collateral constraints, firm size, credit shocks, misallocation

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1 Introduction

The Great Recession has brought financial shocks to the forefront in the study of business cycles. In particular, aggregate dynamics over the recession were in many ways at odds with the predictions of a standard model with shocks to total factor productivity (TFP).¹ Furthermore, small firms suffered relatively more from the disruption in financial markets.² These observations illustrate the distinctive nature of financial shocks which affect firms unevenly, their incidence varying with an individual firm's financing constraint and need for investment. It follows that the aggregate impact of a financial shock critically depends on how many firms are financially constrained, and by how much. This insight suggests that a quantitative assessment of the real effects of financial shocks requires a business cycle model that incorporates realistic firm heterogeneity. In this paper, I focus on firm size heterogeneity and study its importance in propagating aggregate shocks.

Following Jermann and Quadrini (2012), a growing body of work quantitatively investigates the aggregate effects of financial shocks. One common approach is to introduce financial frictions into a model with production heterogeneity, as in Khan and Thomas (2013) and Buera and Moll (2015). Existing business cycle models, however, abstract from evidence on the firm size distribution. This raises a natural question of how aggregate outcomes change with the underlying firm size distribution in an economy. Moreover, firm size is widely regarded as a robust indicator of financing constraints that are not directly observable in the data.³ Thus, reproducing the empirical firm size distribution provides a useful way of nesting unobserved financial heterogeneity into a model. This point is rarely addressed in the literature, partly due to a perception

¹Khan and Thomas (2013) document that the US economy experienced a large and persistent recession from 2007Q4, whereas the fall in measured TFP was relatively small.

²Bloom et al. (2018) report increases in the cross-sectional dispersion of firm growth during the Great Recession. Duygan-Bump, Levkov, and Montoriol-Garriga (2015) find that employment fell relatively more among small firms with financing constraints, while Chodorow-Reich (2014) relates a bank credit channel to the fall in employment by small firms.

³Extensive studies adopt different indicators to measure financing constraints, including firm size, in a small sample of listed firms. See Rajan and Zingales (1995) and Whited and Wu (2006), among others.

that firm heterogeneity has a marginal impact on aggregates. I argue that a model's consistency with firm-level data, *micro-level consistency*, is crucial in quantifying the macroeconomic effects of a financial shock. Consequently, this paper serves as a production-side counterpart to recent studies on the interaction between inequality and aggregate dynamics in Krueger, Mitman, and Perri (2016) and Ahn et al. (2017). By focusing on firm heterogeneity, in addition, I extend the approach of Pugsley, Sedlacek, and Sterk (2021) to an economy with financial frictions and aggregate uncertainty.

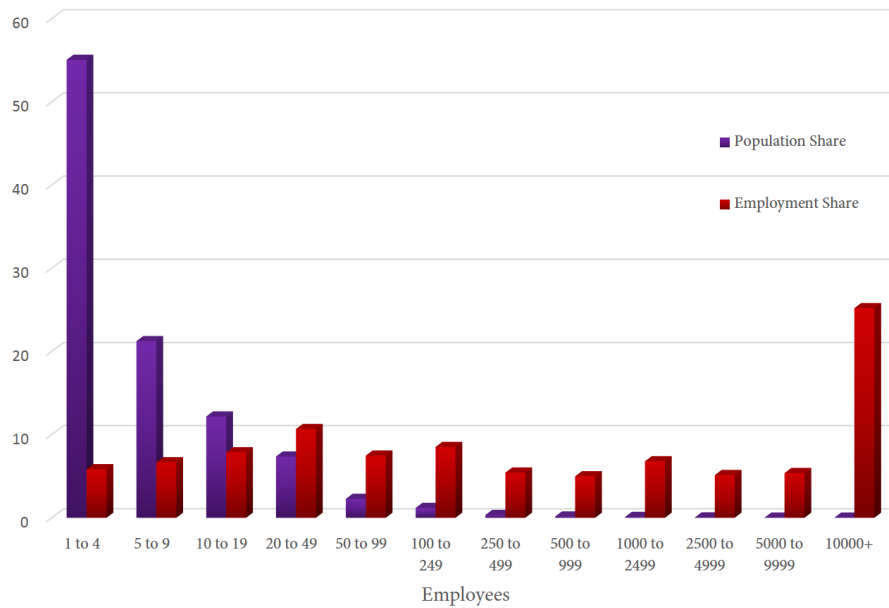
To explore the role of firm heterogeneity over business cycles, I build an equilibrium model with production heterogeneity and financial frictions. Firms are heterogeneous in their productivity, capital, and leverage, and are commonly affected by both aggregate productivity and financial shocks. Two distinctive features differentiate my model from a standard framework. First, I employ a Pareto-distributed stochastic process for firms' idiosyncratic productivity and estimate it by using firm-level data.⁴ This generates substantial differences across firms, and the skewness and the dispersion in the empirical firm size distribution are captured by the model. Second, financial frictions are represented by a forward-looking collateral constraint. Specifically, firms' access to loans is constrained by their capital which serves as collateral, and a credit shock implies an economy-wide change in borrowing capacity. This approach is in keeping with the original model of Kiyotaki and Moore (1997), and it further helps characterize firm-level decision rules in a tractable manner.

The model is consistent with the firm size and leverage distribution in the data. It is well known that US firm distribution is highly skewed and dispersed across different employment size groups in the Business Dynamics Statistics (BDS).⁵ In Figure 1, for instance, about 90 percent of firms are small, hiring fewer than 20 employees, while the largest 0.2 percent of firms

⁴This can be regarded as a simplified version of the time-varying asymmetric shocks in Bloom, Guvenen, and Salgado (2020). I consider different specifications of the assumed process, to establish the robustness of the main results in this paper.

⁵Based on US Census, the BDS covers about 97 percent of private employment, starting from 1977. Firms and establishments are categorized by size and age, and firm size is measured by the number of employees. See Haltiwanger, Jarmin, and Miranda (2009) for details.

Figure 1 : Average firm size distribution in the BDS, 1977 to 2006



Note: I calculate the average firm size distribution from the annual tables in the BDS database from 1977 to 2006. The BDS tables are reported in 12 employment size bins that are fixed over time.

account for 43 percent of total employment. The model closely reproduces this observed firm size heterogeneity, in addition to matching aggregate moments of the US data. Since firm size is an important determinant of leverage, the model further captures the underlying heterogeneity in financial frictions. As a result, the model is tightly aligned with the empirical leverage distribution in the US, and predicts a positive relationship between firm size and leverage.⁶ In contrast, when firms are not sufficiently different in size, and the model is inconsistent with the firm size data, as in existing studies, the size-leverage relationship becomes counterfactual. This validates the importance of micro-level consistency with the empirical size distribution, particularly in a model of heterogeneous firms with financial frictions.

The consistency with the micro-level evidence on firm size distribution and investment dynamics in my model significantly alters the impact of a credit shock with non-trivial and per-

⁶I confirm the empirical size-leverage relationship using a panel of firms in developed economies in Section 3.2. The estimated regression coefficient from model simulated data is close to its empirical counterparts (Table 8).

sistent production heterogeneity. Specifically, I show that a recession triggered by a negative credit shock is more severe when the firm size distribution in the model closely resembles that in the data. To this end, I compare aggregate dynamics across models with different firm distributions. *The baseline model* features Pareto-distributed firm productivity, and has realistic firm heterogeneity in size and leverage. In contrast, *the alternative model* abstracts from the empirical size distribution. In this alternative model, I instead use a conventional log-AR(1) process for firm-level productivity, ignoring the observed skewness in productivity shocks at the firm level.^{7,8} Following a tightening of credit, the baseline economy experiences a deep and persistent recession that is comparable to the Great Recession. Measured TFP gradually falls to 1.5 percent below its steady-state level, while aggregate output drops more than 6 percent at its trough. These responses remain robust when I additionally require the model to incorporate permanent heterogeneity in firm productivity as emphasized by Pugsley, Sedlacek, and Sterk (2021). On the other hand, the alternative model generates *qualitatively* similar aggregate dynamics as in the baseline, but of a substantially smaller magnitude. In particular, the TFP loss due to tightened credit is less than 1 percent and the largest decline in output is about 4 percent, implying a comparatively weaker propagation of the financial shock. Thus, it is obvious that firm heterogeneity *does* interact with aggregates when financial shocks occur.

Conversely, I find that firm heterogeneity does not have large macroeconomic implications following an aggregate TFP shock. The model shows that changes in aggregate series, following a persistent productivity shock, are largely independent of the underlying firm distribution. In particular, their response is similar to that generated in a conventional business cycle model without financial frictions, which is incapable of explaining the Great Recession. Since the shock does not distort borrowing conditions across firms in my model, the corresponding effects on a firm's decisions are independent of its size and the entire firm distribution shifts proportionately.

⁷Given the same aggregate moments across models, this process only matches the empirical standard deviation of firm individual productivity in the data, not its skewness.

⁸This is in the spirit of influential existing studies of production heterogeneity by Khan and Thomas (2008), Bloom (2009), and Bloom et al. (2018).

That is, a productivity shock evenly affects firms of different sizes, without generating distributional effects in the model.

As suggested by the above discussion, a credit shock operates through changes in the distribution of firms. Firms in the model respond to credit tightening by adjusting their investment and employment. These adjustments are not identical across firms, due to differences in their capital held as collateral. In particular, small firms with binding borrowing constraints suffer relatively more than others, because limited credit further stifles their ability to finance investment. This reduces the allocative efficiency of productive factors across firms in the economy, and aggregate productivity falls endogenously. Further, the extent of resource misallocation rises over time, as more firms become financially constrained, in subsequent periods following the credit shock. Khan and Thomas (2013) and Buera and Moll (2015) highlight the importance of such a misallocation channel, but their models are not able to generate a large recession as seen in the US data.⁹ This paper demonstrates that incorporating realistic differences in firm size substantially amplifies the effects of a credit shock. By doing so, it suggests the importance of micro-level consistency for studying aggregate dynamics under resource misallocation.

Intuitively, when highly productive firms are prevented from growing as a result of borrowing constraints, the impact of a negative credit shock is large. This is because the shock not only tightens firms' borrowing limits but also reshapes the distribution of firms. To understand these distributional effects, I decompose the resource misallocation channel into two margins: (i) the average degree of investment distortions among firms with binding borrowing constraints (*the intensive margin*) and (ii) the number of such firms in the distribution (*the extensive margin*).

The above decomposition then allows me to measure the relative importance of each margin through the lens of the model. First, at the steady state of the baseline model, about 9 percent of firms are financially constrained, the extensive margin. Due to binding borrowing constraints, their constrained capital choice averages 32 percent of the optimal level, which represents the

⁹In Khan and Thomas (2013), the largest fall in measured TFP following a credit shock is about 1 percent. In the benchmark model of Buera and Moll (2015), the falls in TFP and output are almost one-to-one.

intensive margin. Consequently, the resource misallocation in this economy is driven by a small number of highly productive but constrained firms that would otherwise become very large, and not by a large number of small firms due to low productivity. This is in stark contrast with the alternative model in which 23 percent of firms are financially constrained but to a lesser degree. These constrained firms are not as highly productive as those in the baseline economy, so the intensive margin distortions are relatively less severe. Accordingly, the aspects of resource misallocation differ with a model's consistency with firm-level data.

The propagation of a credit shock critically depends on the associated changes in the two margins of resource misallocation. Following a credit tightening, the baseline model shows that both margins substantially increase at impact and then gradually revert to their steady-state levels. In particular, the intensive margin adjusts by more than 20 percent, implying that small-constrained firms are relatively more vulnerable to tightened credit. This is consistent with the disproportionate responses among small and young businesses during the Great Recession documented by Siemer (2019). On the other hand, a credit shock in the alternative model leads to relatively small changes in both margins of resource misallocation. This in turn characterizes the weak propagation of the shock through the distribution of firms, when the model is inconsistent with the empirical distributions of firms.

An independent contribution of this paper is to offer the joint estimation of aggregate productivity and credit shocks in a model with heterogeneous firms. This allows me to measure the relative contribution of each shock to the US business cycle. The estimation results indicate that productivity shocks are the main driver of economic fluctuations, while credit shocks explain about 10 percent of output volatility. This is because the estimated credit shocks are infrequent and short-lived, so their contribution is relatively modest in the model. This result is in contrast to the prediction of Jermann and Quadrini (2012), who find the dominant role of financial shocks in an estimated model of a representative firm.

The rest of the paper is organized as follows. Below, I provide a brief review of the most

relevant literature. Section 2 presents the model environment, together with a description of firm-level decision rules. I calibrate and estimate the model in Section 3, while motivating the consistency with the empirical size distribution. Section 4 reports the aggregate dynamics in the model, and Section 5 concludes.

Related Literature This paper builds on recent studies that quantitatively investigate the real effects of financial shocks through the reallocation of production factors over time. In addition to Khan and Thomas (2013) and Buera and Moll (2015) mentioned earlier, Buera, Fattal-Jaef, and Shin (2015) and Gavazza, Mongey, and Violante (2018) focus on frictions in hiring markets to explain unemployment dynamics during the Great Recession. These studies, however, do not closely match the empirical size and leverage distributions.¹⁰ This paper shows that tight consistency with such firm-level data leads to a large amplification of financial shocks. The model that I consider, in addition, allows me to estimate aggregate shocks in a heterogeneous firm model, which is relatively scarce in the literature.¹¹

This paper is closely linked to recently growing research that asks whether micro-level heterogeneity matters for macroeconomic outcomes. Both Krueger, Mitman, and Perri (2016) and Ahn et al. (2017) examine the role of household inequality in propagating real shocks, while Beraja et al. (2019) look at changes in housing equity distribution in response to a stimulus policy. Krueger, Mitman, and Perri emphasize that realistic household heterogeneity is necessary for explaining the changes in aggregate consumption during the Great Recession. The relevance of heterogeneity in these studies, in fact, challenges the perception that aligning a model with micro-level data unnecessarily adds complexity in business cycle studies.¹² I provide further evidence against this view in my analysis of the distributional effects of credit shocks in an econ-

¹⁰Buera, Fattal-Jaef, and Shin (2015) reproduce the employment share of entrepreneurs at the top 10 percent, and Gavazza, Mongey, and Violante (2018) target the BDS distribution in 3 size bins.

¹¹Winberry (2018) and Mongey and Williams (2017) estimate real aggregate shocks, while Ajello (2016) estimates multiple shocks in a New Keynesian model of entrepreneurs. I use a simulation-based method to match moments, in contrast to the Bayesian approach in these studies.

¹²This perception is partly due to the approximate aggregation results in Krusell and Smith (1998). However, their extended model with preference shocks is a useful counterexample, as discussed in Ahn et al. (2017).

omy with heterogeneous firms and collateral constraints. Furthermore, the propagation of such financial shocks is mainly through highly productive but small firms in my model, differently from the transmission of monetary shocks in Ottonello and Winberry (2020).

An extensive literature documents rich firm-level heterogeneity in size and leverage. Among others, Beck, Demiruguc-Kunt, and Maksimovic (2005) examine how firm growth is affected by financial frictions, whereas Whited and Wu (2006) look at the close relationship between firm size and financial variables. Relatedly, the empirical analysis in this paper builds on the finding that small firms are more likely to be financially constrained. To support the idea of using firm size as a proxy of financing conditions, I construct a panel of firms from the Orbis database and analyze the relationship of employment with leverage.¹³ Lian and Ma (2021) show that small firms tend to rely on asset-based financing, consistent with the collateral constraints in my model.

Finally, I study the implications of a model's consistency with firm size and leverage data in the presence of financial frictions. This is motivated by Cabral and Mata (2003), who study the relationship between financing constraints and the firm size distribution.¹⁴ In addition, motivated by Gabaix (2011) and Elsby and Michaels (2013), I assume that firm productivity is Pareto-distributed in the model, which allows for capturing the observed skewness in the empirical distribution of idiosyncratic shocks.

2 Model

Time is discrete in infinite horizon. The economy has a large number of heterogeneous firms that face persistent shocks to aggregate and idiosyncratic productivity. An individual firm's external borrowing is subject to a collateral constraint in the presence of credit shocks. Households

¹³The Orbis covers private firms in addition to listed firms in Compustat. Dinlersoz et al. (2019) construct a representative dataset that combines the Orbis with other sources, while Zetlin-Jones and Shourideh (2017) focus on differences in financing patterns between private and public firms.

¹⁴See Poschke (2018) for cross-country differences in the observed size distribution. Luttmer (2007) highlights the role of selection mechanism in shaping firm size distribution.

are identical and infinitely lived, and markets are perfectly competitive.

2.1 Firms

Production and Financial Frictions A continuum of firms produce a homogeneous good. Each firm owns its predetermined capital stock, k , and hires labor, n , in a competitive labor market. The production technology is given by $y = z\epsilon F(k, n)$, where $F(\cdot)$ exhibits decreasing returns to scale (DRS). The exogenous component of aggregate productivity, z , is common across firms, and it follows a Markov chain with $\pi_{fg}^z \equiv Pr(z' = z_g | z = z_f) \geq 0$ and $\sum_{g=1}^{N_z} \pi_{fg}^z = 1$ for each $f = 1, \dots, N_z$. Next, firm-specific productivity, ϵ , also follows a Markov chain such that $\epsilon \in \mathbf{E} \equiv \{\epsilon_1, \dots, \epsilon_{N_\epsilon}\}$ with $\pi_{ij}^\epsilon \equiv Pr(\epsilon' = \epsilon_j | \epsilon = \epsilon_i) \geq 0$ and $\sum_{j=1}^{N_\epsilon} \pi_{ij}^\epsilon = 1$ for each $i = 1, \dots, N_\epsilon$. Capital accumulation is standard, $k' = (1 - \delta)k + i_k$ with $\delta \in (0, 1)$, where i_k denotes investment.

