Schooling, Skill Demand and Differential Fertility in the Process of Structural Transformation

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Abstract: Along with the structural transformation, I observe that rural fertility rate declined faster than the urban one during the Progressive Era in the U.S. I argue that rural schooling increase, which was mainly driven by the Rural Education Reform, is crucial to understand the observation. Parents at the turn of the twentieth century U.S. started to substitute having more children for more schooling. As young adults acquired skills, they left farms and joined the skill-intensive non-agricultural sector. A quantitative exercise shows that the quality-quantity decision in household accounts for 24% of the decline in the agricultural sector.

JEL Classification: E24, J11, O11, O41

Keywords: Quality-Quantity Trade-off, Structural Transformation

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Declining fertility rates and a shrinking agricultural sector are two of the central features of economic development. Compared with U.S. females born in the 1860s, who reached the peak of their fecundity circa 1900, those born in the 1880s experienced a 34% decrease in the number of children over their lifetime as well as a drastic decline in agricultural employment. While traditional literature treats these two events separately and attributes them as a manifestation of economic progress, I argue that the ability of parents to trade off number of children for their schooling also contributes to a declining fertility rate and expanding skill-intensive non-agricultural sector.

After the U.S. Civil War, the Progressive Era from the 1890s to 1920s saw rapid change in familial and economic structures. Figure 1 illustrates that, when compared to females born in the 1860s, females born in the 1880s who reached the peak of their fecundity at the turn of the twentieth century experienced more than one third drop in fertility.[1] Such a decrease was associated with a decline in the agricultural employment share. To better understand the process, I disaggregate fertility rate into sectoral fertility rates, and two features arise. First, the agricultural fertility rate was higher. Therefore when laborers moved from the agricultural to the non-agricultural sector and followed the fertility pattern in the non-agricultural sector, the aggregate fertility rate declined. Second, the agricultural fertility rate declined much later than the non-agricultural fertility rate. Its decrease further contributed to a rapid decline in aggregate fertility rate at the turn of the twentieth century.

This paper answers two questions: 1) What was the link between the agricultural employment share and demographic factors? 2) Why did the agricultural and non-agricultural fertility rates behave so differently? I propose that education is important to answer these questions. Because of the reduction in educational costs, technological progress and regulatory change, parents at the turn of the twentieth century were encouraged to channel resources from the number of children to their education. This redirection increased the overall skill intensity in the economy and led to an expansion of the more skill-intensive non-agricultural sector. In this way, parental decisions about

[1] Females refer to ever married non-Hispanic white women born in the U.S. who were not residents in a group quarter. The fertility rate documented is the cohort fertility rate. To make the number comparable to the contemporaneous fertility rate (hence the secondary x-axis titled "year"), 27 years are added. This is consistent with the addition of 25 years by Goldin (1990) and the addition of 27 years by Jones and Tertilt (2006) and Albanesi and Olivetti (2014). As a robustness check, Jones and Tertilt (2006) extend the time series of the fertility rate under such a construction to the baby boom period and the peak of such a construction coincides with the realized birth rate. In the subsequent discussion, I use the term fertility rate at time t to refer to the cohort fertility rate of the cohort born in the time interval $[t-29, t-25]$ unless otherwise specified.

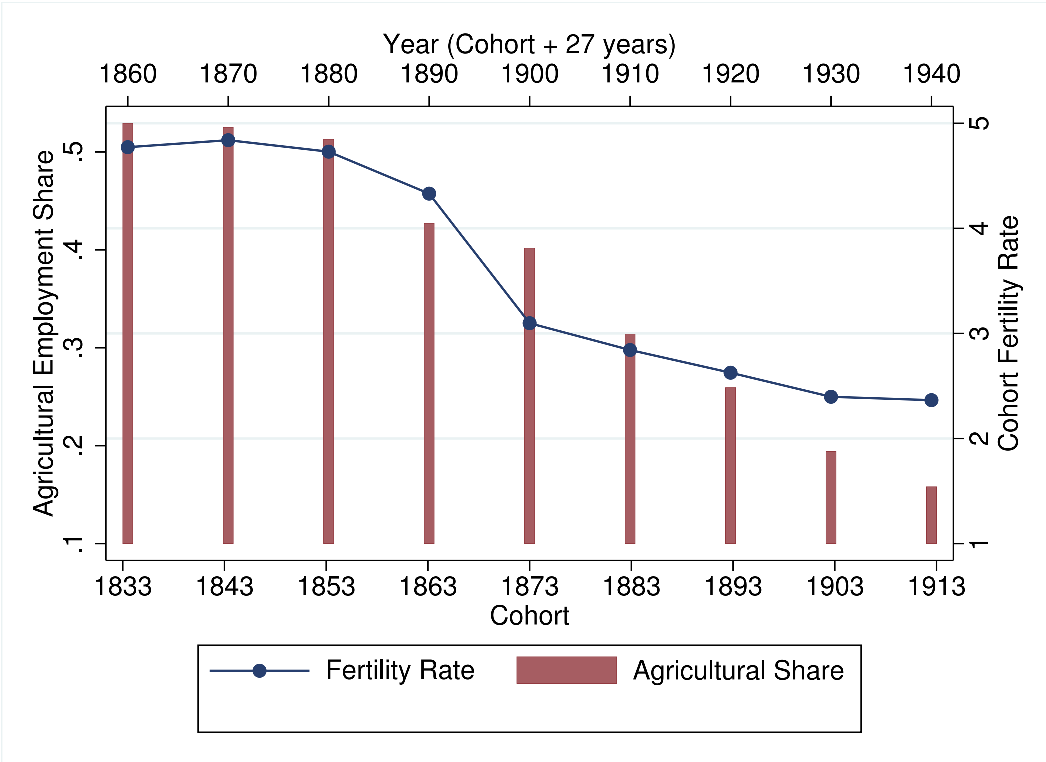

Figure 1: Aggregate Cohort Fertility Rate and Agricultural Employment Share in the U.S. IPUMS (Ruggles et al., 2018) various years.

a quality-quantity trade-off leads to a macroeconomic outcome in the long run. In contrast to the traditional explanations such as the declining mortality rate, increasing female labor participation and delayed marriage, none of which showed any differential sectoral trends during the relevant period, the educational attainment of children, particularly in rural areas, increased drastically around the turn of the century.

In the early stage of U.S. development, only a fraction of people – those who were well-off and lived in cities – had the chance to receive a quality education. Given a certain income, city dwellers would have fewer children, because some resources were devoted to the children's education. For farmers, who faced a higher relative educational cost and a lower relative educational benefit, it was of less interest that their children receive a formal education. Their children, instead, learned field-specific knowledge from them through work in the fields. We therefore observe less intergenerational mobility because farmers' children would be likely to end up being farmers. At the same time, because the children helped their parents in the fields or at home, child-rearing costs were partially subsidized by the children's service. As a result, we observe a higher fertility in the agricultural sector with lower human capital.

Around 1900, nonetheless, the modernization of the educational system, the emergence of non-agricultural work opportunities and the enforcement of compulsory schooling created an incentive for parents, even in the rural settings, to send their children to school. This increased school attendance and lowered the fertility rate in the agricultural sector because the child-rearing cost increased. As young adults received more education, they left rural areas and worked in the non-agricultural sector.

I build a two-sector overlapping-generations model that features joint determination of fertility, education and occupational choice. By using such a model with a higher agricultural educational cost (due to, for example, higher transportation costs, lower density and poor quality of schools), the model predicts a lower human capital investment and a higher fertility in the agricultural sector. During the educational reform, when the cost distortion in the rural area is eliminated, farmer parents started to channel resources from the number of children to their education. When the skill intensity is higher in the economy, the Rybczynski Theorem dictates an expansion in the more skill-intensive non-agricultural sector.

When the model is calibrated to the historic U.S. data, it can predict features that we observed in the data. This model also enable me to answer the two research questions posed through decomposition exercise and counterfactual experiment. The ability that parents can trade off number of children for their skill creates negative correlation between fertility rate and employment in the unskilled agricultural sector in the long run. The Rural Education Reform that eliminated the rural-urban education cost gap led also to the convergent rural-urban fertility rates.

The paper has two major novelties. First, I show that demography is important to structural transformation in the long-run. In the model of which fertility and children's skill levels are jointly determined, when sectoral fertility rates are exogenously fixed at their initial levels, the ability of parents to trade off the quantity of children for their quality is limited. As a result, stationary skill intensity and the extent of structural transformation are reduced. Second, I identify differential sectoral fertility rates in the U.S. and use a consistent model to show that cost distortion in the rural educational sector asymmetrically reduces human capital accumulation and increases the fertility rate in the agricultural sector. This distortion also reduces the extent of industrialization. The elimination of such distortion promotes rural-urban convergence in fertility and schooling as well as accelerates the pace of industrialization.

Literature. The current paper is related to the macroeconomic literature on the economic demography and structural transformation. Most of the literature studies these two sub-disciplines independently.[2] The current paper is mostly related to Greenwood and Seshadri (2002), Doepke (2004) and Bar and Leukhina (2010). Doepke (2004) and Bar and Leukhina (2010) are extensions of Hansen and Prescott (2002) and endogenize fertility choice. Doepke (2004) uses a one-sector model with time and goods costs to explain how the abolition of the child labor law affects the return to having an extra child as well as its implication on fertility. Bar and Leukhina (2010) find that mortality and time cost are important for understanding fertility decline. Greenwood and Seshadri (2002) explore how technological progress leads to declines in fertility and the rural population share. On the empirical side, notable work includes a series of works by Franck and Galor (2015, 2017*a*,*b*), who provide causal evidence on how industrialization promotes human capital accumulation and fertility decline.[3]

This paper contrasts with these previous works in at least two areas. First, most of the previous literature feature only a single fertility rate. As sectoral fertility rates exhibit a large degree of heterogeneity (Guinnane, 2011), a two-sector fertility model is needed. Second, the effect of differential schooling opportunities, which is crucial to understand the differential sectoral fertility behaviors and structural transformation, is not studied in these models.[4]

[2] For example, Becker, Murphy and Tamura (1990), Galor and Weil (2000) Kalemli-Ozcan (2002), Manuelli and Seshadri (2009) and Mookherjee, Prina and Ray (2012) generate a negative relation between the fertility rate and human capital, but they are silent about the structural transformation, in which laborers move from agricultural to non-agricultural sector. Galor (2005, 2011) summarize the relation between human capital and fertility. Caselli and Coleman (2001) use costly skill acquisition as a barrier to structural transformation, but the model does not consider fertility choice. See Herrendorf, Rogerson and Valentinyi (2014) for an extensive review on recent developments in structural transformation.

