Optimal Mirrleesean Taxation and Human Capital Investment†

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Abstract

This paper characterizes optimal capital and labor income taxes in a lifecycle model of human capital investment with heterogeneous agents of continuous types. The key feature of the model is that some private expenses for consumption may be disguised as private expenses for education purposes. The feature leads to positive capital taxes and negative labor taxes in an agent's early lifecycle. The policy of taxes on capital and subsidies on labor serves as a mechanism to alleviate information-induced distortions to learning, as opposed to education subsidies to offset tax-induced distortions to learning in the existing literature.

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1. Introduction

In the existing dynamic Mirrlees literature, agents' heterogeneous skills are private information that change stochastically over time. The benevolent government wishes to redistribute income but cannot observe skills, leading to a non-degenerate tradeoff between equity and efficiency. In the existing literature, the optimal tax system is designed based on redistribution and insurance motives. The capital wedge, an implicit tax on capital, is positive, if there is a motive to insure against lifetime risks on productivity.[1] Moreover, the labor wedge, an implicit tax on labor, is in general positive, in order to avoid agents shirk and work less. The existing dynamic Mirrlees literature analyzed models with exogenous skills, but scholars have already established that skill acquisition is endogenous (e.g., Heckman, 1976, 1999).

Recently, Bovenberg and Jacobs (2005) and Stantcheva (2017) have extended the literature to one that took into account educational decisions. Bovenberg and Jacobs (2005) studied a static model with both verifiable and non-verifiable education expenses.[2] These authors only designed optimal policy in the environment with verifiable education expenses and obtained income taxes and education subsidies. More recently, Stantcheva (2017) has extended Bovenberg and Jacobs (2005) to a lifecycle model that focused on verifiable education expenses. Besides positive optimal labor taxes for redistribution purposes and positive optimal capital taxes for insurance purposes, she uncovered optimal education subsidies that involved less than full deductibility of education expenses on the tax base. These papers are valuable in that they extend the existing Mirrlees literature into one with human capital, and discover optimal subsidies to observable education expenses in order to offset tax-induced distortions to learning.

However, education expenditure may not be all observable.[3] A wide range of goods and services have both consumption and human capital investment component. Computers, equipment, software, home office, books, magazines, radio, TV and private lessons are recreational as well as educational. Most commodities like food, shelter, and clothing are essential for bringing children up and also for maintaining the human capital of adults. Indeed, the literature on human capital has long recognized the difficulty in distinguishing human capital investment from ordinary consumption expenditure. As early as in 1961, in Presidential Address to the American Economic Association, Schultz (1961) had noted the difficulty in estimating the magnitude of human capital investment: "Most relevant activities clearly are …partly consumption and partly (human capital) investment, which is why the task of identifying each component is so formidable and why the measurement of capital formation by expenditures is less useful for human investment." See also Heckman (1976, 1999). This difficulty has been recognized in the ongoing policy debate on how to design the tax system in order to foster human capital accumulation.

If the government cannot distinguish private education expenses from private consumption, the

[1] See, e.g., Werning (2007), da Costa and Masestri (2007), Anderberg (2009), Farhi and Werning (2013), Kopczuk (2013) and Stantcheva (2017).

[2] For other static models, see also Maldonado (2008), Gelber and Weinzierl (2012) and Findeisen and Sachs (2016) which use labor time for education investment.

[3] See Bovenberg and Jacobs (2005), Psacharopoulos (2006), Grochulski and Piskorski (2010), Jacobs and Bovenberg (2011), and Stantcheva (2017) for account of non-observable education expenditure.

policy of a subsidy to unobservable education expenses is not available. Then, a key question that arises is whether non-verifiable education expenses would affect the design of the optimal tax policy on capital and labor over time. This paper attempts to answer this question. Our paper adds values to Bovenberg and Jacobs (2005) and Stantcheva (2017) in that human capital expenses are not distinguishing from are consumption, so consumption expenses are unobservable. As a result, taxing capital income and subsiding labor income in an early lifecycle would counterbalance the distortion to unobservable consumption arising from non-verifiable education expenses. These policies serve as a mechanism that corrects *information-induced distortions to learning*, as opposed to education subsidies that ease *tax-induced distortions* t*o learning* in Bovenberg and Jacobs (2005) and Stantcheva (2017).

We must note that Grochulski and Piskorski (2010) also studied human capital investment in terms of non-verifiable education expenses. In their model, in the initial period endowed physical capital may be consumed or invested that turns into human capital, so agents become a high-skill type. In later periods, bad shocks arrive stochastically, and if agents are hit by bad shocks, they become a low-skill type forever. As a result, their intratemporal wedges are positive for a low type and negative for a high type before terminal dates, so as to reduce leisure for a high-type agent. Similar to the existing literature, they obtain positive intertemporal wedges due to stochastic shocks. Our model is different in that shocks are drawn from a distribution and thus, agents' types may change over time. Moreover, our agents invest in human capital over time. As a result of non-observable consumption, we obtain positive intertemporal wedges and negative intratemporal wedges for agents in an early lifecycle.

Specifically, we study a lifecycle model of human capital investment with heterogeneous agents of continuous types, dubbed the first-order approach. When born, agents are endowed with the same skill but experience different shocks to skill acquisition. These skill acquisition shocks characterize agents' types. In addition to working, agents choose consumption, education expenses and savings in a period. In this framework, agents' types, human capital, non-verifiable education expenses, consumption and work effort are private information. Under asymmetric information, the government (social planner) solves the second-best program: it chooses constrained efficient allocations in order to maximize the utilitarian social welfare subject to resource constraints and incentive-compatibility constraints.

We obtain two novel results from information distortions. First, the capital wedge is positive, and the labor wedge is negative in an early lifecycle and then turns positive before the terminal period of the lifecycle. Second, as compared to a model with only verifiable education expenses, non-verifiable education expenses increase the capital wedge (an implicit capital tax) and decreases the labor wedge (an implicit labor tax).

These wedges arise from non-verifiable education expenses and thus, unobservable consumption. First, as consumption is not observable, the intertemporal marginal rate of substitution in consumption between periods is distorted. A positive capital wedge deters agents from cutting off non-verifiable education expenses for more consumption. Second, high-skill agents may underreport their types by allocating more expenses in consumption and less in education. A negative labor wedge serves as a mechanism to induce agents to work according to their true types and spend more on education.

Our results contribute the following perspectives to the literature. First, our positive capital wedge is obtained based upon unobservable consumption arising from non-verifiable education expenses. This mechanism is in sharp contrast to the existing Mirrlees models, wherein a positive intertemporal wedge is obtained, because skill shocks change stochastically over time. Their positive intertemporal wedge is for an insurance purpose against lifetime risks. By contrast, in our model, even if agents' types do not change stochastically over time, the capital tax is positive, because education expenses and consumption are not observable. Our positive intertemporal wedge is for the purpose of fostering human capital investment. Next, our negative labor tax in an early lifecycle is different from a positive labor tax in the existing literature with verifiable education expenses. While a positive labor tax in the existing literature is to induce agents to work according to their types, a subsidy to labor supply in our model serves as a mechanism to correct distortions that arise from non-verifiable education expenses. This encourages agents to spend more on education and work more. Overall, our capital taxes and labor subsidies serve as devices to counteract information-induced distortions to learning, as opposed to education subsidies as devices to offset tax-induced distortions to learning in the existing literature.

We use linear labor tax rates and linear capital tax rates, along with lump-sum taxes, to implement the intratemporal wedge and the intertemporal wedge. Different from Grochulski and Piskorski (2010), deferred capital taxes are not necessary for linear capital taxes.

Finally, we calibrate our model to match the U.S. data and illustrate the optimal capital and labor income tax policy. We find that capital wedges are positive and labor wedges are negative in an early lifecycle when education expenses are not verifiable, as opposed to zero capital and labor wedges in a model with only verifiable education expenses. These results hold under different distributions of the skill types. Relative to a laissez-faire economy without taxes, our second-best mechanism yields a welfare gain close to the level in the first-best planning economy with a larger gain when the variance of shocks is larger.

1.1 Related literature

Our model is related to human capital and optimal taxes. Research about human capital formation has been a long-standing topic in the literature, starting with Becker (1964), Ben-Porth (1967) and Heckman (1976). The structural branch of the literature emphasizes that human capital acquisition occurs throughout a lifecycle, underscoring the need for a lifecycle model (Cunha and Heckman, 2007). A body of empirical work documents that human capital investment, and thus the earning, is risky (e.g. Palacios-Huerta (2003), Meghir and Pistaferri (2004) and Storesletten et al. (2004)). Our model attempts to embrace some of the literature's main findings in a stylized way.

There is a growing literature named new dynamic public finance which extended the optimal taxation pioneered by Mirrlees (1971) to dynamic settings. As opposed to the Ramsey approach, wherein agents are homogeneous and information is complete, the Mirrlees approach considers agents with heterogeneous earning skills that are private information.[4] The new dynamic public finance literature typically considers exogenously evolving abilities, thus abstracting from endogenous skill acquisition. See Golosov et al. (2003), Kocherlakota (2005), Albanesi and Sleet (2006), Werning (2007), Farhi et al. (2012), and Farhi and Werning (2013), among others.[5] Our paper contributes to this literature by taking into account individuals' skills which change over time based on human capital investment.

Some papers in the dynamic Mirrlees literature have jointly considered the optimal taxation and endogenous human capital. Investment in human capital may take the form of labor effort or education expenses. Our paper focuses on education expenses and is complementary to the strand that uses learning time as education input. See Kapička (2006, 2015), da Costa and Masestri (2007), Boháček and Kapička (2008), Anderberg (2009) and Stantcheva (2015).[6] In particular, Kapička (2006, 2015) studied the dynamic income taxation in a model with only unobservable human capital investment. Yet, his investment is in terms of learning time, and there is no shock during human capital formation. Moreover, agents in his model do not save, and thus capital taxes cannot be studied. Our model is different in that the investment is in terms of education expenses. In particular, our model allows for savings and thus, we can study capital taxes.

Diamond (1980) and Saez (2002) have obtained a negative labor wedge at the bottom of the income distribution in models with an extensive margin of the labor supply. A subsidy to the working poor in their papers is optimal, because the participation effect of the labor force dominates the incentive effect of higher income earners. Our negative labor wedge may be interpreted as a subsidy to the working poor, but our result is based on a different mechanism. Our negative labor wedge aims to induce people to reduce consumption and increase education expenses.

Finally, the feature of unobservable consumption is reminiscent of Allen (1985) and Cole and Kocherlakota (2001). Allen (1985) analyzed whether the optimal long-term contract is better than a series of unrelated short-term contracts when agents can borrow secretly. Cole and Kocherlakota (2001) characterized whether efficient consumption allocation can be decentralized through a competitive asset market in which agents can store asset secretly. In these two existing papers, consumption is unobservable, because agents are allowed to borrow or save secretly. Our model is different. Consumption is unobservable here, because its expenses may be disguised as expenses for education purposes. Moreover, we investigate whether unobservable consumption affects the design of the optimal tax policy on capital and labor over time, which was not studied by these two papers.

We organize this paper as follows. In Section 2, we present a two-period model. The social planner's problem is studied in Section 3, and the signs of capital and labor wedges are analyzed in Section 4. Section 5 provides a tax system to implement the constrained efficient allocation obtained in the social

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[4] For the optimal taxation in the Ramsey approach, readers are referred to Judd (1985), Chamley (1986) and, more recently, Chen and Lu (2013).

[5] Golosov et al. (2006), Kocherlakota (2010), Piketty and Saez (2013) and Kopczuk (2013) provided useful surveys.

[6] Among these papers, learning time is unobservable in Kapička (2006, 2015) and Anderberg (2009) and is observable in the other three studies.

planner's problem as a competitive equilibrium. In Section 6, we offer numerical analysis. In Section 7, we extend the model to T periods. Finally, concluding remarks are offered in Section 8.