Financial frictions are in the form of a collateral constraint. Specifically, firms face borrowing constraints for one-period discount debt, given its price q . Due to the limited enforceability of financial contracts, the amount of newly issued debt in the current period, b' , is limited by a firm's collateral. I assume that the firm's future capital stock, k' , serves as collateral, and the borrowing constraint is given by

$$b' \leq \theta k',$$

where $\theta \in (0, q^{-1})$ captures the level of financial frictions in the economy.¹⁵ Further, θ is assumed to be common across firms and to follow a Markov chain with $\theta \in \Theta \equiv \{\theta_1, \dots, \theta_{N_\theta}\}$. A sudden decrease in θ , thus, corresponds to a (negative) credit shock. Lastly, firms are allowed to accumulate financial wealth held as negative debt. This implies that the collateral constraint may be binding for some, but not all, firms in a given period.

¹⁵When θ approaches the real interest rate, firms are always allowed to finance their desired investment. As in Kiyotaki and Moore (1997), the above collateral constraint is forward-looking while abstracting from the feedback channel of asset prices. This timing assumption further allows me to characterize firms' decisions in a tractable manner in Section 2.3, in contrast to a backward-looking constraint in Khan and Thomas (2013).

Entry, Exit, and Firm Distribution Firms are subject to a fixed probability of exit, $\pi_d \in (0, 1)$, in each period, and exiting firms are replaced by an equal measure of new firms. Entrants are endowed with an initial capital that is a χ fraction of the average capital of the incumbents. This initial endowment is financed by households in exchange for the ownership of firms. The arrival of exit information is known at the beginning of a period, and exiting firms liquidate all of their remaining earnings and assets after production.¹⁶

I describe the state variables of the model and the timing of exogenous shocks and decisions within a given period. At the beginning of a period, a firm is identified by its individual state (k, b, ϵ) ; the predetermined capital, $k \in \mathbf{K} \subset \mathbf{R}_+$; the amount of existing debt to be repaid, $b \in \mathbf{B} \subset \mathbf{R}$; and the current idiosyncratic productivity, $\epsilon \in \mathbf{E}$. The distribution of firms is summarized by a probability measure, $\mu(k, b, \epsilon)$, which is defined on a Borel algebra generated by the open subsets of the product space, $\mathbb{S} \equiv \mathbf{K} \times \mathbf{B} \times \mathbf{E}$. For simplicity, I use $s \equiv (z, \theta)$ to denote the exogenous aggregate state of the model, with its transition probability given by $\pi_{lm}^s \geq 0$. Then the aggregate state is (s, μ) , and the evolution of the firm distribution follows a mapping, $\mu' = \Gamma(s, \mu)$.

Just after the exogenous shocks to s and ϵ are realized, a firm learns whether it will survive to the next period or exit at the end of the current period. Given $(k, b, \epsilon; s, \mu)$, the firm maximizes the expected discounted sum of all dividends. After production is completed, the firm pays the wage bill and clears the existing debt. Conditional on its survival into the next period, the firm chooses its investment for future capital, k' , alongside new debt level, b' . At the same time, the firm must determine the current dividends, D , to be paid to its shareholders. Firms take the wage rate, $w(s, \mu)$, and the discounted debt price, $q(s, \mu)$, as given.

Firm's Problem I reduce the dimension of a firm's individual state (k, b, ϵ) by defining a new

¹⁶This timing ensures that all outstanding debt is repaid, consistent with the assumed collateral constraint. Further, the setting of entry and exit is a simple way to prevent firms from surviving indefinitely as opposed to the evidence on firm dynamics. Note that the model may deliver a stationary distribution of firms in the absence of firm entry and exit, depending on the dispersion of productivity. I numerically check that entry and exit are necessary elements for having financially constrained firms, as kindly suggested by an anonymous referee.

variable called *cash-on-hand*, and then formulate the firm's problem in a recursive way. This approach allows me to derive the decisions of investment and borrowing as functions of the newly defined variable, without altering the equilibrium allocation.¹⁷

Let $n = N^w(k, \epsilon; s, \mu)$ be the static labor choice such that $z\epsilon D_2 F(k, n) = w(s, \mu)$, and define $m(k, b, \epsilon)$ as a firm's cash-on-hand after production and debt repayment.

$$m(k, b, \epsilon; s, \mu) \equiv z\epsilon F(k, N^w) - w(s, \mu)N^w + (1 - \delta)k - b$$

It is clear that all information relevant to a firm's decisions on investment and borrowing is contained in $m(k, b, \epsilon)$, and these decisions made in the current period jointly determine the future holding of cash-on-hand, $m(k', b', \epsilon')$, alongside the realizations of (s', ϵ') .

Next, I define value functions by whether the exit status of a firm is known. Let $v_0(m, \epsilon; s, \mu)$ be the expected discounted value of the firm before the exit shock is realized, at the beginning of a period. If the firm is allowed to move on to the next period, its within-the-period value is given by $v(m, \epsilon; s, \mu)$. The firm's optimization problem is then recursively described by v_0 and v .

$$v_0(m, \epsilon_i; s_l, \mu) = \pi_d \cdot m + (1 - \pi_d) \cdot v(m, \epsilon_i; s_l, \mu) \quad (1)$$

$$v(m, \epsilon_i; s_l, \mu) = \max_{k', b', D} \left[D + \sum_{m=1}^{N_s} \pi_{lm}^s d_m(s_l, \mu) \sum_{j=1}^{N_\epsilon} \pi_{ij}^\epsilon v_0(m'_{jm}, \epsilon_j; s_m, \mu') \right] \quad (2)$$

subject to

$$0 \leq D = m - k' + qb'$$

$$b' \leq \theta k'$$

$$\mu' = \Gamma(s_l, \mu)$$

$$m'_{jm} \equiv m(k', b', \epsilon_j; s_m, \mu')$$

¹⁷I include the original version of the firm's problem in the online appendix, together with its transformation with market clearing prices that are consistent with household decisions.

Equation (1) implies that the firm takes a binary expectation over the values before its exit or survival is known, given the current realizations of (s_t, ϵ_i) . The exit probability, π_d , serves as the weight on the value of exiting, and the exiting firm maximizes its liquidation value of m . Conditional on survival, the firm's problem in Equation (2) involves the intertemporal decisions. The firm optimally chooses its future capital, k' , and new debt level, b' , to maximize the sum of the current dividends, D , and the future expected discounted value, $v_0(m'_{jm}, \epsilon_j; s_m, \mu')$. The dividends correspond to the residual from the firm's budget constraint, and they are restricted to be non-negative.¹⁸ The firm takes as given the stochastic discount factor, $d_m(s_t, \mu)$, which is to be determined in equilibrium given the households' marginal rate of substitution.

2.2 Households and Equilibrium

Representative Household There is a unit measure of identical households. Households earn labor income by supplying some of their time endowment in each period, and the period utility is given by $U(C, 1 - N)$ with the subjective discount factor $\beta \in (0, 1)$. They hold a comprehensive portfolio of assets: firm shares of measure λ and non-contingent discount bonds ϕ . The representative household maximizes the lifetime expected discounted utility by choosing its consumption, C^h , and labor supply, N^h , while adjusting the asset portfolio. Denote $\rho_1(k', b', \epsilon'; s_t, \mu)$ as the ex-dividend prices of firm shares, and $\rho_0(k, b, \epsilon; s_t, \mu)$ as the dividend-inclusive value for

¹⁸This is a common approach in recent studies such as Ottonello and Winberry (2020), based on the finding that firms pay substantial costs for occasionally issuing new equity. In my model, equity financing can be allowed by assuming $\underline{D} \leq D$ with $\underline{D} < 0$ and the equity-issuance costs. When constrained firms in the model mostly rely on debt financing, the results of this paper would remain robust.

current share holding. Then the following describes the household's problem.

$$V^h(\lambda, \phi; s_l, \mu) = \max_{C^h, N^h, \lambda', \phi'} \left[U(C^h, 1 - N^h) + \beta \sum_{m=1}^{N_s} \pi_{lm}^s V^h(\lambda', \phi'; s_m, \mu') \right] \quad (3)$$

subject to

$$\begin{aligned} C^h + q(s_l, \mu)\phi' + \int_{\mathbb{S}} \rho_1(k', b', \epsilon'; s_l, \mu) \lambda'(d[k' \times b' \times \epsilon']) \\ \leq w(s_l, \mu)N^h + \phi + \int_{\mathbb{S}} \rho_0(k, b, \epsilon; s_l, \mu) \lambda(d[k \times b \times \epsilon]) + \Pi_d \\ \mu' = \Gamma(s_l, \mu) \end{aligned}$$

Let $\Phi^h(\lambda, \phi; s, \mu)$ be the household's decision on bond holding, and denote $\Lambda^h(k', b', \epsilon', \lambda, \phi; s, \mu)$ as the new choice of share holding with future state (k', b', ϵ') . The household also finances the initial capital endowment for entrants while receiving the liquidated values of exiting firms. These transfers are made in lumpsum, as captured by Π_d in the budget constraint.

Equilibrium I define recursive competitive equilibrium (RCE) of the model. For the remainder of the paper, I simplify notations for price and policy functions whenever necessary. As discussed in the online appendix, the equilibrium price functions are consistent with the representative household's optimal decisions.

An RCE is a set of functions: prices $(w, q, (d_m)_{m=1}^{N_s}, \rho_0, \rho_1)$, quantities $(N, K, B, D, C^h, N^h, \Phi^h, \Lambda^h)$, and values (v_0, v, V^h) that solve the optimization problems and clear each market, and the associated policy functions are consistent with the aggregate law of motion, as in the following conditions.

1. v_0 and v solve Equations (1) and (2), and (N, K, B, D) are the associated policy functions for firms.
2. V^h solves Equation (3), and $(C^h, N^h, \Phi^h, \Lambda^h)$ are the associated policy functions for households.
3. The labor market clears, $N^h = \int_{\mathbb{S}} N(k, \epsilon; s, \mu) \cdot \mu(d[k \times b \times \epsilon])$.

4. The goods market clears,

$$C^h = \int_{\mathbb{S}} \left[z\epsilon F(k, N) - (1 - \pi_d)(K(k, b, \epsilon; s, \mu) - (1 - \delta)k) + \pi_d(1 - \delta)k \right] \cdot \mu(d[k \times b \times \epsilon]).$$

5. The law of motion for firm distribution is consistent with the policy functions, where Γ defines the mapping from μ to μ' given π_d , $K(k, b, \epsilon; s, \mu)$, and $B(k, b, \epsilon; s, \mu)$.

2.3 Firm Types and Decisions

To derive the firm-level decision rules, I distinguish firm types in the distribution. First, I define a subset of firms whose decisions are not affected by the collateral constraints in any possible future state. These *unconstrained* firms must have accumulated sufficient financial wealth such that their borrowing constraints are never binding and hence become indifferent between paying dividends and saving internally. This is because the shadow value of their retained earnings equals $p(s, \mu) \equiv D_1 U(C^h, 1 - N^h)$, the marginal value of consumption in equilibrium. The remaining firms in the distribution, on the other hand, are defined as *constrained*.¹⁹ A constrained firm may or may not experience a binding borrowing constraint in the current period, while putting non-zero probability weight on having a binding constraint in the future. The firm then optimally chooses not to pay the current dividends, $D = 0$, because its shadow value of retained earnings is greater than the value of dividends.

Next, I derive the intertemporal decisions for unconstrained firms. Since the collateral constraint is not relevant for such firms by definition, their optimal level of future capital, $k' = K^w(\epsilon; s, \mu)$, can be derived as follows. Let $\Pi^w(k, \epsilon; s, \mu) \equiv z\epsilon F(k, N^w) - wN^w$ be current earnings net of the wage bill. In the absence of capital adjustment costs, K^w solves the problem below.

$$\max_{k'} \left[-k' + \sum_{m=1}^{N_s} \pi_{lm}^s d_m(s_l, \mu') \sum_{j=1}^{N_\epsilon} \pi_{ij}^\epsilon (\Pi^w(k', \epsilon_j; s_m, \mu') + (1 - \delta)k') \right]$$

Given N^w and K^w , an unconstrained firm's borrowing choice must ensure that it remains un-

¹⁹This division of firm types is solely to characterize the decision rules, so it does not directly map into the data. Further, a firm may alter its type endogenously over time.

constrained in all future realizations of (s, ϵ) . This leads to the *minimum savings policy*, $b' = B^w(\epsilon; s, \mu)$, that solves the following problem recursively.

$$B^w(\epsilon_i; s_l, \mu) = \min_{(s_m, \epsilon_j)_{j,m}} \left(\tilde{B}(K^w(\epsilon_i; s_l, \mu), \epsilon_j; s_m, \mu') \right) \quad (4)$$

$$\begin{aligned} \tilde{B}(k, \epsilon_i; s_l, \mu) = & z_l \epsilon_i F(k, N^w(k, \epsilon_i; s_l, \mu)) - w(s_l, \mu) N^w(k, \epsilon_i; s_l, \mu) + (1 - \delta)k \\ & - K^w(\epsilon_i; s_l, \mu) + q(s_l, \mu) \min \left\{ B^w(\epsilon_i; s_l, \mu), \theta K^w(\epsilon_i; s_l, \mu) \right\} \end{aligned} \quad (5)$$

In Equation (4), $\tilde{B}(K^w, \epsilon_j; s_m, \mu')$ denotes the level of debt held by an unconstrained firm at the beginning of the next period with an exogenous state (ϵ_j, s_m) . Having chosen K^w in the current period, B^w is the maximum level of debt (or the minimum level of saving) that the firm carries into the next period while still remaining unconstrained in all possible realizations of (s, ϵ) . The threshold function, $\tilde{B}(k, \epsilon)$, can in turn be retrieved using B^w and K^w , as shown in Equation (5), in which the minimum operator reflects the collateral constraint in the current period.

Given unconstrained decisions, I define the threshold level of cash-on-hand to isolate unconstrained firms in the distribution. From the budget constraint in Equation (2), an unconstrained firm pays $D^w \equiv m - K^w + qB^w$ as current dividends. Hence, this firm's cash-on-hand is greater than or equal to a certain threshold level, $\tilde{m}(\epsilon; s, \mu) \equiv K^w(\epsilon; s, \mu) - qB^w(\epsilon; s, \mu)$. Any firms with $m(k, b, \epsilon) \geq \tilde{m}(\epsilon)$ on $\mu(k, b, \epsilon)$ are then recognized as unconstrained, and they adopt the associated policies, K^w and B^w .

On the other hand, firms with $m(k, b, \epsilon) < \tilde{m}(\epsilon)$ are constrained and pay zero dividends. In a given period, some constrained firms experience binding borrowing constraints while others do not. I call the latter constrained firms *Type-1* and the firms with currently binding constraints *Type-2* to distinguish their respective decision rules. Type-1 firms can still adopt the unconstrained capital choice, K^w , but not the minimum savings policy, B^w . Instead, their debt policies are implied by the budget constraint with the zero-dividend policy. Type-2 firms invest only up to the extent their borrowing limits allow, so they are *financially constrained*. The cash-

Table 1 : Decisions by Firm Type

Firm type	Condition	Capital choice, k'	Debt choice, b'
unconstrained	$m \geq \tilde{m}(\epsilon)$	$K^w(\epsilon)$	$B^w(\epsilon)$
constrained, Type-1	$K^w(\epsilon) \leq \bar{K}(m)$	$K^w(\epsilon)$	$\frac{1}{q}(K^w(\epsilon) - m)$
constrained, Type-2	$K^w(\epsilon) > \bar{K}(m)$	$\bar{K}(m)$	$\frac{1}{q}(\bar{K}(m) - m)$

on-hand held by these firms then determines the constrained choice of future capital. That is, given $m = k' - qb'$, the binding collateral constraint in Equation (2) leads to the upper bound for capital choice, $\bar{K}(m) \equiv \frac{m}{1-q\theta}$. Due to the DRS production technology, Type-2 firms then always choose $\bar{K} < K^w$. Firms with more cash-on-hand, therefore, can relax this upper bound to eventually choose K^w at a given level of θ . Table 1 summarizes the decision rules of k' and b' as functions of (m, ϵ) .

3 Parameterization and Model Implications

I calibrate the model and discuss its implications for resource misallocation. Next, I show that the model with a skewed distribution of idiosyncratic shocks closely reproduces the observed heterogeneity in firm size and leverage. Lastly, I estimate aggregate shocks by repeatedly solving the stochastic equilibrium of the model. The numerical methods are summarized in the online appendix.

3.1 Calibration and Model Specifications

The model is annual, and I choose the parameter values to match the conventional aggregate moments in the US data. In addition, I estimate production function using firm-level data, which allows me to measure idiosyncratic productivity shocks, thereby quantitatively disciplining the model.

Table 2 : Parameter Values, Baseline Model

Fixed Parameters			Fitted Parameters		
	Value	Description		Value	Description
β	0.960	Discount factor	α	0.277	Prod. function, k
δ	0.069	Depreciation rate	ψ	2.140	Labor disutility
ν	0.600	Prod. function, n	θ_{ss}	0.750	Collateral constraint
π_d	0.085	Exogenous exit rate	ϵ_m	0.390	Minimum ϵ
χ	0.220	Rel. size of entrants	ϵ_M	1.020	Maximum ϵ
π_ϵ	0.750	Prob. of retaining ϵ	ξ	3.400	Pareto shape

Table 3 : Calibrated Moments, Baseline Model

Description	Data	Model
Average hours worked	-	0.33
Investment-to-capital ratio, BEA	0.069	0.069
Capital-to-output ratio, BEA	2.30	2.24
Debt-to-asset ratio, Flow of Funds	0.22	0.22
Average firm exit rate, BDS	0.085	0.085
Employment share of entrants, BDS	0.023	0.022
Std. dev. of firm prod. shocks, Orbis	0.242	0.238
Skewness of firm prod. shocks, Orbis	1.432	1.471

The functional forms for preferences and production technology are standard in the literature. The period utility function features indivisible labor as in Rogerson (1988), $U(C, 1 - N) = \log C + \psi(1 - N)$, and the production function is $F(k, n) = k^\alpha n^\nu$ with $\alpha, \nu > 0$ and $\alpha + \nu < 1$. Table 2 reports parameter values, and Table 3 summarizes the corresponding model moments and their targets.