[3] Galor and Mountford (2008) discuss how trade changes the demand for human capital and its effects on income and population distribution.

[4] In Greenwood and Seshadri (2002), for example, relative educational costs are the same for all agents: whenever it is optimal for the skilled agents to have skilled children, it is also optimal for the unskilled agents to have skilled children and vice versa. Therefore, the sectoral fertility rates comove in their model. One of the main reasons is due to a data limitation, so they use the number of children under 5 years old to woman ratio as a proxy for the fertility rate. Such a measure does not show a substantial difference in sectoral fertility rates. Hence, their model does not allow an understanding of the observed difference in the trends of sectoral fertility rates, wherein the timing of the fertility decline is different in different sectors.

1 Empirical Evidence

The Progressive Era, spanning from the 1890s to the 1920s, that saw a rapid decline in the fertility rate, an increase in schooling and a structural transformation is the focus of this study. We first explore the patterns of differential sectoral fertility rates and their implications. Then we discuss the plausible cause of such a difference.

1.1 Differential Sectoral Fertility

The fertility rate in the agricultural sector was different from that in the non-agricultural sector in two aspects, as shown in Figure 2. First, the fertility rate in the agricultural sector was higher than that in the non-agricultural sector. The gap was substantial: in the year 1890, for example, an average family in the agricultural sector had 5.5 children, which was 46% more births than for females in the non-agricultural sector.[5]

Assuming the sectoral fertility rates are constant in their 1880 level, a change in composition of the economy to the level in the 1930s (structural change) results in a 21% decline in the aggregate fertility rate from 1880 to 1930. More than 79% of the decline in the aggregate fertility, however, results from the reduction in sectoral fertility rates. Such a decrease in sectoral fertility rates is asymmetric: the second feature from Figure 2 is that, although the cohort fertility rates in both sectors declined, the decrease in the agricultural sector was much later and sharper than the decrease in the non-agricultural sector. These declines may primarily be due to the increase in emphasis on education.

1.2 Alternative Explanations

In this subsection, we discuss the reason why the observed pattern of agricultural and non-agricultural fertility rates behaved so differently cannot be purely due to the decline in mortality rates, the increase in female labor participation and delayed marriage.[6]

[5] Readers may be concerned that the observed gap was due to the correlation between agricultural status and other observables (such as lower literacy and income). After controlling for different variables (such as literacy, income and geographic location), women, or their spouses, working in the agricultural sector still have a higher birth rate than those in other sectors. Other robustness checks consider the fact that females switch industries, reporting errors in occupation and a finer definition of industries. All of the cases result in a qualitatively similar picture. Results can be provided upon request.

[6] However, the discussion in this subsection does not preclude the fact that these changes lead to the change in the aggregate fertility. It simply cannot generate enough of a relative decline in the agricultural fertility rate compared

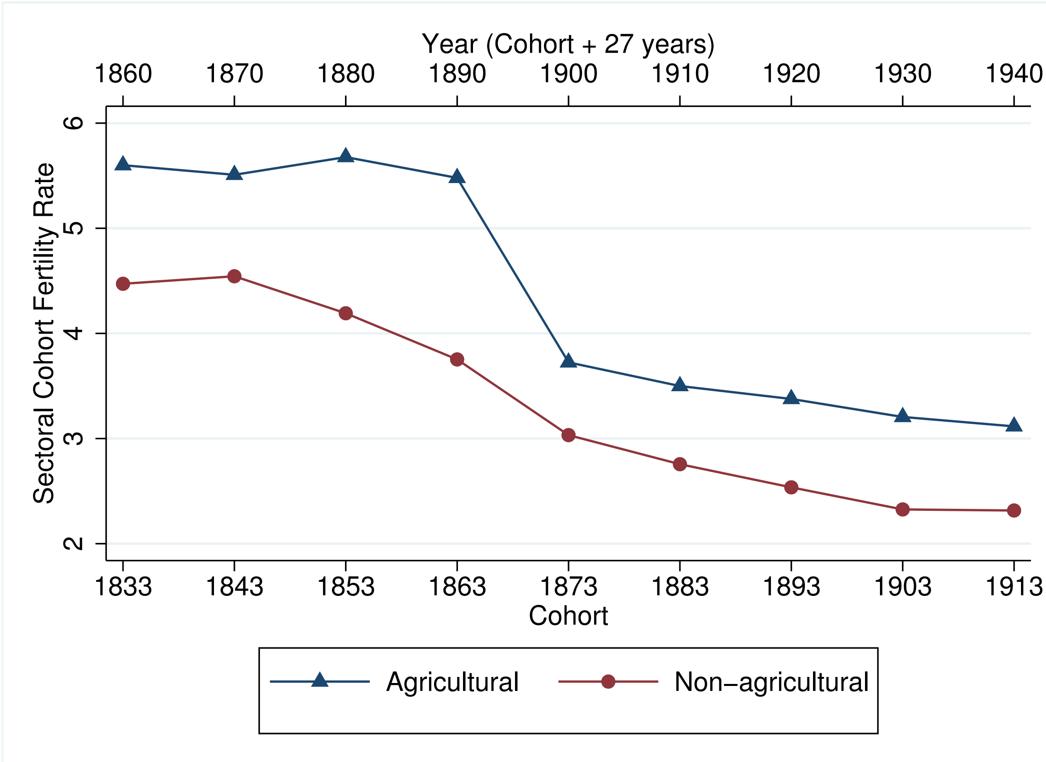

Figure 2: Sectoral Fertility Rates in the U.S. IPUMS (Ruggles et al., 2018) various years.

Mortality Rate. The mortality rate in the rural U.S. was lower than in urban areas because of sparsely populated settlements (Klein, 2012). Clark and Cummins (2015) have a similar observation for England. Calculation from the IPUMS (Ruggles et al., 2018) shows that average mortality rates at the turn of the 20th century were approximately 22% in the agricultural sector and 25% in the non-agricultural sector. So, if the mortality rate is to be positively correlated with fertility rate, we should expect the agricultural sector to have lower level of fertility.

In addition, Albanesi and Olivetti (2014) argue no systematic differences in maternal mortality for urban and rural women. They also find that maternal mortality rates in Western countries only started to decline after the late 1930s. Last but not least, Jones and Tertilt (2006) find that both children born and surviving children born declined for females born between the 1860s and the 1880s, indicating mortality was not the main driving force for the fertility decline in this historical episode. IPUMS data indicate the child mortality rate in the agricultural sector declined from 24% in 1880 to 19% in 1895, while in the non-agricultural sector it decreased by a similar percentage

to the non-agricultural one.

from 27% to 23%.

Female Labor Participation. I use the census data between 1870 and 1960 to investigate the labor market status of females between 15 to 60 years of age. I use two definitions of labor status in IPUMS. The first is the Labor Market Status and the second is to count whether the agent reports market activities by reporting occupations. The two measures are similar for rural and urban females. I found that the female labor participation rate only started to increase substantially after the year 1930, very much aligning with the narrative in Goldin (1990).

Marriage and Birth Patterns. In the early twentieth century, the age at first marriage and the age of first birth were strongly correlated. The average age at first marriage remained very stable at approximately 22.5 years old from 1870 to 1910 cohorts (Bailey, Guldi and Hershbein, 2013). When further restrict our analysis to the white non-Hispanic native born female, IPUMS data indicate that the age at first marriage of the females born in 1860-64 and 1876-80 were both around 23.5 years old. Due to the absence of legal contraceptive methods, the reduction in fertility was mainly due to spacing and stopping (Bailey and Hershbein, 2018).

This phenomenon has an important implication for our analysis: the increase in schooling did not change the nuptial pattern or the age of first birth. Instead, other measures were taken willingly to reduce the number of children.

1.3 Education

At the turn of the twentieth century, as the economy grew faster, more non-agricultural jobs became available. Goldin and Katz (2009) portray an economy demanding more educated workers some decades after the Civil War. This change created a *demand for skills*. On the supply side, Caselli and Coleman (2001, Section III) provide rich evidence to support the argument that educated children joined more skilled non-agricultural occupations through skill acquisition. The modernization of the educational system and the lowering of the educational costs induced parents to put more emphasis on education of the children. The full-time school attendance rate increased from 62% in the years 1870-80 to approximately 90% in 1930-40 (Figure 3, Left Panel).

For various reasons, children in rural areas were less likely to receive full-time schooling when compared to those in urban areas. Goldin and Katz (2009, p. 71) point out that geographical proximity was a determinant for schooling before the year 1900. Using historical data in the year