2. Basic Model

2.1 The Environment

For pedagogical reasons, we start with a two period model. Later, we will extend the model to T periods and show that the results are robust. The economy consists of a continuum of agents who live for two periods. An agent obtains utility from consumption and disutility from working, with a utility function represented by:

$$u(c_1)-\phi(l_1)+\beta\left[u(c_2)-\phi(l_2)\right],$$

where $0<\beta<1$ is the discount factor, c_t is consumption in period t and l_t is work effort (or labor hours) in period t. An agent at most provides $\bar{l}>0$ work effort in a period. We assume that $u(c)$ is continuously differentiable, strictly increasing and concave, and satisfies the Inada condition, and $\phi(l)$ is continuously differentiable, strictly increasing and convex, and satisfies $\phi(0)=0$, $\lim_{l\to 0}\phi'(l)=0$ and $\lim_{l\to \bar{l}}\phi'(l)=\infty$.

At the initial period $t = 1$, an agent's disposable income may be consumed, spent on education to acquire human capital h_2 or saved to form physical capital k_2 in the next period. There are two kinds of education expenses, verifiable x_t and non-verifiable y_t. The human capital technology is $\psi(x_t, y_t)$, which is homothetic, of constant returns, and strictly increasing and strictly concave in x and y; that is, $\psi_{xx}<0<\psi_x$ and $\psi_{yy}<0<\psi_y$. Acquisition of human capital is stochastic. Given the initial human capital level h_1 and its depreciation rate δ_h, the level of human capital in the next period evolves as follows.

Assumption 1. The evolution of human capital in the next period is:

$$h_2=(1-\delta_h)h_1+\psi(x_1,y_1)+\theta,$$

where θ is a stochastic human capital or a "skill acquisition" shock over a fixed support $\Theta=[\underline{\theta},\overline{\theta}]$.

The probability of the realization of the human capital shock θ is denoted by $\pi(\theta)$, with $0\leq\pi(\theta)\leq 1$ for each $\theta\in\Theta$.[7] We assume that the realization of θ is identically and independently distributed (i.i.d.) for each agent. Suppose that the law of large numbers applies; then, $\pi(\theta)$ also means the fraction of agents whose skill acquisition shock is θ. The shock is referred to as an agents' type. An agent's shock is private information and the government does not know it. For simplicity, we assume that all agents are endowed with an identical human capital level h_1 when born, while agents with larger shocks have advantages to acquire human capital more than those with smaller shocks. Human capital characterizes a skill level: an agent with human capital h_t and work effort l_t supplies $z_t = l_t h_t$ units of effective labor.

[7] Skill shocks are time varying, but for simplicity time subscript 1 is dropped in the two-period model here. Later, when the model is extended to T periods, time subscript t will be added.

In each period t =1, 2, the representative firm combines aggregate physical capital K_t and aggregate effective labor Z_t to produce final goods using the technology $F(K_t, Z_t)$. The technology is neoclassical which satisfies constant returns to scale and is strictly increasing and concave in K_t and Z_t. The physical capital depreciates at the rate of δ_k.

In our environment, human capital shocks θ, work effort l_2 and non-verifiable education expenses y_1 are private information. Thus, individual consumption in the first period c_1 and human capital in the second period h_2 both are not publicly observable.

On the other hand, individual physical capital k_t, individual's effective labor z_t, t =1, 2, verifiable education expenses x_1 and initial human capital h_1 are publicly observable. Note that work effort l_1 is inferable from initial human capital and individual's effective labor in the first period, and thus is observable. Although non-verifiable education expenses are not publicly observable, the sum of an agent's consumption and non-verifiable education expenses c_1+y_1 is observable, since it is inferable from verifiable education expenses, capital and labor income. In the special case when there are only verifiable education expenses, then consumption c_1 is observable. In this case, agents have no incentives to invest in human capital for period 2. Yet, as shocks θ are private information, h_2 is still unobservable.

2.2 Resource Feasibility and Incentive Compatibility

In order to maximize the social welfare, the social planner would have chosen to smooth consumption for agents of different types. However, work effort and education expenses are private information. If consumption is equally allocated for agents of all types, this would encourage high-type agents to reduce work effort and education expenses. Thus, the allocation needs to be incentive-compatible.

In the first period, agents report their types, $\theta \in \Theta$.[8] The social planner distributes the resource to agents according to agents' reporting types. Following the notation used in da Costa and Maestri (2007) and Farhi and Werning (2013), we denote $\sigma(\theta)$ as a reporting strategy, specifying a reporting type σ conditional on a true type θ. We use the following notations to distinguish observable allocations from unobservable allocations. If a is observable, $a(\theta)$ denotes what the social planner allocates to type θ. If a is unobservable, then $a^{\sigma}(\theta)$ denotes the choice made by a type θ agent reporting as type σ. When $\sigma=\theta$, type θ is truthfully reported and thus, $a(\theta)$ denotes the optimal choice made by truth-telling agents.

An allocation $A=(c, x, y, l, h, z, k, Z, K)$ specifies consumption $c = \{ c_1^{\sigma}(\theta), c_2(\theta) \}$, verifiable education expenses $x = \{x_1(\theta)\}$, non-verifiable education expenses $y = \{ y_1^{\sigma}(\theta) \}$, work effort $l = \{l_1(\theta), l_2^{\sigma}(\theta)\}$, human capital $h = \{ h_1, h_2^{\sigma}(\theta) \}$, effective labor $z = \{z_1(\theta), z_2(\theta)\}$, and physical capital $k = \{k_1, k_2(\theta)\}$ for all $\sigma, \theta \in \Theta$, and aggregate effective labor $Z = \{Z_1, Z_2\}$ and aggregate physical capital $K =$

[8] It is likely that the allocation in period 2 is based on the history of types in periods 1 and 2. However, the type in period 2 does not affect the economy here, because period 2 is the terminal period when investment to accumulate human capital is not needed. Thus, agents in a two-period model only need to report their types in period 1.

$\{K_1, K_2\}$, given initial h_1 and $k_1 = K_1$.

Given K_1, h_1, G_1 and G_2, an allocation A is ***resource feasible*** if

$$\int_\Theta \pi(\theta)[c_1(\theta)+y_1(\theta)+x_1(\theta)]d\theta + K_2 \le F(K_1,Z_1)+(1-\delta_k)K_1 - G_1, \tag{1a}$$

$$\int_\Theta \pi(\theta)[c_2(\theta)]d\theta \le F(K_2,Z_2)+(1-\delta_k)K_2 - G_2, \tag{1b}$$

$$c_1^\sigma(\theta)+y_1^\sigma(\theta)=c_1(\sigma)+y_1(\sigma), \tag{1c}$$

$$h_2^\sigma(\theta)=(1-\delta_h)h_1+\psi\left(x_1(\sigma),y_1^\sigma(\theta)\right)+\theta, \tag{1d}$$

where $K_2=\int \pi(\theta)k_2(\theta)d\theta,\ Z_t=\int \pi(\theta)z_t(\theta)d\theta$ and G_t is the government expenditure in t = 1, 2.

As mentioned earlier, the sum of an agent's consumption and non-verifiable education expenses $c_1(\theta)+y_1(\theta)$ is observable. Thus, to avoid being caught, an agent with reporting strategy $\sigma(\theta)$ needs to restrict the reported sum of consumption and non-verifiable education expenses $c_1^\sigma(\theta)+y_1^\sigma(\theta)$ to the sum $c_1(\sigma)+y_1(\sigma)$.

The lifetime utility of an agent with reporting strategy $\sigma(\theta)$ is thus

$$W^\sigma(\theta)\equiv u\left(c_1^\sigma(\theta)\right)-\phi\left(l_1(\sigma)\right)+\beta\left[u\left(c_2(\sigma)\right)-\phi\left(l_2^\sigma(\theta)\right)\right]. \tag{2a}$$

An allocation A is ***incentive-compatible*** if

$$W(\theta)\ge W^\sigma(\theta), \forall \sigma,\theta\in\Theta. \tag{2b}$$

In setting up the planning problem, we go along with Stantcheva (2017), which follows from the procedure recently proposed for dynamic Mirrlees models by Farhi and Werning (2013). The procedure goes through two steps to make this problem tractable. First, a relaxed problem is written out based on the first-order approach, which replaces the full set of incentive compatibility constraints by the agent's envelope condition. Then, this relaxed program is turned into a recursive dynamic programing problem through a suitable definition of state variables.

The agent's envelope condition is derived as follows. Incentive compatibility (IC) constraints in (2a) and (2b) imply that, for almost all θ, the temporal incentive constraint holds as follows.

$$W(\theta)=\max_{\sigma\in\Theta}W^\sigma(\theta)=\max_{\sigma\in\Theta}u\left(c_1(\sigma)+y_1(\sigma)-y_1^\sigma(\theta)\right)-\phi\left(\frac{z_1(\sigma)}{h_1}\right)+\beta\left[u\left(c_2(\sigma)\right)-\phi\left(\frac{z_2(\sigma)}{h_2^\sigma(\theta)}\right)\right]. \tag{2c}$$

If we take the derivative with respect to (true) ability shocks, there are two direct effects, namely, on the non-verifiable education expense y_1 and the evolution of human capital, and indirect effects on the allocation through the report. By the envelope condition of the agent, all indirect effects are jointly zero and only the two direct effects remain. This leads to the following envelope condition of the agent:

$$\dot{W}(\theta)\equiv\frac{\partial W(\theta)}{\partial\theta}=-u'\left(c_1(\theta)\right)\frac{\partial y_1(\theta)}{\partial\theta}+\beta\phi'\left(\frac{z_2(\theta)}{h_2(\theta)}\right)\frac{z_2(\theta)}{\left[h_2(\theta)\right]^2}\frac{\partial h_2(\theta)}{\partial\theta}, \tag{2d}$$

where $\frac{\partial h_2(\theta)}{\partial\theta}=\psi_y\left(x_1(\theta),y_1(\theta)\right)\frac{\partial y_1(\theta)}{\partial\theta}+1.$

The envelope condition uncovers how promised utility changes with types at incentive compatible allocations. The first term in (2d) is a static rent, while the second term is a dynamic rent that emerges because the agent has some advance information about his type tomorrow. It has been shown that the envelope condition is a necessary condition for incentive compatibility (Milgrom and Segal, 2002).

3. The Planning Problem

This section envisages the social planner's program. By comparing the social planner's second-best solution to the solution in the agent's decentralization problem, we can understand the distortion in the second-best allocation relative to the laissez-faire allocation.

The social planner chooses allocations that maximize the following utilitarian social welfare:[9]

$$\max \int_{\Theta} \pi(\theta) W(\theta) d\theta,$$

subject to resource constraints (1a)-(1b) and incentive compatibility constraints (2a)-(2b).

The following definition describes the second-best allocation.

Definition 1. An allocation ***A*** is ***constrained efficient*** if it maximizes the welfare of the utilitarian social planner in the class of all feasible incentive-compatible allocations.

The incentive-compatible allocation will be referred to as the constrained efficient allocation. As will be seen, the constrained efficient allocation does not satisfy the standard consumption Euler equation. This leaves a room for the benevolent government to impose the optimal tax in order to replicate the constrained efficient allocation.

3.1 The relaxed planning problem

We now study the relaxed planning problem, which replaces the incentive compatible constraints (2a)-(2b) by the envelope condition (2d).[10] The social planner maximizes the utilitarian social welfare subject to (1a), (1b) and (2d). Let λ_t be the shadow price of the resource constraint in period t and $\mu(\theta)$ be the co-state variable associated with $\dot{W}(\theta)$. Moreover, we use (2c) to replace $c_2(\theta)$ by $u^{-1}\left[\frac{1}{\beta}\left(W(\theta)+\phi(l_1(\theta))-u(c_1(\theta))\right)+\phi(l_2(\theta))\right]$.

Then, the relaxed planning problem is given by

[9] See Diamond (1998) and Tuomala (1990) concerning how the choice of the welfare function affects optimal taxes in a static framework. For more general social welfares, readers are referred to Saez and Stantcheva (2016) as to how the tax policy is reformed under generalized social marginal weights, which is beyond the scope of this paper.

