Nowcasting GDP Growth for Small Open Economies with a Mixed-Frequency Structural Model

Ruey Yau [1] and C. James Hueng [2]

Abstract

This paper proposes a mixed-frequency small open economy structural model, in which the structure comes from a New Keynesian dynamic stochastic general equilibrium (DSGE) model. An aggregation rule is proposed to link the latent aggregator to the observed quarterly output growth via aggregation. The resulting state-space model is estimated by the Kalman filter and the estimated current aggregator is used to nowcast the quarterly GDP growth. Taiwanese data from January 1998 to December 2015 are used to illustrate how to implement the technique. The DSGE-based mixed-frequency model outperforms the reduced-form mixed-frequency model and the MIDAS model on nowcasting Taiwan's quarterly GDP growth.

JEL Classification: C5, E1

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1. Corresponding author. Department of Economics, National Central University, Taoyuan, Taiwan 32001, R.O.C. E-mail address: ryau@mgt.ncu.edu.tw; Tel.: +886 03 4227151; fax: +886 03 4222876.

2. Department of Economics, Western Michigan University, Kalamazoo, MI 49008, U.S.A. and School of Economics, Zhongnan University of Economics and Law, Hubei 430073, China. E-mail address: James.Hueng@wmich.edu

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1 Introduction

Central banks or institutional analysts are often eager to gain access to a country's economic status for timely policy decisions. Real GDP is considered to be one of the most important measures of the aggregate state of an economy. It is, however, only available on a quarterly basis. As an alternative, popular coincident indices of business cycles are estimated. Examples include the composite index of coincident indicators released by the U.S. Conference Board, the coincident indicators developed by Stock and Watson (1989, 1991), and the business condition indicator computed by Aruoba et al. (2009). The main criticism of such coincident indices is that they lack direct economic interpretation.

To overcome such a criticism, a number of economists estimate monthly GDP directly. In terms of modeling methodology, some authors construct monthly GDP based on univariate models for real GDP [e.g. Bernanke et al. (1997) and Liu and Hall (2001)] and others apply a multivariate approach [e.g., Mariano and Murasawa (2003, 2010)]. These studies are in line with the 'common factor' approach proposed by Stock and Watson (1989, 1991). Their basic statistical method is to build state-space models with mixed-frequency series. Being abstract from structural modeling, the common factor approach in the previous studies is essentially a reduced-form method. The coefficients estimated in such a model are not subject to any structural restrictions.

Another popular reduced-form approach to handle data sampled at different frequencies is the mixed-data sampling (MIDAS) regression introduced by Ghysels et al. (2004). It is based on a univariate regression that adopts highly parsimonious lag polynomials to exploit the content in the higher frequency explanatory variables in predicting the lower frequency variable of interest. There is now a substantial literature on MIDAS regressions and their applications; see, for example, Clements and Galvao (2008) on macroeconomic applications, Ghysels et al. (2006) on financial applications, and Foroni et al. (2015) for more flexible specifications. Unlike the Kalman filter state space approach that involves a system of equations, MIDAS regressions involve a single equation. As a consequence, MIDAS regressions might be less efficient, but they are less prone to specification errors.

Differing from the aforementioned studies that build upon reduced-form time series frameworks, some recent studies have considered merging a structural macroeconomic model with the mixed-frequency strategy. Two important contributions are Giannone et al. (2009) and Foroni and Marcellino (2014b). Giannone et al. (2009) develop a framework to incorporate monthly information in quarterly dynamic stochastic general equilibrium (DSGE) models. They take the parameter estimates from the quarterly DSGE as given and obtain increasingly accurate early forecasts of the quarterly variables. Foroni and Marcellino (2014b) demonstrate that temporal aggregation bias, as pointed out in Christiano and Eichenbaum (1987), may arise when economists estimate a quarterly DSGE, while the agents' true decision interval is on a monthly basis. They propose a mixed-frequency strategy to estimate the DSGE model and find that the temporal aggregation bias can be alleviated.[1] However, there is no general rule on to what extent such a complicated framework helps in forecasting or nowcasting real GDP, since it depends on the structure of the DSGE model and on the content of the higher frequency variables.

In this paper we develop a mixed-frequency structural model for a small open economy. The main purpose is to assess the advantage of nowcasting current real GDP using a mixed-frequency model with a structural context. We assume that economic agents make decisions on a monthly basis. Because GDP is a quarterly series, the mixed-frequency technique is adopted to provide early estimates of the real GDP growth. By building on a monthly small open DSGE model, we derive the mixed-frequency state-space representation. The Kalman filter estimation technique of Durbin and Koopman (2001) is used in our mixed-frequency econometric framework to take account of missing monthly values of quarterly variables.

A few other studies are related to this paper. Boivin and Giannoni (2006) incorporate

[1]Some other recent studies with the mixed-frequency strategy include the fixed-frequency VAR model in Schorfheide and Song (2015) and Rondeau (2012). The latter combines quarterly series with annual series in an effort to estimate a DSGE model for emerging economies.

a large data set that contains additional variables (i.e. non-core variables) that are not considered in a DSGE model. Their approach is conceptually appealing, because it exploits information contained in the other indicators when making inferences about the latent state of the economy. The DSGE model parameters as well as the factor loadings for the non-core variables are jointly estimated using Bayesian methods. Nevertheless, their study solely employs data at the quarterly frequency level. In reality, higher frequency information may arrive and central banks or institutional forecasters like to include the additional information in their forecasting framework. Rubaszek and Skrzypczynski (2008) and Edge et al. (2008) have surveyed the literature on evaluating the forecasting properties of the DSGE model in a real-time environment. Schorfheide et al. (2010) examine whether a DSGE model could be used to forecast non-core variables that are not included in a structural model. Instead of jointly estimating all the parameters in the system, they suggest using a two-step Bayesian method to reduce the computational burden.

For a demonstration, we apply our model to the Taiwanese data over the sample period from January 1998 - December 2015 and evaluate the model's performance on nowcasting real GDP growth rates. For purposes of comparison, we also estimate a reduced form of the mixed-frequency model and a basic MIDAS model. We find that the DSGE-based mixed-frequency model produces better results on nowcasting GDP growth than the two alternative models.

The next section lays out the model. Section 3 presents the empirical findings using Taiwanese data and the final section concludes.

2 The Model

2.1 A Small Open Economy DSGE Model

The goal of this paper is to evaluate the advantage of nowcasting real GDP growth with a structural model that allows us to include more frequently arriving monthly observations.

We assume that the agents in the economy make decisions on a monthly basis. By using a small open economy DSGE model, we derive its mixed-frequency state-space representation and estimate the model using the maximum likelihood method.

The basis of the structural model is similar to the one in Gali and Monacelli (2005), in which they derive a New Keynesian DSGE model that consists of households, firms and a central bank. The decision rules of agents form a system of nonlinear difference equations with rational expectations. Then a log-linearization approximation to this system around its steady state is derived and can be characterized by the equations summarized below.