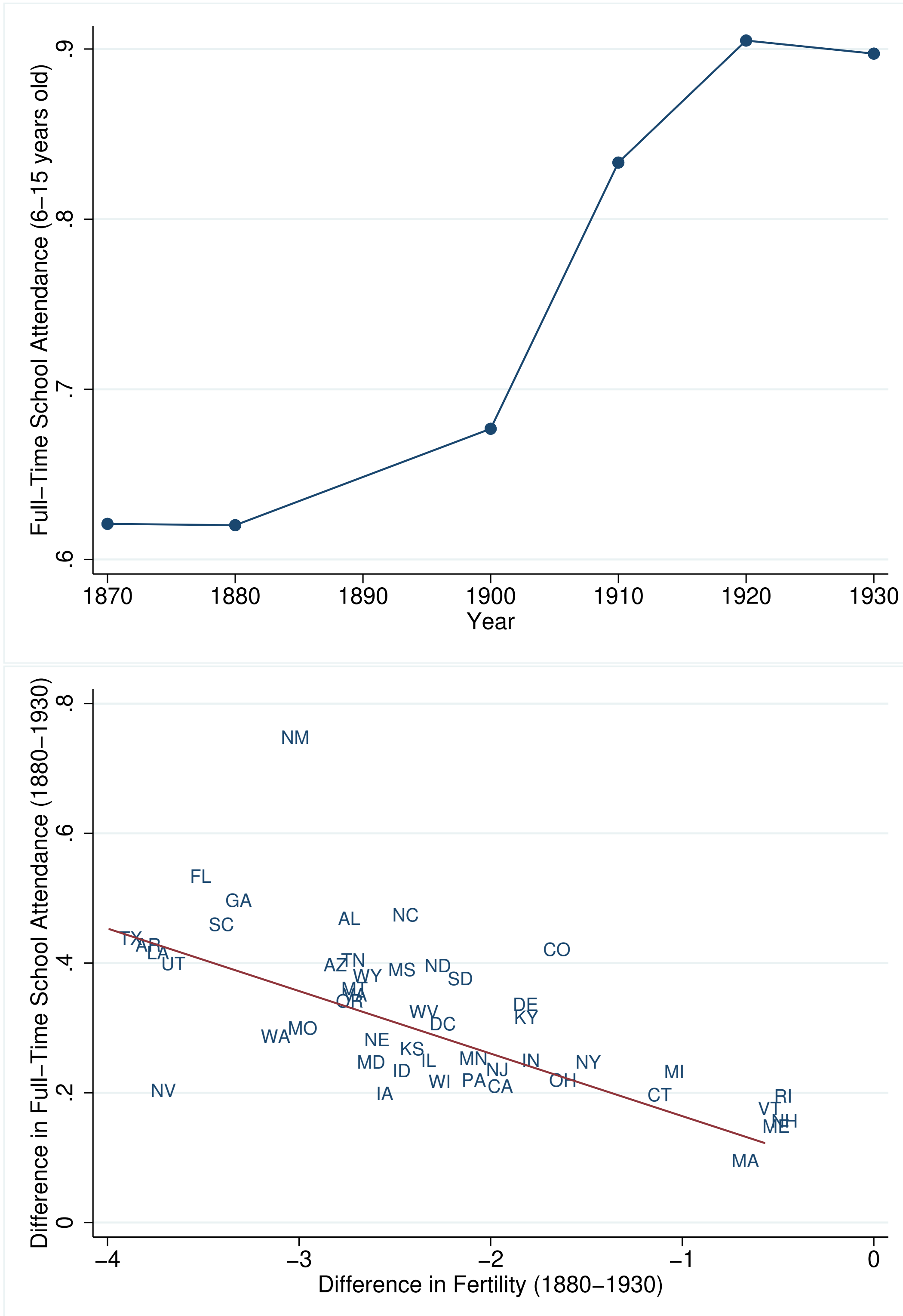

Figure 3: Left: School Attendance of 6-15 years old children who were not in the labor force; Right: Difference in Fertility Rate and Difference in Education Attainment (1880-1930). IPUMS (Ruggles et al., 2018) various years.

1880, I find that states with 1 unit less in urban population share were associated with 0.3 fewer schools per square miles and 1 less school per square miles is translated to 0.19 percentage point reduction in full-time school attendance for those who were between 6 to 15 years of age.[7]

Long and Ferrie (2013) shows that approximately 65% of farmers' sons ended up being farmers in the late 19th century. Schultz (1964) points out, when (agricultural) technological progress is slow, the children of farmers will only need to learn from their parents to accumulate the field-specific knowledge needed to work in the fields.[8] The lack of schooling in the rural area has also been aggravated by higher direct costs relative to income because less public fund per capita has been directed into the rural area (Go and Lindert, 2010). Goldin and Katz (2009, p. 158) find that the full-time schooling fractions are lower in the areas where agricultural employment was greater. From the calculation using IPUMS data (Ruggles et al., 2018), it had been 16.6% less likely for the children between 6 to 15 years of age with farmer parents to receive full-time schooling. This was the rural education problem (Steffes, 2008).

At the turn of the twentieth century, governments in different localities stepped in to solve the rural education problem (Steffes, 2008). As discussed in Caselli and Coleman (2001, Section III), the advancement of modern transportation technology complemented by road paving increased the quality of education and changed school curricula, making schooling more accessible to lower income children and even those living in rural areas. In addition to their arguments, the implementation of compulsory schooling and the abolishment of child labor also played a key role in understanding the increase in schooling. Grouping states by the status of compulsory schooling legislation shows that school attendance for states with compulsory schooling is 75%, which is higher than in the states without legislation (average 59%). I follow Steffes (2008) and refer to this change in educational policy that particularly benefited those in rural areas as the *Rural Education Reform*. It became much more likely for agricultural descendants to receive an education. It was only 6.1% less likely for children between 6 to 15 years old with parents working in the agricultural sector to receive full-time schooling when compared to their non-agricultural counterparts in 1930-40.[9]

[7] Information on school density is defined as number of schools in unit land area. Both are from Report of the Commissioner of Education (United States Office of Education, 1880). I restrict the data on the regions with land area larger than 5000 square miles in the year 1880. So, Washington D.C. and Rhode Island are excluded from the calculation. Both of the statistics are significant in 10% confidence interval.

[8] Laband and Lentz (1983) found empirical evidence that farmers' followers (e.g., sons of farmers) earn 110% more than non-followers.

[9] This observation is robust in age. For example, if the 6-18 years old children are chosen, the disadvantage of the

2 Model

In this section, I build a model that features the joint determination of sectoral fertility rates and structural transformation. Because many factors changed during our study period, not only would a model be useful to understand the interaction of these factors but also would it enable us to single out the relative contribution of each factor.

The economy consists of two sectors (i): the agricultural and the non-agricultural (or manufacturing) sectors ($i = a, m$). These two sectors differ in two dimensions. First, on the supply side, the non-agricultural sector is more skill-intensive. Second, on the demand side, the income elasticity of agricultural goods is less than unity. Two types of agents (j) occur in the economy: skilled (s) and unskilled (u). Therefore, the model is a two-sector-two-skill model. The main reason for such a model is the restriction on detailed data available to map the model.[10]

The economy is populated with N_d working adults, and among them, N_s are skilled.[11] I define the endogenous share of unskilled labor allocated to the agricultural sector as ξ^a, so that we have:

$$\begin{aligned} N_a &= \xi^a (N_d - N_s) \\ N_m &= (1 - \xi^a)(N_d - N_s) \end{aligned}$$

where N_a and N_m are the endogenous number of agents in the agricultural sector (farmers) and unskilled workers in the manufacturing sectors, respectively, or we can define η^s as skill endowment, so $\eta^s = \frac{N_s}{N_d}$. I also denote η^{ij} as the endogenous share of agents in sector i with skill type j.

2.1 Production

The production in the agricultural sector (Y_a) requires only unskilled labor (L^{au}). This result is consistent with the fact documented by Caselli and Coleman (2001) that shows the percentage of workers who received only a primary education was substantially higher in the agricultural sector

farmer descendants changed from -14.4% in the years 1870-80 to -6.6% in the years 1930-40.

[10]The model here can be easily extended into a general model with I sectors (goods) and J skill levels. The main idea, however, is qualitatively the same as this simplified model presented.

[11]I use adults and agents interchangeably, as adults are the only agents that make economic decisions. However, they differ from workers or labors because the labor supply decision also considers the time cost of child-rearing.

when compared to non-agricultural sector. Thus, the agricultural output (Y_a) is given as:

$$Y^a = AL^{au}$$

The non-agricultural output (Y_m) requires a higher skill intensity:

$$Y^m = A\zeta_m \left(L^{mu}\right)^{\alpha} \left(L^{ms}\right)^{1-\alpha}$$

where A is the economy-wide productivity and ζ_m measures the productivity between non-agricultural and agricultural production.[12] The skill intensity in the non-agricultural sector is higher. Therefore, when the skill intensity in the economy increases, the non-agricultural sector will expand relative to the agricultural sector.

I refer to agricultural agents as farmers in situations without ambiguity and refer to non-agricultural skilled and unskilled agents as skilled agents and unskilled agents, respectively. Although capital is abstracted from the model, the introduction of capital does not change the qualitative result in this paper. In the calibration exercise, the sectoral level difference in capital will be absorbed by the sectoral productivity difference. I denote l^{ij} as the extensive form notation of labor in sector i with skill type j, and y_i as the output per adults. Let w^s and w^u be the skilled and unskilled wage, respectively. The representative firms in agricultural and non-agricultural sectors solve the following maximization problems. Manufacturing goods are treated as numeraire, and p is the price of agricultural goods relative to manufacturing goods.

$$\max_{l^{au}} py_a - w^{au}l^{au} \tag{1}$$

$$\max_{l^{au},l^{ms}} y_m - w^{mu}l^{mu} - w^{ms}l^{ms} \tag{2}$$

The wage premium is given by $\frac{w^s}{w^u} = \frac{1-\alpha}{\alpha}\left(\frac{l^{mu}}{l^{ms}}\right)$, which is high when the relative supply of skilled labor is lower – a downward-sloping demand function. Throughout the exposition, I require $\frac{w_s}{w_u}$ to be larger than unity so that agents have an incentive to educate their children. This result is consistent with Goldin and Margo (1992) and Goldin and Katz (2009): even when wage compression appeared in the early 20th century, the wage rate of the skilled workers was still higher than that of

[12] The relative efficiency ζ_m measures the productivity difference between the two production technologies. The higher productivity difference may capture the fact that the non-agricultural sector is more capital intensive.

the unskilled agents. Note that the skill intensity η^s in the economy is always weakly higher than that of the skilled agents (i.e. $\eta^{ms} \leq \eta^s$), and the equality holds whenever the wage premium is as high as the unity.

2.2 Preference and Endowment

All agents live for two periods: childhood and adulthood. Adults are endowed with 1 unit of time that can be allocated between work and child-rearing. Only adults make relevant economic decisions. In particular, the adults make consumption, fertility and child education decisions. The utility of an adult of skill i works in sector j with c_a^{ij} units of agricultural consumption, c_m^{ij} units of non-agricultural consumption, $n^{ij,s}$ skilled children and $n^{ij,s}$ unskilled children is given as:

$$\frac{\nu\left[(c_a^{ij}-\bar{c})\right]^{1-\sigma}}{1-\sigma}+\frac{(1-\nu)\left(c_m^{ij}\right)^{1-\sigma}}{1-\sigma}+\frac{\psi\left[n^{ij,s}V(w'^s,i')+n^{ij,u}V(w'^u,i')\right]}{(n^{ij,s}+n^{ij,u})^{\varepsilon}}$$

where $\bar{c}$ is the subsistence level and is only defined in agricultural goods; ψ is the coefficient of altruism, and ε is the curvature for the total number of children – we follow the literature and assume that $0 < \varepsilon < 1$. In other word, the more children an adult already has, the smaller is the additional utility from another child. Finally, $V(w'^s, i') > 0$ and $V(w'^u, i') > 0$ are the utility that skilled children and unskilled children will enjoy as adults working in sector i, respectively.[13] The utility function is otherwise standard when compared to the joint works of Barro-Becker (Becker and Barro (1988), and Barro and Becker (1989)) and a number of models following Doepke (2004), the only difference is the introduction of two sectoral goods in view of the fact that sectoral fertility rates are very different.