[10] As noted earlier, the solution of the relaxed planning problem is only a necessary condition, unless we restrict some conditions to force the second order condition of (2c) to be smaller than or equal to zero. Instead of doing that, in section 6 below we will follow Farhi and Werning (2013) and Stancheva (2017) and directly verify that the solution of our relaxed problem is incentive compatible.

$$
\begin{aligned}
\mathcal{H} &= \pi(\theta)W(\theta) \\
&+ \lambda_1\left[F\left(K_1,\int_\Theta \pi(\theta)z_1(\theta)d\theta\right)+(1-\delta_k)K_1-G_1-\pi(\theta)c_1(\theta)-\pi(\theta)x_1(\theta)-\pi(\theta)y_1(\theta)-K_2\right] \\
&+ \lambda_2\left[F\left(K_2,\int_\Theta \pi(\theta)z_2(\theta)d\theta\right)+(1-\delta_k)K_2-G_2-\pi(\theta)u^{-1}\left[\tfrac{1}{\beta}\left(W(\theta)+\phi\left(\tfrac{z_1(\theta)}{h_1}\right)-u\left(c_1(\theta)\right)\right)+\phi\left(\tfrac{z_2(\theta)}{h_2(\theta)}\right)\right]\right] \\
&+ \mu(\theta)\left[-u'\left(c_1(\theta)\right)\tfrac{\partial y_1(\theta)}{\partial\theta}+\beta\phi'\left(\tfrac{z_2(\theta)}{h_2(\theta)}\right)\tfrac{z_2(\theta)}{\left[h_2(\theta)\right]^2}\tfrac{\partial h_2(\theta)}{\partial\theta}\right],
\end{aligned}
$$

along with boundary conditions

$$\mu(\underline{\theta})=\lim_{\theta\to\underline{\theta}}\mu(\theta)=0 \quad \text{and} \quad \mu(\overline{\theta})=\lim_{\theta\to\overline{\theta}}\mu(\theta)=0. \tag{3a}$$

The first-order conditions with respect to $c_1(\theta)$, $z_1(\theta)$, $z_2(\theta)$, K_2 and $x_1(\theta)$ are, respectively,

$$\frac{\partial\mathcal{H}}{\partial c_1(\theta)}=-\lambda_1\pi(\theta)+\lambda_2\pi(\theta)\frac{u'\left(c_1(\theta)\right)}{\beta u'\left(c_2(\theta)\right)}-\mu(\theta)u''\left(c_1(\theta)\right)\tfrac{\partial y_1(\theta)}{\partial\theta}=0, \tag{3b}$$

$$\frac{\partial\mathcal{H}}{\partial z_1(\theta)}=\lambda_1F_z(K_1,Z_1)\pi(\theta)-\lambda_2\pi(\theta)\frac{\phi'\left(\tfrac{z_1(\theta)}{h_1}\right)\tfrac{1}{h_1}}{\beta u'\left(c_2(\theta)\right)}=0, \tag{3c}$$

$$\frac{\partial\mathcal{H}}{\partial z_2(\theta)}=\lambda_2\pi(\theta)\left[F_z(K_2,Z_2)-\frac{\phi'\left(\tfrac{z_2(\theta)}{h_2}\right)\tfrac{1}{h_2}}{u'\left(c_2(\theta)\right)}\right]+\frac{\beta\mu(\theta)\tfrac{\partial h_2(\theta)}{\partial\theta}}{\left[h_2(\theta)\right]^2}\left[\phi''\left(\tfrac{z_2(\theta)}{h_2(\theta)}\right)\tfrac{z_2(\theta)}{h_2(\theta)}+\phi'\left(\tfrac{z_2(\theta)}{h_2(\theta)}\right)\right]=0, \tag{3d}$$

$$\frac{\partial\mathcal{H}}{\partial K_2}=-\lambda_1+\lambda_2\left[F_k(K_2,Z_2)+1-\delta_k\right]=0, \tag{3e}$$

$$
\begin{aligned}
\frac{\partial\mathcal{H}}{\partial x_1(\theta)} &= -\lambda_1\pi(\theta)+\frac{\lambda_2\pi(\theta)}{u'\left(c_2(\theta)\right)}\phi'\left(\tfrac{z_2(\theta)}{h_2(\theta)}\right)\tfrac{z_2(\theta)}{\left[h_2(\theta)\right]^2}\psi_x\left(x_1(\theta),y_1(\theta)\right) \\
&+\mu(\theta)\beta\left\{\begin{array}{l}\tfrac{d}{dh_2(\theta)}\left[\phi'\left(\tfrac{z_2(\theta)}{h_2(\theta)}\right)\tfrac{z_2(\theta)}{\left[h_2(\theta)\right]^2}\right]\tfrac{\partial h_2(\theta)}{\partial\theta}\psi_x\left(x_1(\theta),y_1(\theta)\right) \\ +\phi'\left(\tfrac{z_2(\theta)}{h_2(\theta)}\right)\tfrac{z_2(\theta)}{\left[h_2(\theta)\right]^2}\psi_{yx}\left(x_1(\theta),y_1(\theta)\right)\tfrac{\partial y_1(\theta)}{\partial\theta}\end{array}\right\}=0.
\end{aligned} \tag{3f}
$$

The law of motion for the co-state $\mu(\theta)$ is

$$\frac{d\mu(\theta)}{d\theta}=-\frac{\partial\mathcal{H}}{\partial W(\theta)}=-\pi(\theta)\left[1-\frac{\lambda_2}{\beta u'\left(c_2(\theta)\right)}\right]. \tag{3g}$$

Note that condition (3e) for capital is the same as the corresponding condition in the Ramsey model. Moreover, if the IC constraint is not binding and thus $\mu(\theta)=0$, conditions (3b), (3d) and (3f) reduce to standard conditions for consumption, effective labor and education investment in both periods in the Ramsey model, wherein the discounted marginal utility of consumption and effective labor for each type is equal to the marginal cost. Then, (3g) does not apply. However, with a binding IC constraint and thus $\mu(\theta)\neq 0$, conditions (3b)-(3g) differ from those in the Ramsey model.

Based on (3a) and (3g), in the Appendix we have shown the following lemma.

Lemma 1. *Suppose that $c_2(\theta)$ is monotone increasing in θ. Then, $\lambda_2>0$ and $\mu(\theta)<0$ for $\theta\in(\underline{\theta},\overline{\theta})$.*

As is standard, there is a positive co-state λ_2 for the resource constraint. Moreover, according to (3a), for those at the boundary of the type distribution, $\mu(\underline{\theta})=\mu(\bar{\theta})=0$ and the shadow price of information friction is zero. However, for those inside the boundary of the type distribution, $\mu(\theta)<0$. A negative co-state $\mu(\theta)$ of an incentive-compatibility constraint measures the magnitude of the welfare cost due to information frictions.[11]

With $\mu(\theta)<0$, we can rearrange conditions (3b)-(3g) to obtain the following three conditions, which characterize the constrained efficient allocation determined by the social planner.

$$\frac{u'(c_1(\theta))}{\beta u'(c_2(\theta))}=\left[F_k(K_2,Z_2)+1-\delta_k\right]+\frac{\mu(\theta)}{\lambda_2\pi(\theta)}u''(c_1(\theta))\tfrac{\partial y_1(\theta)}{\partial\theta}, \tag{4a}$$

$$\frac{\phi'\left(\frac{z_1(\theta)}{h_1}\right)\frac{1}{h_1}}{u'(c_1(\theta))}=F_z(K_1,Z_1)-\frac{\mu(\theta)}{\lambda_2\pi(\theta)}\frac{\beta u'(c_2(\theta))u''(c_1(\theta))F_z(K_1,Z_1)\frac{\partial y_1(\theta)}{\partial\theta}}{u'(c_1(\theta))}, \tag{4b}$$

$$\frac{\phi'\left(\frac{z_2(\theta)}{h_2}\right)\frac{1}{h_2}}{u'(c_2(\theta))}=F_z(K_2,Z_2)+\frac{\mu(\theta)}{\lambda_2\pi(\theta)}\frac{\beta\left[\phi''\left(\frac{z_2(\theta)}{h_2(\theta)}\right)\frac{z_2(\theta)}{h_2(\theta)}+\phi'\left(\frac{z_2(\theta)}{h_2(\theta)}\right)\right]\frac{\partial h_2(\theta)}{\partial\theta}}{\left[h_2(\theta)\right]^2}. \tag{4c}$$

In (4a), the social planner equalizes the household's marginal rate of substitution (henceforth, MRS) between consumption today and tomorrow to the marginal rate of transformation (henceforth, MRT) between consumption and investment today. As investment today accumulates capital tomorrow, the MRT between consumption and investment today is the firm's marginal product of capital (henceforth, MPK) tomorrow. Moreover, (4b) and (4c) suggest that the social planner equalizes the household's MRS between leisure and consumption to the MRT between labor and consumption. The MRT between labor and consumption is a firm's marginal product of labor (henceforth, MPL).

3.2 Properties of the Optimum and Wedges

In the second-best optimum, distortions in agents' choices may be described by wedges. A wedge measures distortions on the second-best allocation relative to the laissez-faire allocation. Agents' work effort, consumption and non-verifiable education expenses are private information, which causes distortions. The government can correct the distortion by levying taxes on two observables: labor income and capital income. There are two marginal distortions on the allocation, defined as the labor wedge and the capital wedge as follows.

$$\tau_{z_t}(\theta)\equiv 1-\frac{\phi'\left(\frac{z_t(\theta)}{h_t(\theta)}\right)\frac{1}{w_t h_t(\theta)}}{u'(c_t(\theta))}, \tag{5a}$$

$$\tau_{k_t}(\theta)\equiv 1-\frac{u'(c_{t-1}(\theta))}{\beta R_t E\left[u'(c_t(\theta))\right]}, \tag{5b}$$

[11] A negative co-state $\mu(\theta)$ may be viewed as the marginal welfare loss in order for the social planner to choose allocations that are incentive-compatible. If the IC constraint is not binding, which arises when agents have no incentives to cheat, then $\mu(\theta)=0$ and the social planner does not have to sacrifice the welfare.

where $w_t = F_Z(K_t, Z_t)$ and $R_t = F_k(K_t, Z_t) + (1-\delta_k)$

In the dynamic taxation literature, the labor wedge is an i*ntratemporal* wedge, which is defined as the difference of the household's MRS between labor and consumption today from the firm's MPL (i.e., the wage rate). In a similar fashion, the capital wedge is an *intertemporal* wedge, which is defined as the difference of the household's MRS in consumption between today and tomorrow from the firm's MPK tomorrow (i.e., the rental rate). The labor and capital wedges serve to measure the distortion of the second-best allocation relative to laissez-faire allocation. In a laissez-faire economy, there is no information distortion, and thus the labor and capital wedges are zero. However, in an economy where consumption today is not observable due to non-verifiable education expenses, the information asymmetry causes the labor and capital wedge to deviate from zero in the social planner's problem. A positive labor wedge indicates that labor is distorted downwards and thus too much labor, while a negative labor wedge means that labor is distorted upwards. Similarly, a positive capital wedge reveals that consumption is distorted downwards and thus too much savings, and a negative capital wedge suggests that consumption is distorted upwards and thus too little savings.

Substituting (4a)-(4c) into (5a) and (5b), it is clear that the intertemporal wedge and the intratemporal wedge both are zero for the agents at the top and the bottom of the type distribution: $\tau_{k_t}(\bar{\theta}) = \tau_{k_t}(\underline{\theta}) = 0$ and $\tau_{z_t}(\bar{\theta}) = \tau_{z_t}(\underline{\theta}) = 0.$ This result confirms the property of "no distortion at the top and the bottom" in static models of Mirrlees (1971) and Stiglitz (1982), which states that the consumption-labor decision made by agents of the top and the bottom types should be undistorted. The result is valuable in that the property in the existing new public finance literature with exogenous skills (e.g., Farhi and Werning, 2013) is robust to the environment with endogenous skills and private consumption.