The first equation is an open-economy IS curve derived from combining the consumption Euler equation and the goods market clearing conditions, where the representative household consumes both domestic goods and imported goods and chooses optimal consumption and labor hours:

$$y_t = E_t y_{t+1} - \frac{1}{\sigma}(R_t - E_t \pi_{t+1}) - \frac{\alpha\omega}{\sigma} E_t \Delta s_{t+1}, \tag{1}$$

where y_t is real output, R_t is the gross return on a risk-free one-period discount bond paying one unit of domestic currency, π_t is the domestic CPI inflation rate, and s_t is the terms-of-trade, which is defined as the relative price of import in terms of export (in logarithm).[2] The coefficients in (1) are functions of the deep parameters of the DSGE model: $1/\sigma$ is the intertemporal elasticity of substitution, α is the index of openness, with $\alpha = 0$ corresponding to a closed economy and $\alpha = 1$ to a fully open economy, and $\omega = \sigma + (1-\alpha)(\sigma - 1)$.[3] This equation describes how aggregate output (y_t) is related to its future expected value ($E_t y_{t+1}$), the expected real interest rate ($R_t - E_t \pi_{t+1}$), and the expected change in the terms-of-trade ($E_t \Delta s_{t+1}$). For a small open economy, as its terms-of-trade is expected to improve (i.e., negative $E_t \Delta s_{t+1}$), the global demand for domestic goods is expected to increase, which in

[2]The constant term in (1) is ignored because we demean all the variables in the empirical work.

[3]The coefficient ω is obtained under the assumption that the elasticities of substitution between domestic and foreign goods and between goods produced in different foreign countries are both equal to one, i.e., the functions have a Cobb-Douglas form.

turn has a positive effect on the domestic real output.[4]

The second equation is an open economy New Keynesian Phillips curve derived from the optimal price setting behavior of domestic firms:

$$\pi_t = \beta E_t \pi_{t+1} - \alpha\beta E_t \Delta s_{t+1} + \alpha \Delta s_t + \frac{(1-\beta\phi)(1-\phi)}{\phi}(\sigma_\alpha + \varphi)(y_t - \bar{y}_t), \tag{2}$$

where $\bar{y}_t$ is the potential output, β is the discount rate, ϕ is the percentage of firms with sticky prices within a period, $1/\varphi$ is the Frisch elasticity of labor supply, and $\sigma_\alpha = \sigma/(1 - \alpha + \alpha\omega)$.[5] The potential output is the real output in the absence of nominal rigidities and it is determined by $\bar{y}_t = -\alpha[\frac{\sigma_\alpha(\omega-1)}{\sigma_\alpha+\varphi}]y_t^*$, where y_t^* is exogenous foreign real output. As the level of price rigidity increases (i.e., the higher the value of ϕ), the coefficient of the output gap in (2) becomes smaller and the New Keynesian Phillips curve becomes flatter. The labor supply elasticity is another structural parameter that affects the slope of the Phillips curve. When real wages rise by 1%, the household is willing to increase working hours by $1/\varphi$ units. As labor supply becomes more elastic (i.e., the lower the value of φ), the New Keynesian Phillips curve becomes flatter.

Assuming the Law of One Price, the dynamics of the nominal exchange rate is:

$$\Delta e_t = \pi_t - \pi_t^* + (1-\alpha)\Delta s_t + \varepsilon_{e,t}, \tag{3}$$

where π_t and π_t^* are the CPI inflation rates of the home country and foreign country, respectively; and e_t is the nominal exchange rate with a positive value of Δe_t indicating a depreciation in the domestic currency. This equation states that, other than the exchange rate shock, $\varepsilon_{e,t}$, the nominal exchange rate is explained by the purchasing power parity adjusted by a fraction of changes in the terms-of-trade when the economy is not completely

[4]In the model, technology is not separately specified and is therefore imbedded in the real output.

[5]In this DSGE model, the firms' staggered price-setting scheme is adopted from Calvo (1983). That is, each intermediate firm faces a constant probability $(1-\phi)$ to re-optimize its price within a period. The index of openness, $0 \leq \alpha \leq 1$, is the ratio of domestic consumption allocated to imported goods. In equilibrium, the domestic CPI is a CES function of the price level of domestic goods and the price level of imported goods.

open to the world economy. As the terms-of-trade condition deteriorates (i.e., when Δs_t becomes positive as import prices increase faster than export prices), the domestic currency depreciates.

Under the assumption that the international financial markets are complete, the Euler equation in each country holds. The goods market clearing condition then implies an equilibrium condition that determines the dynamics of the terms-of-trade as:

$$\Delta s_t = \sigma_\alpha(\Delta y_t - \Delta y_t^*).$$

When foreign output growth (Δy_t^*) is higher than domestic output growth (Δy_t), the demand for the domestically produced goods rises so that the terms-of-trade condition improves (Δs_t becomes negative). However, this structural equation creates a very strong restriction between the dynamics of real output growth and the terms-of-trade to be matched in the estimation. This restriction is found to create a conflict with other endogenous variables' dynamics and results in highly implausible estimates.[6] Following Lubik and Schorfheide's (2007) suggestion, we assume that changes in the terms-of-trade follow an AR(1) process:

$$\Delta s_t = \rho_s \Delta s_{t-1} + \varepsilon_{s,t}, \tag{4}$$

where ρ_s is the autoregressive coefficient and $\varepsilon_{s,t}$ is a terms-of-trade shock. In addition, we assume that the central bank's monetary policy reaction function is forward-looking and the interest rate is adjusted in a gradual fashion (interest-rate inertia) [see Clarida et al. (2000)]:

$$R_t = \rho_R R_{t-1} + (1-\rho_R)[\,\psi_\pi E_t \pi_{t+1} + \psi_y E_t(\,y_{t+1} - \bar{y}_{t+1})] + \varepsilon_{R,t}, \tag{5}$$

where ρ_R is the parameter of interest rate smoothing, the ψ_i's are the policy reaction coefficients, and $\varepsilon_{R,t}$ is the exogenous shock to monetary policy. Finally, the structural model is completed by adding exogenous AR(1) processes with autoregressive coefficients ρ_j's on the foreign inflation rate (π_t^*) and the foreign output growth (Δy_t^*):

$$\pi_t^* = \rho_{\pi^*} \pi_{t-1}^* + \varepsilon_{\pi^*,t}\ , \tag{6}$$

[6] A similar problem was previously diagnosed in Lubik and Schorfheide (2007) when they estimated Gali and Monacelli's (2005) model based on data for Australia, Canada, New Zealand and the U.K.

$$\Delta y_t^* = \rho_{y^*} \Delta y_{t-1}^* + \varepsilon_{y^*,t} \ , \tag{7}$$

where $\varepsilon_{\pi^*,t}$ and $\varepsilon_{y^*,t}$ are structural shocks to π_t^* and Δy_t^*, respectively. Together with the shock to monetary policy, there are five structural shocks in the model. These shocks are assumed to be mutually independent and distributed as $\varepsilon_{j,t} \sim iidN(0, \sigma_j^2)$ for $j = R, e, s, \pi^*$, and y^*.

2.2 State-Space Representation of the DSGE Model

The log-linearized rational expectations model can be solved with a numerical method and the solution is a state transition equation that describes the law of motion of the endogenous variables and driving forces in the model.[7] Next, we show how to transform the state transition equation into a state-space representation, in which a measurement equation relates the DSGE model's variables to the observable data.