Because only two skill types and laborers are mobile across sectors, the (potential) income of an adult takes two values: w^u and w^s. The effective cost to raise a child ($\widetilde{\tau}^{ij,k}$) depends on the skill type of the child k. If an agent chooses not to educate her children (i.e. $k = u$), child-rearing only incurs time cost (τ_u). Otherwise, raising a child requires a time cost (τ_s) and an educational cost γ_i in terms of non-agricultural goods, which takes two values. Notice that I assume $\tau_s > \tau_u$ to capture the fact that raising a skilled child requires a higher time cost. It also captures the fact that some

[13]The utility for agents in different sectors i are potentially different because they face different cost. However, as labors are mobile and equilibrium migration cost is introduced, the utility will be independent of sectors in equilibrium analysis.

of the child-rearing cost is subsidized by the child working in the fields or helping with household chores should the child not be required to attend schools. Therefore, if laws were enforced to reduce the extent of child labor, we should expect such a time cost to increase. When the agent was in the agricultural sector, the educational cost was $\gamma_i = \gamma_a = \overline{\gamma}$. Otherwise, $\gamma_i = \gamma_m = \underline{\gamma}$.

In Section I, we discussed that it is more costly or less beneficial for children in rural areas to attend school before the educational reform. Therefore, I assume that $\overline{\gamma} > \underline{\gamma}$, meaning that the schooling cost in the agricultural sector is higher than in the non-agricultural sectors. This assumption can also be thought of as a distortion of the educational cost in the agricultural sector. During and after the reform, measures were taken to help equalize the schooling opportunities (such as advancement in transportation systems and improvement of rural schools). I assume in the model that the modernization of the educational system is captured by the elimination of cost distortion (i.e. $\overline{\gamma} = \underline{\gamma}$). More discussion on modeling educational reforms are discussed in Section II.D. The overall cost of having a type k child $\left(\widetilde{\tau}^{ij,k}\right)$ is:

$$\widetilde{\tau}^{ij,k} = \begin{cases} \widetilde{\tau}^{ij,s} = \tau_s w^j + \left[\overline{\gamma}\Gamma_{i=a} + \underline{\gamma}(1 - \Gamma_{i=a})\right] \\ \widetilde{\tau}^{ij,u} = \tau_u w^j \end{cases}$$

where $\Gamma_{i=a}$ is an indicator function equal to 1 if the agent is in the agricultural sector $(i = a)$ and 0 otherwise. The budget constraint is:

$$pc_a^{ij} + c_m^{ij} + \textstyle\sum_{k=u,s} \widetilde{\tau}^{ij,k} n^{ij,k} = w^j \tag{3}$$

The household problem of the agent is given by:

$$V(w^j, i) = \max_{c_a^{ij}, c_m^{ij}, n^{ij,s}, n^{ij,u}} \left\{ \begin{array}{c} \frac{\nu\left[(c_a^{ij} - \bar{c})\right]^{1-\sigma}}{1-\sigma} + \frac{(1-\nu)\left(c_m^{ij}\right)^{1-\sigma}}{1-\sigma} \\ + \frac{\psi}{(n^{ij,s} + n^{ij,u})^{\varepsilon}} \sum_{k=u,s} n^{ij,k} V(w'^k, i') \end{array} \right\} \tag{4}$$

subject to (3)

The above problem can be simplified by a few steps. I state the proposition first, and the mathematical details are relegated to the appendix.

PROPOSITION 1 *The household problem to maximize* (4) *subject to* (3) *can be written as:*

$$V(w^j, i) = \max_{E^{i,j},k} \left\{ \Omega \frac{\left(w^j - E^{i,j} - p\bar{c}\right)^{1-\sigma}}{1-\sigma} + \psi \left(E^{i,j}\right)^{1-\varepsilon} \frac{V(w'^k, i')}{\left(\widetilde{\tau}^{i,jk}\right)^{1-\varepsilon}} \right\}$$

where $\Omega = \left[\nu^{\frac{1}{\sigma}} \left(\frac{1}{p}\right)^{\frac{1-\sigma}{\sigma}} + (1-\nu)^{\frac{1}{\sigma}}\right]^{\sigma}$ and $E^{ij} = \sum_{k=u,s} \widetilde{\tau}^{ij,k} n^{ij,k}$

PROOF: *See Appendix.*

The proposition can be proven in two steps. To have some insights about the properties of the model, the details are summarized below.

Firstly, two-sector consumptions and the fertility choice problem can be reduced into a problem choosing the consumption composite and the allocation of child-rearing expenditure.[14] This calculation simplifies the problem by reducing a two-sector model to an effectively one-sector model with only one consumption index, $C^{ij} = pc_a^{ij} + c_m^{ij}$. After the consumption is determined, the child-rearing expenditure E^{ij} is narrowed by the budget constraint (3). The child-rearing expenditure E^{ij} is split into expenditures on skilled and unskilled children.

Note that we can express agricultural and non-agricultural consumption expenditure in terms of the consumption index C^{ij}:

$$pc_a^{ij} = \frac{C^{ij}}{1 + p^{\frac{1-\sigma}{\sigma}} \left(\frac{1-\nu}{\nu}\right)^{\frac{1}{\sigma}}} + p\bar{c} \left[1 - \frac{1}{1 + p^{\frac{1-\sigma}{\sigma}} \left(\frac{1-\nu}{\nu}\right)^{\frac{1}{\sigma}}}\right]$$

and

$$c_m^{ij} = p^{\frac{1-\sigma}{\sigma}} \left(\frac{1-\nu}{\nu}\right)^{\frac{1}{\sigma}} \left[\frac{C^{ij} - p\bar{c}}{1 + p^{\frac{1-\sigma}{\sigma}} \left(\frac{1-\nu}{\nu}\right)^{\frac{1}{\sigma}}}\right]$$

In the agricultural sector, pc_a^{ij} exhibits a non-linear income effect for positive price and positive subsistence level $\bar{c} > 0$. Specifically, when compared to non-agricultural goods, the *relative* expenditure share in agricultural consumption decreases as income increases. However, when $\bar{c} = 0$, the relative expenditure share between the two sectors is constant as income increases.

Secondly, define f^{ij} as the child-rearing expenditure share on skilled children. Then, we have $n^{ij,u} = \frac{E^{ij}(1-f^{ij})}{\widetilde{\tau}^{ij,u}}$ and $n^{ij,s} = \frac{E^{ij} f^{ij}}{\widetilde{\tau}^{ij,u}}$. Substituting the two expressions into the utility function (4).

[14] This characteristic is not uncommon. For example, Voigtländer and Voth (2013) shows that a class of Stone-Geary preference can be reduced to an indirect utility depending only on one's disposable income. In our case, disposable income is the potential wage net of child-rearing cost.

Focusing on the terms regarding fertility choice yields the following expression:

$$\left(E^{i,j}\right)^{1-\varepsilon}\frac{\psi}{\left(\frac{f^{ij}}{\widetilde{\tau}^{ij,s}}+\frac{1-f^{ij}}{\widetilde{\tau}^{ij,u}}\right)^{\varepsilon}}\left[\frac{f^{ij}}{\widetilde{\tau}^{ij,s}}V(w'^{s},i')+\frac{1-f^{ij}}{\widetilde{\tau}^{ij,u}}V(w'^{u},i')\right]$$

Note that without the term $\frac{\psi}{(\frac{f^{ij}}{\widetilde{\tau}^{ij,s}}+\frac{1-f^{ij}}{\widetilde{\tau}^{ij,u}})^{\varepsilon}}$, the expression above is linear in f^{ij}. When the term is presented, and given that $\varepsilon > 0$, the value function is convex in f^{ij}, and the agent has only one type of child in optimization.[15]

Note that agents only choose child-rearing expenditures E^{ij} and the type of children k to maximize their value. Moreover, the types of children only enters the parent's utility through the discounted future utility $\frac{V(w'^{k},i')}{\left(\widetilde{\tau}^{i,jk}\right)^{1-\varepsilon}}$. This calculation implies that the agent compares the discounted future values of both types of children to make a choice between educating their children or not (i.e., a quality-quantity trade-off decision).

2.3 Equilibrium

In what follows, I focus on the stationary competitive general equilibrium, which is defined as follows.

DEFINITION 1 *The stationary competitive equilibrium is a vector of price* $\left\{p, w^{j}\right\}$, *an individual allocation* $\left\{c_{a}^{ij}, c_{m}^{ij}, n^{ij,k}\right\}$, *an aggregate allocation* $\left\{\eta^{ij}, \lambda^{ij,k}, y^{i}\right\}$, *a set of value functions* $V(w^{u}, a)$, $V(w^{u}, m)$ $V(w^{s}, m)$ *and migration cost* B *such that given productivity* A, ζ_{m} *and initial skill endowment* η_{0}^{s} , *the following conditions hold:*

1. *Given the price vector, the firms maximize profit* (1) *and* (2)
2. *Given the price vector, the agents maximize utility* (4) *subject to* (3)
3. *The labor markets for sector* i *and agent* j *clear:*

$$l^{ij}=\eta^{ij}\left(1-\sum\nolimits_{k}\lambda^{ij,k}\widetilde{\tau}^{ij,k}n^{ij,k}\right), \forall i,j$$

[15] Because the preference is similar to Doepke (2004), the property of corner solution is also inherent from such a preference assumption. This, however, is only an individual behavior and does not preclude the economy to have a distribution of skill even within the same sector-skill pair.

4. The goods markets in sector $i \in \{a, m\}$ clear:

$$
\begin{aligned}
y^a &= \sum\nolimits_{ij} \eta^{ij} c_a^{ij} \\
y^m &= \sum\nolimits_{ij} \eta^{ij} c_m^{ij} + \sum\nolimits_{ij} \eta^{ij} \lambda^{ij,s} n^{ij,s} \gamma_i
\end{aligned}
$$

5. The equilibrium skill intensity η^s is time invariant

$$
\eta^s = \frac{\sum_{ij} \eta^{ij} \lambda^{ij,s} n^{ij,s}}{\sum_{ij} \eta^{ij} \lambda^{ij,s} n^{ij,s} + \sum_{ij} \eta^{ij} \lambda^{ij,u} n^{ij,u}}
$$

6. The value for unskilled agents equalized up to a migration cost B

The migration cost absorbs the necessary educational cost difference between agricultural and non-agricultural sectors, and the wages of unskilled workers are the same due to free mobility. This implies the values of unskilled agents are the same. Thus, the following corollary follows directly from Proposition 1.