Other than the top and the bottom types, we use (4a)-(4c) to rewrite the intertemporal and the intratemporal wedge in (5a) and (5b), respectively, as follows:

$$\tau_{k_2}(\theta) = \frac{-\mu(\theta)}{\lambda_2 \pi(\theta)} \frac{u''(c_1(\theta)) \frac{\partial y_1(\theta)}{\partial \theta}}{F_k(K_2, Z_2) + 1 - \delta_k}, \tag{6a}$$

$$\tau_{z_1}(\theta) = \frac{\mu(\theta)}{\lambda_2 \pi(\theta)} \frac{\beta u'(c_2(\theta)) u''(c_1(\theta)) \frac{\partial y_1(\theta)}{\partial \theta}}{u'(c_1(\theta))}, \tag{6b}$$

$$\tau_{z_2}(\theta) = \frac{-\mu(\theta)}{\lambda_2 \pi(\theta)} \frac{\beta \left[\phi''\left(\frac{z_2(\theta)}{h_2(\theta)}\right) \frac{z_2(\theta)}{h_2(\theta)} + \phi'\left(\frac{z_2(\theta)}{h_2(\theta)}\right) \right] \frac{\partial h_2(\theta)}{\partial \theta}}{\left[h_2(\theta)\right]^2 F_z(K_2, Z_2)}. \tag{6c}$$

To determine the sign of those wedges, we need to analyze agents' behavior. In the next subsection, we study the optimal choice of the agent with a reporting strategy $\sigma(\theta)$.

3.3 Optimal Behavior of the Agent

Because some education expenses are non-verifiable, agents with a reporting strategy $\sigma(\theta)$ can reallocate expenses between consumption and education without being caught by the social planner, as long as the sum of these two expenses is consistent with the announced type.

Given the constrained efficient allocation $\{c_1 + y_1, x_1, c_2, k, z_1, z_2\}$ that the social planner allocates for truth-telling agents, an agent with a reporting strategy $\sigma(\theta)$ will choose the allocation $\{c_1, y_1, h_2\}$ to maximize the following problem:

$$\max_{c_1, y_1, h_2} u(c_1) - \phi\left(\frac{z_1(\sigma)}{h_1}\right) + \beta\left[u(c_2(\sigma)) - \phi\left(\frac{z_2(\sigma)}{h_2}\right)\right], \tag{7}$$

s.t. $c_1 + y_1 = c_1(\sigma) + y_1(\sigma)$ and $h_2 = (1-\delta_h)h_1 + \psi(x_1(\sigma), y_1) + \theta$, with h_1 given.

According to Definition 1, the constrained efficient allocation $\{c_1^\sigma(\theta), y_1^\sigma(\theta), h_2^\sigma(\theta)\}$ must solve the above problem. We denote $\phi_h\left(\frac{z_t}{h_t}\right) \equiv -\phi'\left(\frac{z_t}{h_t}\right)\frac{z_t}{(h_t)^2} < 0$ and $\phi_{hh}\left(\frac{z_t}{h_t}\right) \equiv \phi''\left(\frac{z_t}{h_t}\right)\frac{(z_t)^2}{(h_t)^4} + 2\phi'\left(\frac{z_t}{h_t}\right)\frac{z_t}{(h_t)^3} > 0$. Then, the following theorem can be proved directly from the first-order conditions of the above problem.[12]

Theorem 1. *The constrained efficient allocation A satisfies*

$$u'(c_1^\sigma(\theta)) = -\beta\phi_h\left(\frac{z_2(\sigma)}{h_2^\sigma(\theta)}\right)\psi_y(x_1(\sigma), y_1^\sigma(\theta)).$$

Furthermore, $\frac{\partial y_1^\sigma(\theta)}{\partial\theta} < 0$ *and* $\frac{\partial h_2^\sigma(\theta)}{\partial\theta} > 0$.

The theorem says that, an agent with a reporting strategy σ(θ) spends on non-verifiable education y_1 until the marginal utility cost of foregone consumption today equal to the resulting lower discounted marginal disutility of labor resulting from higher human capital tomorrow. Moreover, given the same reporting type σ, high-type agents tend to invest less expenses in non-verifiable education y_1 today but they still have higher human capital level h_2 tomorrow than those with lower types.

Note that if the model reduces to one with only verifiable education expenses $x_1(\theta)$, then $y_1(\theta)$=0. In this situation, Theorem 1 does not apply except for $\frac{\partial h_2(\theta)}{\partial\theta} > 0$.

4. The Signs of Capital and Labor Wedges

We are ready to analyze the wedge. First, we've proved the sign of for the intertemporal wedge.

Proposition 1. *If there are no non-verifiable education expenses,* $\beta R_2 u'(c_2(\theta)) = u'(c_1(\theta))$ *for* $\theta \in [\underline{\theta}, \overline{\theta}]$. *But, if there are non-verifiable education expenses,* $\beta R_2 u'(c_2(\theta)) > u'(c_1(\theta))$ *for* $\theta \in (\underline{\theta}, \overline{\theta})$.

Thus, if there are only verifiable and no non-verifiable education expenses, even with skill shocks, the standard consumption Euler equation holds and thus, the intertemporal wedge is zero. Yet, when

[12] All the proofs for the theorems, propositions and lemmas below are relegated to the Appendix.

there are non-verifiable education expenses, then the consumption Euler equation does not hold and the intertemporal wedge is positive; i.e, $\tau_{k_2}(\theta) > 0$ for $\theta \in (\underline{\theta}, \overline{\theta})$.

The reason goes as follows. With non-verifiable education expenses, the marginal rate of substitution for consumption in different periods is distorted. The agent may underreport his type by substituting away from non-verifiable education expenses toward consumption. While an agent under-reports his type by choosing a high level of consumption today, he would want to save more for tomorrow in order to smooth the consumption. However, since savings are observable, an under-reporting agent would have to save as much as his reporting type in order to avoid being caught. A positive capital wedge would reduce the incentive to save and discourages agents from under-reporting. The policy is an efficient way to deter agents from cutting off non-verifiable education expenses for more consumption and to foster investment in human capital.

Next, we state the proposition that establishes the sign of the intratemporal wedge.

Proposition 2. *If there are no non-verifiable education expenses,* $\tau_{z_1}(\theta) = 0$ *and* $\tau_{z_2}(\theta) > 0$ *for* $\theta \in (\underline{\theta}, \overline{\theta})$. *But, if there are non-verifiable education expenses,* $\tau_{z_1}(\theta) < 0$ $\tau_{z_1}(\theta) < 0$ *and* $\tau_{z_2}(\theta) > 0$ $\tau_{z_2}(\theta) > 0$ *for* $\theta \in (\underline{\theta}, \overline{\theta})$.

Thus, if there are only verifiable and no non-verifiable education expenses, the intratemporal wedge is zero in the first period and is positive in the second period. With only verifiable education expenses, agents cannot cut down education expenses without being caught. In this case, given that initial human capital is the same for all skill types, there is no distortion between consumption and labor in the first period and thus, the intratemporal wedge is zero in the first period.

Yet, when there are non-verifiable education expenses, the intratemporal wedge is negative in the first period. Intuitively, agents with higher skill shocks have an advantage in education and learning. As education expenses, and thus consumption, are not observable, agents with higher skill shocks may underreport their types. If agents underreport their types, owing to negative labor wedge they would have to work even more hours and allocate less expenses in education and even more in consumption. However, increasing an already high consumption would raise their utility only by a small margin, but working even more hours would decrease their utility by a large margin. A negative labor tax serves as a mechanism to induce agents to work according to their true types and spend more on education.

In the second period, and thus the terminal period, everybody has no incentives to accumulate human capital. This goes back the standard Mirrlees literature. Thus, the labor tax is positive.

We must emphasize that our positive intertemporal wedge is obtained, not because there are skill shocks but because consumption is not observable. This mechanism is in sharp contrast to the existing Mirrlees models without human capital (e.g., Goloslov et al. 2006; Werning, 2007; Farhi and Werning, 2013) and with human capital (e.g., da Costa and Maestri, 2007; Anderberg, 2009; Stantcheva, 2017), wherein a positive intertemporal wedge is obtained, because skill shocks change stochastically over time. Thus, their positive intertemporal wedge is for an insurance purpose against lifetime risks.

Moreover, our negative labor income tax in the first period. In the extension to a T-period model later, we will show that the labor income tax is negative in an agent's early lifecycle. This is a new result in the dynamic Mirrlees literature. In the existing literature, in order to prevent the agents from misreporting, the labor wedge is positive and constant over time (e.g., Golosov et al., 2006; Werning, 2007; Farhi and Werning, 2013). Even in the studies with verifiable education expenses by Bovenberg and Jacobs (2005) and Stantcheva (2017), their labor wedges are positive. Our results of negative labor income taxes are different, because some education expenses are non-verifiable. In this situation, a subsidy to labor supply in an early lifecycle serves as a mechanism to correct distortions that arise from non-verifiable education expenses. This prevents agents from misreporting, and induces them to invest in human capital.

Elsewhere, Diamond (1980) and Saez (2002) also obtained a negative labor wedge at the bottom of the income distribution in models that considered an extensive labor supply. In their papers, a subsidy to the working poor is optimal, because the participation effect of the labor force dominates the incentive effect of higher income earners. A negative labor wedge in our paper may be interpreted as a subsidy to the working poor, but the result is based on a different mechanism. Our negative labor wedge aims to encourage people to reduce consumption and increase education expenses.

Finally, our positive capital taxes and negative labor taxes in an early lifecycle is due to unobservable consumption. Unobservable consumption was the key feature in Allen (1985) and Cole and Kocherlakota (2001). In their models, consumption is not observable, because agents are allowed to save or borrow secretly. Our model is different. Consumption is unobservable here, because the government cannot tell an agent's education expenses from consumption expenses. Moreover, the analysis is different. Allen (1985) analyzed whether the optimal long-term contract is better than a series of unrelated short-term contracts and Cole and Kocherlakota (2001) explored whether the efficient allocation of consumption can be decentralized through a competitive asset market. Our paper investigates whether unobservable consumption affects the design of the optimal tax policy on capital and labor over time, which was not studied by these two papers.

5. Implementation

While it is tempting to interpret these capital and labor wedges defined in (5a) and (5b) as capital and labor taxes, the relationship between wedges and taxes is not straightforward, because there is a *double deviation* problem.[13] In this section, we propose a tax system that implements the constrained efficient

[13] Intuitively, each wedge controls only one aspect of a worker's behavior (labor in a period, or savings) taking all other choices fixed *at the optimal level.* For example, assuming that an agent supplies the socially optimal amount of labor, a capital tax defined by an intertemporal wedge would ensure that the agent also makes a socially optimal amount of savings. However, agents choose labor and savings jointly; if an agent considers changing her labor, then, in general, she also considers changing her savings. Thus, there are *double deviations*. Kocherlakota (2005), Albanesi and Sleet (2006) and Golosov and Tsyvinski (2006) showed that such double deviations would give an agent a higher utility than the utility from the socially optimal allocations, and therefore the optimal tax system must be enriched with additional elements in order to implement the optimal allocations.

allocation as a competitive equilibrium.

There are different tax systems that can implement constrained efficient allocations as a competitive equilibrium. Albanesi and Sleet (2006) implemented constrained efficient allocations in terms of nonlinear taxes in a competitive equilibrium. They showed that optimal taxes on agents' wealth and labor income are non-linear and non-separable and depend on accumulated wealth and current labor income and not on past history. By restricting his model to linear capital and arbitrarily nonlinear labor income taxes, Kocherlakota (2005) implemented constrained efficient allocations by separating capital from labor income taxes and both taxes are history-dependent. Following the tax structure in Kocherlakota (2005), Grochulski and Piskorski (2010) found that deferred capital taxes are the necessary condition for linear capital taxes, with negative expected capital taxes in early lifecycle and positive expected capital taxes in later lifecycle, so the *ex ante* expected present value of lifetime capital taxes is zero. Parallel to these studies, our paper proposes a tax system to implement constrained efficient allocations.