Let $X_t = [\pi_t, y_t - \bar{y}_t, R_t, \Delta e_t, \Delta s_t, \pi_t^*, \Delta y_t^*]'$ denote the vector that contains endogenous state variables and exogenous driving force variables, and let $\varepsilon_t = [\varepsilon_{R,t}, \varepsilon_{e,t}, \varepsilon_{s,t}, \varepsilon_{\pi^*,t}, \varepsilon_{y^*,t}]'$ denote the vector of exogenous structural shocks. The rational expectations model (1)-(7) can be expressed as[8]

$$B(\theta) X_t = C(\theta) X_{t-1} + D(\theta) E_t X_{t+1} + F(\theta) \varepsilon_t, \tag{8}$$

where $\theta \equiv \{\sigma, \alpha, \varphi, \beta, \phi, \rho_s, \rho_R, \rho_{\pi^*}, \rho_{y^*}, \psi_\pi, \psi_y\}$ is the vector of the model parameters, and $B(\theta), C(\theta)$ and $D(\theta)$ are conformable matrices of coefficients from the model described in the previous section. The unique stable solution for this model is given by

$$A_0(\theta) X_t = A_1(\theta) X_{t-1} + F(\theta) \varepsilon_t, \tag{9}$$

[7] The most popular solution methods are Blanchard and Kahn (1980), Klein (2000), Sims (2002), and Uhlig (1999).

[8] In the state-space representation, (1) is rewritten as: $y_t - \bar{y}_t = E_t(y_{t+1} - \bar{y}_{t+1}) - \frac{1}{\sigma}[R_t - E_t \pi_{t+1}] - \left[\frac{\alpha \sigma_\alpha (\omega - 1)}{\sigma_\alpha + \varphi}\right] E_t \Delta y_{t+1}^*$.

where $A_1(\theta) = C(\theta)$ and $A_0(\theta)$ satisfies: $A_0(\theta) = B(\theta) - D(\theta)A_0(\theta)^{-1}A_1(\theta)$. The solution to the log-linearized rational expectations model then has the following form of transition for the state variables:

$$X_t = \Phi_1(\theta)X_{t-1} + \Phi_2(\theta)\varepsilon_t, \tag{10}$$

where $\Phi_1(\theta) = A_0(\theta)^{-1}A_1(\theta)$ and $\Phi_2(\theta) = A_0^{-1}(\theta)F(\theta)$.

To estimate the DSGE model, a measurement equation based on a set of observables, Y_t, is specified as

$$Y_t = \Lambda X_t + u_t, \tag{11}$$

where Λ defines the relationship between the observed variables and the unobservable state variables, and u_t is the measurement error. Jointly, equations (10) and (11) form a state-space representation.

In a conventional DSGE model estimation, (10) is usually timed at the quarterly frequency. This assumption is imposed mainly because a real output measure such as GDP does not have monthly observations. However, as argued in Aadland and Huang (2004), Kim (2010), and Foroni and Marcellino (2014b), if the true decision period is a month, then a quarterly frequency model may lead to misspecification error or temporal aggregation bias. In the following subsection, we describe how to estimate the monthly DSGE model of (10)-(11) within a mixed-frequency framework.

2.3 The Structural Mixed-Frequency (DSGE-MF) Model

In this subsection, we build a mixed-frequency model that is based on the DSGE framework introduced in the previous subsection, and is denoted as the **DSGE-MF** model. We assume that the economic agents make decisions on a monthly basis, that is, the subscript t in (1)-(11) denotes a month. The observable variables in the measurement equation (11), however, include both quarterly and monthly data. Specifically, GDP data for both the domestic and foreign countries are quarterly observations, while inflation rates, interest rates, terms-of-trade, and exchange rates are all monthly observations. To incorporate the quarterly observations in this monthly model, we use the following aggregation rule. For the domestic real output growth, we define the aggregator Q_t as:

$$Q_t = \Delta y_t + \Delta y_{t-1} + \Delta y_{t-2} + \xi_t \, Q_{t-1}, \tag{12}$$

where

$$\xi_t = \begin{cases} 0 & \text{if } \ t = \text{January, April, July, October,} \\ 1 & \text{otherwise.} \end{cases}$$

The resulting $Q_{March}, Q_{June}, Q_{September}$, and $Q_{December}$ are quarter-on-quarter growth rates of output.

Denote $GDPGR_t$ as the observed quarter-on-quarter real GDP growth rate in the months of March, June, September, or December, and as missing values in the other months. We approximate the logarithm of real GDP in a specific quarter by summing over three months the logarithm of real GDP in that quarter to obtain

$$GDPGR_t \approx Q_t + \text{measurement error,}$$

for t = March, June, September, or December. In other words, aside from measurement errors, Q_t is the (quarter-on-quarter, qoq) growth rate of output from the previous quarter. We can use the estimate of Q_t to make inferences about the qoq real GDP growth. Similarly, the aggregation rule for the foreign real output growth rate is

$$Q_t^* = \Delta y_t^* + \Delta y_{t-1}^* + \Delta y_{t-2}^* + \xi_t \, Q_{t-1}^*. \tag{13}$$

Apparently the aggregators Q_t and Q_t^* are latent because their components are all latent variables.

We add these aggregators into the latent vector X_t and link the real GDP growth rate data in Y_t to the aggregators. Define $\tilde{X}_t$ as the new vector of state variables, which includes the two aggregators and necessary lagged variables: $\tilde{X}_t = [X_t, y_{t-1} - \bar{y}_{t-1}, y_{t-2} - \bar{y}_{t-2}, \Delta y^*_{t-1}, Q_t, Q^*_t]'$. The state space model (10)-(11) becomes

$$\tilde{X}_t = \tilde{\Phi}_1(\theta;\xi_t)\tilde{X}_{t-1} + \tilde{\Phi}_2(\theta;\xi_t)\varepsilon_t, \tag{10'}$$

$$Y_t = \tilde{\Lambda}\tilde{X}_t + u_t, \tag{11'}$$

where $\tilde{\Phi}_1(\theta;\xi_t)$, $\tilde{\Phi}_2(\theta;\xi_t)$, and $\tilde{\Lambda}$ are conformable matrices of coefficients when the state vector is extended from X_t to $\tilde{X}_t$. The structural shocks ε_t's and the measurement errors u_t's are assumed to be iid normally distributed and are mutually independent of each other. Note that in this monthly frequency model, the quarterly output variables in the vector Y_t contain missing values. The state transition equation (10') and the measurement equation (11') can be jointly estimated using the contemporaneous Kalman filter to yield maximum likelihood estimates; see Durbin and Koopman (2001) for further details on the Kalman filter's smoother and treatment for missing observations. Since we are interested in the state variables themselves, the Kalman smoother is used to derive the estimates of the state variables.