Aggregate Moments The subjective discount factor, β , is set to imply an annual real interest rate of 4 percent, and the labor coefficient in the production function, ν , is chosen to imply an average share of labor income of 0.6, as in Cooley and Prescott (1995). The depreciation rate, δ , is set to match the average investment-to-capital ratio during the postwar US period. Given the

value of δ , the capital coefficient, α , is determined to yield the observed average capital-to-output ratio. I use the time-series data of output, private fixed investment, and private capital stock between 1954 and 2006 from the National Income and Product Accounts (NIPA) and the Fixed Asset Tables of the Bureau of Economic Analysis (BEA). The steady-state level of the financial parameter, θ_{ss} , is set to closely match the aggregate debt-to-asset ratio of 0.22 for non-financial corporate businesses in the Flow of Funds from 1954 to 2006.²⁰ The preference parameter for the disutility from labor, ψ , is set to imply that total hours worked equal to one-third. I then set the exogenous exit probability, π_d , to be consistent with the average firm exit rate in the BDS from 1993 to 2006. The parameter for the initial capital stock of new firms, χ , is set to 0.22, so that entrants account for about 2 percent of aggregate employment as observed in the BDS data.

Firm Productivity Shocks In the *baseline* model, I assume that firm-specific productivity ϵ is drawn from a bounded Pareto distribution $G(\epsilon; \epsilon_m, \epsilon_M, \xi)$ with $0 < \epsilon_m < \epsilon_M$ and $\xi \geq 1$.²¹ Further, firms are assumed to retain their productivity with a fixed probability π_ϵ in each period. This probability is set to 0.75 so that the implied persistence is close to the estimated persistence of traditional TFP in Foster, Haltiwanger, and Syverson (2008). A new draw of ϵ from $G(\cdot)$ corresponds to a skewed idiosyncratic shock, and its persistence and skewness imply that most firms in the model have relatively low productivity over time.

I discretize the productivity process with a grid of 13 even-spaced points and determine the values of its bounds (ϵ_m, ϵ_M) and the shape parameter ξ . Specifically, I estimate firm-level productivity shocks in the data, and target their distribution moments, the standard deviation and the skewness, in the calibration. The estimation involves 2 steps: (i) panel data construction and production function estimation, and (ii) fixed-effects regression of measured firm productivity.

²⁰To be consistent with the model only with short-term debt financing, I calculate the aggregate debt as the sum of debt securities and loan liabilities in the Flow of Funds.

²¹Similar settings are used in Buera and Shin (2013) and Buera, Fattal-Jaef, and Shin (2015). The bounded Pareto distribution has a finite upper bound. Given an interval for its support, the cumulative density normalizes the density of the corresponding standard Pareto. My approach is also in line with that in Bloom, Guvenen, and Salgado (2020) who assume time-varying volatility and skewness of firm productivity with aggregate uncertainty.

I briefly describe these steps below, and the details of sample selection and estimations are in the online appendix.

First, I construct a panel of firms from the Orbis database and supplement it with the output and capital deflators in the OECD Structural Analysis Database (STAN). The sample covers non-financial and non-public administrative companies in the Group of Seven (G-7) countries from 2013 to 2018.²² The key variables of production function estimation are real operating revenue (y), real tangible fixed assets (k), and employment (n). The resulting dataset contains 1,525,656 observations of 254,276 firms.

I estimate the empirical production function given in

$$\log y_{i,j,t} = \beta_k \cdot \log k_{i,j,t} + \beta_n \cdot \log n_{i,j,t} + \log x_{i,j,t} + \delta_j \cdot \text{country} + \delta_t \cdot \text{year} + e_{i,j,t}, \quad (6)$$

where $x_{i,j,t}$ denotes the firm i 's revenue-based productivity in country j at year t , after controlling country-year-fixed effects. I employ the method suggested by Olley and Pakes (1996) to account for the endogeneity issue in this estimation. The estimates of β_k and β_n are 0.20 and 0.73 respectively, similar to those reported in existing studies such as Imrohoroglu and Tuzel (2014). I retrieve the residuals from the above estimation as the measured firm-level productivity $x_{i,j,t}$.

Let $\tilde{x}_{i,j,t}$ be the percentage deviation of $x_{i,j,t}$ from the mean productivity of firm i , and assume that it follows

$$\tilde{x}_{i,j,t} = \beta_1 \cdot \tilde{x}_{i,j,t-1} + \tilde{\eta}_{i,j,t} + \tilde{e}_{i,j,t}. \quad (7)$$

Estimating the above with firm/industry-, country-, and year-fixed effects, the residuals $\tilde{\eta}_{i,j,t}$ are identified as the idiosyncratic shocks to firm productivity. The resulting distribution of $\tilde{\eta}_{i,j,t}$ is dispersed and positively skewed; the standard deviation is 0.24 and the skewness is 1.43. These

²²Since the Orbis coverage of US firms is non-representative, I include observations from other developed economies to extend the sample size and reduce potential bias. Rajan and Zingales (1995) also consider public firms in the G-7 countries (US, Canada, United Kingdom, France, Germany, Italy, and Japan). Moreover, my estimation results do not significantly change when the sample covers 11 more countries in Western Europe (Austria, Belgium, Denmark, Finland, Greece, Ireland, Luxembourg, Netherlands, Portugal, Spain, Sweden).

Table 4 : Comparison of Firm Size Distribution

Bin: employees	Population Share (%)			
	BDS Data	Baseline	Alternative	Base-ex
1: 1 to 4	55.06	51.30	32.27	49.13
2: 5 to 19	33.42	33.38	27.99	31.06
3: 20 to 99	9.64	9.80	16.34	12.78
4: 100 to 499	1.53	2.85	7.42	3.91
5: 500 to 2,499	0.26	1.29	4.33	1.63
6: 2,500+	0.09	1.38	6.65	1.48
MSE	-	0.000	0.012	0.001

Note: *BDS Data* is the average value calculated from 1977 to 2006. *Baseline* is the model with calibrated Pareto ϵ shocks, *Alternative* is the model with a log-AR(1) process, and *Base-ex* is an extension of the baseline model with permanent heterogeneity. *Size bin* is in terms of employment, and model employment shares are exactly matched with those in the BDS. *MSE* is the mean-squared-error of the model moments.

moments are tightly matched in the baseline model using a Pareto-distributed ϵ , as shown in the bottom panel of Table 3.

Lastly, I consider an alternative specification of idiosyncratic shocks in the model. Following the conventional approach in the literature, I assume that the logarithm of ϵ follows an AR(1) process: $\log \epsilon' = \rho_\epsilon \log \epsilon + \eta'$ with $\eta' \sim N(0, \sigma_\eta^2)$. To pin down the value of σ_η in this *alternative model* using the above dataset, I instead regress the measured log-productivity on its lagged value with fixed effects. The calibrated value of σ_η is 0.167 such that the standard deviation of $\log \epsilon$ is 0.238, given its persistence of 0.75 as in the baseline model.

Firm Size Distribution As mentioned in the introduction, the empirical distribution of firm employment size in the US is highly skewed and dispersed. The baseline model closely reproduces the empirical size distribution in the BDS, whereas the alternative model does not. This stark contrast is summarized in Table 4.

In Table 4, I report the average distribution in the BDS from 1977 to 2006, across 6 employment size bins: 1 to 4, 5 to 19, 20 to 99, 100 to 499, 500 to 2,499, and 2500+. Given these size bins, each model exactly matches the employment shares in the BDS, so the table only presents

the model-implied population shares.²³

Clearly, the baseline model generates a realistic firm size distribution. That is, the model size distribution is highly skewed, as it is in the data. This consistency is obtained by closely matching the population shares at the lower tail (rows 1 and 2). On the other hand, the model is not equally successful at the upper tail of firm size distribution. The table shows that there are relatively more firms in the largest group in the model when compared to the data. This is mainly because of truncation of the productivity shocks, implying that the largest firms are smaller than those in the BDS.²⁴

When the observed skewness in firm productivity is ignored, however, the model does not deliver realistic heterogeneity in firm size. In the alternative economy (column 3 of Table 4), there are relatively fewer small firms and more large firms, broadly at odds with the observed data; about 32 percent of firms are in the smallest size group, whereas the largest firms account for more than 6 percent of the business population. This makes the model's overall fit to the BDS data, measured by mean-squared-error (MSE), significantly poorer than the baseline model. Moreover, the dispersion in firm size is not sufficiently large. The highest level of productivity in the alternative economy is only about 63 percent of that in the baseline model, so that the largest firm hires less than one-fourth of its counterpart in the latter.

I further check whether the model still delivers an empirically-consistent size distribution when permanent heterogeneity across firms exists. Note that firms in the baseline model are only driven by transitory shocks that are estimated with the controls for firm-fixed effects. As emphasized by Pugsley, Sedlacek, and Sterk (2021), however, ex-ante heterogeneity plays an im-

²³This method of measuring relative size in a model is from Jo and Senga (2019). By setup, all models exactly match the BDS employment shares that remain stable over time, and only population shares need to be compared. For this reason, actual size of employment is not directly comparable across different models. The details of the numerical method are in the online appendix.

²⁴The size distribution in the baseline model still exhibits a power law at its right tail. The p-value of power-law test is 0.58 and the slope of the fitted line is -2.15 in the log-log plot of density and employment, consistent with the example in Gabaix (2016). To generate a heavier-tail, the model may additionally include extremely large firms that do not face financing constraints as assumed in Khan and Thomas (2013). I thank the editor and an anonymous referee for their helpful comments about the truncation issue and the Pareto-tail.

portant role in explaining the observed size differences in the data. Thus, I consider an extension of the baseline model by distinguishing the permanent and transitory components of idiosyncratic productivity.²⁵ Specifically, in the *base-ex* model, I assume that there are N_ϵ^p different types of firms, and for each type, ϵ is drawn from a type-specific distribution which is bounded-Pareto with N_ϵ^t values. This simple extension introduces permanent heterogeneity in mean productivity into the model, reducing the role of productivity shocks in determining firm size dispersion. Given $N_\epsilon^p = 3$ and $N_\epsilon^t = 5$, I calibrate the remaining parameters of ϵ to match the empirical moments of idiosyncratic shocks.²⁶

Table 4 shows that the base-ex model is largely consistent with the empirical evidence on cross-sectional differences in firm size (column 4). The population shares at the two smallest size groups are reasonably close to their empirical values, but the model’s performance at the upper tail is less than satisfactory. In Section 4, I show that the aggregate dynamics of the base-ex model are isomorphic to those from the baseline economy, highlighting the importance of micro-level consistency with firm size data, regardless of the source of differences in productivity, in the propagation of a financial shock.

3.2 Financial Frictions and Resource Misallocation

I discuss the cross-sectional implications of financial frictions and examine the aspects of resource misallocation across models with different idiosyncratic shocks. The baseline model features a small set of highly productive but financially constrained firms. It also exhibits different patterns of firm growth that substantially vary with initial productivity.

Firm-level Decisions Collateral constraints in the model imply that a firm’s investment and

²⁵My approach is similar to that of Elsby and Michaels (2013). Studies with other approaches include Kaas and Kircher (2015), and Gavazza, Mongey, and Violante (2017). Cao et al. (2022), on the other hand, focus on the Pareto tail of establishment distribution.

²⁶Holding the persistence parameter π_ϵ fixed, I re-calibrate the model to closely match the empirical moments reported in Table 3. I fix the population share of each type at 0.55, 0.30, and 0.15, and calibrate the bounds of ϵ and the common shape parameter ξ to match the volatility and skewness in productivity shocks in the Orbis data.

borrowing decisions are subject to its individual state, cash-on-hand and exogenous productivity. It follows that substantial differences across firms in their cash-on-hand, $m(k, b, \epsilon)$, lead to cross-sectional heterogeneity in both real and financial variables.

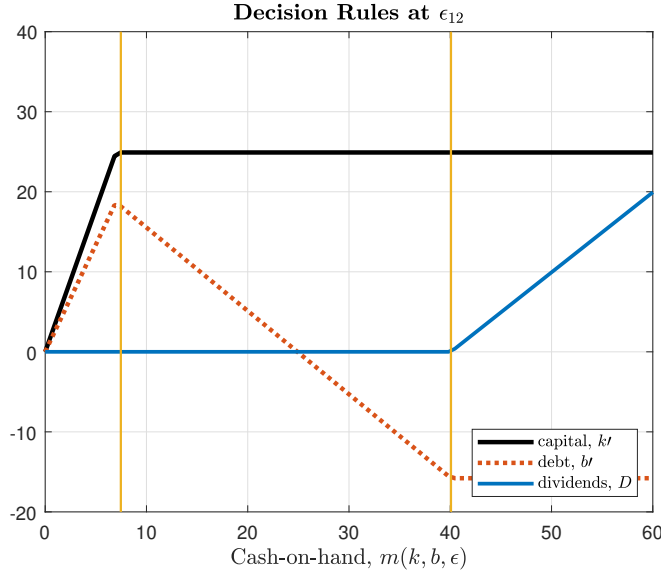
Figure 2 clearly illustrates this point by showing the decision rules for future capital, k' , debt, b' , and dividends, D , at a given productivity level in the baseline model.²⁷ As a firm's cash-on-hand becomes larger, investment and borrowing decisions change nonlinearly due to the presence of financial frictions. In the figure, the two vertical lines distinguish firm types, and the associated decisions, discussed in Section 2. The vertical line on the left separates Type-1 and Type-2 firms, with the latter being those with lowest m and to the left of the line. Due to binding constraints, Type-2 firms adopt constrained capital choices (*black solid line*), which are smaller than the efficient level implied by their productivity, and maintain positive debt (*red dotted line*). Type-1 firms, on the other hand, are now able to finance their desired investment, and their debt gradually falls as m increases. The vertical line on the right distinguishes unconstrained firms with $m \geq \tilde{m}$; these firms choose K^w and B^w while paying positive dividends.

Since cash-on-hand is a function of ϵ , it is clear that the distribution of idiosyncratic productivity crucially affects the distribution of cash-on-hand and hence firm-level decisions in the model. Thus, depending on the underlying individual productivity shocks, the model may predict significantly different implications of financial frictions. I examine this point by comparing the aspects of resource misallocation between the baseline and alternative models below.

Resource Misallocation While firms with insufficient cash-on-hand tend to adopt the constrained capital choice, small firms are not necessarily financially constrained in my model. That is, some firms become small just because they have relatively low productivity and remain at their optimal production scale, whereas other small firms have had to choose a constrained level of capital due to binding borrowing constraints. The latter firms are Type-2 in the model, account-

²⁷The patterns of decision rules are similar across different productivity. In the figure, hence, I show the decisions rules at ϵ_{12} for clear visibility.

Figure 2 : Decisions as functions of cash-on-hand



Note: For a clear illustration, I plot the decision rules in the baseline model at a given productivity level, ϵ_{12} . The two vertical lines distinguish firm types by m : Type-2, Type-1, and unconstrained firms from left to right.

ing for the misallocation of productive factors in the aggregate economy.

Given the Pareto-distributed productivity, many firms in the baseline model are small not because of collateral constraints, but because of low productivity. The first column of Table 5 presents the share of each firm type, and it turns out that only 9.5 percent of firms are financially constrained as their collateral constraints are binding. At the same time, the skewed size distribution implies that more than 85 percent of firms are small. Thus, most small firms in the baseline model operate at their efficient production scale, as implied by their low exogenous productivity, consistent with the observation in Hurst and Pugsley (2011). However, as discussed below, the extent of resource misallocation can be non-trivial in my model when financial frictions substantially restrict the allocation of resources to a small number of highly productive firms. To understand this, it is useful to distinguish the margins of misallocation by looking at how many firms are financially constrained and by how much.

The key idea is that not only the number of financially constrained firms (*the extensive mar-*

Table 5 : Comparison of Type Distribution and Misallocation

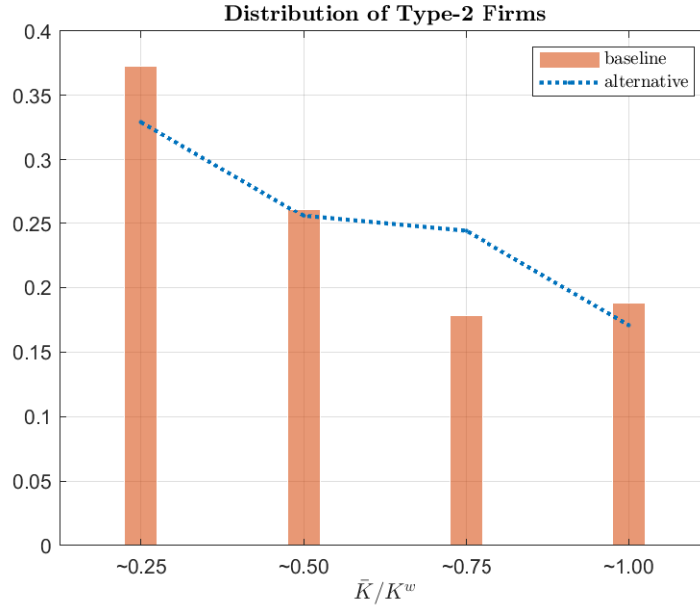
	Baseline	Alternative
<i>Pop. share (%)</i>		
unconstrained	0.18	2.43
type-1	90.29	74.11
type-2	9.53	23.45
<i>Avg. \bar{K}/K^w (%)</i>	32.38	50.01
<i>ΔTFP (%)</i>	2.62	0.95

Note: *Baseline* is the model with calibrated Pareto ϵ shocks, and *Alternative* is the model with a log-AR(1) process. *Avg. \bar{K}/K^w* is the average ratio of constrained to unconstrained capital, and ΔTFP is the change in measured TFP when eliminating collateral constraints in each model.

gin) determines the aggregate effect of financial frictions, but also does the degree of financial frictions faced by such firms (*the intensive margin*). The bottom panel of Table 5 reports the average ratio of constrained-to-unconstrained capital choices among Type-2 firms. In the baseline model, this ratio is about 32 percent. Consequently, while only 9.5 percent of firms are constrained, those that are experience substantial distortions in their investment. I also compute the corresponding loss of aggregate productivity by comparing the measured TFP in the model with or without collateral constraints. The last row of Table 5 reports a productivity loss of 2.6 percent in the baseline economy in the presence of financial frictions. Thus, simply having many small firms in the model does not necessarily magnify the extent of resource misallocation. Only those firms with high exogenous productivity, and binding constraints, contribute to the lowering of aggregate productivity. Given the skewed firm productivity distribution, the share of such Type-2 firms is relatively small in the baseline model, while their corresponding distortions in the intensive margin are substantial.