Golosov et al. (2006) pointed out that the simplest method of implementation is to assign arbitrarily high punishments if agents' observable allocation in any period is different from the constrained efficient allocation. Yet, this way severely limits an agent's choices and may be unrealistic. To relax the limitation and to create a direct connection between wedges and taxes, we provide a tax system to implement the constrained efficient allocation in a competitive equilibrium. Different from Grochulski and Piskorski (2010), deferred capital taxes are not necessary for linear capital taxes. We show that these optimal linear capital and labor tax rates are exactly the same as the wedges established by the social planner in subsection 3.2.

5.1 A Class of the Tax System

In the tax system, if observable allocations (z_1, k_2, c_2) satisfy the following two conditions, then a tax system of linear factor income tax rates along with lump-sum taxes can implement constrained efficient allocations.

$$S_1^\theta(z_1, k_2) \equiv \left(1 - \tau_{z_1}(\theta)\right) w_1\left(z_1(\theta) - z_1\right) - \left(k_2(\theta) - k_2\right) = 0,$$

$$S_2^\theta(c_2, k_2) \equiv \left(1 - \tau_{k_2}(\theta)\right)\left(1 + r_2\right)(k_2(\theta) - k_2) - \left(c_2(\theta) - c_2\right) = 0.$$

If these two above conditions are not met, agents will be punished.

The tax system is $T = \{T_1, T_2\}$ described as follows.[14] In the first period, the taxes are

$$T_1 = \tilde{T}_1(\theta) \equiv \Gamma_1(\theta) + \tilde{\tau}_{z_1}(\theta) w_1 z_1,$$

if there is some $\theta \in \Theta$ such that the condition $S_1^\theta(z_1, k_2) = 0$ holds and $x_1 = x_1(\theta)$; otherwise, $T_1 = \infty$.

In the second period, the taxes are

[14] By simply restricting $x_1 = x_1(\theta)$ for some $\theta \in \Theta$, when the agent chooses allocations $\{ c_1, c_2, y_1, h_2, z_1.z_2, k_2 \}$ to maximize the lifetime utility in Subsection 5.2, without further restrictions it is impossible for the resulting allocation to be the same as the constrained efficient allocation. By adding these additional constraints $S_1^\theta(z_1, k_2) = 0$ and $S_2^\theta(c_2, k_2) = 0$, the resulting allocation is the same as the constrained efficient allocation. See Appendix A5.

$$T_2=\tilde{T}_2(\theta)\equiv\Gamma_2(\theta)+\tilde{\tau}_{z_2}(\theta)w_2z_2+\tilde{\tau}_{k_2}(\theta)r_2k_2,$$

if there is some $\theta\in\Theta$ such that the condition $S_1^\theta(z_1,k_2)=S_2^\theta(k_2,c_2)=0$ holds; otherwise, $T_2=\infty$.

The tax system is explained as follows. Linear labor tax rates and linear capital tax rates $(\tilde{\tau}_{z_t}(\theta),\tilde{\tau}_{k_2}(\theta))$ and lump-sum taxes $\Gamma_t(\theta)$, $t=1, 2$ and $\theta\in(\underline{\theta},\overline{\theta})$, are designed for agents who meet the two conditions $S_1^\theta(z_1,k_2)=S_2^\theta(k_2,c_2)=0$. If any one of these two conditions is not met, then agents will be punished sufficiently severely.

5.2. Agent's Problem

Given the price $\{r_1, r_2, w_1, w_2\}$ and initial physical and human capital (k_1, h_1), the agents face the tax system $T=\{T_1, T_2\}$. The problem of an agent of type θ is:

$$max\ u(\tilde{c}_1(\theta))-\phi\left(\tfrac{\tilde{z}_1(\theta)}{h_1}\right)+\beta\left[u(\tilde{c}_2(\theta))-\phi\left(\tfrac{\tilde{z}_2(\theta)}{\tilde{h}_2(\theta)}\right)\right],$$

subject to after-tax budget constraints and human capital accumulation:

$$\tilde{c}_1(\theta)+\tilde{x}_1(\theta)+\tilde{y}_1(\theta)+\tilde{k}_2(\theta)\le w_1\tilde{z}_1(\theta)+(1+r_1)k_1-T_1,$$

$$\tilde{c}_2(\theta)\le w_2\tilde{z}_2(\theta)+(1+r_2)\tilde{k}_2(\theta)-T_2,$$

$$\tilde{h}_2(\theta)=(1-\delta_h)h_1+\psi(\tilde{x}_1(\theta),\tilde{y}_1(\theta))+\theta,$$

where the maximization is made over $\left\{\tilde{c}_1(\theta),\tilde{c}_2(\theta),\tilde{z}_1(\theta),\tilde{z}_2(\theta),\tilde{x}_1(\theta),\tilde{y}_1(\theta),\tilde{h}_2(\theta),\tilde{k}_2(\theta)\right\}$.

Moreover, given the price $\{r_1, r_2, w_1, w_2\}$, the problem of the representative firm is:

$$\max_{K_t,Z_t}F(\tilde{K}_t,\tilde{Z}_t)-w_t\tilde{Z}_t-(r_t+\delta_k)\tilde{K}_t,\ t=1, 2.$$

5.3 Implementation with the Tax System

We define the competitive equilibrium and the optimal tax system.

Given a tax system $T=\{T_1, T_2\}$ and government expenditure $\{G_1, G_2\}$, a ***competitive equilibrium*** is an allocation $\tilde{A}=(\tilde{c},\tilde{x},\tilde{y},\tilde{h},\tilde{l},\tilde{z},\tilde{k},\tilde{Z},\tilde{K})$ and prices $\{r_1, r_2, w_1, w_2\}$ such that,

(1) given prices $\{r_1, r_2, w_1, w_2\}$, the allocation $\tilde{A}$ for type θ solves the individual problem of type θ;

(2) prices $\{r_1, r_2, w_1, w_2\}$ are solved by the firm's problem: $w_t=F_z(\tilde{K}_t,\tilde{Z}_t)$ and $r_t=F_k(\tilde{K}_t,\tilde{Z}_t)-\delta_k$;

(3) markets are clear and thus, $\tilde{A}$ is a feasible allocation;

(4) the government balances the budget in each period: $\int_\Theta\pi(\theta)\tilde{T}_1(\theta)\ge G_1$ and $\int_\Theta\pi(\theta)\tilde{T}_2(\theta)\ge G_2$.

Definition 2. A tax system $T=\{T_1, T_2\}$ is **optimal** if it implements the constrained efficient allocation A as a competitive equilibrium allocation $\tilde{A}$.

Then, we can establish the following theorem.

Theorem 2. *There exists an optimal tax system T = {T_1, T_2} such that these two linear factor income tax rates are consistent with the wedges. That is,* $\tilde{\tau}_{k_2}(\theta) = \tau_{k_2}(\theta)$ *and* $\tilde{\tau}_{z_t}(\theta) = \tau_{z_t}(\theta)$ *for t =1, 2.*

Theorem 2 says that our tax system can implement the constrained efficient allocation as a competitive equilibrium. Moreover, these linear capital and labor tax rates $(\tilde{\tau}_{z_t}(\theta), \tilde{\tau}_{k_2}(\theta))$ in competitive equilibrium are consistent with the wedges $(\tau_{z_t}(\theta), \tau_{k_2}(\theta))$ in the planning problem in Subsection 3.2.

6. Numerical Analysis

In this section, we offer numerical analysis to highlight the quantitative importance of our results. Our numerical analysis takes a middle position between a simple demonstration of the optimal mechanism and a careful calibration of quantitative implications for the optimal wedge. As we will see, the results in Propositions 1 and 2 are robust and the solution of the relaxed planning problem indeed satisfies incentive compatible constraints. Our quantitative exercises also shed light on new insights.

6.1 Calibration

We calibrate the model economy based on the US data and then quantitatively solve the constrained efficient allocation. The calibration proceeds as follows.

First, we construct a baseline economy, which is otherwise the same as our model in Section 2, except that the economy is decentralized and the tax system is not optimal. The capital and labor income taxes are linear and set to their current average levels in the US.

Agents are set to have two periods of lives. Each period represents 20 years.[15] In the baseline, some parameter values are set based on the existing literature, normalization or assumptions. Table 1 lists these parameter values. The rest of the parameter values are endogenously calibrated to match the data. Table 2 summarizes these endogenously calibrated parameter values.

[Insert Tables 1 and 2 here]

For the tax system in the baseline economy, according to McDaniel (2007), the average tax rates in the US during 1960-2007 for the capital and the labor income are around 0.3 and 0.2, respectively. Thus, in our baseline economy, we set $\tau_k^b = 30\%$ and $\tau_z^b = 20\%$. We assume zero government expenditure G_t=0 in every period, so the tax revenue is equally redistributed to agents as a lump-sum transfer LS_t. An agent's budget constraints are as follows:

$$c_1 + x_1 + y_1 + k_2 \le \left(1 - \tau_z^b\right) w_1 z_1 + R_1 k_1 + LS_1, \tag{8a}$$

$$c_2 \le \left(1 - \tau_z^b\right) w_2 z_2 + \left(1 - \tau_k^b\right) R_2 k_2 + LS_2. \tag{8b}$$

As for the human capital accumulation, under construction, initial human capital level is equal for

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[15] We assume that agents enter the economy at age 20 and retire at age 61. The first period represents ages between 20 and 40, and the second period ages between 41 and 60.

all agents; hence, we normalize the initial human capital level at h_1=1. For simplicity, we assume that the human capital is fully depreciated in 20 years, and thus set δ_h=1. Hence, based on δ_h=1 and Assumption 1, an agent's human capital in period 2 takes the following form

$$h_2 = \psi(x_1, y_1) + \theta. \tag{8c}$$

Following Farhi and Werning (2013), the skill shock θ is log normally distributed so that

$$\log\theta \overset{i.i.d}{\sim} N\left(-\tfrac{\hat{\sigma}^2}{2}, \hat{\sigma}^2\right).$$

Moreover, following Farhi and Werning (2013), we assume that the distribution moves proportionally over a finite interval $[\underline{\theta}, \overline{\theta}]$ with the value of the degree of uncertainty at $\hat{\sigma}^2 = 0.0095$.[16] The bottom of the type distribution is normalized at $\underline{\theta}=1$, and the top of the type distribution $\overline{\theta}$ is calibrated to target the wage premium.[17]

For human capital investment, following Ewijk and Tang (2000) we use the Cobb-Douglas form:

$$\psi(x, y) = B\left(x^{1-\rho} y^{\rho}\right)^{\eta}.$$

Note that the form reduces to the case with only verifiable education expenses if ρ = 0. For parameter values, following Ewijk and Tang (2000), we set η =0.4 and ρ=0.667, and the technology level B is calibrated to target the capital-output ratio.

The periodic utility function takes the following form:

$$u(c_t) - \phi\left(\frac{z_t}{h_t}\right) = \frac{c_t^{1-\chi}}{1-\chi} - \frac{1}{\kappa}\left(\frac{z_t}{h_t}\right)^{\kappa}.$$

Following Farhi and Werning (2013), we set κ=3, which implies the Frisch elasticity for labor of 0.5. Following Conesa et al. (2009), we set χ=2, implying the intertemporal elasticity of substitution of 0.5. The discount factor is set at 4% per annum, which gives $\beta=(0.96)^{20}=0.442$ for 20 years.

The technology of the final good is assumed to be the Cobb-Douglas form:

$$Y = F(K, Z) = AK^{\xi} Z^{1-\xi}. \tag{8d}$$

Following Conesa et al. (2009), we normalize A=1 and set ξ=0.36. The initial capital is normalized at $K_1 = 1$, and we assume that the physical capital is fully depreciated after 20 years, that is δ_k =1.