COROLLARY 1 *The agent is indifferent between either type of child if and only if the following equation holds:*

$$
\frac{V(w^s)}{(\widetilde{\tau}^{ij,s})^{1-\varepsilon}} = \frac{V(w^u)}{(\widetilde{\tau}^{ij,u})^{1-\varepsilon}} \tag{5}
$$

The left hand side of (5) is the value of having a skilled child discounted by the price of raising one, while that on the right hand side is the discounted value of an unskilled child. I denote the proportion of agents in sector i with skill level j who decided to have children k as $\lambda^{ij,k}$. If the LHS of (5) is larger than the RHS, the agent prefers to have skilled children ($\lambda^{ij,s} = 1$ and $\lambda^{ij,u} = 0$). Otherwise, the agent prefers unskilled children ($\lambda^{ij,s} = 0$ and $\lambda^{ij,u} = 1$). If the two expressions are equal, individuals are indifferent between the type of child ($0 \leq \lambda^{ij,k} \leq 1$). Although not every household considers the mixture of children, we still expect distribution of skill intensity across the economy in aggregate.

We explore the equilibrium characteristics in what follows. These characteristics give us an idea of how the model can be used to explain the sectoral difference in fertility rates. Also, I discuss the problem of equilibrium selection to see why some equilibria match the historical data better than the others. This discussion then guides our numerical exercise.

Sectoral Fertility Difference. The farmer has the highest relative cost of raising a skilled child before the educational reform. To see this fact, note that farmers face higher schooling costs (i.e., $\overline{\gamma} > \underline{\gamma}$), and the relative cost of having skilled children decreases with income (i.e. $\partial \frac{\tau_s w^j + \gamma^i}{\tau_u w^j} / \partial w^j < 0$). Also, notice that the quality-quantity equation (5) can be rewritten as $\frac{V(w^s)}{V(w^u)} \gtreqless \left(\frac{\tilde{\tau}^{ij,s}}{\tilde{\tau}^{ij,u}} \right)^{1-\varepsilon}$.

$$\underbrace{\frac{\tau_s w^s + \underline{\gamma}}{\tau_u w^s} < \frac{\tau_s w^u + \underline{\gamma}}{\tau_u w^u}}_{non-agricultural} < \underbrace{\frac{\tau_s w^u + \overline{\gamma}}{\tau_u w^u}}_{agricultural} \tag{6}$$

Specifically, this cost structure (6) suggests that agents with higher incomes and better access to education (i.e., skilled workers in urban areas) have a lower relative cost of educating their children. On the opposite end of the spectrum, farmers have the highest relative cost to raise skilled children. As a result, similar to what we have observed, farmers would have a larger number of children with a lower average education.[16]

Fertility Trend. Fertility rates in the agricultural sector declined much later than fertility rates in the non-agricultural sector. By (5) and (6), it is found that when farmers (weakly) prefer skilled children over unskilled children $\frac{V(w^s)}{V(w^u)} \geq \left(\frac{\tau_s w^u + \underline{\gamma}}{\tau_u w^u} \right)^{1-\varepsilon}$, agents in non-agricultural sector strictly prefer to have skilled children over unskilled children $\left(\frac{\tau_s w^s + \underline{\gamma}}{\tau_u w^s} \right)^{1-\varepsilon} < \left(\frac{\tau_s w^u + \underline{\gamma}}{\tau_u w^u} \right)^{1-\varepsilon} < \frac{V(w^s)}{V(w^u)}$. The converse, however, is not true.

So the model captures the fact that skilled agents start to trade-off the number (quantity) of their children for quality, followed by middle class agents and then by farmers. This phenomenon explains the observed data showing that agents in non-agricultural sectors started to experience a fertility decline before farmers did. It is also true historically that we usually observe skilled agents or higher social class workers starting to limit births before lower-class workers (see Clark and Cummins 2015 for the case in England in the late 18th century). Therefore, analytically, our model generates the level and trend differences observed in data.

Equilibrium Types. Given equations (5) and (6), and the fact that the value function of agents is uniquely defined, at most one type of agent is indifferent between types of children. For example, if unskilled agents are indifferent between either type of child (i.e., $\frac{V(w^s)}{V(w^u)} = \left(\frac{\tau_s w^u + \underline{\gamma}}{\tau_u w^u} \right)^{1-\varepsilon}$, which corresponds to Equilibrium Type III in Table 1), skilled agents must strictly prefer to have skilled

[16]Development economists use cross-country evidence to show that urban proximity increases school attendance, while children who live far from urban areas tend to work more and go to school less (see, e.g., Fafchamps and Wahba, 2006).

children while farmers must strictly prefer to have unskilled children. Moreover, neither $\frac{V(w^s)}{V(w^u)} < \left(\frac{\tau_s w^s + \underline{\gamma}}{\tau_u w^s}\right)^{1-\varepsilon}$ nor $\frac{V(w^s)}{V(w^u)} > \left(\frac{\tau_s w^u + \overline{\gamma}}{\tau_u w^u}\right)^{1-\varepsilon}$ is admissible, as neither skilled nor unskilled workers occur in the next period and cannot be in equilibrium. Thus, five equilibrium types are possible as shown in Table 1.

Table 1: Possible Equilibrium Types

Type I	Type II	Type III	Type IV	Type V
$\frac{V(w^s)}{V(w^u)} = SC$	$SC < \frac{V(w^s)}{V(w^u)} < UC$	$\frac{V(w^s)}{V(w^u)} = UC$	$UC < \frac{V(w^s)}{V(w^u)} < FC$	$\frac{V(w^s)}{V(w^u)} = FC$

Table 2: Denote skilled agent's relative cost, $SC = \left(\frac{\tau_s w^s + \underline{\gamma}}{\tau_u w^s}\right)^{1-\varepsilon}$; unskilled agent's relative cost $UC = \left(\frac{\tau_s w^u + \underline{\gamma}}{\tau_u w^u}\right)^{1-\varepsilon}$; and farmer's relative cost $FC = \left(\frac{\tau_s w^u + \overline{\gamma}}{\tau_u w^u}\right)^{1-\varepsilon}$

We would see in what follows that some equilibria are more plausible than the others. First, if Equilibrium Type I is a stationary equilibrium, skilled agents (who are the minority of the economy) need to give birth to a sufficiently large number of children to sustain the stationary equilibrium. Otherwise, the skill intensity of the economy decreases, eventually hitting the 0 lower bound. In the historical content, however, as discussed in Clark and Cummins (2015), the case is usually the opposite. Therefore, I omit Equilibrium Type I from our analysis because it cannot be used to fit the historical data.

Equilibrium Types II and IV are knife-edge equilibria, in which the probability of attainment is zero. To see this fact, note that when one type of individual j in sector i is indifferent between either type of child, we have $0 \leq \lambda^{ij,k} \leq 1$. In such a case, given sectoral fertility rates, the proportion $\lambda^{ij,k}$ adjusts to sustain the invariance of the skill ratio η^s. However, in the situation with no indifferent agents, $\lambda^{ij,k}$ is either 0 or 1. Therefore, the ratio of equilibrium fertility rates generated from the household maximization problem needs to be *exactly* those required to make the skill distribution time invariant. Given that the fertility decision of agents derives from the utility maximization problem, such coordination seems unlikely.

2.4 Modeling Rural Education Reform

The Rural Education Reform aimed to create an equal educational opportunity for children as discussed in Section I Steffes (2008). In modeling, I take it seriously and assume the educational cost distortion in the agricultural sector is eliminated $\overline{\gamma} = \underline{\gamma}$. Therefore, all the agents in the economy face the same *absolute* educational cost. This is a strong assumption. However, it is necessary because this model features only two skill levels. To see this, consider the cost structure (6) for unskilled agents and farmers, and assume that the unskilled agents are originally indifferent as to whether or not to educate their children:

$$\frac{V(w^s)}{V(w^u)} = \frac{\tau_s w^u + \underline{\gamma}}{\tau_u w^u} < \frac{\tau_s w^u + \overline{\gamma}}{\tau_u w^u}$$

When educational reform is not drastic, which means the new agricultural educational cost $\overline{\gamma}\prime$ satisfies $\overline{\gamma}\prime > \underline{\gamma}$, farmers would still strictly prefer to have unskilled children. Notice that if the choice of skill level is continuous, a small reduction in educational cost would lead to an increase in educational attainment (e.g. Caselli and Coleman, 2001). However, we do not have sector-specific educational data necessary to determine the sector-specific educational cost.

Although the assumption may seem stringent, it is not implausible when we consider education as a form of lump-sum investment. Using the 1915 Iowa State Census, Goldin and Katz (2009, Table 5.2) show that 72% of workers 25 to 34 years old in the professional and managerial category had some high school training and, on average, they had 12 years of schooling. Even among sales and clerical workers, 62% received some high school education. Only 13% of white collar workers without any years of high school were employed. Thus, a certain threshold of education is needed (in their case, high-school education) for the workers to join the non-agricultural sector. In our model, because the only reason for education is to enter into the non-agricultural sector, a slight reduction in educational costs might not result in a drastic increase in schooling to, say, high school.

In the educational reform, as $\overline{\gamma} = \underline{\gamma}$, Equilibrium Types III through V collapse into a single equilibrium type because $\frac{\tau_s w^u + \underline{\gamma}}{\tau_u w^u} = \frac{\tau_s w^u + \overline{\gamma}}{\tau_u w^u}$. This calculation means that the relative costs of having skilled children are equalized for unskilled agents and farmers despite their industries.

In pre-reform Equilibrium Type V, unskilled agents in the non-agricultural sector strictly prefer skilled to unskilled children $\left(\lambda^{mu,s} = 1 \text{ if } \overline{\gamma} > \underline{\gamma}\right)$. After the Rural Education Reform, however, unskilled agents and farmers become indifferent between skilled and unskilled children $\big(0 \leq \lambda^{mu,s} = \lambda^{au,s} \leq 1$ if $\overline{\gamma} = \underline{\gamma}\big)$

Thus, $\lambda^{mu,s}$ is non-increasing in the Rural Education Reform. This result is not supported by the data because all the major occupations increased their children's school attendance at the turn of the 20th century. Therefore, I do not consider Equilibrium Type V in the numerical analysis.[17]

I define Equilibrium Type III and equilibrium after the Rural Education Reform as Regimes I and II. Regime I is the most plausible equilibrium type to have existed before the Rural Education Reform because we clearly observe different occupational mobility rates among each group of agents. According to Long and Ferrie (2013), in the late 19th century, approximately 70% of farmers' sons worked as unskilled workers and farmers. Only 50% and 30% of the sons of unskilled workers and skilled workers, respectively, ended up being unskilled workers and farmers. The model can qualitatively match this calculation. More of the discussion on occupational mobility in Section V.