On the other hand, the two margins of resource misallocation are different in the alternative model with log-normally distributed productivity. In Table 5, the population share of Type-2 firms is about 23 percent and the average level of constrained capital is at 50 percent of the optimal choice. That is, relatively more firms are financially constrained but their investment is closer to the optimal level when compared to the baseline model. As a result, the contribution

Figure 3 : Distribution of capital ratio, Type-2 firms



of the intensive margin is relatively smaller and the corresponding efficiency loss is less than 1 percent. Importantly, we see that it is possible to underestimate the extent of resource misallocation arising from financial frictions when a model is inconsistent with the empirical firm size distribution. As the propagation of financial shocks relies on this resource misallocation channel, moreover, a business cycle model with financial frictions should arguably be consistent with the distribution of distortions.

I further highlight the differences in the intensive margin across the two models, by looking at the distribution of \bar{K}/K^w . Figure 3 shows that the distribution of idiosyncratic productivity in my model leads to different composition of financially constrained firms. Specifically, about 63 percent of Type-2 firms have their capital ratio below 50 percent in the baseline economy (*red bars*), whereas such firms are about 57 percent in the alternative model (*blue dotted line*). Thus, constrained firms' investment is relatively more distorted in an economy with skewed productivity shocks and a realistic size distribution.²⁸

²⁸I thank an anonymous referee for the suggestion of looking at the distribution of capital gap among constrained firms, instead of the entire distribution of cash-on-hand.

The above discussion suggests that financial frictions can have different implications on resource misallocation and aggregate productivity conditional on the underlying idiosyncratic productivity distribution in a model. The two models considered—the baseline and the alternative—not only differ in the extent of misallocation but also in its composition. This further implies that the model can predict different cross-sectional heterogeneity, as will be discussed in the next subsection. Lastly, in the online appendix, I compare the growth patterns of young firms conditional on their initial productivity.

3.3 Model Predictions on Firm Heterogeneity

I examine the model's consistency with the empirical moments that are not targeted in my calibration. These include firm-level investment moments and size-leverage relationships in the data. In this way, I can externally validate my approach of reproducing the BDS size distribution for nesting the unobserved financial heterogeneity in the baseline model.

Investment Moments Business cycle studies of heterogeneous firms typically target micro-level investment moments in the data. I summarize the empirical moments of investment rate (i/k) and compare them with model-generated moments.

In Table 6, I report the investment moments calculated from the Orbis dataset, used in my estimation in Section 3.1. Although the dataset is relatively short, the pooled empirical moments are consistent with those in the literature. For instance, the mean investment rate is 0.117, close to the value in Cooper and Haltiwanger (2006).²⁹

First, while not a target in the calibration, both the baseline and base-ex models show rela-

²⁹Cooper and Haltiwanger (2006) document the moments of investment rate in a balanced panel of large-manufacturing plants in the Longitudinal Research Database (LRD). The standard deviation from my sample is a bit larger than their value (0.444 vs. 0.337), whereas the autocorrelation becomes significantly larger when I estimate a size-weighted regression with fixed effects (0.337 vs. 0.058). The share of firms with positive lumpy investments is also similar (0.170 vs. 0.156). Further, these moments do not change much when 11 other developed economies are additionally considered.

Table 6 : Comparison of Firm-level Investment Dynamics

	Investment Moments				
	Orbis Data	Baseline	Alternative	Base-ex	Alt-lv
$\mu(i/k)$	0.117	0.142	0.388	0.198	0.213
$\sigma(i/k)$	0.444	0.365	0.921	0.371	0.454
$\rho(i/k)$	0.337	0.293	-0.196	0.352	-0.173
lumpy inv.	0.170	0.103	0.447	0.215	0.537
MSE	-	0.003	0.165	0.003	0.101

Note: *Orbis Data* is computed from the balanced panel of firms in the G-7 countries. $\mu(i/k)$ is the average investment rate (i/k), $\sigma(i/k)$ is the standard deviation, $\rho(i/k)$ is the first order autocorrelation, and *lumpy inv.* is the share of firms with $i/k > 0.2$. *Baseline* is the model with calibrated Pareto ϵ shocks, *Alternative* is the model with a log-AR(1) process, *Base-ex* is an extension of the baseline model with permanent heterogeneity, and *Alt-lv* is the model with a log-AR(1) process with lower volatility. *MSE* is the mean-squared-error of the model moments. All model moments are calculated from a large panel of firms simulated at each steady state.

tively good fits to the empirical moments of the investment rate (columns 2 and 4). The average investment rate in the baseline model is about 20 percent larger than the empirical value, whereas the investment volatility is somewhat smaller. Further, firms' investment in these models exhibits substantial persistence and lumpiness as seen in the data. These results suggest that the models with skewed idiosyncratic shocks generate plausible investment dynamics at the firm level, without relying on real adjustment costs.³⁰

Next, in the alternative model, firm-level investment dynamics are not similar to the empirical moments (column 3 of Table 6). Due to the extremely large volatility in productivity shocks, the mean and standard deviation of investment rates are far away from those in the Orbis data. In addition, as commonly seen in models without capital adjustment costs, the autocorrelation of investment rates is negative. When I reduce the volatility of idiosyncratic shocks to 0.09, to particularly match the empirical value of $\sigma(i/k)$, the model-implied investment dynamics still remain unsatisfactory (the *alt-lv* model in column 5).³¹

These discrepancies indicate that the model may predict counterfactual heterogeneity in the

³⁰The moments of employment dynamics in the baseline and base-ex models are also relatively more closer to those reported in Sedlacek and Sterk (2017) and Gavazza, Mongey, and Violante (2017). For instance, the standard deviation of employment growth ranges between 0.47 and 0.53 in these models.

³¹The resulting volatility in ϵ in the alt-lv model is similar to that in the baseline model. However, the model also significantly fails to explain the empirical firm size distribution.

cross-section, depending on how idiosyncratic productivity shocks are calibrated. Further, it is clear that the model's consistency with firm size data is obtained at no cost to its implications on investment moments at the firm level.

Size-Leverage Relationship In the model, firms make borrowing decisions subject to their collateral constraints, and the corresponding distribution of financial frictions can be measured by using the ratio of constrained-to-optimal capital, as shown earlier in Table 5 and Figure 3. However, there exist practical challenges for empirically evaluating the model's predictions on financing constraints at the firm level. This is because such constraints are not directly observed in the data and firms' decisions are only captured by their leverage ratio. Hence, I examine the model's consistency with the observed firm leverage distribution and the reduced-form evidence on firm size and leverage in the data.

I begin by comparing the leverage distribution across models with different productivity shocks and firm size distributions. Table 7 summarizes the model-generated leverage distributions, together with the empirical distribution reported by Crouzet and Mehrotra (2020).³² The baseline and base-ex models are largely consistent with the observed leverage values by asset size quantile (rows 2 and 4). In particular, the mean leverage tends to remain relatively high as firm size increases in the model (columns 1 to 3). This pattern, on the other hand, does not exist in the alternative model (row 3). There, leverage is significantly larger among the firms at the bottom 90 percent of asset distribution. The top 1 percent of firms save using financial assets in this model, so their leverage becomes zero.

Next, I establish the empirical relationship between firm employment size and leverage, and then verify whether the model predicts a similar result. In the following, I find that the baseline model with realistic heterogeneity in firm size delivers a positive size-leverage relationship as

³²They provide the descriptive statistics of US firms in the Quarterly Financial Report (QFR) of the US Census Bureau. The asset size bins and the empirical moments in Table 7 are directly taken from their work.

Table 7 : Mean Leverage by Asset Size

	0-90th	90-99th	99-99.5th	>99.5th	MSE
QFR Data	0.20	0.19	0.23	0.22	-
Baseline	0.20	0.26	0.23	0.14	0.003
Alternative	0.26	0.13	0.00	0.00	0.027
Base-ex	0.18	0.22	0.21	0.16	0.001

Note: *QFR Data* reproduces the values of net leverage in Crouzet and Mehrotra (2020). *Baseline* is the model with calibrated Pareto ϵ shocks, *Alternative* is the model with a log-AR(1) process, and *Base-ex* is an extension of the baseline model with permanent heterogeneity. *MSE* is the mean-squared-error of the model moments. Assets in the model are defined as the sum of capital and financial savings.

seen in the data.

For the empirical analysis, I separately construct a balanced panel of firms in the Orbis database, covering the G-7 countries from 2010 to 2015. I focus on firms that are non-financial and non-public administration companies distinguished by NAICS 2017 codes. The resulting dataset is a panel of 184,565 firms with 1,060,143 observations, and the online appendix contains the details of data construction, sample selection, and descriptive statistics.

I consider a standard leverage regression that accounts for industry-country-time fixed effects. In the regression, I use short-term debt leverage (*stlev*) as the main dependent variable for each firm-year observation.³³ The regression equation is given by

$$leverage_{i,t} = \beta_1 \cdot emp_{i,t-1} + \beta_2 \cdot collateral_{i,t-1} + \beta_3 \cdot profit_{i,t-1} + \beta_4 \cdot prod_{i,t-1} + \alpha + \delta_t \cdot year + e_{i,t},$$

where *emp* is the logarithm of employment, *collateral* is the ratio of tangible fixed assets to total assets, *profit* is the earnings to asset ratio, *prod* is a measure of labor productivity, α is industry/country fixed effects, and δ_t is a time fixed effect.³⁴ The coefficient on employment size, β_1 , then represents the empirical relationship between firm size and leverage. To check the

³³This is to make direct comparisons with the model results, since I assumed one-period discount debt in the model. Tables 20 and 21 in the online appendix contain the results for total and financial leverages (*tlev*, *flev*).

³⁴This specification is almost identical to those in Dinlersoz et al. (2019) and Chatterjee and Eyigungor (2022). These studies also report positive and significant relationships between firm size and leverage among US private and public firms. To reduce the sample bias towards large firms, I use asset-size weights in the regression as thankfully suggested by an anonymous referee.

Table 8 : Fixed-effects Leverage Regression, Data and Model

Dep. variable	<i>stlev_{i,t}</i>				
	G-7	US	Baseline	Alternative	Base-ex
Sample	(1)	(2)	(3)	(4)	(5)
<i>emp_{i,t-1}</i>	0.012*** (17.60)	0.018*** (13.59)	0.013*** (16.77)	-0.042*** (-140.47)	0.091*** (46.73)
<i>collateral_{i,t-1}</i>	-0.189*** (-29.16)	-0.094*** (-5.74)	-0.011*** (-3.86)	-0.063*** (-43.17)	-0.314*** (-54.41)
<i>profit_{i,t-1}</i>	-0.340*** (-16.86)	-0.216*** (-7.30)	2.224*** (205.95)	2.941*** (433.90)	1.380*** (114.47)
<i>prod_{i,t-1}</i>	0.024*** (13.10)	0.003 (0.64)	0.785** (2.13)	-2.308*** (-19.50)	3.968*** (2.66)
industry FE	✓	✓			
country FE	✓				
year FE	✓	✓	✓	✓	✓
obs.	867,458	8,427	325,195	325,625	301,625
adj. <i>R</i> ²	0.453	0.656	0.739	0.780	0.902

Note: *t*-statistics in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. *stlev* is short-term leverage, *emp* is employment size in logs, *collateral* the ratio of tangible fixed assets to total assets (ratio of capital to assets in model), *profit* is net income over total assets (revenue less wage bill over assets in model), and *prod* is a measure of labor productivity that divides turnover by employment (output over employment in model). All variables are winsorized at the 1 percent level.

model's consistency with the empirical estimate of β_1 , I simulate a large panel of firms in each model and run the above regression without industry- and country-fixed effects.

Table 8 summarizes the results from the empirical and model-simulated data. First, the coefficient on employment is positive and statistically significant for firms in the G-7 countries and in the US (columns 1 and 2). Specifically, the estimate is 0.012 for all firms in the sample, which implies that a firm's leverage ratio tends to increase by about 2.8 percentage point when its employment size becomes 10 times larger ($0.0276 = \log 10 \cdot 0.012$), after controlling for other firm characteristics. This result is also analogous to the fact that leverage is relatively higher among the largest firms in the top 10 percent, in Table 7. Moreover, the above employment coefficient is in line with the estimates in the recent works by Dinlersoz et al. (2019) and Chatterjee and Eyigungor (2022); they respectively find the coefficient of 0.007 for private firms and 0.035 for listed firms in the US.

Next, columns 3 and 5 of Table 8 indicate that leverage increases in lagged employment in the models with a realistic firm size distribution and skewed productivity shocks. That is, the employment coefficients in the baseline and base-ex models are positive and significant as seen in the first two columns of the table. Further, their values are within a reasonable range of the empirical estimates although untargeted. Especially, the value from the baseline model is close to that in the pooled sample of firms across countries. Other regression coefficients are also largely consistent with the empirical estimates, with the exception of firms' profitability.³⁵ As productivity is persistent in the model, leverage tends to increase when either a firm's collateral ratio is low or its productivity is high in the preceding period.

The alternative model, in contrast, yields a counterfactual prediction. Column 4 of Table 8 shows that the size-leverage relationship is reversed, implying that large firms tend to be less leveraged. As discussed earlier, the underlying productivity distribution in this model is not sufficiently skewed and dispersed. The resulting size distribution has relatively fewer firms in the smallest group, while 23 percent of firms have binding collateral constraints (Tables 5 and 4). Since the latter firms are close to their efficient production scale, when compared to those in the baseline economy, a marginal increase in their employment would allow them to choose the unconstrained capital and reduce their debt. This pattern appears to be dominant in the alternative model, which explains the negative effect of lagged employment on leverage.

It is clear that the reduced-form evidence in Table 8 suffers from potential endogeneity problems. However, analyzing the effects of financial shocks requires a business cycle model that is consistent with real and financial heterogeneity seen in data. In this paper, I achieve such micro-level consistency in a parsimonious way by introducing skewed firm productivity shocks in the model, which further leads to a realistic firm size distribution. This approach is based on the insight of Krueger, Mitman, and Perri (2016), in particular. In their work, an empirically-

³⁵The online appendix reports additional results from the leverage regression. Depending on type of firms (private and listed) or leverage ratio (total, financial, and short-term leverage), the coefficients on other variables significantly change. However, the sign of employment coefficient is robustly positive.

consistent wealth distribution of households brings realistic heterogeneity in the marginal propensity of consumption, which in turn amplifies the distributional effects of an aggregate shock. Further, my approach is also closely related with Pugsley, Sedlacek, and Sterk (2021) by emphasizing the model’s consistency with the relevant empirical moments at the firm level. While they focus on the autocovariance structure of firm employment dynamics, I confirm the importance of accounting for the empirical size and leverage distributions in a heterogeneous-firm model in the presence of financial frictions and aggregate uncertainty.

3.4 Estimation of Aggregate Shocks

I jointly estimate the parameters of the aggregate shock processes. For aggregate productivity shocks, I assume that the exogenous aggregate TFP component follows a log-AR(1) process.

$$\log z' = \rho_z \log z + \eta'_z, \quad \eta_z \sim N(0, \sigma_{\eta_z}^2)$$

Credit shocks, on the other hand, directly alter the value of the financial parameter, θ , in firms’ borrowing constraints. I assume that θ follows a two-state Markov switching process with $N_\theta = 2$, $\theta \in \Theta = \{\theta_{ss}, \theta_l\}$ and $\theta_l < \theta_{ss}$. The ordinary period of having θ_{ss} continues with probability p_{oo} , and the transition from the period of reduced credit to an ordinary period is governed by p_{lo} . Hence, a sudden decrease in θ with a probability of $1 - p_{oo}$ corresponds to a credit shock.

Given the assumed stochastic processes, let $\gamma \equiv (\rho_z, \sigma_{\eta_z}, \theta_l, p_{oo}, p_{lo})$ be the set of parameters to be estimated. I estimate γ by using a minimum-distance-estimator, holding other parameter values fixed in the baseline model.³⁶ I use the real and financial time-series data from NIPA and the Flow of Funds between 1954 and 2018: HP-filtered real GDP (Y), total hours worked (N), and total real debt (B).³⁷ Empirical moments include the first-order autocorrelations and

³⁶The objective function is, $\min_\gamma (M_{data} - M_{model}(\gamma))' W_\gamma (M_{data} - M_{model}(\gamma))$, where W_γ is a weighting matrix, M_{data} is a vector of empirical targets, and $M_{model}(\gamma)$ is a vector of model-simulated moments. Following Gavazza, Mongey, and Violante (2018), I set $W_\gamma = \text{diag}(1/M_{data})^2$.

³⁷As documented in Jermann and Quadrini (2012), the cyclicalty of US financial variables significantly changes

Table 9 : Estimated Parameters

Estimated Aggregate Shocks				
	Estimate		Data Moment	Model Moment
ρ_z	0.9213	ρ_Y	0.5512	0.5446
σ_{η_z}	0.0152	σ_Y	0.0206	0.0237
θ_l	0.6101	ρ_N	0.5569	0.5509
p_{oo}	0.9162	σ_N	0.0202	0.0154
p_{lo}	0.3004	σ_B	0.0497	0.0511

Note: All data series are annual from 1954 to 2018. Y is the real GDP, N is total hours worked, and B is the total debt. Model moments are computed from a 5,000-period simulation of the baseline model. I use HP-filter with a smoothing parameter of 100.

standard deviations of these target variables, $(\rho_Y, \sigma_Y, \rho_N, \sigma_N, \sigma_B)$. Table 9 summarizes the estimation results, and I discuss their business-cycle implications in the online appendix.

4 Aggregate Dynamics and Firm Size Heterogeneity

I now turn to investigating the role of firm heterogeneity in propagating aggregate shocks. To this end, I compare impulse responses across models with different idiosyncratic shocks and hence with different heterogeneity in size and leverage, and examine how the margins of resource misallocation change over time.