2.4 The Reduced-Form Mixed-Frequency (RE-MF) Model

For comparison purposes, a reduced-form mixed-frequency empirical model, denoted as **RE-MF**, is estimated. The model does not impose any cross-equation restrictions. The RE-MF model includes the aggregation rules (12)-(13) and the following transition equations:

$$\pi_t = [\pi_{t-1}, y_{t-1}, R_{t-1}, \Delta e_{t-1}, \Delta s_{t-1}, \pi^*_{t-1}, \Delta y^*_{t-1}]'\, b_\pi + \eta_{\pi,t}, \tag{14}$$

$$y_t = [\pi_{t-1}, y_{t-1}, R_{t-1}, \Delta e_{t-1}, \Delta s_{t-1}, \pi^*_{t-1}, \Delta y^*_{t-1}]'\, b_y + \eta_{y,t}, \tag{15}$$

$$R_t = b_R\, R_{t-1} + \eta_{R,t}, \tag{16}$$

$$\Delta e_t = b_e\, \Delta e_{t-1} + \eta_{e,t}, \tag{17}$$

$$\Delta s_t = b_s\, \Delta s_{t-1} + \eta_{s,t}, \tag{18}$$

$$\pi^*_t = b_{\pi^*}\, \pi^*_{t-1} + \eta_{\pi^*,t}, \tag{19}$$

$$\Delta y_t^* = b_{y^*}\ \Delta y_{t-1}^* + \eta_{y^*,t}, \tag{20}$$

where the b_j's are coefficient vectors or scalars and the $\eta_{j,t}$'s are mutually uncorrelated exogenous errors, for $j = \pi, y, R, e, s, \pi^*$, and y^*. Equations (14) and (15) specify that the domestic inflation rate and real output growth are dependent on all of the lagged variables. Due to high persistence in the remaining five variables, they are specified as AR(1) processes.

Define $\tilde{S}_t = [\pi_t, y_t, R_t, \Delta e_t, \Delta s_t, \pi_t^*, \Delta y_t^*, y_{t-1}, y_{t-2}, \Delta y_{t-1}^*, Q_t, Q_t^*]'$ as the vector of state variables in the RE-MF model. The corresponding state space model can be written compactly as

$$\tilde{S}_t = \Psi_1 \tilde{S}_{t-1} + \Psi_2 \eta_t, \tag{21}$$

$$Y_t = \Psi_3 \tilde{S}_t + v_t. \tag{22}$$

The matrix Ψ_3 defines the relationship between the observables in Y_t and state variables in $\tilde{S}_t$. The entries in the matrices Ψ_1 and Ψ_2 are free parameters that remain to be estimated. The reduced-form shocks in η_t and the measurement errors in v_t are assumed to be iid normally distributed and are mutually independent of each other.

3 Application to Taiwanese Data

3.1 Data

Taiwanese data from January 1998 to December 2015 are used to estimate the structural model. The sample starts in 1998 because it is believed that Taiwan's monetary policy is better described by an interest rate rule after 1998. The U.S. data are used for foreign variables. The observable vector includes the monthly CPI inflation rate (INF_t), quarterly real GDP growth rate ($GDPGR_t$), monthly rate of the overnight interbank call rate ($RATE_t$), monthly nominal exchange rate of the Taiwan dollar against the U.S. dollar (ΔEXG_t), the monthly percentage change in the ratio of the import price index to the export price index (ΔTOT_t), the monthly U.S. CPI inflation rate (INF_t^*), and the quarterly U.S. real GDP growth rate

($GDPGR_t^*$), i.e., $Y_t = [INF_t, GDPGR_t, RATE_t, \Delta EXG_t, \Delta TOT_t, INF_t^*, GDPGR_t^*]'$. The quarterly observations for domestic and foreign real GDP growth are quarter-on-quarter growth rates. All other observables are monthly observations of year-on-year percentage changes.[9] Figure 1 shows the time plot of the observables.

[Figure 1 here]

3.2 Estimation Results of the DSGE-MF and RE-MF Models

The state transition equation and measurement equation (10')-(11') are jointly estimated using the Kalman filter to yield maximum likelihood (ML) estimates. In order for the Kalman filter estimation to reach reasonable and robust estimates, we calibrate four structural parameters based on the results reported in the previous literature and estimate the remaining parameters using the ML method. In column (A) of Table 1, the calibrated values are listed in the upper panel and the ML estimates and their associated standard errors (S.E.'s) are reported in the lower panel.

[Table 1 here]

Among the calibrated parameters, the discount rate (β) for the monthly frequency model is set at 0.998 to match a sample average of 1.91% for the annualized interbank call rate in Taiwan. Following Teo (2009), a DSGE study for the Taiwanese economy, we set the inverse of the Frisch labor supply elasticity (φ) at 5. The value of the degree of openness (α) is set at 0.53, which is calibrated from the historical average of Taiwan's import share of GDP. For the price stickiness parameter, ϕ is calibrated at 0.875, which implies that on average each intermediate firm waits 8 months before resetting its prices. This calibrated

[9]The monthly observations in this paper are year-on-year percentage changes. As an alternative, we have estimated models with these observables constructed as month-on-month percentage changes. However, these series contain high degrees of noise and cause the maximum likelihood estimates to be poor. In the paper, the real GDP growth is constructed as the quarter-on-quarter growth rate to avoid further enlarging the dimension of the state-space model. If instead year-on-year real output growth is used, we need to include additional lagged variables in the state vector.

value is consistent with the estimated stickiness duration of 2.7 quarters in Teo (2009) for the Taiwanese economy.[10]

As for the inverse of the intertemporal elasticity of substitution (IES), the ML estimate $\hat{\sigma} = 1.169$ is statistically significant at the 5% level. The estimates of the first-order autoregressive coefficients in the exogenous AR(1) processes are all highly significant. The degree of policy inertia is high and statistically significant ($\hat{\rho}_R = 0.980$), but the policy reaction parameters ($\hat{\psi}_\pi$ and $\hat{\psi}_y$) are not significant at the conventional significance levels. The insignificant policy response to the expectations of inflation or the output gap might have resulted from the fact that the central bank of Taiwan rarely acted hawkishly during the sample period because inflation was tame. Figure 1(g) confirms the tendency for the nominal interest rate to decline, which is in line with more rate cuts than rate hikes, as shown in Figure 1(h).

Figure 2 plots the estimated state variables of the DSGE-MF model and the linked variables in the data. In the cases of interest rates and exchange rates, the estimated states and observables are almost identical, as is evident from Table 1(A), which shows that the standard deviations of the measurement errors of R_t and Δe_t are essentially zero. In the case of the domestic CPI inflation rate, Figure 2(a) shows a good in-sample fit, which can be explained by the small standard deviation in the measurement error ($\hat{\sigma}_\pi = 0.564$). On the other hand, the changes in the terms-of-trade are more volatile than the model estimates, which results in a large standard deviation in the measurement error ($\hat{\sigma}_{u,s} = 5.037$). Moreover, a large standard deviation in the structural exchange rate shocks ($\hat{\sigma}_e = 6.629$) indicates that the dynamics of the exchange rates often deviates from the law of one price.