During the transition from Regime I to Regime II, the aggregate fertility rate declines. The agricultural fertility rate declines faster, and labor is reallocated from the agricultural sector into the non-agricultural sector. However, the reduction in educational costs facing agricultural agents does not cause a major change in the non-agricultural fertility rate. As a result, the gap in fertility rates between the agricultural and the non-agricultural sectors shrinks. When more agents choose to educate their children, skill intensity in the economy increased. Therefore, labor is reallocated to a more skill-intensive non-agricultural sector.

3 Numerical Analysis

3.1 Calibration

I calibrate the model to fit two steady states in 1880 and 1930. I take sectoral productivity levels, the agricultural employment share, the agricultural consumption share, sectoral fertility rates and school fee estimates in 1880 as the main data moments for calibration.[18] Although data on other moments are available, their measured values were imprecise or not exact. In this study, I do not

[17]In Equilibrium Type V, because the unskilled agents strictly prefer to have skilled children, numerical analysis shows that their fertility rates would be too low and educational attainment too high when compared to our data. So, such equilibrium is not selected by data as well.

[18]Sectoral fertility rates use married white non-Hispanic women who were born in the U.S. and were not resident in a group quarter at the time of interview. Doing so under-represents the agricultural employment share. Therefore, I use the agricultural employment share from Herrendorf, Rogerson and Valentinyi (2014) as a weight (together with sectoral fertility rates) to compute the aggregate fertility rate. The constructed aggregate fertility rate is consistent with that implied by the data. See Data Appendix.

arbitrarily pick a value within a certain imprecise data range as my data moment. Instead, I save the data to evaluate the performance of the model by showing that my model's prediction is within the possible range.

I pick 1880 as the initial value for calibration and assume that the economy starts in Regime I (i.e., farmers strictly prefer to have unskilled children). Eleven parameters must be calibrated: three technology parameters $\{A, \zeta_m, \alpha\}$, five preference parameters $\{\psi, \varepsilon, \nu, \sigma, \overline{c}\}$, two parameters governing fertility cost $\{\tau_s, \tau_u\}$ and one parameter governing educational cost $\{\underline{\gamma}\}$.[19] Five of them $\{\psi, \varepsilon, \sigma, \tau_s, \underline{\gamma}\}$ are normalized or can be found in the literature. The remaining six $\{A, \zeta_m, \alpha, \nu, \overline{c}, \tau_u\}$ are *jointly* determined to match six moments using data in the year 1880. Although all six parameters were chosen jointly, some parameters are more important for some particular moments.

Table 3: Summary of Parameter Value

Category	Parameter	Value	Description
Preference	ψ	0.132	Altruistic Coefficient
	ϵ	0.50	Curvature of Number of Children
	ν	0.37	Agricultural Expenditure Share
	σ	0.50	Curvature of Consumption
	$\bar{c}$	0.47	Subsistence Consumption
Technology	A	1.33	Farm Output per Labor
	ζ_m	6.23	Non-Farm Output per Labor
	α	0.735	Unskilled Income Share
Fertility Cost	τ_u	0.051	Unskilled Time Cost
	τ_s	0.15	Skilled Time Cost
Educational Cost	$\underline{\gamma}$	0.52	Non-agricultural Educational Cost

Preferences: $\{\psi, \varepsilon, \nu, \sigma, \overline{c}\}$. The model's preference side is similar to Doepke's model (Doepke, 2004), so I borrow his parameter values and set $\psi = 0.132$, $\sigma = 0.5$ and $\varepsilon = 0.5$. He picks these values to match the fertility differential between skilled and unskilled parents to have unskilled children and a total fertility rate in the U.S. Because Doepke (2004) presents a one-sector model, no information on the asymptotic sectoral expenditure shares ν or the subsistence level of agricultural consumption $\overline{c}$ are available. I choose the subsistence level of consumption $\overline{c}$ to be 0.47, so the agricultural employment share in the U.S. is 0.51. The choice of ν warrants more discussion because ν affects the reductions in both the agricultural consumption share and the employment share. When the

[19]Notice that as farmers strictly prefer to have unskilled children, the educational cost to farmers $\overline{\gamma}$ is irrelevant. I show that data imply $\overline{\gamma} > \underline{\gamma}$.

ν is lower, the decline is faster in both the agricultural consumption and the employment shares. However, if the ν is chosen to match the average decline in the agricultural employment share, the decline in the consumption share would be too large, and vice versa. One of the reasons for such tension can be international trade: the import of agricultural goods reduces the agricultural employment required but still enables the maintenance of high agricultural consumption. In view of this fact, because I want my model to explain the decline of the agricultural employment share, I choose the agricultural consumption weight ν to be 0.37 to match the average decline in agricultural consumption.

Technologies: $\{A, \zeta_m, \alpha\}$. The value of A is set to 1.33 to match the output per laborer in the farm sector in the first steady state. Then, ζ_m is calibrated to fit the non-farm output per laborer. The non-farm output per worker was 2.35 times the farm output per worker in 1880. Moreover, the relative price for non-agricultural goods is 1.19 times higher. Considering also the difference in number of workers, ζ_m is set to be 6.23. Thus, the non-agricultural output per worker was significantly higher. This result can be justified by the fact that the non-agricultural sector was skill-intensive and capital-intensive, and those two factors would be absorbed by ζ_m.

The value of α is difficult to determine. First, the real world has more than two skill levels, so the model concept is difficult to map onto data. Second, the definition of skill has changed drastically throughout our long period of study. For example, high school graduates were rare in the late 19th century and the number largely increased after the high school movement (1910-30). According to the U.S. Bureau of the Census (Series H 599), only 2% of 17-year-olds graduated from high school in 1880 while the number increased to 28.8% by 1930. In view of this fact, I use another definition of α for calibration. Because α is the unskilled income share in the economy, I set α to be 0.735 so the unskilled workers' expenditures on children ensure that the number of non-agricultural children is 1.35 times lower than that in the agricultural sector. This result gives us 4.20 children per household.[20]

Fertility cost: $\{\tau_u, \tau_s\}$. τ_u is set to be 0.051, so the agricultural fertility rate generated by the model is 5.67 children per family. The time cost for skilled child-rearing is set to be 0.15. From Haveman and Wolfe (1995) and Knowles (1999), the time devoted to child-rearing ranges from 10%

[20]As our model includes only females, this value represents an average of 2.10 children per female in the non-agricultural sector. This finding is common in the literature; e.g., Manuelli and Seshadri (2009) use the same method.

to 20% of the parental time endowment. Because the model focuses only on female fertility choice, I choose $\tau_s = 0.15$. This parameter choice is consistent with the fact that the opportunity cost of a child is equivalent to approximately 15% of the parents' time endowment on average (De La Croix and Doepke, 2003 and Cordoba, Liu and Ripoll, 2016). Notice that the unskilled child-rearing cost is lower than the skilled one. This result can be due to, among other reasons, child labor. When the child-rearing cost is lower, the household income is high, as can be seen from examining the budget constraint equation (3) is examined. So, the child labor can be interpreted as a form of household subsidy.

Educational cost: $\{\underline{\gamma}\}$. Consistent with (Goldin and Katz, 2009, p. 140)'s documentation that the "*school teacher was the most important expense, and often the only one, families in rural areas and... cities often paid for services in-kind with the provision of room and board, in addition to a stipend*", Doepke (2004) matches the educational cost with the pupil/teacher ratio. The pupil/teacher ratio in late 1870 was approximately 34 (Snyder, 1993). This ratio requires the educational cost to be approximately 3% of a high-skill worker's (teacher's) wage. This result is also consistent with the findings in Go and Lindert (2010) that 2.1-3.5 teachers occurred per 100 pupils across the U.S. in school year 1849/50. Also, despite the backward transportation system in the rural area, the rural educational cost would be approximately 20% higher using the class size estimate. This result is consistent with our assumption that the educational cost is higher in the agricultural sector. In equilibrium, the educational cost $\underline{\gamma}$ is set to 0.52 and it makes $\bar{\gamma} = 0.63$ Table 2 summarizes the parameters.

Table 4: Parameter Change

Parameter	1880 Value	1930 Value	Note
A	1.33	2.32	Farm Output per Worker
ζ_m	6.23	4.06	Non-Farm Output Premium
τ_u	0.051	0.073	Agricultural Fertility Rate
$\bar{\gamma}$	0.63	0.52	Snyder (1993); Go and Lindert (2010)

In the second steady state, I assume that the educational reform is enforced and $\overline{\gamma} = \underline{\gamma}$. The cost of schooling after the reform cannot be estimated using the method of class size because many states imposed free schooling at the turn of the twentieth century (Goldin and Katz, 2009). Therefore, in the main analysis, I assume the educational cost remains constant in the non-agricultural sector. A realistic picture is that the cost of education reduced throughout the period that I study. Nonetheless, assuming a declining educational cost only strengthens the argument: a larger fertility decline, a higher skill intensity and a greater extent of structural transformation.

I also allow three parameters to change between the two time points: productivity levels $\{A, \zeta_m\}$ and unskilled child-rearing costs τ_u. The two productivity parameters are backed out using sectoral output per laborer, so $A = 2.32$ and $\zeta_m = 4.06$. This calculation shows that the productivity growth in the agricultural sector is faster, which is consistent with Caselli and Coleman (2001). $\tau_u = 0.073$ is chosen to match the change in the agricultural fertility rate. The increase in τ_u is interpreted as a change in cost that lumps all of the other factors that reduce the fertility rate. This calculation includes child labor, among others. If one thinks of child labor as an income source for a household, reduction in the extent of the child labor means that the income of a family is reduced. States without restrictions on child labor and compulsory schooling laws had higher fertility rates on average in the year 1900. I use τ_u to explore the exogenous changes that lead to declines in the fertility rate. Hazan and Berdugo (2002), Doepke (2004) and Doepke and Zilibotti (2005) explore the link between the introduction of child labor laws and the rise in time cost of raising children. This calculation allows the importance of Rural Education Reform to stand out in the subsequent numerical exercise.[21] The details are shown in Table 3.

3.2 Model Performance

The model prediction is reported in Table 4. The first two columns match well because we target most of the data moments in the calibration exercise. To evaluate model fitness, we consider the model performance in 1930, which is summarized in the last two columns of Table 4. The model gives four correct predictions that we can observe in the data. Firstly, both sectoral and aggregate fertility rates decline in the model, and secondly, agricultural fertility declines more than in the non-agricultural sector. Doing so shrinks the gap in fertility rates between the agricultural and non-agricultural sectors.