4.1 Impulse Responses: Credit Shock

Aggregate Effects I consider a credit shock that hits the baseline economy at the steady state, holding exogenous TFP fixed. Specifically, the value of θ falls by 19 percent from θ_{ss} , and gradually recovers after 3 periods with the estimated persistence p_{lo} . This magnitude of the shock is chosen to imply a 26 percent drop in aggregate borrowing at the steady state, consistent with the

in the 1980s, so I choose to include just one financial moment for aggregate debt in the estimation. For total hours worked, I use the time-series data constructed by Cociuba, Prescott, and Ueberfeldt (2018). The estimates change when I exclude observations after 2006, and I thank an anonymous referee for suggesting to include the post-Great Recession data in the estimation.

empirical observations during the Great Recession.³⁸

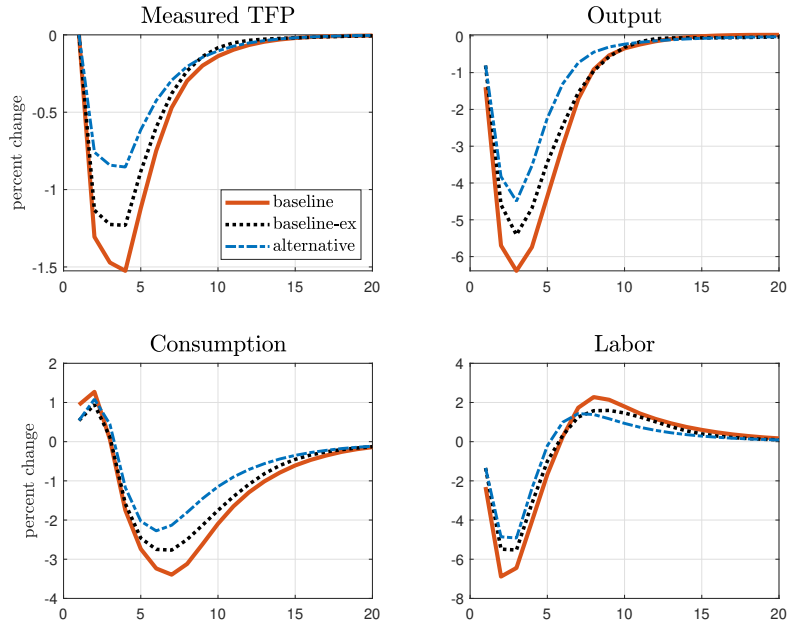
Figure 4 plots the dynamics of aggregate variables following the calibrated shock. The credit shock creates a persistent and prolonged recession (*solid red line*). From the impact date, measured TFP gradually falls, reaching its trough at 1.5 percent below its steady-state level. This fall in TFP mainly stems from the slow reallocation of capital across firms during the recession. That is, firms that were Type-2 at the steady state become more financially constrained as credit tightens; the number of such firms also increases. This lowers the allocative efficiency of factors of production in the economy, and hence measured TFP falls endogenously. Importantly, the quantitative magnitude of such increased misallocation critically depends on the underlying distribution of firm productivity and financing constraints in an economy. I return to this point later, decomposing the distributional effects of the credit shock.

In Figure 4, the response of output illustrates that a persistent recession triggered by a financial shock is intrinsically different from that following a real shock. Output initially drops by 1.2 percent, then sees its largest decline at date 3. On the other hand, consumption rises slightly at impact, then gradually falls during the recession. Together, these responses characterize a financial recession driven by the endogenous fall in aggregate productivity.

As discussed in Section 3, the aspects of resource misallocation arising from financial frictions vary with the underlying firm productivity distribution in a model. This suggests that the propagation of a financial shock can be significantly different across models with or without realistic firm heterogeneity. To highlight this point, I conduct the same experiment in both the alternative and base-ex models. For comparability with the results from the baseline model, I control the size of the credit shock in each model. That is, the value of θ_{ss} respectively falls by 21 percent in the alternative model and by 20 percent in the base-ex model, so that the resulting

³⁸Various measures show a massive reduction in business lending during the recession. Among others, Ivashina and Scharfstein (2010) report that the newly issued volume of syndicated loans fell more than 50 percent in 2008, while Khan and Thomas (2013) show that the fall in real lending from commercial banks is about 26 percent between 2008 and 2011. I follow the latter conservative value in my experiment.

Figure 4 : Aggregate dynamics following a credit shock



Note: The stochastic impulse responses to a sudden credit tightening. The size of the shock implies an endogenous decrease of aggregate borrowing by 26 percent in all models, and θ start recovering in period 4 with persistence of p_{lO} . *Baseline* is the model with calibrated Pareto ϵ shocks, *Alternative* is the model with a log-AR(1) process, and *Base-ex* is an extension of the baseline model with permanent heterogeneity.

decreases in aggregate lending remain the same at 26 percent.³⁹

Figure 4 also compares the aggregate responses in different models. First, when the credit shock hits the alternative economy, the subsequent aggregate dynamics (*blue line with dash-dots*) are similar to those in the baseline economy. However, the recession is not as severe as in the baseline economy. In particular, the greatest decline in measured TFP is 0.8 percent. Thus, the increased misallocation following the credit shock is less pronounced in the model with counterfactual distributions of firm size and leverage. Because of this small TFP loss, it is natural to have a relatively quick recovery thereafter. That is, the half-life of the recession is about 5 years, which is 1.5 years shorter than in the baseline model.

This finding does not change when the model incorporates ex-ante heterogeneity in firm pro-

³⁹I thank the editor and anonymous referees for their suggestion of clarifying the size of a credit shock in this experiment.

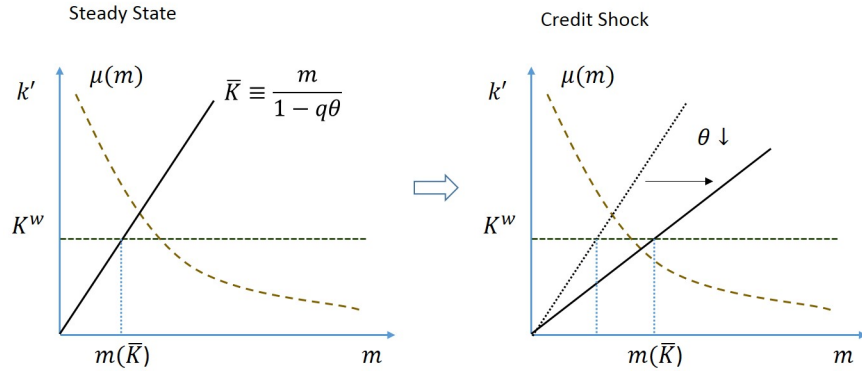
ductivity. The base-ex model shows aggregate responses to a credit shock are marginally smaller than those in the baseline model (*black dotted line*). For instance, the drop in output is about 90 percent of that in the baseline model, suggesting that these models with realistic heterogeneity lead to a relatively deep recession and its slow recovery following a credit shock. Thus, this result is not driven by the presence of permanent heterogeneity across firms, but rather the model's consistency with the observed firm-level data. Such micro-level consistency, therefore, is crucial for quantitatively evaluating the transmission of a credit shock through the distribution of firms.

Inspecting the Mechanism From the discussion in Section 3.2, the extent of resource misallocation in the model can be decomposed into two margins: the number of financially constrained firms (the extensive margin) and the tightness of their borrowing limits (the intensive margin).

Depending on the distribution of productivity and financing constraints in the model, these two margins may potentially change in different ways following a credit shock. This is because the shock widens the gap between constrained and efficient capital choices, and raises the number of such firms at the same time. Moreover, some firms in the model may suffer relatively more, while others are still able to optimally adjust their production factors in response to changes in equilibrium prices. These differential responses across firms highlight the distinct nature of a financial shock, and they collectively emerge as a change in measured TFP at the aggregate level. Thus, incorporating a skewed distribution of idiosyncratic shocks in the model not only delivers plausible cross-sectional results in steady state, but also amplifies the extent of misallocation through time-varying changes in the distribution of firms. The resulting aggregate dynamics are substantially larger in the baseline model, as already seen in Figure 4.

I first illustrate the above mechanism in Figure 5 by using a graphical analysis of the model in partial equilibrium. In the left panel, I plot a skewed distribution of cash-on-hand m at the steady state (*red dashed line*). The straight line from the origin represents the upper bound of capital choice as a function of m , so that firms with $m < m(\bar{K})$ choose constrained capital $\bar{K} \leq K^w$. The right panel of Figure 5 shows the changes in decision rules following a credit

Figure 5 : Comparative statics following a credit shock, in partial equilibrium



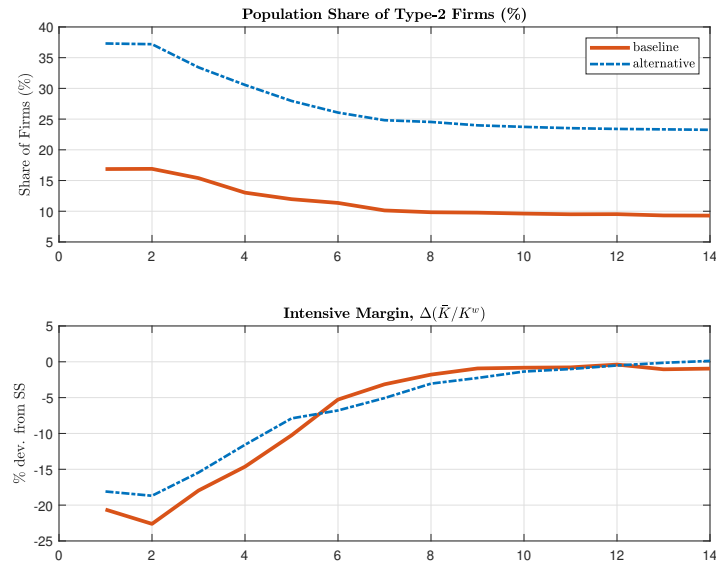
Note: The above graphically illustrates the changes in capital choices following a fall in θ , holding the distribution of cash-on-hand and factor prices fixed.

shock. As θ falls, the upper bound schedule becomes flatter and the threshold level of cash-on-hand rises, holding the distribution fixed. Firms hence respond differently to the shock, based on whether or not they face binding borrowing constraints. Note that the flattened upper bound in Figure 5 leads to the increased gap between K^w and \bar{K} for financially constrained firms. This represents a change in the intensive margin. At the same time, the credit shock raises the number of constrained firms due to the increased threshold of cash-on-hand $m(\bar{K})$. This indicates that the firm-type distribution varies over time, corresponding to a change in the extensive margin.

While the graphical analysis provides a useful insight on the resource misallocation channel, it is unclear how each margin contributes to equilibrium changes in aggregate productivity. For this reason, Figure 6 plots the dynamics of the intensive and the extensive margins in the baseline model following a credit shock (*red solid line*). The top panel of the figure shows that a credit tightening immediately raises the number of Type-2 firms in the distribution at impact, from 9 to 17 percent, implying that the extensive margin sharply increases. At the same time, in the bottom panel of Figure 6, the average ratio of constrained-to-unconstrained capital falls by more than 20 percent and slowly returns to the steady-state level.⁴⁰ Such changes in the inten-

⁴⁰This makes the marginal product of capital more dispersed across firms, raising the potential benefits of capital reallocation during the recession. This is in line with the finding in Eisfeldt and Rampini (2006).

Figure 6 : Comparison of extensive and intensive margins across models



Note: *Baseline* is the model with calibrated Pareto ϵ shocks, and *Alternative* is the model with a log-AR(1) process. The top panel presents the share of firms with binding constraints, measuring the extensive margin of resource misallocation. The bottom panel shows the average ratio of constrained to unconstrained capital choices among Type-2 firms, representing the intensive margin.

sive margin imply that the size of financially constrained firms significantly decreases, given the same level of idiosyncratic productivity. Hence, the adjustment in the intensive margin induces small, constrained firms to respond disproportionately more to tightened credit. Specifically, the model-implied gap in employment between Type-2 and other firms is 7.5 percentage point, similar to the estimate in Siemer (2019).⁴¹

In the alternative model, the population share of Type-2 firms changes from 23 to 37 percent, while such firms' capital choice shrinks by 18 percent at the impact of the shock. So, the adjustment in the extensive margin is relatively larger and that in the intensive margin is smaller. Although the number of Type-2 firms in the alternative model is still at a higher level, such firms in the baseline model face tougher borrowing conditions during the recession. It follows that the

⁴¹Siemer (2019) look at the Census data and estimate the differential responses of small firms during the Great Recession. He finds that small firms in financially-dependent sectors reduced their employment relatively more by 4.4 percentage point. I thank an anonymous referee for suggesting this comparison.

intensive margin adjustments amplify the effects of a credit shock in the baseline economy.

From the above discussion, it is clear that incorporating realistic heterogeneity in firm size and leverage matters for aggregate dynamics in the presence of financial frictions. The baseline model, employing Pareto-distributed productivity, is capable of generating an empirically-consistent firm size distribution. Given the importance of firm size in determining the extent of unobserved financing constraints, the real effects of a financial shock is amplified in a model with an idiosyncratic productivity distribution that reproduces the observed skewness and dispersion in firm size data.

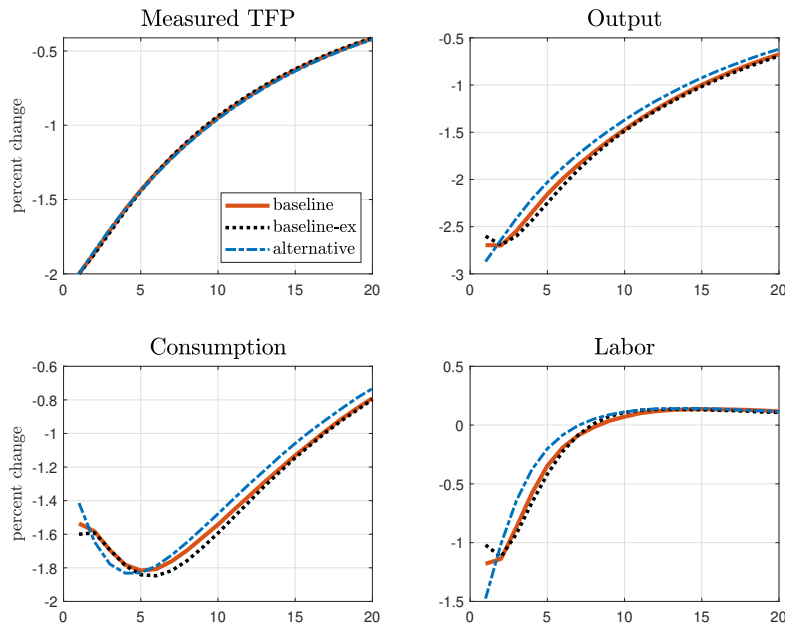
4.2 Impulse Responses: Productivity Shock

Figure 7 presents the aggregate dynamics following a persistent productivity shock in the baseline model. The shock reduces z by 2 percent at impact and gradually disappears over time. Its magnitude is chosen to match the observed decline in measured TFP in the US from 2007 to 2009. While this reproduces the fall in TFP following a credit shock, the aggregate quantities now display different patterns. In Figure 7, output, consumption, and employment all drop immediately and then gradually recover to their steady-state levels. Measured TFP closely follows the changes in z over time, but other variables fall by significantly less when compared to the financial recession in Figure 4. For instance, the real shock generates a 2.7 percent drop in output, whereas it is more than 6 percent following a credit shock with modest falls in TFP. Thus, it is clear that a TFP shock alone does not reproduce the observed changes in macro variables during the Great Recession.

The results in Figure 7 also indicate that the productivity shock does not significantly interact with the resource misallocation in the model. That is, productivity shocks have little distributional effects, unlike credit shocks.⁴² This is because all firms are evenly affected by a productivity

⁴²In models with household heterogeneity and incomplete markets, on the other hand, a productivity shock can have significant distributional effects. See Krueger, Mitman, and Perri (2017) among others.

Figure 7 : Aggregate dynamics following a productivity shock



Note: The stochastic impulse responses to a persistent productivity shock. The size of the shock is to lower z by 2 percent at the impact in all models. *Baseline* is the model with calibrated Pareto ϵ shocks, *Alternative* is the model with a log-AR(1) process, and *Base-ex* is an extension of the baseline model with permanent heterogeneity.

shock, and there are no changes in borrowing conditions that would have affected constrained firms disproportionately. Hence, the margins of resource misallocation almost stay at the steady-state levels in response to a real shock. It is then intuitive that the underlying firm heterogeneity does not significantly alter the macroeconomic implications of the model when only productivity shocks are considered. This point is illustrated in Figure 7, in which I also compare the aggregate responses across different models. Clearly, there is no pronounced difference in each aggregate series following the same TFP shock. Since firm heterogeneity does not matter for productivity shocks, standard models without financial shocks are equally successful in explaining the US business cycles.

It is notable that the extensive margin barely changes in response to a productivity shock. In the baseline model, the share of Type-2 firms only varies by less than 0.4 percentage point. Hence, about 90 percent of firms (unconstrained and Type-1) can flexibly adjust their investment and

employment during the recession. Since the productivity shock evenly depresses unconstrained capital choice at each level of individual productivity, these firms shrink at the same rate regardless of their size. That is, the responses of small and large firms are not substantially different from each other. Given that my estimated aggregate productivity shocks are dominant in driving business cycles in relative to credit shocks, firms of different sizes, on average, exhibit broadly similar dynamics over business cycles in the model. This is consistent with the finding in Crouzet and Mehrotra (2020).

4.3 The Great Recession

I show that the baseline model closely reproduces the observed aggregate dynamics during the Great Recession, when both credit and productivity shocks are jointly considered. For this, I conduct a peak-to-trough analysis of the model in comparison to the US data.

Table 10 reports the peak-to-trough changes of aggregates seen in Figures 4 and 7, with the empirical values since 2006. As seen in the second row, the baseline model slightly over-predicts the responses of aggregate output and employment to a credit shock, while the largest declines in measured TFP and consumption are smaller. In the third row, I report the changes in the model following a sudden decrease in z by 2 percent. While the model exactly matches the observed decline in TFP by setup, the aggregate changes are at Together, these discrepancies indicate that the US economy might have experienced multiple shocks at the onset of the Great Recession.