The technology level of human capital accumulation B and the top of the type distribution $\overline{\theta}$ are calibrated. We calibrate these two parameters to match the data of the capital-output ratio and the wage premium, respectively. Following Peterman (2016), the capital-output ratio in the US is 2.7 per annum,

[16] The degree of uncertainty $\hat{\sigma}^2$ is empirically estimated by matching the increase in the cross-sectional variance of wages or earnings in a given cohort as this cohort ages. The estimate depends on whether time fixed effects (smaller estimates) or cohort fixed effects (larger estimates) are imposed, and on the time period (larger estimates in the 1980's). Using cohort fixed effects over the period 1967–1996, Heathcote *et al.* (2005) find $\hat{\sigma}^2 = 0.0095$ for the wage of male individuals, which value was used by Farhi and Werning (2013), and we follow suit.

[17] Different from Farhi and Werning (2013), wherein each agent's skill exogenously follows an AR(1) process induced by i.i.d. productivity shocks, in our paper each agent's skill evolves via both endogenous human capital investment and i.i.d. skill shocks θ.

which implies a 20-year capital-output ratio of 0.135. The estimated value for the wage premium in the literature lies within 1.2 and 2.4, such as 1.26-1.74 in Murphy and Welch (1992), 1.37-1.75 in Autor et al. (1998), 1.7-2.4 in Heathcote et al. (2010), and 1.2-2.2 in James (2012). Our calibration targets a medium value of 1.8. To match the wage premium, we go along Stancheva (2017) and compute the labor income of the top 42 percent relative to the bottom 42 percent in the population.

We calibrate the values of B and $\bar{\theta}$ in the following way. First, we derive the individual's problem in the decentralized economy. In Appendix A7, we have set up the problem of an agent with skill type θ. Based on the parametric functional forms, we employed the first-order conditions lead to derive five necessary conditions.

Next, with these five necessary conditions, along with equations (8a)-(8c), and based on the parameter values in Table 1, we solve the allocation $\{c_1(\theta), c_2(\theta), z_1(\theta), z_2(\theta), x_1(\theta), y_1(\theta), k_2(\theta), h_2(\theta)\}$ for each skill type θ. Specifically, we guess initial values for B and $\bar{\theta}$, and use these nine equations above to solve the allocation for each skill type θ. The resulting allocation is then used to compute the capital-output ratio and the wage premium. If the resulting capital-output ratio and wage premium are different from target numbers, we adjust the values of B and $\bar{\theta}$ and re-compute the allocation using these nine equations. Then, we compute the resulting capital-output ratio and the wage premium. The process is repeated, until the resulting capital-output ratio and the wage premium reach the target values of 0.135 and 1.8, respectively. The resulting calibrated value for B is 4.03 and the resulting calibrated values of $\bar{\theta}$ is 2.85. See Table 2.

Finally, we envisage the numerical results. We apply these parameter values to the second-best economy and calculate the constrained efficient allocation, capital wedges and labor wedges. In addition, we compare the capital and labor wedge in models with and without non-verifiable education expenses.

6.2 Results

6.2.1 Capital and labor wedges under different distributions of skill shocks

To start, we quantify the signs of the capital wedge and the labor wedge for agents of different skill types. Under the log normal distribution of skill shocks, the resulting wedges are shown by solid lines in the top panel of Figure 1. It is clear to see that the capital wedge is positive, and the labor wedge is negative in the first period and positive in the second period.

[Insert Figure 1 here]

To see whether the wedges are affected by different distributions of the skill type, we also illustrate the results when the skill shock θ is drawn from the uniform distribution. See the solid lines in the bottom panel of Figure 1. As Figure 1 shows, although different distributions of skill shocks may affect the shape of the wedges for different skill types, they do not affect the sign of the wedges. No matter whether the skill shock is log normally distributed or uniformly distributed, the capital wedge is positive, and the labor wedge is negative in the first period and positive in the second period, as predicted by the model.

6.2.2 Role of non-verifiable education expenses

Next, we investigate why non-verifiable education expenses, and thus unobservable consumption, are crucial in determining the sign of the wedges in our model. To understand this, we shut down non-verifiable education expenses in our model for now and thus, $\rho=0$. The resulting capital and labor wedges are reported in terms of dotted lines in both the top and the bottom panel of Figure 1. As the results show, when there are only verifiable education expenses, even with skill shocks, the capital wedge and labor wedge in the first period are both zero.

A comparison of solid lines and dotted lines in Figure 1 thus indicate that non-verifiable education expenses, and thus unobservable consumption, change the capital wedge from zero to a positive value and the labor wedge in period 1 from zero to a negative value. Moreover, while it is standard in the existing literature that the labor wedge is positive in the terminal period, when there are non-verifiable education expenses, the positive labor wedge in the terminal period is smaller than that when there are only verifiable education expenses. Thus, non-verifiable education expenses tend to lower the labor wedge.

6.2.3 Constrained efficient allocation

Now, we report constrained efficient allocations as planned by the social planner. These allocations are presented in terms of solid lines in Figures 2. For comparisons, we also compute the first-best planning allocations in an otherwise the same economy except for informational frictions. The results are illustrated in terms of dotted lines in Figure 2.

[Insert Figure 2 here]

First, for consumption, in the second-best planning problem, the social planner allocates consumption that is increasing with skill types in order to correct incentives. This is in contrast to the first-best planning problem wherein the social planner allocates consumption equally to all skill types.

Next, for effective labor, in the second-best planning problem, the social planner corrects the incentive by asking higher skill types to provide less effective labor in period 1 and more effective labor in period 2. Along with the fact that consumption is increasing with skill types, agents' lifetime utility is increasing with skill types, as shown in the last chart in the bottom panel of Figure 2. By contrast, in the first-best planning problem, the social planner allocates the same effective labor to all skill types in period 1 since agents are endowed with the same initial human capital, and asks higher skill types to offer more effective labor in period 2, which causes agents' lifetime utility to decrease with skill types.

Finally, for the human capital investment, the results show that both verifiable and non-verifiable education expenses are increasing with skill types in both the first-best and the second-best planning problems. As a result, the human capital is strictly increasing with skill types in period 2. In particular, it is clear to see that agents with lower skill types spend more verifiable and non-verifiable education expenses in the second-best planning problem than those in the first-best planning problem. The result indicates that the second-best planning program encourages human capital accumulation for lower skill types more than the first-best planning program.

6.2.4 Welfare

We now calculate the welfare gain of the constrained efficient allocation in the second-best planning economy. In particular, we compare the welfare gain of the second-best economy from the laissez-faire economy without taxes.

Let the welfare in the laissez-faire economy (*LF*) be denoted by $W^{LF}\left(c_1^{LF}(\theta),c_2^{LF}(\theta),l_1^{LF}(\theta),l_2^{LF}(\theta)\right)$, where $c_t^{LF}(\theta)$ and $l_t^{LF}(\theta)$ are, respectively, consumption and the labor supply of type θ in the first-best laissez-faire economy in time *t*. Let the welfare of the second-best economy (*SB*) be denoted by W^{SB}. The welfare gain of the second-best planning economy from the laissez-faire economy is defined in terms of consumption equivalence: the percentage increase in consumption in the second-best economy relative to the laissez-faire economy. Denote ω as the percentage increase in consumption. Then, the following condition is met:

$$W^{LF}\left((1+\omega_{SB})c_1^{LF}(\theta),(1+\omega_{SB})c_2^{LF}(\theta),l_1^{LF}(\theta),l_2^{LF}(\theta)\right)=W^{SB}.$$

In Farhi and Werning (2013), they compare the welfare gain with respect to three different estimated values of $\hat{\sigma}^2$: a low risk with $\hat{\sigma}^2=0.0061$, a medium risk with $\hat{\sigma}^2=0.0095$ and a high risk with $\hat{\sigma}^2=0.0161$. Following their work, we also compute the welfare gain with respect to these three different values. The results are reported in Table 3. As expected, the welfare gain is increasing with the value of $\hat{\sigma}^2$ (cf. top row in Table 3.) Intuitively, the more the risk is, the higher the welfare gain is in our second-best economy.

[Insert Table 3 here]

For comparisons, we compute the welfare gain of the first-best planning economy (*FB*) relative to the laissez-faire economy as follows:

$$W^{LF}\left((1+\omega_{FB})c_1^{LF}(\theta),(1+\omega_{FB})c_2^{LF}(\theta),l_1^{LF}(\theta),l_2^{LF}(\theta)\right)=W^{FB},$$

where W^{FB} denotes the welfare of the first-best planning economy. See the bottom row of Table 3.

It is clear that the overall welfare gain in our second-best planning economy is close to that in the first-best planning economy. Even so, for different agents, their welfare gain in our second-best planning economy may be different from that in the first-best planning economy. To see this, we compute the welfare gain for all individuals according to their skill types. The results are illustrated in Figure 3.

[Insert Figure 3 here]

It is clear from the figure that, in the first-best planning economy, in spite of a large welfare gain for the whole economy relative to that in the laissez-faire economy, agents with high skill types have a lower utility level than those with low skill types (cf. the dotted line). In particular, high skill types have negative welfare gains. This renders high types incentives to misreport their types in the first-best planning economy. By contrast, our second-best planning economy provides correct incentives for agents to work and learn. As a result, agents of all types have positive welfare gains (cf. the solid line in Figure 3), while the welfare gain for the whole economy almost reaches the level in the first-best planning economy (cf. Table 3). In other words, the cost for providing correct incentives to work and learn is low in our model.

6.2.5 IC verification

Finally, in the relaxed planning problem, we have replaced the incentive compatible constraint by the envelope condition. Yet, the solution of the relaxed planning problem may not be the solution of the original social planning problem. Following the way used by Farhi and Werning (2013) and Stancheva (2017), we have verified that our solution satisfies the incentive compatibility constraint (2b). The results are in Figure 4.

[Insert Figure 4 here]

As seen in Figure 4, for agents of all skill types, if they report their types truly, they acquire the highest lifetime utility (cf. the dotted line in Figure 4). Therefore, the solution that we characterize in section 3.1 is indeed the solution of the original social planning problem.

7. Extension: T-period Model

We have characterized human capital investment and optimal capital and labor taxes in a two-period model. To see whether these results obtained are robust for more than two periods, in this section we extend the model to T periods.

Although an extension to T periods makes our results robust, the extension makes the model complicated. In a T-period model, all skill shocks that agents have experienced in the past will affect human capital formation. Thus, in period t, an agent's decision is affected not only by shocks θ_t realized in period t but also by shocks encountered in the past. Denote by $\theta^{t-1}=(\theta_1,\theta_2,...\theta_{t-1})$ the past history up to period t-1. Then, the evolution of the human capital in Assumption 1 is revised as follows.

$$h_{t+1}(\theta^t)=(1-\delta_h)h_t(\theta^{t-1})+\psi(x_t(\theta^t),y_t(\theta^t))+\theta_t.$$

As the shock history affects human capital formation, the evolution of the human capital is more complicated than a two-period model. To make the model tractable, following Stantcheva (2017),[18] we focus on the partial equilibrium wherein the interest rate R_t and the wage rate w_t are treated as given. To further simplify the model, we assume that human capital is completely depreciated in one period. In so doing, we can consider a family of related problems that admits a recursive representation.

The lifetime utility of an agent with the history of types $\theta^t=(\theta_1,\theta_2,...,\theta_t)$ can be express as the following Bellman equation:

$$W(\theta^t)=u(c(\theta^t))-\phi\left(\frac{z(\theta^t)}{h_t(\theta^{t-1})}\right)+\beta\int W(\theta^{t+1})\pi(\theta_{t+1})d\theta_{t+1}\ .$$

[18] In our two-period model, the interest and the wage rate are derived from aggregate production function which depend on marginal returns to capital and labor. To be consistent when extending to T periods, we have to treat the interest and the wage rate as given. Then, the wage rate time human capital is equal to the marginal return to labor and is endogenous. In Stantcheva (2017), there is no production function, and it does not need to treat the wage rate as given, as her wage rate equals the marginal return to labor, which depends on human capital.