Thirdly, the skill intensity in the model increases. Although, I take no stance on what "skill"

[21] Although τ_s is likely changed in the second steady state due to the advancement of home production technology, I do not have concrete data to back out such a change. Thus, in the baseline exercise, I assume that τ_s is a constant to ensure that the effect generated from the model is not driven by unobservables without data counterparts. The linkage of rising τ_u to the child labor regulation can be used to justify that a constant τ_s throughout the model period: skilled children were not affected by child labor regulation because they were not exposed to the labor market.

means in the current exercise (because the definition of skill likely changed during our long period of study), the model prediction for skill intensity resembles the high school graduation rate in the data well.[22] Due to the increase in skill intensity and income, lastly, the agricultural employment share and the agricultural consumption share have declined. Although the agricultural employment share represents only approximately half of the observed reduction, the predicted average agricultural consumption share fits well with the data. This result is related to our choice of ν as discussed above. These four observations show that the model, albeit stylized, gives reasonable predictions.

Table 5: Model Prediction

	Year 1880		Year 1930	
Variable	Data	Model	Data	Model
Agricultural Fertility	5.68	5.67*	3.21	3.21*
Non-Agricultural Fertility	4.19	4.20*	2.33	2.68
Fertility Gap	1.49	1.47	0.88	0.53
Aggregate Fertility	4.96	4.96*	2.50	2.87
Agricultural Employment Share	0.51	0.51*	0.20	0.36
Agricultural Consumption Share	0.45	0.45**	0.34	0.34
Skill Intensity	.	2.6%	.	12.5%
High School Grad.	< 2.0%	.	18.9%	.

* indicates target value

** indicates target decline

4 Experiment

4.1 Decomposition

In this subsection, I argue that the Rural Education Reform was important for understanding the decline in the fertility rate, the reduction in the sectoral fertility rate gap, the increase in skill intensity and the decline in the agricultural employment share. The effect of the Rural Education Reform is robust even if the productivity progress and the increase in child-rearing cost are considered. To this end, I carry out a decomposition exercise.

[22] The schooling data on high school graduate is from the 1940 census, which is the first year that information on educational attainment is available. To calculate the share of high school graduates (Grade 12) in a population that is comparable to our model, I restrict my attention to the population aged 40-70 (who would be 30-60 years old in the year 1930) because our model period is 30 years. In that age group, 18.9% of the population graduated from high school in 1930. I interpret this result as the upper bound for the estimate because lower education may be associated with earlier mortality.

Four parameter values change among the two steady states of the model. They are the technological progress in agricultural and non-agricultural sectors $\{A_a, \zeta_m\}$, the increase in the child-rearing cost $\{\tau_u\}$, and the educational reform $\{\underline{\gamma} = \overline{\gamma}\}$. The counter-factual experiment assumes that the groups of parameters resume their original values individually. In particular, the counter-factual experiments involve a) no Rural Education Reform ($\underline{\gamma} < \overline{\gamma}$); b) no productivity progress ($A_{a,1930} = A_{a,1880}$ and $\zeta_{m,1930} = \zeta_{m,1880}$); and c) no change in unskilled child-rearing costs ($\tau_{u,1930} = \tau_{u,1880}$).

Table 6: Relative Contribution

	Agr. Fertility ↓	Non-Agr. Fertility ↓	Agr. Fertility ↓	Agr. Share ↓	Skill Intensity ↑	Birth Gap ↓
Educational Reform	29.9%	20.0%	24.1%	20.3%	49.1%	115.2%
Productivity ↑	-33.3%	-31.2%	-29.6%	59.4%	14.6%	-51.5%
Unskilled Cost ↑	103.5%	111.2%	105.5%	20.3%	36.4 %	36.4%
Total	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%

Three major features are apparent in Table 5. First, while the increase in productivity levels explains the majority of the decline in the agricultural employment share, as the traditional structural transformation model predicts, the Rural Education Reform affects the structural transformation in a very different way. In the traditional literature, productivity progress pushes and pulls labor away from the agricultural sector (Alvarez-Cuadrado and Poschke, 2011). It has no implication on education, and the skill intensity of the economy changes slightly (around 15%). However, when the cost of education decreased during the educational reform, more agents educate their children. Hence the skill intensity of the economy increases and the skill-intensive non-agricultural sector expands.

Second, an increase in child-rearing costs explains a significant proportion of the decrease in sectoral and aggregate fertility rates. This result is because τ_u is calibrated to match aggregate fertility rates, so altering τ_u should have large implications for fertility rates. However, it is secondary to explaining the fertility rate gap between the agricultural and non-agricultural sectors, because the increase in τ_u affects both sectors.

Third, the Rural Education Reform closed the fertility gap between the agricultural and non-agricultural sectors. This result is mainly due to the fact that Rural Education Reform has a different effect on different sectoral fertility rates. Originally, no farmers' children received an education due to high educational costs. Because the educational reform results in a relative increase in farmers' children receiving an education, the agricultural fertility rate decreases more than the non-agricultural one. However, we still observe a sizable reduction in the non-agricultural fertility rate. This result is mainly due to the income effect. When educational reform increases, the relative income of the skilled decreases because the increase in skill intensity reduces the wage premium (wage compression in Goldin and Katz, 2009). Because the income effect is positive, the non-agricultural fertility also declines.

4.2 Demography and Structural Transformation

Endogenous fertility is important for the structural transformation through the quality-quantity trade-off. The idea is that when the fertility rate is kept constant, the ability of agents to trade-off the number of their children for education is reduced. Agents in the economy cannot reduce their births and channel their resulting budgets to education to the same extent as in the case of endogenous fertility. Therefore, the skill intensity is lowered. Because of the positive link between the non-agricultural employment share and the aggregate skill intensity, the magnitude of structural transformation is lower.

The following exercise formalizes this idea. I exogenously fix sectoral fertility rates to their year 1880 values and do not allow them to change across the two steady states. The result is shown in Table 6. Two points are worth notice.

First, the increases in skill intensity in endogenous and exogenous fertility regimes are 9.8 and 3.7 percentage points, respectively. Thus, the increase in skill intensity decreases by more than 62% when the quality-quantity trade-off channel is shut down. More unskilled workers occur in non-agricultural sectors in the exogenous fertility case. This result is mainly due to the fact that exogenous technological progress drives workers away from the agricultural sector. If the skill intensity cannot be increased to the level in the case of endogenous fertility, workers can only work as unskilled non-agricultural workers.

Second, because of the lower skill intensity, the extent of the structural transformation is lowered. The reduction in the agricultural employment share in endogenous and exogenous fertility regimes are 15.1 and 11.5 percentage points, respectively. Thus, the quality-quantity trade-off channel accounts for 24% of the reduction. Notice that the reduction in increase in skill intensity does not

Table 7: Demography and Structural Transformation

	1880	Endogenous 1930	Exogenous 1930
Aggregated Fertility	4.96	2.87	4.11
Skill Intensity	2.7%	12.5%	6.4%
Agricultural Share	51.2%	36.1%	39.7%
Non-Agricultural Unskilled Share	46.1%	51.4%	53.9%

translate one-to-one to the reduction in the extent of structural transformation.[23]

5 Further Discussion

In this subsection, I argue that the model can also predict three events in the earlier twentieth century, namely, compression of skill wage premium, nearly constant agricultural price and increasing intergenerational mobility.

Wage Premium. Wage compression occurred in the early twentieth century (Goldin and Margo 1992). For example, the annual earnings of full professors had been 4.16 times that of the average worker in manufacturing in 1908, and this number decreased by 28.8% to only 2.96 in the year 1930 Goldin and Katz (2009). This case was not isolated: (Goldin and Katz, 2009, Chapter 2) document declining wages in a wide range of skilled occupations relative to that of unskilled ones.

The supply and demand for human capital shaped the distribution of earnings in the U.S. labor market. In the early twentieth century, the supply of skilled labor was plentiful due to the rapid increase in schooling, while the relative skill-biased technological progress was not high enough (Acemoglu and Autor, 2012). Caselli and Coleman (2001) find a similar observation: from 1880 to 1920, the non-agricultural wage premium decreased by 37.5%.

In our model, the non-agricultural wage premium decreases because of the increase in the number of skilled workers in the economy. Consider the equation of the current model $\frac{w^s}{w^u} = \frac{1-\alpha}{\alpha}\left(\frac{l^{mu}}{l^{ms}}\right)$, an increase in the relative number of skilled laborers drives the wage premium down. The predicted

[23] In the model, two effects mainly determine the agricultural employment share. I coin them as demand and supply factors. The demand-side factor is mainly driven by the demand for agricultural goods, which depends on preference parameter ν. The supply of agricultural employment is inversely related to the skill intensity. When ν is large, the demand factor cannot drive down much of the agricultural employment share. Thus, the supply of the agricultural employment plays a more important role. Therefore, in the case of my baseline calibration, because ν is moderate, both of the factors play some role in driving down the agricultural employment share.

decline of the non-agricultural wage premium is approximately 23.5% between the years 1880 and 1930, which is close to the historical narratives.

Although we abstract from the relative labor augmented technological progress in this stylized model, the effect of technology on the wage premium in this historical period should be minimal. From historical data, we see an almost six times increase in high school graduates in the work force from the year 1880 to 1930 while the increase in farm and non-farm output per worker had been relatively smaller.

Agricultural Price. The relative price of agricultural goods is determined by the demand from households and by supply in the agricultural production. The less-than-unitary income elasticity in agricultural goods relatively reduces demand for farm goods when income increases. The price of farm goods should then decrease when compared to non-agricultural ones. The increase in supply of the farm goods because of the productivity progress in the farm sector also caused downward pressure on the agricultural price.

However, the effect is roughly offset by the decline in the supply of farmworkers in the model: during the educational reform, more children received an education and work in the non-agricultural sector. Thus, the relative price of agricultural goods was roughly constant in data: the ratio of the price level in the years 1880 and 1930 is approximately 1.00 (Caselli and Coleman, 2001), while in the current model, the predicted agricultural price ratio ($\frac{p_{a,1880}}{p_{a,1930}}$) is 1.05.

Table 8: Intergenerational Mobility

Panel A: Model		Skilled	Unskilled	Farmer
Before Reform	Share of Agents	0.03	0.46	0.51
	% of children skilled	100%	27%	0%
After Reform	Share of Agents	0.13	0.51	0.36
	% of children skilled	100%	31%	31%
Panel B: Data		**Skilled**	**Unskilled**	**Farmer**
1850-1880	% of children skilled	66%	54%	29%
1950-1973	% of children skilled	92%	86%	75%

Table 9: Skilled Workers in the data are defined as white collar, skilled and semi-skilled workers.