I now consider a combination of productivity and credit shocks that can better explain the aggregate dynamics during the 2007 Recession. Specifically, I calibrate the magnitude of such combined shocks to exactly reproduce the drop in real GDP by 5.3 percent. The last row of Table 10 reports the corresponding results when I reduce the values of z and θ respectively by 1 and 17 percent. Though untargeted, the largest decline in measured TFP also closely matches its empirical value, and the combined shock explains about 75 percent of the decreases both in

Table 10 : Peak-to-trough Changes, Great Recession and Baseline Model

	Y	TFP	C	N
Data	5.34	2.01	4.13	6.40
θ shock	6.39	1.57	3.39	6.88
z shock	2.70	2.00	1.82	1.18
$(\theta + z)$ shock	5.33	1.95	3.09	4.79

Note: Data is for the relative changes of HP-filtered annual US data from their levels in 2006. θ shock is for the responses of the baseline economy following a credit shock and z shock is for those following a negative TFP shock. $(\theta + z)$ shock considers a combination of both shocks that reproduces the fall in real GDP.

consumption and employment. In addition, I can measure the relative contribution of the credit tightening by isolating the impact of the drop in z in the model economy. That is, for the observed GDP fall of 5.3 percent, the credit shock appears to account for 74 percent. This suggests that the resource misallocation across firms, resulting from a credit shock, was a major factor of the aggregate dynamics during the Great Recession.

5 Concluding Remarks

Evidence on the size and leverage distributions of firms is often ignored in the business cycle literature. In this paper, I focus on their role in propagating aggregate shocks. I build an equilibrium business cycle model with heterogeneous firms and forward-looking collateral constraints. I estimate idiosyncratic productivity shocks and discipline the model to be consistent with the data on the distribution of firm size and leverage. Unlike productivity shocks, financial shocks operate through the distribution of firms, and the differential firm-level responses characterize the distributional impacts.

I find that the macroeconomic effects of financial shocks are significantly different conditional on whether a model incorporates realistic firm heterogeneity. Previous studies that fail to capture the data on firm size and leverage may under-predict the real effects of financial shocks. The model in this paper shows that the misallocation channel following a financial shock is signif-

ificantly stronger with realistic heterogeneity. Therefore, consistency with firm-level heterogeneity is crucial in studying business cycles with financial shocks.

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Online Appendix

A Firm Problem and Numerical Methods

I solve the model numerically to obtain the quantitative results in this paper. In the following, I present the original firm's problem in the absence of cash-on-hand in the model. I then describe the numerical methods used to reproduce the empirical firm size distribution, solve the equilibrium, and estimate the aggregate shocks.

A.1 Original Firm's Problem and Equilibrium Prices

In the model, cash-on-hand is defined after imposing the static labor demand condition to the original firm's problem described below. At the beginning of a period, a firm with $(k, b, \epsilon_i; s_l, \mu)$ solves

$$\begin{aligned} v_0^f(k, b, \epsilon_i; s_l, \mu) &= \pi_d \cdot \max_n \left[z_l \epsilon_i F(k, n) - w(s_l, \mu)n + (1 - \delta)k - b \right] \\ &\quad + (1 - \pi_d) \cdot v^f(k, b, \epsilon_i; s_l, \mu) \\ v^f(k, b, \epsilon_i; s_l, \mu) &= \max_{n, k', b', D} \left[D + \sum_{m=1}^{N_s} \pi_{lm}^s d_m(s_l, \mu) \sum_{j=1}^{N_\epsilon} \pi_{ij}^\epsilon v_0^f(k', b', \epsilon_j; s_m, \mu') \right] \end{aligned}$$

subject to

$$\begin{aligned} 0 &\leq D = z_l \epsilon_i F(k, n) - w(s_l, \mu)n + (1 - \delta)k - b - k' + q(s_l, \mu)b' \\ b' &\leq \theta k' \\ \mu' &= \Gamma(s_l, \mu), \end{aligned}$$

where v_0^f and v^f are analogous to Equations (1) and (2) respectively.

Next, as frequently used in the literature, it is convenient to modify the firm's value functions by using the equilibrium prices implied by the market clearing quantities. Let C and N be the

equilibrium quantities for aggregate consumption and labor. Then the prices in the model can be expressed using the marginal utility of consumption and leisure in equilibrium. The output value in equilibrium can be expressed using the marginal utility of consumption, $D_1U(C, 1 - N)$. The real wage, $w(s, \mu)$, is equal to the marginal rate of substitution between leisure and consumption, and the inverse of the discounted bond price, q^{-1} , equals the expected gross real interest rate. Lastly, the stochastic discount factor, $d_m(s, \mu)$, is the household's intertemporal marginal rate of substitution across states. I summarize these equilibrium prices in terms of marginal utilities as below.

$$\begin{aligned}
 w(s, \mu) &= \frac{D_2U(C, 1 - N)}{D_1U(C, 1 - N)} \\
 q(s, \mu) &= \frac{\beta \sum_{m=1}^{N_s} \pi_{lm}^s D_1U(C'_m, 1 - N'_m)}{D_1U(C, 1 - N)} \\
 d_m(s, \mu) &= \frac{\beta D_1U(C'_m, 1 - N'_m)}{D_1U(C, 1 - N)}
 \end{aligned}$$

Given $p(s, \mu) \equiv D_1U(C, 1 - N)$, the firm's problem can be rewritten in terms of this utility price without carrying the stochastic discount factor. That is, by defining $V_0^f \equiv p \cdot v_0^f$ and $V^f \equiv p \cdot v^f$, I can solve the equilibrium allocations solely from the firm's problem, in a manner consistent with the household's optimal decisions.

A.2 Matching Firm Size Distribution

I employ the method of matching the empirical firm size distribution in a model which is used in Jo and Senga (2019).⁴³ I use the average employment shares in the tabulated size bins in the BDS as proxies for distinguishing firm-size groups at the steady state of the model. Given a stationary distribution of firms, $\mu(k, b, \epsilon)$, I construct a cumulative employment distribution

⁴³There are various ways to reproduce the size distribution of firms in the data. For instance, Restuccia and Rogerson (2008) numerically find the time-invariant distribution of productivity in the absence of distortions. More importantly, not many business cycle studies in the literature focus on reproducing the observed size dispersion in the BDS data, which typically requires a skewed and dispersed distribution of idiosyncratic productivity.

by using N^w at each (ϵ, k) . Then, for each firm size bin, I bisect the threshold level of model employment size, \bar{n} , that exactly matches the cumulative employment share in the BDS. This relative way of measuring firm size then aligns the model employment shares exactly with their empirical counterparts. I calculate the measure of firms at each size bin distinguished by the thresholds, which yields the distribution of population shares in the model.

A.3 Solving for Stationary Equilibrium

Given parameter values, I solve the model and find a stationary equilibrium in which individual decisions are consistent with the market clearing prices. That is, I compute the equilibrium price, p , such that the excess demand, $ED(p)$, equals to zero, as in the definition of recursive competitive equilibrium. In particular, the value function in Equation (2) doesn't have to be solved, since the firm-level decisions are already derived as functions of cash-on-hand, m . It is then straightforward to find a stationary distribution of firms, $\mu(k, b, \epsilon)$, by iteratively updating the decisions at a given price. I use the bisection method to find the equilibrium price, and the weighted grid method to update the distribution. The algorithm involves the following steps.

1. Initiate the algorithm by setting parameter values and grids on state space.
2. Given $p \in [p_l, p_r]$, solve the policy functions, (N^w, K^w, B^w) , using the implied wage rate, $w(p)$, from the marginal rate of substitution.
3. At each grid point of the distribution, (k, b, ϵ) , calculate the corresponding cash-on-hand, m .
4. Given m , update the distribution by using the intertemporal decision rules for k' and b' , conditional on survival.
5. Repeat steps 3 and 4 until the distribution converges, while replacing exiting firms with entrants in each period.
6. Given a stationary distribution, check if the excess demand at p is zero and repeat steps 2 to 5.

A.4 Solving for Stochastic Equilibrium

A standard approach to solving an equilibrium business cycle model with heterogeneous agents is to approximate the endogenous aggregate state—the distribution of individuals. Since the model distribution is a high-dimensional object, the method typically replaces it with a finite set of moments. Under aggregate uncertainty with $s \equiv (z, \theta)$, I use the method of approximate aggregate state in Krusell and Smith (1998), following its application to a heterogeneous-firm model in Khan and Thomas (2008). That is, $\mu(k, b, \epsilon)$ is approximated by the first moment of the distribution of capital across firms, $K \equiv \int_{\mathbb{S}} k d\mu$. The aggregate law of motion for the distribution, $\Gamma(s, \mu)$, is in turn replaced by a simple forecasting rule, $\hat{\Gamma}(s, K)$. As in Khan and Thomas (2013), I also introduce two dummy variables, ζ_1 and ζ_2 , into the forecasting rule. $\zeta_1 = 1$ if the economy was hit by a credit shock last period, and $\zeta_2 = 1$ if it happened 2 periods ago.

I discretize the stochastic process of z using the Rouwenhorst method with $N_z = 5$, and expand the exogenous state space by including credit shocks and the dummies. I separate the forecasting rule, respectively for aggregate future capital, $\hat{\Gamma}_K$, and equilibrium marginal utility, $\hat{\Gamma}_p$, conditional on s realizations. Given $N_s = N_z \cdot N_\theta$, the forecasting rule is then given by,

$$\log x = \beta_0^i + \beta_1^i \log K_t + \beta_2^i \zeta_{1,t} + \beta_3^i \zeta_{2,t}, \quad i = 1, \dots, N_s,$$

where $x \in \{\tilde{K}_{t+1}, \tilde{p}_t\}$. I simulate the model for T periods, and collect the simulated time series, $\{p_t, K_t\}_{t=1}^T$, along with the realized shocks and dummy values, $\{s_t, \zeta_{1,t}, \zeta_{2,t}\}_{t=1}^T$. Until convergence, $\hat{\Gamma}_K$ and $\hat{\Gamma}_p$ are updated by estimating the above equations with the simulated data for 10,000 periods.

Table 11 reports the conditional forecasting rule in the baseline model. The forecasting rules for future capital and current output valuation are reasonably accurate such that R^2 values are mostly higher than 0.99 in all specifications. I also find that the accuracy measure suggested by Den Haan (2010), the maximum forecast error, is also lower than 1 percent in each regression

Table 11 : Forecasting Rule in Stochastic Equilibrium

Forecasting Rule for Future Capital, $\hat{\Gamma}_K$							
s	β_0	β_1	β_2	β_3	S.E.	Adj. R^2	max. error
(z_1, θ_{ss})	-0.14261	0.79872	-0.00535	-0.00058	0.00178	0.99744	0.00889
(z_1, θ_l)	-0.14943	0.80109	-0.00526	-0.00067	0.00216	0.99514	0.00591
(z_2, θ_{ss})	-0.12841	0.79650	-0.00549	-0.00069	0.00145	0.99812	0.00804
(z_2, θ_l)	-0.13585	0.79826	-0.00543	-0.00081	0.00231	0.99602	0.01001
(z_3, θ_{ss})	-0.11496	0.79265	-0.00553	-0.00078	0.00157	0.99766	0.00982
(z_3, θ_l)	-0.12127	0.79665	-0.00556	-0.00087	0.00241	0.99571	0.00791
(z_4, θ_{ss})	-0.09964	0.79160	-0.00560	-0.00079	0.00180	0.99711	0.00822
(z_4, θ_l)	-0.10693	0.79445	-0.00559	-0.00085	0.00246	0.99525	0.00826
(z_5, θ_{ss})	-0.08367	0.79124	-0.00576	-0.00081	0.00152	0.99754	0.00648
(z_5, θ_l)	-0.09145	0.79367	-0.00576	-0.00087	0.00253	0.99549	0.00637
Forecasting Rule for Marginal Utility, $\hat{\Gamma}_p$							
s	β_0	β_1	β_2	β_3	S.E.	Adj. R^2	max. error
(z_1, θ_{ss})	1.34921	-0.40890	0.00213	-0.00014	0.00138	0.99415	0.00612
(z_1, θ_l)	1.34241	-0.40771	0.00217	-0.00016	0.00178	0.98719	0.00509
(z_2, θ_{ss})	1.32365	-0.40710	0.00210	-0.00019	0.00110	0.99586	0.00562
(z_2, θ_l)	1.31670	-0.40573	0.00215	-0.00018	0.00182	0.99039	0.00783
(z_3, θ_{ss})	1.29763	-0.40560	0.00212	-0.00016	0.00121	0.99479	0.00769
(z_3, θ_l)	1.28924	-0.40640	0.00213	-0.00022	0.00193	0.98945	0.00638
(z_4, θ_{ss})	1.27251	-0.40209	0.00214	-0.00014	0.00140	0.99332	0.00565
(z_4, θ_l)	1.26395	-0.40305	0.00216	-0.00020	0.00194	0.98859	0.00686
(z_5, θ_{ss})	1.24744	-0.39829	0.00212	-0.00015	0.00117	0.99428	0.00490
(z_5, θ_l)	1.23931	-0.39798	0.00216	-0.00020	0.00202	0.98860	0.00480

Note: S.E. is the standard error in each regression, and *max. error* is the maximum forecast error in the simulation.

result. When compared to those in Khan and Thomas (2013), these measures are relatively less accurate, implying that the highly skewed firm distribution in my model generates such differences. As Krueger, Mitman, and Perri (2016) discussed, the method of approximate aggregate state still works in my model since the distribution of firms systematically moves along with changes in aggregate state, (s, K) .

A.5 Estimating Aggregate Shocks

Given the assumed aggregate shock processes in Section 3.4, I estimate the set of parameters, γ , by repeatedly solving the stochastic equilibrium of the model. To evaluate the objective function of the minimum-distance-estimator at a specific γ , I use the model moments generated from a separate simulation for 5,000 periods. I use the Nelder-Mead simplex method to find the minimum within the parameter space, which does not rely on the differentiability of the objective. I set the initial value of θ_l to be 10 percent lower than θ_{ss} , and other initial values are taken from Khan and Thomas (2013).

B Business Cycles with Estimated Aggregate Shocks

I discuss the properties of the estimated aggregate shocks. Table 9 shows that the estimated productivity shocks are consistent with those in recent studies of production heterogeneity. Given the annual frequency in the model, the productivity shocks are highly persistent ($\rho_z = 0.92$) and the volatility parameter, $\sigma_{\eta_z} = 0.015$, is close to the calibrated value in Khan and Thomas (2013).

The estimated credit shocks, however, appear to rarely occur while being moderately persistent. The conditional probability of experiencing a credit shock during an ordinary period, $1 - p_{oo}$, is 8.4 percent, and a credit tightening on average lasts for about 3 years. A long simulation of the model further shows that the fraction of time with $\theta = \theta_l$ is about 20 percent in the economy. These results are comparable to the evidence of historical banking crises in the US, as documented by Reinhart and Rogoff (2009).⁴⁴ When limited to explaining the observed banking crises only, the estimated shocks are slightly more often and persistent. This is because the assumed credit shocks in the model are stylized and intended to incorporate any disturbances that affect firms' collateralized borrowing in financial markets.

While rare, a credit shock has a large impact on firms' borrowing limits once it is realized.

⁴⁴Since 1800, the time share of 13 US banking crises is 13 percent, and the average duration is about 2.1 years.

Following the shock, the value of θ falls about 19 percent from its steady-state value. The implied standard deviation of θ is 5.7 percent, which is greater than that estimated by Jermann and Quadrini (2012).⁴⁵ These results indicate that credit shocks are essentially different from productivity shocks in their implications for business cycle dynamics.

Given the estimated shocks, I simulate the baseline economy for 5,000 periods and report the business cycle statistics in Table 12, together with the empirical moments. The unconditional moments of aggregate variables imply that the model with both productivity and credit shocks exhibits plausible business cycle dynamics. The standard deviation of output is about 2.2 percent, while the relative volatility of consumption is roughly a half of the output volatility. Due to the absence of explicit adjustment costs of capital, the investment volatility is somewhat larger than is typical. The volatility of employment is relatively small in the model, whereas the total hours worked are almost as volatile as the real GDP.⁴⁶

The table also shows that the cross-correlation of consumption with output, $\rho(C, Y)$, and that of employment with output, $\rho(N, Y)$, are closer to their empirical counterparts when compared to a standard real business cycle model that is only driven by exogenous TFP shocks. This is a natural result since I consider two types of aggregate shocks, and the correlations substantially increase when credit shocks are eliminated in the model. Nonetheless, the overall business cycle implications from the baseline model remain consistent with the standard models in the literature.

More importantly, the estimated aggregate shocks yield a notable finding on their relative contributions in shaping business cycles in the US. The inference on the importance of financial shocks, in particular, can be obtained by comparing the standard deviation of output in the baseline model with that in the absence of credit shocks. That is, about 10 percent of output volatility

⁴⁵In their baseline model, the estimated standard deviation of the collateral parameter is 4 percent at quarterly frequency, while financial shocks are highly persistent.

⁴⁶The relative standard deviation of US non-institutional employment is 0.684, which is much smaller than that of total hours worked in the data.