7.1 Incentive compatibility constraint

Given the history of types $\theta^t = (\theta_1, \theta_2, ..., \theta_t)$, we consider one particular deviation strategy $\sigma(\theta^t) = (\theta^{t-1}, \sigma) = (\theta_1, \theta_2, ..., \theta_{t-1}, \sigma)$. The expected lifetime utility in period t is:

$$W^\sigma(\theta^t) = u\left(c(\theta^{t-1}, \sigma) + y(\theta^{t-1}, \sigma) - y^\sigma(\theta^t)\right) - \phi\left(\frac{z(\theta^{t-1}, \sigma)}{\psi\left(x(\theta^{t-1}), y(\theta^{t-1})\right) + \theta_{t-1}}\right) + \beta \int W^\sigma(\theta^{t-1}, \theta_t, \theta_{t+1}) \pi(\theta_{t+1}) d\theta_{t+1}.$$

Since $W^\sigma(\theta^{t-1}, \theta_t, \theta_{t+1}, \theta_{t+2}) = W(\theta^{t-1}, \sigma, \theta_{t+1}, \theta_{t+2})$, this expected lifetime utility can be rewritten as

$$W^\sigma(\theta^t) = u\left(c(\theta^{t-1}, \sigma) + y(\theta^{t-1}, \sigma) - y^\sigma(\theta^t)\right) - \phi\left(\frac{z(\theta^{t-1}, \sigma)}{\psi\left(x(\theta^{t-1}), y(\theta^{t-1})\right) + \theta_{t-1}}\right)$$
$$+ \beta \int \left[u\left(c(\theta^{t-1}, \sigma, \theta_{t+1})\right) - \phi\left(\frac{z(\theta^{t-1}, \sigma, \theta_{t+1})}{\psi\left(x(\theta^{t-1}, \sigma), y^\sigma(\theta^t)\right) + \theta_t}\right) + \int W(\theta^{t-1}, \sigma, \theta_{t+1}, \theta_{t+2}) \pi(\theta_{t+2}) d\theta_{t+2}\right] \pi(\theta_{t+1}) d\theta_{t+1}.$$

Incentive compatibility indicates that

$$W(\theta^t) = \max_\sigma W^\sigma(\theta^t).$$

Hence, the envelope condition is

$$\dot{W}(\theta^t) \equiv \frac{\partial W(\theta^t)}{\partial \theta_t} = -u'\left(c(\theta^t)\right) \frac{\partial y(\theta^t)}{\partial \theta_t} + \beta \int \phi'\left(\frac{z(\theta^{t+1})}{h_{t+1}(\theta^t)}\right) \frac{z(\theta^{t+1})\left[\psi_y(\theta^t) \frac{\partial y(\theta^t)}{\partial \theta_t} + 1\right]}{\left[h_{t+1}(\theta^t)\right]^2} \pi(\theta_{t+1}) d\theta_{t+1},$$

where $\psi_y(\theta) = \psi_y(x(\theta), y(\theta))$.

In the social planning problem, we express the expected lifetime utility and the envelope condition as the following recursive formulation:

$$W(\theta^t) = u\left(c(\theta^t)\right) - \phi\left(\frac{z(\theta^t)}{\psi\left(x(\theta^{t-1}), y(\theta^{t-1})\right) + \theta_{t-1}}\right) + \beta v(\theta^t), \tag{9a}$$

$$\dot{W}(\theta^t) = -u'\left(c(\theta^t)\right) \frac{\partial y(\theta^t)}{\partial \theta_t} + \beta \Delta(\theta^t), \tag{9b}$$

where

$$v(\theta^t) = \int W(\theta^{t+1}) \pi(\theta_{t+1}) d\theta_{t+1}, \tag{9c}$$

$$\Delta(\theta^t) = \beta \int \phi'\left(\frac{z(\theta^{t+1})}{\psi\left(x(\theta^t), y(\theta^t)\right) + \theta_t}\right) \frac{z(\theta^{t+1})\left[\psi_y\left(x(\theta^t), y(\theta^t)\right) \frac{\partial y(\theta^t)}{\partial \theta_t} + 1\right]}{\left[\psi\left(x(\theta^t), y(\theta^t)\right) + \theta_t\right]^2} \pi(\theta_{t+1}) d\theta_{t+1}. \tag{9d}$$

As will be clear, the new variables $\Delta(\theta^t)$ and $\Delta(\theta^t)$ will serve as state variables. The social planner's objective is to minimize the expected discounted cost of providing an allocation, subject to the incentive compatibility condition and the expected lifetime utility of each (initial) type θ being above a threshold $\underline{W}(\theta)$. The relaxed planning problem will replace the incentive constraint by the envelope condition. Formally, define the expected resource cost as

$$\mathcal{K}(v, \Delta, \theta_{s-1}, s) = \min \sum_{t=s}^{T} \left(\frac{1}{R}\right)^{t-1} \int_{\Theta^t} \left[c_t(\theta^t) + x_t(\theta^t) + y_t(\theta^t) - w_t z_t(\theta^t)\right] \pi(\theta_t) \pi(\theta_{t-1}) ... \pi(\theta_s) d\theta_t ... d\theta_s.$$

7.2 The relaxed social planning problem

Now, we consider a family of related problems that admit a recursive representation. For any date t and past history θ^{t-1}, following Farhi and Werning (2013) and Stantcheva (2017), we study the continuation problem that minimizes the remaining discounted expected costs while taking as given some previous values for $v(\theta^{t-1})$ and $\Delta(\theta^{t-1})$; denoted by v and Δ, respectively. For any date t, once we condition on the past shock θ_{t-1}, the entire history of shocks θ^{t-1} is redundant. Then, using recursive formulation in (9a)-(9d), we write the relaxed social planning problem in terms of a recursive Bellman equation.

In any period t =2, 3,… T -1, the expected resource cost minimization problem of the social planner is as follow:[19]

$$\mathcal{K}(v,\Delta,\theta_-,t)=\min\int_{\underline{\theta}}^{\bar{\theta}}\left[c_t(\theta)+x_t(\theta)+y_t(\theta)-w_t z_t(\theta)+\tfrac{1}{R}\mathcal{K}\left(v(\theta),\Delta(\theta),\theta,t+1\right)\right]\pi(\theta)d\theta,$$

subject to

$$W(\theta)=u\left(c(\theta)\right)-\phi\left(\frac{z(\theta)}{\psi\left(x(\theta_-),y(\theta_-)\right)+\theta_-}\right)+\beta v(\theta),$$

$$\dot{W}(\theta)=-u'\left(c(\theta)\right)\frac{\partial y(\theta)}{\partial\theta}+\beta\Delta(\theta),$$

where $v=\int W(\theta)\pi(\theta)d\theta$ and $\Delta=\int\phi'\left(\frac{z(\theta)}{h_t(\theta_-)}\right)\frac{z(\theta)\left[\psi_y(\theta_-)\frac{\partial y(\theta_-)}{\partial\theta_-}+1\right]}{\left[h_t(\theta_-)\right]^2}\pi(\theta)d\theta,$ with θ_- denoting past shocks and the minimization is taken over $c(\theta),x(\theta),z(\theta),W(\theta),v(\theta)$ and $\Delta(\theta)$.[20]

In period 1, the problem needs to be reformulated. The problem in t = 1 is indexed by $\left(\underline{W}(\theta)\right)_\Theta$, the set of target lifetime utilities $\underline{W}(\theta)$ for skill type θ:

$$\mathcal{K}\left(\left(W(\theta)\right)_\Theta,1\right)=\min\int\left[c(\theta)+x(\theta)+y(\theta)-w_1 z(\theta)+\tfrac{1}{R}\mathcal{K}\left(v(\theta),\Delta(\theta),\theta,2\right)\right]\pi(\theta)d\theta,$$

subject to

$$W(\theta)=u\left(c(\theta)\right)-\phi\left(\tfrac{z(\theta)}{h_1}\right)+\beta v(\theta),$$

$$\dot{W}(\theta)=-u'\left(c(\theta)\right)\frac{\partial y(\theta)}{\partial\theta}+\beta\Delta(\theta),$$

$$W(\theta)\geq\underline{W}(\theta),$$

where the minimization is taken over $c(\theta),x(\theta),z(\theta),W(\theta),v(\theta)$ and $\Delta(\theta)$.

7.3 Optimal private non-verifiable human capital investment

Suppose that an agent with the true type $\theta^t=(\theta_1,\theta_2,...,\theta_t)$ chooses to report $(\theta_1,\theta_2,..,\theta_{t-1},\sigma)$. Then,

[19] The details of solving the social planning problems presented in this subsection can be found in the Appendix.
[20] Similar to the two-period model in Section 3, we will solve this relaxed planning problem here and then verify that the solution satisfies the incentive compatibility numerically.

he will choose non-verifiable education expenses to maximize the following problem

$$W^{\sigma}\left(\theta^{t}\right)=\max_{y^{\sigma}\left(\theta^{t}\right)} u\left(c^{\sigma}\left(\theta^{t}\right)\right)-\phi\left(\frac{z\left(\theta^{t-1},\sigma\right)}{h_{t}\left(\theta^{t-1}\right)}\right)+\beta\int W^{\sigma}\left(\theta^{t+1}\right)\pi\left(\theta_{t+1}\right)d\theta_{t+1},$$

subject to

$$c^{\sigma}\left(\theta^{t}\right)+y^{\sigma}\left(\theta^{t}\right)=c\left(\theta^{t-1},\sigma\right)+y\left(\theta^{t-1},\sigma\right),$$

$$W^{\sigma}\left(\theta^{t+1}\right)=u\left(\theta^{t-1},\sigma,\theta_{t+1}\right)-\phi\left(\frac{z\left(\theta^{t-1},\sigma,\theta_{t+1}\right)}{h_{t+1}^{\sigma}\left(\theta^{t}\right)}\right)+\beta\left[\int W\left(\theta^{t-1},\sigma,\theta_{t+1},\theta_{t+2}\right)\pi\left(\theta_{t+2}\right)d\theta_{t+2}\right].$$

From the first-order conditions of the above problem, one can prove the following theorem.

Theorem 3. *The constrained efficient allocation A satisfies*

$$u'\left(c^{\sigma}\left(\theta^{t}\right)\right)=-\beta\int \phi_{h}\left(\frac{z\left(\theta^{t-1},\sigma,\theta_{t+1}\right)}{h_{t+1}^{\sigma}\left(\theta^{t}\right)}\right)\psi_{y}\left(x\left(\theta^{t-1},\sigma\right),y^{\sigma}\left(\theta^{t}\right)\right)\pi\left(\theta_{t+1}\right)d\theta_{t+1}.$$

Furthermore, $\frac{\partial y^{\sigma}\left(\theta^{t}\right)}{\partial\theta_{t}}<0$ *and* $\frac{\partial h_{t+1}^{\sigma}\left(\theta^{t}\right)}{\partial\theta_{t}}>0$.

It is obvious that Theorem 3 extends the results in Theorem 1 from 2 periods to T periods.

7.4 The Signs of Capital and Labor Wedges

Finally, we analyze the signs of the intertemporal wedge and the intratemporal wedge in a *T*-period model. First, we establish the sign of the intertemporal wedge as follows.

Proposition 3. *If there are only verifiable education expenses, then*

$$\frac{1}{u'(c(\theta^{t-1}))}=\frac{1}{\beta R_{t}}E\left[\frac{1}{u'(c(\theta^{t}))}\right] \quad \textit{for } \theta^{t}\in\Theta^{t}, \quad t=2,3,..T.$$

However, if there are non-verifiable education expenses, the above Inverse Euler equation does not hold for $\theta^{t}\in\Theta^{t}$, *t=2,3,..T-1. Moreover, in the terminal period, the relation is*

$$\frac{1}{u'(c(\theta^{T-1}))}>\frac{1}{\beta R_{T}}E\left[\frac{1}{u'(c(\theta^{T}))}\right],$$

which induces a higher intertemporal wedge than that when there are only verifiable education expenses.