Intergenerational Mobility. Last but not least, the model has implications on intergenerational mobility. The equilibrium skill intensity η^s has increased from 2.7% from the first steady state to 13%. This increase is mostly contributed from two factors. First, the share of farmers receiving an education increased by 31 percentage points. Second, the share of skilled agents (who have 100% skilled children) increased from 3% to 13%.

The prediction of the model aligns qualitatively with occupational mobility data adopted from Long and Ferrie (2013), as shown in the last two rows of Table 7. In particular, the numbers of skilled children born to unskilled workers and farmers were very distinct in the late nineteenth century but became relatively similar after the Rural Education Reform.

6 Conclusion

The data show a rapid decline in the fertility rate at the turn of the twentieth century in the U.S. This decline was mainly caused by the reduction in the agricultural fertility rate. The rapid decline in fertility and the relative decrease in rural fertility cannot be explained by traditional factors such as a decline in the child mortality rate, an increase in female labor participation or a delayed marriage. The decline in the aggregate fertility rate was also coupled with a decrease in the agricultural employment share and an increase in school attendance. I argue that education played an important role in these observed changes.

Originally, the higher educational cost for farmers discouraged them from sending their children to school. Therefore, they had a lower child-rearing cost. As a result, they had more children with lower human capital. The educational reform, productivity progress and the change in regulations reduced the educational cost for the farmers and encouraged them to educate their children. This change promotes schooling. Switching from not educating to educating their children on the extensive margin increased the marginal child-rearing cost. Hence, the fertility rate is reduced. As the skill intensity of the economy increases, the more skill-intensive non-agricultural sector expands.

Two results that can be learned from the paper. First, the educational reform is important for explaining the patterns of declining fertility rates, structural transformation and the increase in skill intensity. Second, considering endogenous fertility along with structural transformation is by itself important because exogenously fixing the fertility rate and not allowing the agent to choose between the number of children and their educational levels reduces the increase in skill intensity and the decline in the agricultural employment share.

Understanding the educational reform is important even currently. This understanding can be a policy recommendation for developing economies that want to rapidly reduce population growth, upgrade human capital, and industrialize.

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A Data Appendix

Fertility and Mortality Rates. Children Ever Born (CHBORN), children surviving (CHSURV), gender (SEX), age (AGE), marital status (MARST), race (RACE), Hispanic origins (HISP), birth place (BPL) and group quarters status (GQ) from IPUMS-USA (Ruggles et al., 2018) are used to determine the fertility and mortality rates.

From 1900-90 (except 1920-30), U.S. census investigated the number of children ever given birth to a female (CHBORN). I restrict the attention to ever married native white non-Hispanic women not residing in group quarter (henceforth, "subgroup of female studied"). I focus on those who were above 45 years old and use different census years to maximize my data sample. Moreover, I restrict my sample to those younger than 65 years old to minimize selection issue.

In the year 1900 and 1910 (but not other years), the census asks a female how many children they had ever born who were still living in the census day (CHSURV). Using such information together with CHBORN to the "subgroup of female studied", the information on sectoral specific mortality rate can be obtained.

Sectoral Composition. Industry (IND1950) and spouse's location in household (SPLOC) in IPUMS-USA, consumption share (Historical Statistics series G495 to 581) and U.S. sectoral employment in Herrendorf, Rogerson and Valentinyi (2014, HRV henceforth) are used to determine the sectoral composition.

Both of IPUMS-USA and HRV defined "Agricultural" as "Agricultural, Forestry and Fishing" which are predominately rural industries. I define a family with either the female or her spouse working in agricultural sector an agricultural family. Robustness check have been done using different definition (e.g. rural versus urban and farm versus non-farm) and using only the female's or only her spouse's industry to define the household. The result is quantitatively similar. Separating the economy into finer industries does not change the qualitative result. The information on agricultural consumption share is from Historical Statistics (series G495 to 581). The agricultural consumption share is extrapolated from average food consumption share in 1900 and 1920.

Although the sample used to derive sectoral composition is restricted to the "subgroup of female studied", it tracks well with the measure of HRV. The imputed agricultural employment share for the years 1880 and 1930 and those reported in HRV differ by less than 5%. I use HRV as the sectoral measure because they provide number of workers in different sector. This will provide information on the number of workers in each sector for my construction of sectoral productivity levels. One of the potential problem by using HRV is that the aggregate fertility rate computed using HRV and sectoral fertility would not be consistent with the data. However, when compared the imputed aggregate fertility and data for the years 1880 and 1930, the difference is around 5%.

Productivity. Real GDP (Historical Statistics series F3) and farm and non-farm nominal GDP (Historical Statistics series F127-8) and HRV are used to compute the average labor productivity.

The labor productivity computed is the real output per worker. As the sectoral real GDP is not reported in the Historical Statistics, real GDP is divided into farm and non-farm real GDP by their nominal share. The real sectoral GDP is then divided by the number of employment in that sector as reported in HRV to obtain the real sectoral output per worker. As productivity is yearly data and more volatile than the other variables, moving average is applied to smooth the series.

Schooling. School attendance (SCHOOL), employment status (EMPSTAT), industry (IND1950), age (AGE), state (STATEFIP), education attainment (EDUC), father's and mother's location in a household (POPLOC and MOMLOC) in IPUMS-USA and Historical Statistics series H599 are used to determine the education status.

School Attendance is defined as the number of school-attending children between 6-15 years old in a particular state, each contributing to their own sampling weight. I also map children to their parents using POPLOC and MOMLOC. By using the parents' industry to define the household, the children can be defined as agricultural versus non-agricultural decedents. By using the self-reporting employment status and industry information, I can also infer in a child was studying full time or not.

Fraction of potential workers with high school diploma in 1880 is based on the information that the high school graduate is around 2% (Historical Statistics series H599). As high school graduates were increasing in the late nineteenth century, the fraction of workers with high school diploma must be less than 2%. The fraction in 1930 is by using the data in 1940 (the first year with information on education attainment). I count the number of workers who had at least high school degree between 40-70 years old population (as the one model period is 30 years) weighed by the sampling weight.

B Mathematical Proof

Proof of Proposition 1

Consider the household problem that maximizes (4) subject to (3). The FOC on c_a^{ij} reads:

$$c_m^{ij} = p^{\frac{1}{\sigma}} \left(\frac{1-\nu}{\nu}\right)^{\frac{1}{\sigma}} \left(c_a^{ij} - \bar{c}\right)$$

Denote $C^{ij} \equiv \; pc_a^{ij} + c_m^{ij}$ and combine with FOC yield

$$\begin{aligned} pc_a^{ij} &= \frac{C^{ij} - p\bar{c}}{1 + p^{\frac{1-\sigma}{\sigma}} \left(\frac{1-\nu}{\nu}\right)^{\frac{1}{\sigma}}} + p\bar{c} \\ c_m^{ij} &= p^{\frac{1-\sigma}{\sigma}} \left(\frac{1-\nu}{\nu}\right)^{\frac{1}{\sigma}} \left[\frac{C^{ij} - p\bar{c}}{1 + p^{\frac{1-\sigma}{\sigma}} \left(\frac{1-\nu}{\nu}\right)^{\frac{1}{\sigma}}}\right] \end{aligned}$$

The consumption part of the utility is then

$$\begin{aligned} & \frac{\nu \left[\left(c_a^{ij} - \bar{c}\right)\right]^{1-\sigma}}{1-\sigma} + \frac{(1-\nu)\left(c_m^{ij}\right)^{1-\sigma}}{1-\sigma} \\ = \;& \nu \left(\frac{1}{p}\right)^{1-\sigma} \left[1 + p^{\frac{1-\sigma}{\sigma}} \left(\frac{1-\nu}{\nu}\right)^{\frac{1}{\sigma}}\right]^{\sigma} \frac{\left(C^{ij} - p\bar{c}\right)^{1-\sigma}}{1-\sigma} \\ = \;& \Omega(p;\nu) \frac{\left(C^{ij} - p\bar{c}\right)^{1-\sigma}}{1-\sigma} \end{aligned}$$

where $\Omega(p;\nu) = \nu \left(\frac{1}{p}\right)^{1-\sigma} \left[1 + p^{\frac{1-\sigma}{\sigma}} \left(\frac{1-\nu}{\nu}\right)^{\frac{1}{\sigma}}\right]^{\sigma}$. Then we put $E^{ij} = \sum_{k=u,s} \widetilde{\tau}^{ij,k} n^{ij,k}$ and notice that:

$$\begin{aligned} n^{ij,s} &= \frac{E^{ij} f^{ij}}{\widehat{\tau}^{s}} \\ n^{ij,u} &= \frac{E^{ij}\left(1 - f^{ij}\right)}{\widehat{\tau}^{u}} \end{aligned}$$

where f^{ij} is the fraction of child-rearing expenditure devoting to raising skilled children. The second part of the value function can be written as:

$$\begin{aligned} & \frac{\psi}{(n^{ij,s} + n^{ij,u})^{\varepsilon}} \left[n^{ij,s} V(w'^{s}, i') + n^{ij,u} V(w'^{u}, i')\right] \\ = \;& \left(E^{ij}\right)^{1-\varepsilon} \frac{\psi}{(\frac{f^{ij}}{\widehat{\tau}^{s}} + \frac{1-f^{ij}}{\widehat{\tau}^{u}})^{\varepsilon}} \left[\frac{f^{ij}}{\widehat{\tau}^{s}} V(w'^{s}, i') + \frac{1-f^{ij}}{\widehat{\tau}^{u}} V(w'^{u}, i')\right] \end{aligned}$$

As a result, the value function reads:

$$
\begin{aligned}
V(w^j, i) &= \max_{E^{ij}, f^{ij}} \left\{ \begin{array}{c} \Omega(p;\nu)\frac{\left(C^{ij}-p\bar{c}\right)^{1-\sigma}}{1-\sigma} \\ +\left(E^{ij}\right)^{1-\varepsilon} \frac{\psi}{(\frac{f^{ij}}{\widehat{\tau}^{ij,s}}+\frac{1-f^{ij}}{\widehat{\tau}^{ij,u}})^{\varepsilon}} \left[\frac{f^{ij}}{\widehat{\tau}^{ij,s}} V(w'^{s}, i') + \frac{1-f^{ij}}{\widehat{\tau}^{ij,u}} V(w'^{u}, i')\right] \end{array} \right\} \\
s.t. \quad w^j &= C^{ij} + E^{ij}
\end{aligned}
$$

which is clearly convex in f^{ij} for all $\varepsilon \in (0,1)$. So in optimization problem, either $f^{ij} = 1$ or $f^{ij} = 0$. By applying either $f^{ij} = 1$ $(k = s)$ or $f^{ij} = 0$ $(k = u)$, the result follows immediately.