Table 12 : Business Cycles, Data and Model

	Y	C	I	N	K
<i>US, 1954-2018</i>					
σ_x/σ_Y	(2.063)	0.602	3.880	0.979	0.799
$\rho(x, Y)$	-	0.887	0.819	0.863	-0.003
<i>Baseline Model</i>					
σ_x/σ_Y	(2.239)	0.514	4.548	0.639	0.565
$\rho(x, Y)$	-	0.833	0.931	0.895	0.109
<i>Baseline Model, no credit shock</i>					
σ_x/σ_Y	(2.028)	0.515	4.009	0.552	0.496
$\rho(x, Y)$	-	0.933	0.965	0.942	0.086
<i>Alternative Model</i>					
σ_x/σ_Y	(2.096)	0.540	4.386	0.613	0.524
$\rho(x, Y)$	-	0.848	0.927	0.884	0.112

Note: This table reports the business cycle moments of output Y , consumption, C , investment, I , employment, N , and capital, K . σ_x/σ_Y is the relative standard deviation of x to that of Y , and $\rho(x, Y)$ is the contemporaneous correlation of x with Y . The moments are obtained from a 5,000-period simulation, and each series is log HP-filtered with a smoothing parameter of 100. For US data, I use the HP-filtered series of real per-capita GDP, non-durable consumption, private investment, total hours worked, and private capital for each variable.

is explained by credit shocks in the long-run, while productivity shocks account for the rest. This is in contrast to the finding of Jermann and Quadrini (2012) that financial shocks explain the largest shares of the variations in GDP, investment, and hours worked. Since the borrowing constraint in their model is always binding for a representative firm, a financial shock naturally implies a large aggregate effect. However, my estimates imply that credit shocks are relatively scarce and less persistent, limiting their contribution to an average business cycle. This paper hence finds a role for credit shocks that is substantially different from that in a representative agent model by incorporating rich firm-level heterogeneity.⁴⁷

Lastly, the bottom panel of Table 12 summarizes the simulated moments of aggregates in the alternative economy. For comparability in the simulation, I set the value of θ_l at 0.536 as in the impulse responses in Figure 4 while holding other estimated parameters of aggregate shocks fixed. In overall, the model delivers similar patterns of business cycle dynamics as in the baseline

⁴⁷Another possible approach is to estimate a representative- agent model with occasionally binding constraints, as in Guerrieri and Iacoviello (2017). However, Zetlin-Jones and Shourideh (2017) emphasize the essential role of heterogeneity by showing that an average firm in the Flow of Funds can finance its investment without relying on external borrowing.

model, but at a smaller magnitude. Since the aggregate responses to a TFP shock are almost identical between the two models (Figure 7), the above differences in the business cycle moments are largely driven by the relatively modest propagation of credit shocks in the alternative model. This further implies that the inferences on aggregate shocks would be inaccurate when I use a model without realistic firm heterogeneity for the estimation.

C Additional Model Results

I report additional results from the model in the main text, with those from other specifications of idiosyncratic shocks. In particular, I show that the version of the model that directly targets the empirical firm size distribution is analogous to the baseline model. In each model, I re-calibrate parameter values to closely reproduce the aggregate moments in Table 3.

C.1 Other Model Specifications

First, I consider a variant of the baseline model with Pareto-distributed productivity shocks. Instead of using the estimated idiosyncratic shocks from the Orbis data, I set the parameter values of ϵ in the *base-sz model*, $(\epsilon_m, \epsilon_M, \xi)$, to closely reproduce the firm size distribution in the BDS. I particularly focus on whether this model yields similar predictions on non-targeted cross-sectional heterogeneity and aggregate dynamics as in the baseline model.

Next, I adjust the persistence of idiosyncratic shocks. In the baseline model, the persistence in productivity is determined by the fixed probability of retaining ϵ , $\pi_\epsilon = 0.75$, to be consistent with the evidence from US firms. On the other hand, in the *base-hp model*, I instead examine the case with $\pi_\epsilon = 0.9$. This allows me to check whether the impulse responses in Section 4 are driven by imposing relatively frequent changes in firm-level productivity over time.

I also provide additional results from the *alt-lv model* in which idiosyncratic shocks follows a log-AR(1) process with lower volatility ($\sigma_\eta = 0.09$). As mentioned in the main text, the resulting

moments of investment rate becomes closer to their empirical counterparts in the alt-lv model, when compared to the alternative model (Table 6). However, this comes at the cost of generating a counterfactual firm size distribution.

C.2 Firm Heterogeneity

Following the organization of Section 3 in the main text, I compare cross-sectional heterogeneity in firm size, investment moments, and size-leverage relationships across different models.

Table 13 reports the model-generated firm size distribution. As targeted, the base-sz model delivers a realistic firm size distribution in which the population shares at the smallest and largest size bins are closer to the empirical targets. This is because the calibrated idiosyncratic shocks imply a larger support of ϵ with a fatter tail. In contrast, the model's fit to the empirical data becomes slightly worse when the persistence of ϵ rises. The base-hp model predicts relatively more firms at both smallest and largest groups in the size distribution. Still, this model is able to capture the observed skewness in firm size data when compared to the models with log-AR(1) shocks. These results indicate that employing a non-Gaussian distribution for firm productivity can be a parsimonious way of nesting realistic firm size heterogeneity in a model, as pointed by Elsbey and Michaels (2013).

Next, Table 14 reports the moments of firm-level investment dynamics. In overall, the newly considered models in this appendix are broadly consistent with the corresponding empirical moments in the Orbis dataset (columns 3 and 4). The base-sz model, in particular, is almost as successful as the baseline model in explaining the observed patterns of investment at the firm level. It follows that directly targeting the empirical size distribution does not significantly damage the model's consistency with the investment moments in the data. Due to highly persistent firm productivity, on the other hand, the mean and standard deviation of investment rate are

Table 13 : Comparison of Firm Size Distribution, Other Specifications

Bin: employees	Population Share (%)			
	BDS Data	Baseline	Base-sz	Base-hp
1: 1 to 4	55.06	51.30	53.81	58.22
2: 5 to 19	33.42	33.38	33.63	28.36
3: 20 to 99	9.64	9.80	8.52	8.26
4: 100 to 499	1.53	2.85	2.22	2.52
5: 500 to 2,499	0.26	1.29	0.91	1.18
6: 2,500+	0.09	1.38	0.91	1.46
MSE	-	0.000	0.000	0.007

Note: *BDS Data* is the average value calculated from 1977 to 2006. *Baseline* is the model with calibrated Pareto ϵ shocks, *Base-sz* is the model that targets the BDS size distribution, and *Base-hp* is the model with high persistence of ϵ . *Size bin* is in terms of employment, and model employment shares are exactly matched with those in the BDS. *MSE* is the mean-squared-error of the model moments.

Table 14 : Comparison of Firm-level Investment Dynamics, Other Specifications

	Investment Moments			
	Orbis Data	Baseline	Base-sz	Base-hp
$\mu(i/k)$	0.117	0.142	0.149	0.102
$\sigma(i/k)$	0.444	0.365	0.377	0.170
$\rho(i/k)$	0.337	0.293	0.321	0.635
lumpy inv.	0.170	0.103	0.093	0.056
MSE	-	0.003	0.003	0.093

Note: *Orbis Data* is computed from the balanced panel of firms in the G-7 countries. $\mu(i/k)$ is the average investment rate (i/k), $\sigma(i/k)$ is the standard deviation, $\rho(i/k)$ is the first order autocorrelation, and *lumpy inv.* is the share of firms with $i/k > 0.2$. *Baseline* is the model with calibrated Pareto ϵ shocks, *Base-sz* is the model that targets the BDS size distribution, and *Base-hp* is the model with high persistence of ϵ . *MSE* is the mean-squared-error of the model moments. All model moments are calculated from a large panel of firms simulated at each steady state.

relatively low in the base-hp model. The model's fit remains slightly better than the alt-lv model that directly targets the empirical volatility of investment rate (column 5 of Table 6). Together, these models with skewed idiosyncratic shocks generate realistic investment dynamics at the firm level.

I now report the quantile distribution of firm leverage in Table 15. When compared to the baseline model, the base-sz model predicts slightly higher leverage ratio at the top 0.5 percent and a lower value at the bottom 90 percent of asset distribution. In relative to the QFR data,

Table 15 : Mean Leverage by Asset Size, Other Specifications

	0-90th	90-99th	99-99.5th	>99.5th	MSE
QFR Data	0.20	0.19	0.23	0.22	-
Baseline	0.20	0.26	0.23	0.14	0.003
Base-sz	0.17	0.24	0.23	0.17	0.001
Base-hp	0.15	0.27	0.21	0.16	0.003
Alt-lv	0.28	0.24	0.46	0.00	0.028

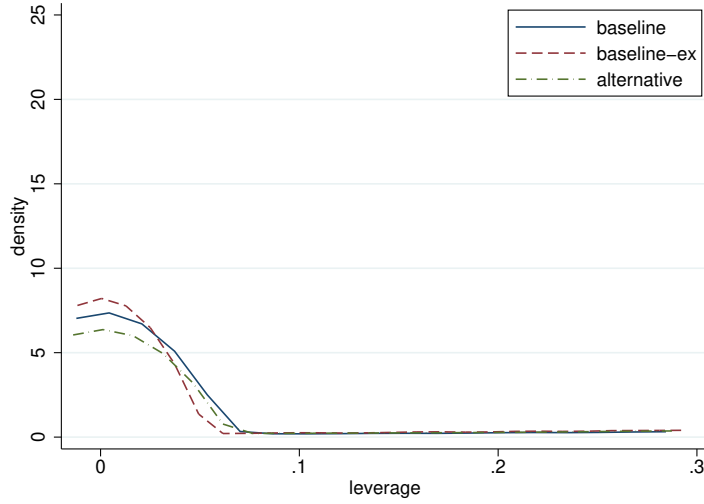
Note: *QFR Data* reproduces the values of net leverage in Crouzet and Mehrotra (2020). *Baseline* is the model with calibrated Pareto ϵ shocks, *Base-sz* is the model that targets the BDS size distribution, *Base-hp* is the model with high persistence of ϵ , and *Alt-lv* is the model with a log-AR(1) process with lower volatility. *MSE* is the mean-squared-error of the model moments. Assets in the model are defined as the sum of capital and financial savings.

however, the overall fit is better in this model, suggesting that directly matching the empirical size distribution additionally achieves the consistency with the heterogeneity in firm leverage. Moreover, this result does not vary with the assumed persistence of firm productivity, as the base-hp also performs well. In contrast, the alt-lv model exhibits large discrepancies with the observed financial data.

I also estimate the kernel density of firm leverage across models with different idiosyncratic shocks. As Figure 8 shows, the estimated distribution exhibits higher density at low levels of leverage in each model. Although the leverage distribution is slightly flatter in the alternative model, there is no significant difference across the models. This implies that simply checking the leverage distribution, without taking into account the size-leverage relationships, may lead to a conclusion that these models are equally successful in generating realistic heterogeneity in firm leverage. Thus, the evidence in Tables 7 and 15 serves as a useful benchmark for a macro model with financial frictions.

Lastly, I check the employment coefficient in the leverage regression in Section 3.3. Consistent with the previous results, the size-leverage relationship is positive and significant in the models with Pareto-distributed productivity. The value of coefficient varies; 0.022 in the base-sz model and 0.002 in the base-hp model. The alt-lv model predicts a negative relationship, on the

Figure 8 : Distribution of Leverage by Model



Note: The distributions are generated using kernel density estimation, and zoomed at the lower tail for comparability with Figure 6 in Dinlersoz et al. (2019). *Baseline* is the model with calibrated Pareto ϵ shocks, *Alternative* is the model with a log-AR(1) process, and *Base-ex* is an extension of the baseline model with permanent heterogeneity

other hand, as in the alternative model.

From the above results, it follows that a heterogeneous-firm model with financial frictions may generate misleading predictions in the cross-section when the empirical distribution of idiosyncratic shocks is not carefully captured, especially its skewness. Moreover, the base-sz model is almost isomorphic to the model with estimated productivity shocks, implying its usefulness for incorporating the unobserved financial heterogeneity across firms.

C.3 Firm Dynamics

Another important source of heterogeneity in the model arises through firm lifecycle dynamics in the presence of collateral constraints. To see this point, Figure 9 illustrates the average patterns of firm growth and leverage upon entry in the baseline model. At age 0, firms are born with a small initial capital, $k_0 \equiv \chi \int_{\mathbb{S}} k d\mu$, that is much less than that of an average firm in the economy. Conditional on survival, these young firms gradually accumulate capital by externally

Table 16 : Fixed-effects Leverage Regression, Other Specifications

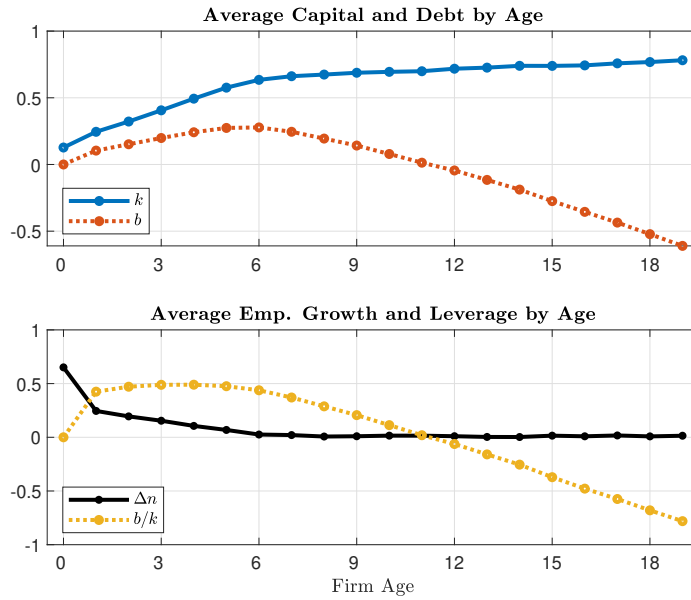
Dep. variable	$stlev_{i,t}$				
Sample	G-7	Baseline	Base-sz	Base-hp	Alt-lv
	(1)	(2)	(3)	(4)	(5)
$emp_{i,t-1}$	0.012*** (17.60)	0.013*** (16.77)	0.022*** (26.29)	0.002*** (2.69)	-0.055*** (-149.08)
$collateral_{i,t-1}$	-0.189*** (-29.16)	-0.011*** (-3.86)	-0.011*** (-3.48)	-0.123*** (-37.56)	-0.068*** (-46.13)
$profit_{i,t-1}$	-0.340*** (-16.86)	2.224*** (205.95)	2.046*** (171.62)	3.320*** (208.15)	3.109*** (441.23)
$prod_{i,t-1}$	0.024*** (13.10)	0.785** (2.13)	6.183*** (8.45)	-14.010*** (-14.08)	11.247*** (21.71)
industry FE	✓				
country FE	✓				
year FE	✓	✓	✓	✓	✓
obs.	867,458	325,195	324,360	324,515	325,840
adj. R^2	0.453	0.739	0.753	0.694	0.737

Note: t -statistics in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. $stlev$ is short-term leverage, emp is employment size in logs, $collateral$ the ratio of tangible fixed assets to total assets (ratio of capital to assets in model), $profit$ is net income over total assets (revenue less wage bill over assets in model), and $prod$ is a measure of labor productivity that divides turnover by employment (output over employment in model). All variables are winsorized at the 1 percent level.

borrowing. As they approach the efficient production scale over time, the collateral constraint becomes less relevant and they gradually start saving in financial assets. When the borrowing condition becomes tighter, these lifecycle patterns are prolonged and hence raise the number of financially constrained firms in the distribution.

Due to the persistence and dispersion of idiosyncratic productivity, firms in the baseline economy exhibit different growth profiles when young. That is, the differences in initial (or ex-ante) productivity at entry persistently affect the growth patterns of young firms, as emphasized by Pugsley, Sedlacek, and Sterk (2021). To see this point, in the top panel of Figure 10, I distinguish firm employment dynamics by initial productivity, after normalizing the employment size to 1 at age 9. First, at age 0, firms have the same level of capital k_0 , so employment is entirely determined by their initial productivity ϵ_0 . Thereafter, only a small number of highly productive entrants grow at a faster rate, far exceeding that of the average firm in the economy (*red line with circles*). This is because they tend to remain relatively more productive in the subsequent periods

Figure 9 : Firm dynamics in the baseline model



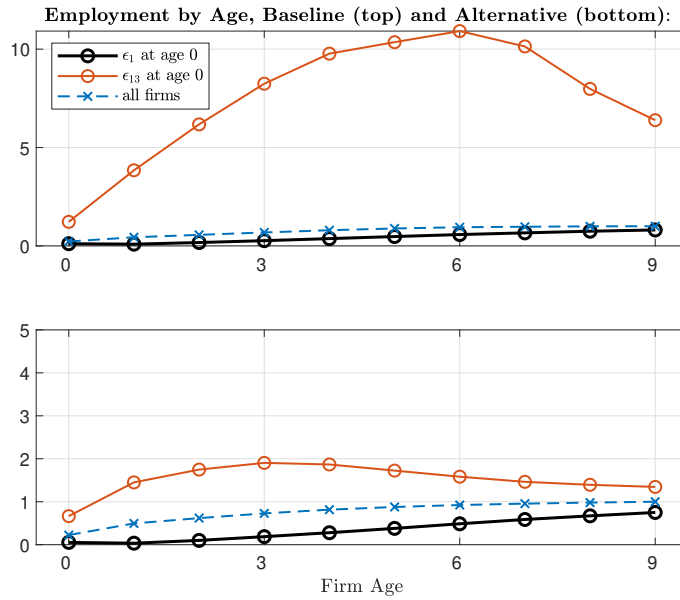
Note: The average firm dynamics are constructed from a simulation of 100,000 firms for 160 periods at the steady state of the baseline model. The top panel displays the cohort average capital and debt, and the bottom panel presents the employment growth-age and the leverage-age profiles.

given the persistence of ϵ . These high-growth firms easily achieve an efficient production scale, remaining relatively larger at age 9. In contrast, other young firms with low initial productivity grow slowly over time. These differences in growth profile would eventually disappear, because there is no permanent component in firm productivity and ex-post shock realizations become dominant in determining the cross-sectional heterogeneity.⁴⁸

In the alternative model, on the other hand, young firms display similar growth patterns across different initial productivity (bottom panel of Figure 10). In particular, the employment size of firms with the highest ϵ is at most about 3 times larger than the average in earlier periods, and such difference almost disappears around age 9. In this economy, it appears that the ex-ante heterogeneity in productivity is not an important factor of determining size differences over firm lifecycle. Thus, the role of financial frictions is relatively small, as earlier shown in Table 5.

⁴⁸The collateral constraints thus work as a selection mechanism among incumbents in the model, affecting the shape of size distribution jointly with the assumed productivity. This is in line with the idea of Luttmer (2007), although I abstract from the endogenous margins of firm entry and exit.