[Figure 2 here]

Figure 2(e) plots the estimated and the actual quarterly GDP growth (i.e., Q_t versus $GDPGR_t$) at the quarterly frequency. These two series comove with a correlation coefficient

[10] Estimates in Teo (2009) are obtained using the Bayesian method for the sample period of 1992Q1 - 2004Q4.

of 0.25. The large standard deviation in the measurement error ($\hat{\sigma}_{u,y} = 1.77$) results in the imperfect performance of the model in capturing the volatility of the actual GDP growth. In approximation, $\text{var}(GDPGR_t) = \text{var}(Q_t) + \text{var}(u_{y,t})$. The standard deviation of the actual GDP growth is 1.83; however, the standard deviation of Q_t in this model is merely 0.42. A more satisfactory model would generate an estimate of $\sigma_{u,y}$ low enough for the estimated GDP growth to mimic the fluctuations in the actual data.

In a sensitivity analysis, we experiment with three alternative sets of parameters that are chosen to be fixed by calibration. Table 1(B) sets one additional parameter (the intertemporal elasticity of substitution, IES) as fixed, i.e. $\sigma = 1$. Calibrating the IES parameter at the log-utility specification is a common setup in DSGE modeling and has been adopted in Teo (2009). The remaining ML estimates in Table 1(B) are similar to those in Table 1(A). The policy reaction parameters continue to be statistically insignificant. The second alternative set is to have the price stickiness parameter estimated using the ML method (see Table 1(C)). Other than the high estimate of stickiness ($\hat{\phi} = 0.945$) and that the policy reaction parameters become statistically significant, the remaining ML estimates show evidence of robustness.[11] In the third alternative set, the openness parameter is freely estimated to yield $\hat{\alpha} = 1.000$, inferring that the Taiwanese economy is completely open. Even with such an extreme value for α, the other ML estimates are close to those reported in columns (A), (B), and (C). In general, the ML estimation results are proved to be quite robust. For the nowcasting performance evaluation in the next subsection, we report the results based on the DSGE specification that was used to produce Table 1(A).

[Table 2 here]

In Table 2, we report the ML estimates of the reduced-form mixed-frequency model. The coefficients in these AR(1) state transition equations all exhibit high persistence. For the

[11] With $\hat{\phi} = 0.945$, the estimated pricing adjustment behavior is highly sluggish for it implies that on average each firm waits 18 months before resetting prices. As discussed in Kim (2010), the estimates of price stickiness could be very sensitive to the estimation strategies and model specifications. The range in the estimates of the price stickiness duration in some U.S. studies is wide too, being as short as 8 months and as long as 24 months.

reduced-form equation of y_t, the coefficients that are statistically significant correspond to lagged output growth (y_{t-1}) and lagged terms-of-trade changes (Δs_{t-1}). In the measurement equation, the most volatile measurement error series is associated with the actual GDP ($\hat{\sigma}_{u,y} = 1.376$). Given that this estimate is smaller when compared to the DSGE-MF models in Table 1, the RE-MF model has a satisfactory overall in-sample fit.

3.3 Nowcasting Real GDP Growth

In this subsection, we evaluate the structural model's ability to nowcast the real GDP growth rates against the reduced-form models. We estimate these models recursively over the period from 2012M1 - 2015M12. The nowcast evaluation is exercised based on a pseudo real-time dataset, which is a final vintage dataset that takes the ragged-edge data structure.[12] For instance, when we have data available up to 2015M12 and are interested in nowcasting GDP growth for the fourth quarter of 2015, the data used in the estimation include all the monthly series up to 2015M12 and all the quarterly series up to 2015M9. This is due to the publication lag of the GDP figures. The 2015Q4 GDP data will not be published until late January or early February of the following year. See Table 3 for an illustration of the data structure in nowcasting the GDP growth for 2015Q4.

[Table 3 here]

The state-space approaches of the DSGE-MF and RE-MF models are system approaches that jointly describe the dynamics of all the variables considered in the models. Their computational burden may greatly increase as the models include more variables and the dimension of the parameter set rises. As an alternative approach that allows one to deal with data sampled at different frequencies, the MIDAS regression is a popular reduced-form model that can be found in many empirical applications. We consider a basic MIDAS regression for

[12] In Taiwan, no real-time data on quarterly national accounts are available. Given that policy evaluation is not the main purpose of the current paper, our second-best choice is to use the pseudo real-time data. A number of empirical studies have conducted model comparisons based on the pseudo real-time data; see, for example, Schumacher and Breitung (2008), Giannone et al. (2008), and Foroni and Marcellino (2014a).

the purpose of nowcasting comparisons. Let τ index the quarter, Y_τ^Q denote the quarterly GDP growth we are interested in nowcasting, and $X_{k,\tau}^M$ denote the monthly explanatory variable that is dated as the k-th month of quarter τ, with $k = 1, 2$, and 3. A basic MIDAS model for a single monthly explanatory variable is given by:

$$Y_\tau^Q = \mu + \gamma \sum_{j=0}^{2} w_{3-j}(\theta_1, \theta_2)\ X_{3-j,\tau}^M + \varepsilon_\tau, \tag{23}$$

where

$$w_k(\theta_1, \theta_2) = \frac{\exp(\theta_1 k + \theta_2 k^2)}{\sum_{j=1}^{3} \exp(\theta_1 j + \theta_2 j^2)},$$

and μ, γ, θ_1, and θ_2 are regression parameters. The design of the normalized exponential Almon lag polynomial helps to prevent the proliferation of the parameters set. We estimate an extended version of (23) that includes all five monthly variables in our structural model as the skip-sampled explanatory variables in our MIDAS regression.[13] According to the unit root test, only the interest rate series appears to be nonstationary.[14] Therefore, the monthly variables included in the MIDAS regression are the first-difference of the interest rate, the percentage change in the exchange rate, the percentage change in the terms-of-trade, the domestic inflation rate, and the U.S. inflation rate. The MIDAS regression parameters are estimated by using the Nonlinear Least Squares method.

[Table 4 here]

The nowcasting comparison results based on the DSGE-MF, the RE-MF and the MIDAS models are presented in Table 4, with bold figures indicating the model that produces the

[13]The mixed-frequency model is a flexible model that can incorporate any timely information that proves to be useful for nowcasting, such as survey data from experts or a monthly industrial production index. Because the current paper focuses on a structural model, it only includes variables that are considered in the DSGE framework. For comparison purposes, it is only fair if we use the same set of observables as in other models.

[14]The Augmented Dickey Fuller (ADF) test is applied to the dependent variable (GDP growth rate) and the null of a unit root is rejected at the conventional significance level. For the explanatory variables, the ADF tests reject the null of a unit root for the domestic and foreign inflation rates, the changes in the terms-of-trade, and the changes in the exchange rate. The test shows that the skip-sampled monthly interest rate series contains a unit root. The ADF test results are available from the authors upon request.

smallest squared nowcast error for a particular quarter. In the 16 quarters under evaluation, the DSGE-MF model has the smallest squared error in 9 cases, the RE-MF model has that in 4 cases, and the MIDAS regression has that in 3 cases. A comparison based on the RMSE (root-mean-squared-error) also favors the DSGE-MF model (0.944 from the DSGE-MF versus 1.266 from the RE-MF and 1.462 from the MIDAS). When scrutinizing the outcome more closely, we find that the MIDAS model tends to underestimate the true GDP growth in most quarters. Consequently, the three cases with the lowest squared errors from the MIDAS model merely emerge by chance. When we limit our attention to compare the two mixed-frequency state-space models, the DSGE-MF beats the RE-MF model in 11 quarters and loses to it in 5 quarters. This leads us to conclude that the DSGE-based mixed-frequency model outperforms both the reduced-form mixed-frequency model and the MIDAS regression.