Thus, as in Proposition 1 in a two-period model, if there are no non-verifiable education expenses, even with skill shocks, the Inverse Euler equation holds and thus, $\tau_{k_t}(\theta^{t})=0$ and the intertemporal wedge is zero. Yet, when there are non-verifiable education expenses, the Inverse Euler equation does not hold and the intertemporal wedge may be positive. In particular, in the terminal period, the intertemporal wedge is unambiguously positive. A positive capital wedge would reduce the incentive to save and discourages agents from under-reporting.

Next, we establish the sign of the intratemporal wedge in the following proposition, which extends the results in Proposition 2 from 2 periods to *T* periods.

Proposition 4. *If there are only verifiable education expenses,* $\tau_{z_1}(\theta_1)=0$ $\tau_{z_1}(\theta_1)=0$ *for* $\theta_1 \in [\underline{\theta},\bar{\theta}]$ $\theta_1 \in [\underline{\theta},\bar{\theta}]$ *and* $\tau_{z_t}(\theta^t)>0$ *for* $\theta \in \Theta^t$, *t≥2. But, if there are non-verifiable education expenses, then* $\tau_{z_1}(\theta_1)<0$ *for* $\theta_1 \in (\underline{\theta},\bar{\theta})$*,* $\tau_{z_t}\left(\theta^t\right)$ *is ambiguous for* $\theta^t \in \Theta^t$, *t=2,3,..T-1, and* $\tau_{z_T}(\theta^T)>0$ *for* $\theta^T \in \Theta^T$

If there are no non-verifiable education expenses, then intratemporal wedge is zero in the first period and positive in all other periods. If there are non-verifiable education expenses, a subsidy to labor supply in an early lifecycle is optimal. A subsidy to labor supply in an early lifecycle serves as a mechanism to induce agents to work according to their true types and spend more on education.

8. Concluding Remarks

The existing dynamic Mirrlees literature analyzed optimal income tax policies in models with exogenous skills. Recently, some authors have extended the literature into models with educational decisions. These authors proposed subsidies to observable education expenses in order to alleviate tax-induced distortions on learning. As a wide range of goods and services have both consumption and human capital investment component, education expenses may not be all verifiable. Thus, the policy of a subsidy to education expenses may not be available. This paper studies whether non-verifiable education expenses affect the design of optimal taxation policies on capital and labor income.

Our paper extends Bovenberg and Jacobs (2005) and Stantcheva (2017) to a setting with a twist that education expenses may not be all verifiable. As a result, consumption is not directly observable. Our model follows Farhi and Werning (2013) and posits shocks to agents' skills. Agents undergo shocks to skill acquisitions over time. In addition to working and savings, agents choose consumption and education expenses in each period. The social planner chooses constrained efficient allocations that maximizes the utilitarian social welfare subject to resource constraints and incentive-compatibility constraints.

We obtain two novel results concerning wedges that arise from constrained efficient allocations. First, the capital wedge is positive, even if skill shocks do not change over time. Second, the labor wedge is negative in an early lifecycle. These results emerge from unobservable consumption. Optimal capital wedges are positive, because the policy offers incentives to discourage agents from cutting non-verifiable education expenses for more consumption. Optimal negative labor wedges serve as a mechanism to induce agents not to cut work hours so as not to allocate less expenses for education and more for consumption.

To implement these wedges as an outcome in a market equilibrium, we propose a tax system. In our tax system, capital and labor income are taxed linearly, along with lump-sum taxes, if an agent's history of capital and effective labor satisfies some conditions; otherwise, the agent would be taxed severely. Deferred capital taxes are not necessary in order to tax capital income linearly. Compared to the laissez-faire economy without taxes, our second-best optimal mechanism gives rise to a welfare gain, which is close to the level in the first-best planning economy. This suggests that, relative to the first-best planning economy, the welfare cost for providing correct incentives to work and learn is very low in our second-best planning economy.

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Table 1. Exogenously calibrated parameters

Definition	Symb	Value	Source/Note
Population			
The bottom type	$\underline{\theta}$	1	Normalization
Degree of uncertainty	$\hat{\sigma}^2$	0.0095	Farhi and Werning (2013)
Preference			
CRRA parameter	χ	2	Conesa et al. (2009)
Disutility elasticity	κ	3	Farhi and Werning (2013)
Discount factor	β	0.442	0.96 annual ; Kapička and Neira
Final good production: Cobb-Douglas			
Productivity level	A	1	Normalization
Capital share	ξ	0.36	Conesa et al. (2009)
Initial physical capital	K_1	1	Normalization
Depreciation	δ_k	1	By assumption
Human capital Technology: Ben-Porath			
Education degree	η	0.4	Ewijk and Tang (2000)
Share of non-verifiable education	ρ	0.667	Ewijk and Tang (2000)
Initial human capital	h_1	1	Normalization
Depreciation	δ_h	1	By assumption
Tax system			
Capital income tax rate	τ_k^b	0.3	McDaniel (2007)
Labor income tax rate	τ_z^b	0.2	McDaniel (2007)
Government expenditure	G_t	0	By Assumption

Table 2. Endogenously matched parameters

Calibrated parameter		Value	Target	Value	Source
Technology level of HC	B	4.03	Capital-output ratio	0.135	Peterman (2016)
The top of type distribution	$\overline{\theta}$	2.85	Wage premium	1.8	Various sources

Table 3. Welfare gains over laissez-faire no-tax economy

Economies	$\hat{\sigma}=0.0061$	$\hat{\sigma}=0.0095$	$\hat{\sigma}=0.0161$
Second-best (our model)	1.4370%	1.7483%	1.9998%
First-best	1.4608%	1.7671%	2.0154%

Note: Welfare gains are in terms of consumption equivalence.

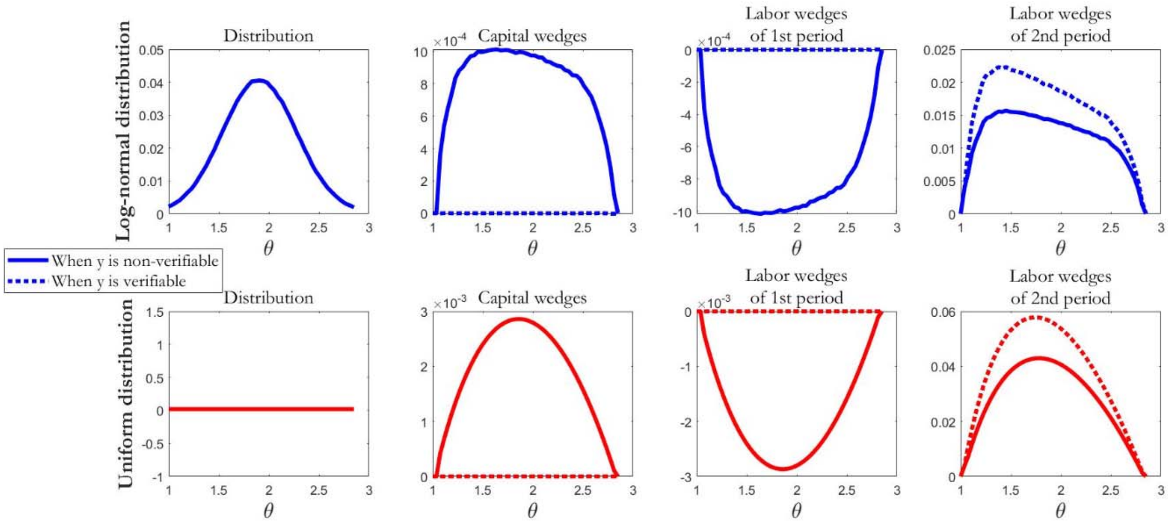

Figure 1. Wedges under log-normal distribution and uniform distribution

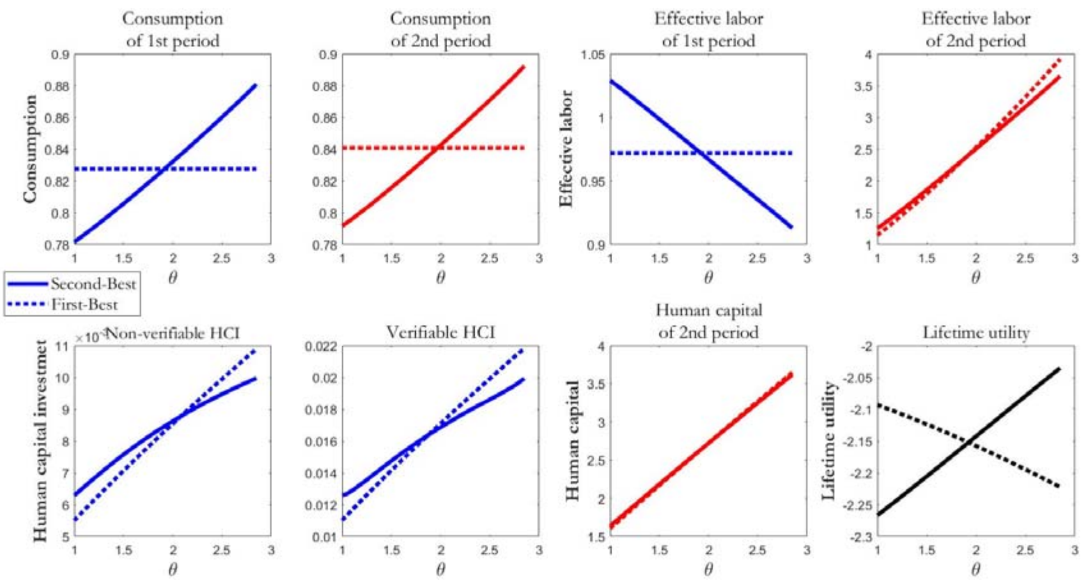

Figure 2. Constrained efficient allocation and allocation in the first-best.

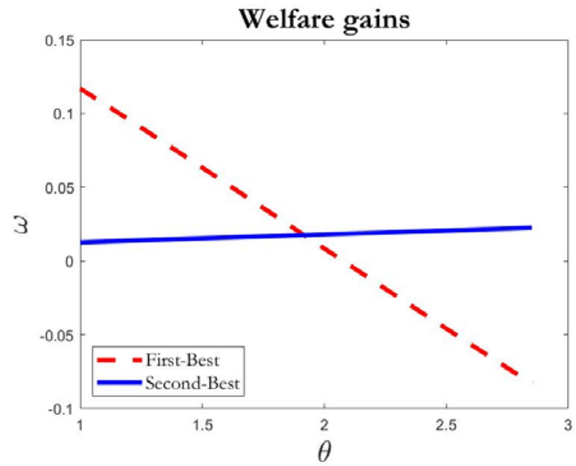

Figure 3. Welfare gains for each type.

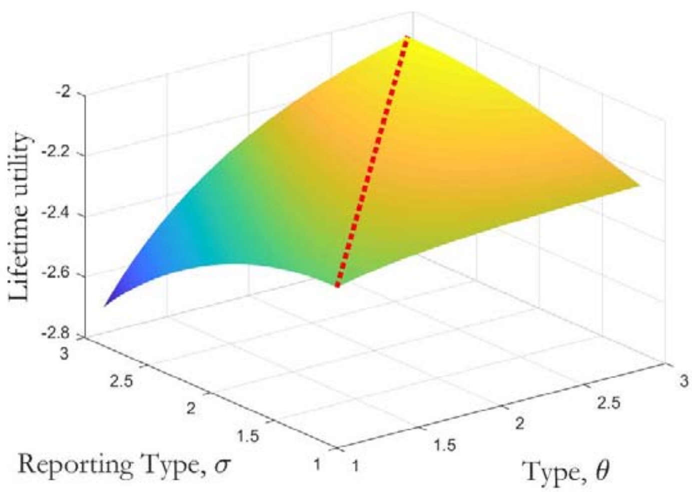

Figure 4. Verifying IC constraints.