Figure 10 : Employment by firm age and initial productivity



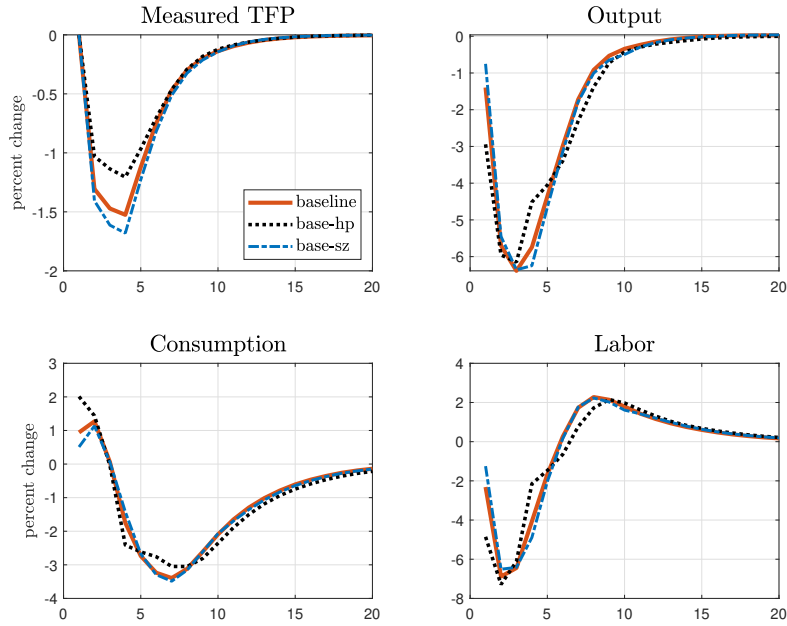
Note: The average firm dynamics are constructed from a simulation of 100,000 firms for 160 periods in the baseline (left) and the alternative (right) models. ϵ_{13} is the largest firm productivity draw (blue line with circles). All values are normalized by the employment size at age 9.

C.4 Aggregate Responses to a Credit Shock

I compare the aggregate dynamics of the models discussed above, while controlling the size of a credit shock in each model economy. First, Figure 11 shows the impulse responses in models with realistic firm size heterogeneity. In the base-sz model, the endogenous fall in measured TFP is slightly more than 1.5 percent at its trough (*dash-dotted line*). Nonetheless, the corresponding changes in other aggregate variables are very close to those in the baseline model. This confirms the finding in the main text that a recession triggered by a sudden credit tightening is relatively larger in an economy with an empirically-consistent size distribution.

In addition, the above finding remains robust when firms face highly persistent idiosyncratic shocks in the model. As shown in Figure 11, the base-hp model also generates a deep and persistent recession following a credit shock (*black dotted line*). The aggregate responses are comparable to those in the baseline model, but the largest drop in TFP is slightly modest. This verifies

Figure 11 : Aggregate dynamics following a credit shock, other specifications 1

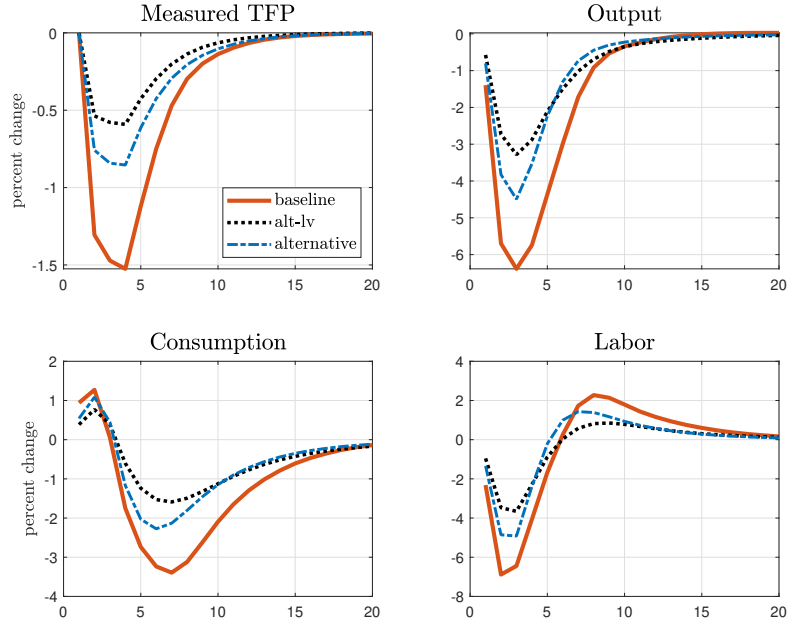


Note: The stochastic impulse responses to a sudden credit tightening. The size of the shock implies an endogenous decrease of aggregate borrowing by 26 percent in all models, and θ start recovering in period 4 with persistence of p_{lO} . *Baseline* is the model with calibrated Pareto ϵ shocks, *Base-sz* is the model that targets the BDS size distribution, and *Base-hp* is the model with high persistence of ϵ

that the amplification of the shock in the baseline model does not arise from imposing lower persistence in productivity.

I conduct the same experiment in the alt-lv model. Figure 12 shows that the magnitude of aggregate responses is substantially smaller in this economy when compared to those in the alternative model. Specifically, with lower volatility of productivity shocks, a credit shock leads to about 0.6 percent drop in measured TFP at its trough, while the dynamics of other variables remain close to their counterparts in the alternative model. It follows that increasing idiosyncratic volatility alone does not necessarily generate a deep financial recession as observed in the US data, in addition to the inconsistency with the evidence on firm heterogeneity.

Figure 12 : Aggregate dynamics following a credit shock, other specifications 2



Note: The stochastic impulse responses to a sudden credit tightening. The size of the shock implies an endogenous decrease of aggregate borrowing by 26 percent in all models, and θ start recovering in period 4 with persistence of p_{lO} . *Baseline* is the model with calibrated Pareto ϵ shocks, *Alternative* is the model with a log-AR(1) process, and *Alt-lv* is the model with a log-AR(1) process with lower volatility.

D Empirical Appendix

I describe the data source and the sample construction that are related with the empirical works in Section 3 of the main text, and then report additional results.

D.1 Productivity Shock Estimation

I use the Orbis database to construct the dataset in which firm-level information is collected and provided by Bureau van Dijk (BvD). Since the online platform only allows for downloading the last 10 observations for each firm, I construct a balanced annual panel from 2013 to 2018 in the G-7 countries (US, Canada, United Kingdom, France, Germany, Italy, and Japan), and extend it with 11 other economies in the EU area (Austria, Belgium, Denmark, Finland, Greece, Ireland, Luxemburg, Netherlands, Portugal, Spain, Sweden). Below, I describe the sampling criteria and

the estimation method for measuring productivity shocks at the firm level.

I focus on public and private limited firms only. I exclude firms in agriculture, utility, finance, insurance, real estate, and public administration, based on the industry classification of NACEv2-2 digit codes. The panel dataset includes firms that report the variables of interest in each year: employment (n), fixed assets (k_f), tangible fixed assets (k), and operating revenue (y). I drop observations with missing or negative values of these variables. To deflate the variables for capital and value-added by sector, I merge the dataset with the Structural Analysis Database (STAN) in the OECD and convert (y, k, k_f, y) respectively into real terms. Each firm's investment in period t is defined as the net difference of its capital between period t and $t + 1$. All variables used in the regression analysis are winsorized at the top and the bottom 1 percent. The final dataset contains 254,276 firms with 1,525,656 observations in the G-7 countries, and 636,796 firms with 3,820,776 observations in the extended sample.

To estimate the production function using Equation (6), I follow the Olley-Pakes method, a conventional approach of structurally estimating the relative shares of factors of production. The method uses a two-step procedure to address selection and simultaneity biases and to control for within-firm serial correlations. Specifically, the method assumes that each firm observes its own productivity before making the decisions of employment and investment in each period, whereas capital remains fixed within the period. This is in line with the timing assumption in my model, so the estimates of β_k and β_n can be directly used for retrieving firm-level productivity from the data. I estimate these coefficients with country- and year-fixed effects, repeating 40 times for bootstrapping.⁴⁹

In Table 17, the estimate of β_n ranges from 0.68 to 0.81 across different samples or capital variables. This is consistent with those in the existing empirical works that particularly study balanced panels, although the value is typically assumed to be 0.6 in many macro models. The value of β_n slightly falls when I use total fixed assets instead of tangible fixed assets in the regres-

⁴⁹I use the Stata package *prodest* provided by Ravigatti and Mollisi (2018).

Table 17 : Comparison of Production Function Estimates

Dep. variable	log y			
	G-7		G-7+EU11	
Sample	(1)	(2)	(3)	(4)
log n	0.732*** (334.22)	0.677*** (265.33)	0.811*** (668.30)	0.767*** (478.55)
log k	0.199*** (99.85)		0.176*** (90.59)	
log k_f		0.246*** (145.45)		0.219*** (87.76)
obs.	486,874	520,477	1,142,439	1,255,291
firms	217,076	220,717	526,242	544,586
Wald-p	0.00	0.00	0.00	0.00

Note: t -statistics in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. y is operating revenue, n is employment, k is tangible fixed asset, and k_f is total fixed assets. *Wald-p* is the p-value of the Wald statistic for constant-returns-to-scale. All variables are winsorized at the 1 percent level.

sion (columns 2 and 4). Lastly, the capital coefficients are also within a reasonable range of the values reported in previous studies.

From the fitted values of $y_{i,j,t}$ in the above regression, the residuals are the measured (revenue-based) firm productivity $x_{i,j,t}$ for firm i in country j at year t . I then eliminate the firm-fixed effects by demeaning the measured productivity and convert it into percentage terms $\tilde{x}_{i,j,t}$. The resulting distribution of \tilde{x} is positively skewed and dispersed in all specifications considered in Table 17. For instance, in the first column of the table, the estimates imply that the standard deviation of \tilde{x} is 0.26 and the skewness is 1.63. In the baseline model, I estimate Equation (7) with asset-size weights and fixed effects, and use the empirical moments of idiosyncratic shocks to calibrate the parameter values of the assumed Pareto distribution.⁵⁰ In the alternative model, I instead use the demeaned log \tilde{x} for the regression and retrieve the corresponding idiosyncratic shocks which exhibits the skewness of 0.23. Ignoring this skewness, I only calibrate the value of σ_η , the standard deviation of Gaussian innovations in the assumed log-AR(1) process for ϵ .

⁵⁰To control for multiple fixed effects in the regression, I use the Stata package *reghdfe* provided by Correia (2017).

D.2 Leverage Regression

Data Construction I separately construct a balanced panel of firms in the Orbis database. This is because the sample size drastically becomes smaller in recent years, possibly due to the reporting and collecting time lags of the financial variables that are mainly used in my estimation. As before, I focus on public and private limited firms only. I exclude firms in finance, insurance, real estate, medical sector, and public administration, based on the industry classification of NACIS 2017 primary codes. The panel dataset includes firms that report the variables of interest for all years, and I winsorize all variables used in the regression at the top and the bottom 1 percent. The financial data are in US dollars based on the exchange rate reported by the Orbis. Lastly, I use US SIC primary codes of individual firms to control the industry fixed effects up to 3 digits. The resulting dataset contains 184,565 firms with a total of 1,060,143 observations from 2010 to 2015, and Table 18 presents the summary statistics. Consistent with the existing studies, I define the following financial ratios to be used in the regression analysis.

- *collateral ratio*: asset tangibility measured as the ratio of tangible fixed assets to total assets.
- *profit ratio*: firm profitability measured as the ratio of net income to total assets.
- *total leverage*: defined as total liabilities (excluding shareholders' funds) over total assets.
- *financial leverage*: defined as short-term borrowing (current liabilities) plus long-term debt over total assets.
- *short-term leverage*: defined as short-term borrowing over total assets.

In the empirical analysis, I establish the robust relationship between employment and leverage at the firm level. Since the Orbis coverage of US firms is limited and non-representative, I follow the approach of Rajan and Zingales (1995) by including observations in other developed

Table 18 : Descriptive Statistics, Orbis G-7 Sample

	Mean	Std. Dev
operating revenue (mil. USD)	54,202.0	231,906.6
total assets (mil. USD)	52,024.5	236,863.0
tangible fixed assets (mil. USD)	11,556.7	55,133.6
employment	148.4	625.1
collateral ratio	0.214	0.222
profit ratio	0.030	0.073
total leverage	0.664	0.227
financial leverage	0.571	0.234
short-term leverage	0.475	0.231

Note: Statistics from a balanced panel of 184,565 firms from 2010 to 2015. All variables are winsorized at the 1 percent level.

Table 19 : Firm Shares by country, Orbis G-7 Sample

	US	Canada	UK	France	Germany	Italy	Japan
share of firms (%)	0.99	0.01	8.38	12.63	3.89	61.30	12.80
share of pvt. firms (%)	11.58	0.00	93.78	84.73	87.67	87.76	11.20

Note: There are 136,276 private limited firms out of 184,565 firms. *share of firms* is the population share of firms in each country, and *share of pvt. firms* is the country-specific share of private firms.

economies. My dataset contains private companies that typically rely on limited financing options. Further, this type of firms in the US plays a key role in shaping the skewed size distribution in the BDS.⁵¹ Given that my focus is on the empirical size-leverage relationship in an aggregate economy, it is important to incorporate such firms as many as possible. As reported in Table 19, the share of private firms is more than 77 percent in my dataset from the Orbis, and I further show that the empirical result is robust when only such firms are considered. By controlling industry-country-year fixed effects in the leverage regression, I reduce the potential bias arising from different financial institutions and business environment across countries. In addition, I use asset size as the weight for the panel fixed-effects regression.

Additional Empirical Results First, I consider potential differences between private and listed

⁵¹Cabral and Mata (2003) show that financing constraints significantly affect the shape of firm size distribution.

Table 20 : Fixed-effects Leverage Regression, Private and Listed Firms

Dep. variable	Private Firms			Listed Firms		
	$tlev_{i,t}$	$flev_{i,t}$	$stlev_{i,t}$	$tlev_{i,t}$	$flev_{i,t}$	$stlev_{i,t}$
	(1)	(2)	(3)	(4)	(5)	(6)
$emp_{i,t-1}$	0.020*** (16.02)	0.012*** (9.29)	0.014*** (11.48)	0.025*** (14.45)	0.012*** (8.09)	0.016*** (19.90)
$collateral_{i,t-1}$	0.019* (1.87)	-0.026*** (-2.64)	-0.223*** (-22.53)	-0.006 (-0.45)	-0.035*** (-2.81)	-0.163*** (-19.85)
$profit_{i,t-1}$	-0.731*** (-25.92)	-0.684*** (-25.40)	-0.314*** (-13.48)	-0.840*** (-16.40)	-0.722*** (-17.19)	-0.353*** (-13.30)
$prod_{i,t-1}$	0.033*** (13.83)	0.026*** (10.37)	0.025*** (9.91)	0.014*** (2.99)	0.016*** (3.91)	0.020*** (8.42)
obs.	670,387	670,387	670,387	197,071	197,071	197,071
adj. R^2	0.182	0.182	0.338	0.372	0.319	0.542

Note: t -statistics in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. Each regression includes terms for industry-country-year fixed effects. $tlev$ is total leverage, $flev$ is financial leverage that includes both short-term and long-term debt, $stlev$ is short-term leverage. emp is employment size in logs, $collateral$ the ratio of tangible fixed assets to total assets, $profit$ is net income over total assets, and $prod$ is a measure of labor productivity that divides turnover by employment. All variables are winsorized at the 1 percent level.

firms in their financing decisions. This is motivated by the work of Dinlersoz et al. (2019) who find a positive relationship between firm leverages and employment size especially for private firms in the US. Table 20 reports the leverage regression results respectively for private and listed firms in the G-7 countries.

The table shows that all coefficients on both private and listed firms' employment are positive and significant. Their values range from 0.012 to 0.025, possibly reflecting differences in financing decisions by firm type and borrowing method. While the employment coefficient is robust to the changes in the dependent variable, other firm controls do not exhibit systematic patterns. This motivates additional studies for identifying firm-level determinants of leverage.

Next, Table 21 shows the regression results with different measures of firm leverage, respectively from the sample of the G-7 countries and that of the US. Again, all regression coefficients on employment are positive and significant, implying that firms are likely to be more leveraged as they grow and their financing constraints are relaxed. While the majority of firms in the sample are listed in the US, the above result still confirms the importance of firm size as a determinant

Table 21 : Fixed-effects Regression, G-7 and US Firms

Dep. variable	G-7			US		
	$tlev_{i,t}$	$flev_{i,t}$	$stlev_{i,t}$	$tlev_{i,t}$	$flev_{i,t}$	$stlev_{i,t}$
	(1)	(2)	(3)	(4)	(5)	(6)
$emp_{i,t-1}$	0.022*** (18.76)	0.010*** (10.33)	0.012*** (17.60)	0.030*** (9.99)	0.009*** (3.51)	0.018*** (13.59)
$collateral_{i,t-1}$	0.002 (0.24)	-0.027*** (-3.26)	-0.189*** (-29.16)	-0.031 (-0.98)	-0.013 (-0.44)	-0.094*** (-5.74)
$profit_{i,t-1}$	-0.823*** (-22.54)	-0.729*** (-24.01)	-0.340*** (-16.86)	-0.605*** (-10.94)	-0.488*** (-9.38)	-0.216*** (-7.30)
$prod_{i,t-1}$	0.022*** (7.07)	0.021*** (7.47)	0.024*** (13.10)	-0.013 (-1.46)	-0.006 (-0.78)	0.003 (0.64)
obs.	867,458	867,458	867,458	8,427	8,427	8,427
adj. R^2	0.289	0.256	0.453	0.493	0.456	0.656

Note: t -statistics in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. Each regression includes terms for industry-year fixed effects. $tlev$ is total leverage, $flev$ is financial leverage that includes both short-term and long-term debt, and $stlev$ is short-term leverage. emp is employment size, $collateral$ the ratio of tangible fixed assets to total assets, $profit$ is net income over total assets, and $prod$ is a measure of labor productivity that divides turnover by emp . All variables are winsorized at the 1 percent level.

of firms' financing decisions.