It is noted, however, that the DSGE-MF model performs disappointingly with larger nowcast errors for quarters with sudden output contraction (i.e., 2012Q2, 2013Q1, 2015Q3 and 2015Q4). Several possibilities could have contributed to these underperforming moments. First, the structural model selected for the small open economy in Section 2 needs to be revised in order to closely mimic the true economic structure of Taiwan. The structural model we adapt is abstracted from specifying capital goods, money, and wages, and from distinguishing between tradable and non-tradable goods. A more realistic DSGE model that incorporates investment and the financial sector may benefit our nowcasting task. Secondly, the insignificant monetary policy reaction function may suggest that the Taylor rule has not been adopted in Taiwan during the sample period. If this is true, instant changes in the interest rate will fail to give the model a correct inference on the expected inflation or expected future output growth.

With the above-mentioned model limitations in mind, working with a structural model will certainly pay off in terms of seeking a way to improve it. In our application, the estimation results lead us to look for a different policy rule that better suits the Taiwanese economy. In brief, the mixed-frequency structural modeling approach proposed in this paper generates nowcasting gains because of the model's superiority in two respects. The mixed-frequency approach allows us to use timely available information and the structure helps in shedding light on possible ways to improve the empirical model.

4 Conclusion

The major contribution of this paper is that it builds a mixed-frequency small open economy structural model. Based on a monthly small open dynamic stochastic general equilibrium (DSGE) model, we demonstrate how to develop a state-space representation that incorporates both monthly and quarterly observations. The key innovation in our method is that it introduces an aggregation rule in that the latent aggregator is linked to the observed quarterly output growth via aggregation. The mixed-frequency structural model is jointly estimated by the Kalman filter. Finally, we use the Kalman smoother to estimate the current latent aggregator and use it to nowcast real GDP growth.

Taiwanese data from January 1998 to December 2015 are used to assess whether the mixed-frequency structural model has an advantage over the reduced-form mixed-frequency model and the MIDAS model in nowcasting real GDP growth. We find that the DSGE-based mixed-frequency model produces better results than the other two reduced-form alternatives. This suggests a promising information superiority based on the structural model and the mixed-frequency estimation strategy.

The proposed DSGE-MF method, however, is subject to a few limitations. First, a structural model is more prone to specification errors. The small open DSGE model we consider is rather simple relative to reality. For example, the model is abstracted away from capital goods and monetary aggregates. Wage stickiness is not taken into account, either. The insignificant Taylor reaction function may indicate that the interest rate rule is not adopted in Taiwan. A continuous search for structural models that can better characterize a specific small open economy is required before we can make the DSGE-based mixed-frequency framework more valuable in practice.

Second, the computational cost of a mixed-frequency structural model may increase rapidly as the structure becomes more realistic and complex. The maximum likelihood estimates of the parameters could become very sensitive to model specifications as the state-space model involves more variables and the dimension of the parameter set increases. A possible solution for this difficulty is to impose a common factor structure in the state-space representation, which can effectively reduce the number of parameters to be estimated.

Another extension to our current framework that may prove to be fruitful is to develop a more efficient and informative measurement equation. This may be achieved by including relevant monthly indicators that contain important messages about real output. For example, a monthly business sentiment measure such as the Purchasing Managers' Index (PMI) may provide valuable information for nowcasting real GDP growth. We leave this task to future research.

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Table 1: Full sample estimation results of the DSGE-MF model

Parameter		**(A)**		**(B)**		**(C)**		**(D)**	
		Calibrated Value		Calibrated Value		Calibrated Value		Calibrated Value	
Discount rate	β	0.998		0.998		0.998		0.998	
Inv. of IES	σ			1.000		1.000		1.000	
Inv. of labor supply elast.	φ	5.000		5.000		5.000		5.000	
Openness	α	0.530		0.530		0.530			
Price stickiness	ϕ	0.875		0.875				0.875	
		Estimate	S.E.	Estimate	S.E.	Estimate	S.E.	Estimate	S.E.
Inv. of IES	σ	1.169**	0.099						
Inv. of labor supply elast.	φ								
Openness	α							1.000**	0.000
Price stickiness	ϕ					0.945**	0.003		
AR(1) coef. of Δs_t	ρ_s	0.877**	0.051	0.886**	0.034	0.864**	0.001	0.854**	0.042
AR(1) coef. of Δy_t^*	ρ_{y*}	0.880**	0.062	0.879**	0.020	0.879**	0.058	0.879**	0.066
AR(1) coef. of π_t^*	$\rho_{\pi*}$	0.938**	0.031	0.938**	0.031	0.938**	0.031	0.938**	0.031
Interest rate smoothness	ρ_R	0.980**	0.008	0.977**	0.007	0.979**	0.011	0.976**	0.011
Policy reaction to inflation	ψ_π	-0.094	0.447	0.007	0.006	-0.115**	0.056	0.012	0.024
Policy reaction to output gap	ψ_y	0.537	1.514	0.512	1.141	0.485**	0.107	0.510	1.911
Std. dev. of structural shocks:									
	σ_R	0.033**	0.015	0.033**	0.015	0.033**	0.015	0.033**	0.015
	σ_e	6.629**	0.450	6.752**	0.436	6.731**	0.438	6.277**	0.428
	σ_s	0.868**	0.157	1.099**	0.130	1.055**	0.131	0.698**	0.092
	$\sigma_{\pi*}$	0.456**	0.044	0.456**	0.044	0.456**	0.044	0.456**	0.044
	σ_{y*}	0.027**	0.009	0.027**	0.008	0.027**	0.009	0.027**	0.009
Std. dev. of measurement errors:									
	$\sigma_{u,\pi}$	0.564**	0.078	0.580**	0.083	0.587**	0.084	0.462**	0.097
	$\sigma_{u,y}$	1.770**	0.190	1.814**	0.197	1.763**	0.189	1.814**	0.197
	$\sigma_{u,R}$	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	$\sigma_{u,e}$	0.000	0.179	0.000	0.036	0.000	0.007	0.000	0.063
	$\sigma_{u,s}$	5.037**	0.201	4.944**	0.205	4.948**	0.203	5.174**	0.202
	$\sigma_{u,\pi*}$	0.000	0.024	0.000	0.004	0.000	0.002	0.000	0.026
	$\sigma_{u,y*}$	0.449**	0.058	0.447**	0.052	0.447**	0.057	0.447**	0.059
ML value		-1570.72		-1573.98		-1572.30		-1562.78	

Note: ** indicates significance at the 5% level.

Table 2: Full sample estimation results of the RE-MF model

Regressors	State Transition Equations						
	eq (14)	eq (15)	eq (16)	eq (17)	eq (18)	eq (19)	eq (20)
	π_t	y_t	R_t	Δe_t	Δs_t	π_t^*	Δy_t^*
π_{t-1}	0.688**	-0.117					
	(0.071)	(0.120)					
y_{t-1}	-0.481	-0.674**					
	(0.337)	(0.102)					
R_{t-1}	0.367	1.735	0.979**				
	(0.920)	(1.969)	(0.005)				
Δe_{t-1}	-0.008	-0.047		0.922**			
	(0.020)	(0.038)		(0.028)			
Δs_{t-1}	0.068*	0.134*			0.964**		
	(0.038)	(0.073)			(0.019)		
π_{t-1}^*	0.049	-0.043				0.938**	
	(0.181)	(0.338)				(0.031)	
Δy_{t-1}^*	3.740	11.030					0.903**
	(5.941)	(14.273)					(0.056)
Std. dev. of state equation shocks:							
	0.870**	0.166	0.033**	2.469**	1.558**	0.456**	0.023**
	(0.059)	(0.303)	(0.015)	(0.482)	(0.124)	(0.044)	(0.008)
Std. dev. of measurement errors:							
	0.000	1.376**	0.000	0.000	0.000	0.000	0.472**
	(0.094)	(0.390)	(0.000)	(0.022)	(0.061)	(0.012)	(0.075)
ML value:			-1085.02				

Notes: Figures in parentheses are standard errors. * indicates significance at the 10% level; ** indicates significance at the 5% level; 0.000 indicates an estimate that is smaller than 0.001.

Table 3: Mixed-frequency data structure

YR/MO	CPI Inflation (%) INF_t	Interest Rate (%) $RATE_t$	Exchange Rate Change (%) ΔEXG_t	TOT Change (%) ΔTOT_t	Foreign CPI Inflation (%) INF_t^*	GDP Growth (%) $GDPGR_t$	Foreign GDP Growth (%) $GDPGR^*$
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
2014 M07	1.764	0.387	-0.137	0.000	2.202		
2014 M08	2.074	0.386	0.025	-1.404	1.955		
2014 M09	0.711	0.387	1.249	-2.496	1.821	1.259	1.218
2014 M10	1.050	0.387	3.188	-3.695	1.702		
2014 M11	0.854	0.387	4.069	-6.489	1.430		
2014 M12	0.602	0.387	5.494	-9.243	1.261	0.227	0.573
2015 M01	-0.939	0.387	4.696	-11.528	0.782		
2015 M02	-0.204	0.388	3.914	-10.280	1.017		
2015 M03	-0.621	0.387	3.547	-10.444	1.035	0.472	0.508
2015 M04	-0.819	0.387	2.848	-10.135	0.696		
2015 M05	-0.731	0.387	1.788	-9.701	0.633		
2015 M06	-0.565	0.387	3.561	-9.921	0.678	-1.145	0.647
2015 M07	-0.632	0.387	4.476	-9.979	0.786		
2015 M08	-0.439	0.367	7.826	-11.022	0.906		
2015 M09	0.296	0.320	9.067	-9.993	0.604	-0.302	0.493
2015 M10	0.315	0.301	7.543	-8.565	0.609		
2015 M11	0.529	0.301	6.524	-6.613	0.806		
2015 M12	0.135	0.275	4.946	-5.250	0.906		

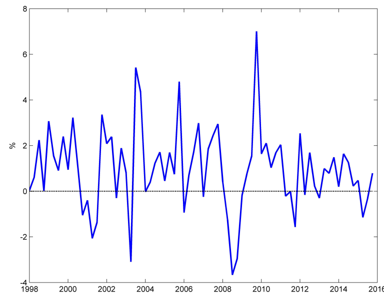
Table 4: Results of nowcasting real GDP growth (qoq)

YR/MO	Real GDP Growth	DSGE-MF Model		RE-MF Model		MIDAS Model	
		Nowcast value	Sq. error	Nowcast value	Sq. error	Nowcast value	Sq. error
	(A)	(B)	(C)	(D)	(E)	(F)	(G)
2012 M03	2.536	**1.010**	**2.331**	-0.032	6.594	-0.038	6.630
2012 M06	-0.157	1.065	1.493	**-0.026**	**0.017**	-0.944	0.620
2012 M09	1.692	**1.538**	**0.024**	0.390	1.695	-0.618	5.337
2012 M12	0.233	**0.555**	**0.104**	2.213	3.921	-1.535	3.125
2013 M03	-0.300	0.858	1.342	**0.400**	**0.491**	-1.445	1.310
2013 M06	0.993	0.505	0.239	**1.231**	**0.057**	-0.432	2.031
2013 M09	0.786	**0.563**	**0.050**	1.527	0.548	-0.201	0.973
2013 M12	1.487	**1.420**	**0.005**	0.641	0.716	-0.190	2.813
2014 M03	0.200	1.104	0.818	1.513	1.724	**0.194**	**0.000**
2014 M06	1.641	**1.576**	**0.004**	1.061	0.336	-0.080	2.962
2014 M09	1.259	1.023	0.056	**1.086**	**0.030**	-0.772	4.128
2014 M12	0.227	**0.529**	**0.091**	-0.189	0.173	-1.172	1.957
2015 M03	0.472	**0.395**	**0.006**	-0.523	0.991	-0.665	1.294
2015 M06	-1.145	1.018	4.677	1.540	7.210	**-0.554**	**0.349**
2015 M09	-0.302	1.306	2.588	0.529	0.692	**-0.475**	**0.030**
2015 M12	0.790	**1.436**	**0.418**	1.450	0.436	0.001	0.623
RMSE			0.944		1.266		1.462

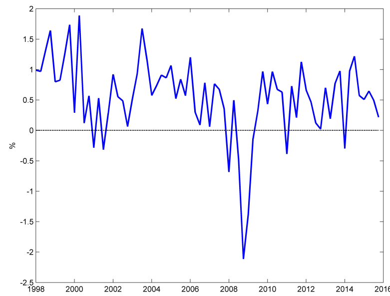
Note: Entries in bold are associated with the model that produces the smallest squared nowcast error for a particular quarter.

Figure 1: Observables

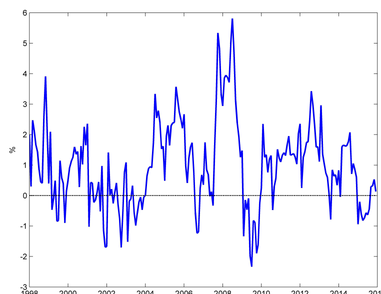
(a) Taiwan real GDP growth



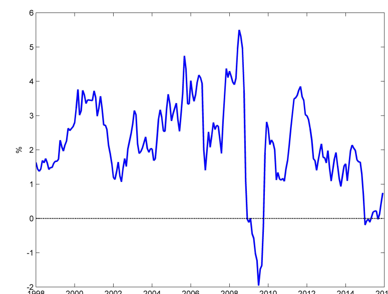
(b) U.S. real GDP growth



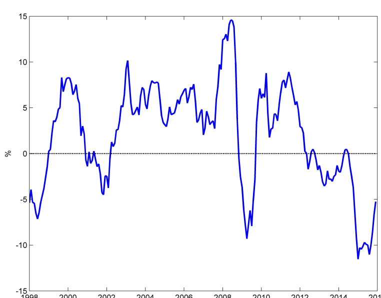
(c) Taiwan CPI inflation



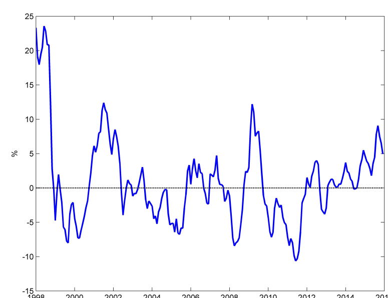
(d) U.S. CPI inflation



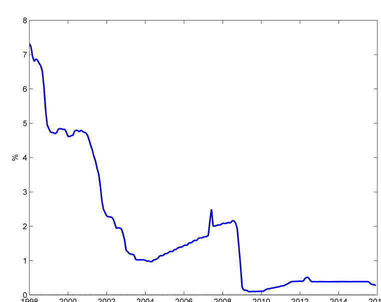
(e) Changes in terms-of-trade



(f) Changes in exchange rate



(g) Interest rate (annual rate)



(h) Changes in interest rate

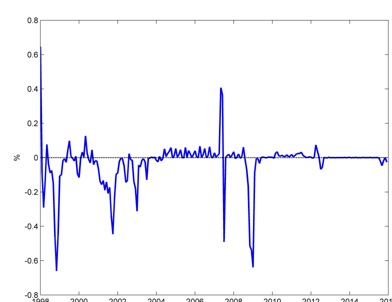
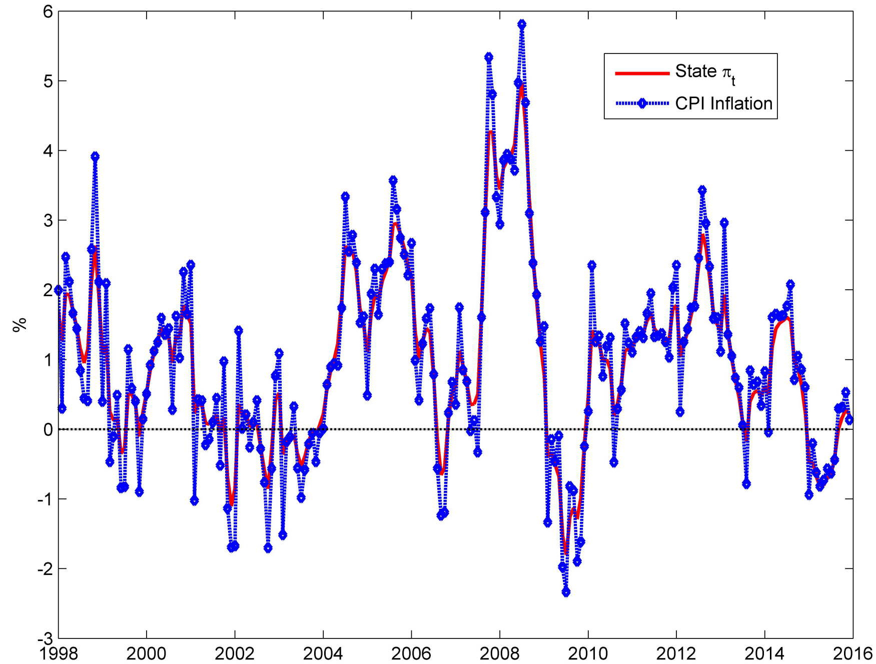
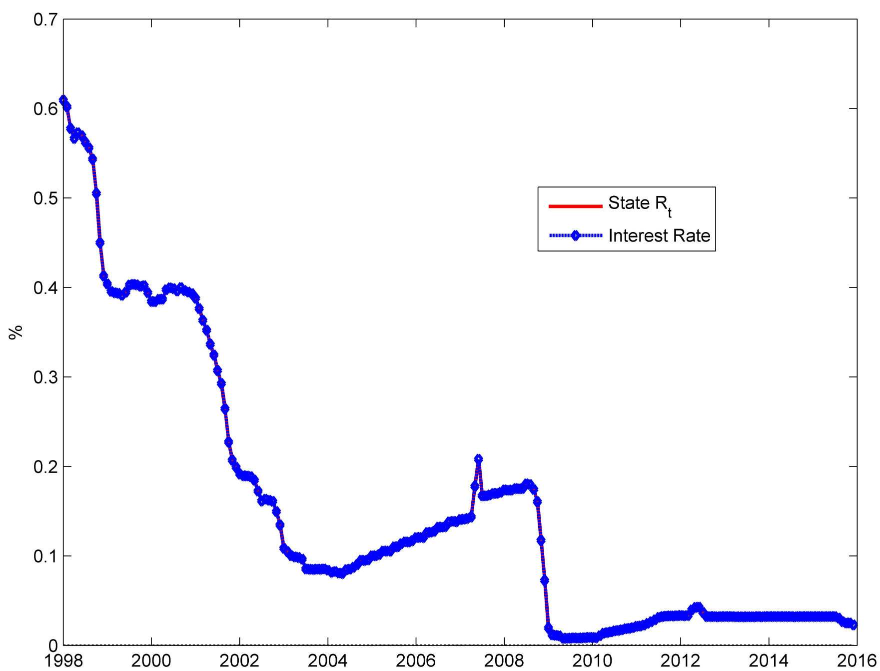


Figure 2: Estimated state variables (from the DSGE-MF model) vs. observed data

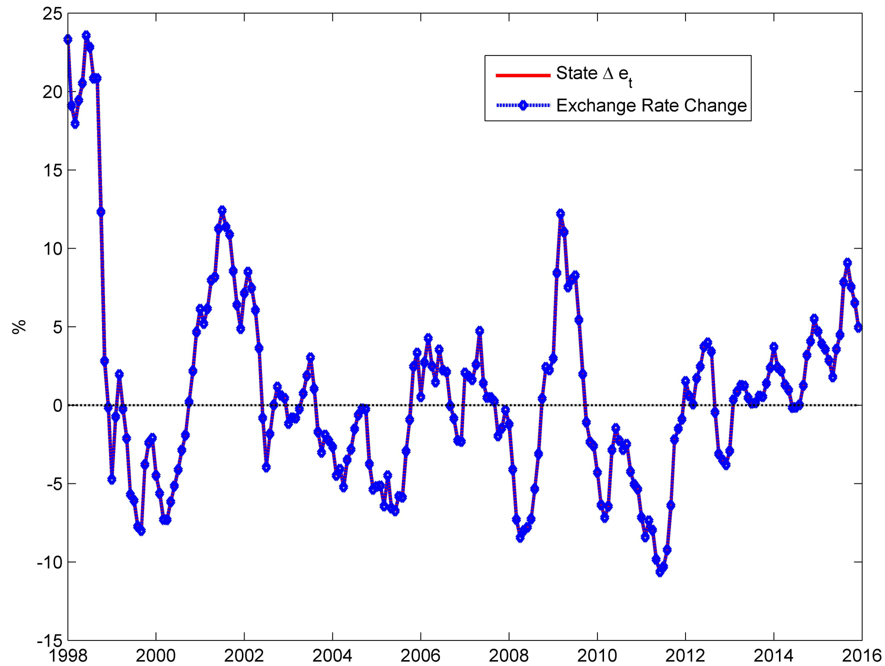
(a) State variable π_t vs. CPI inflation rate



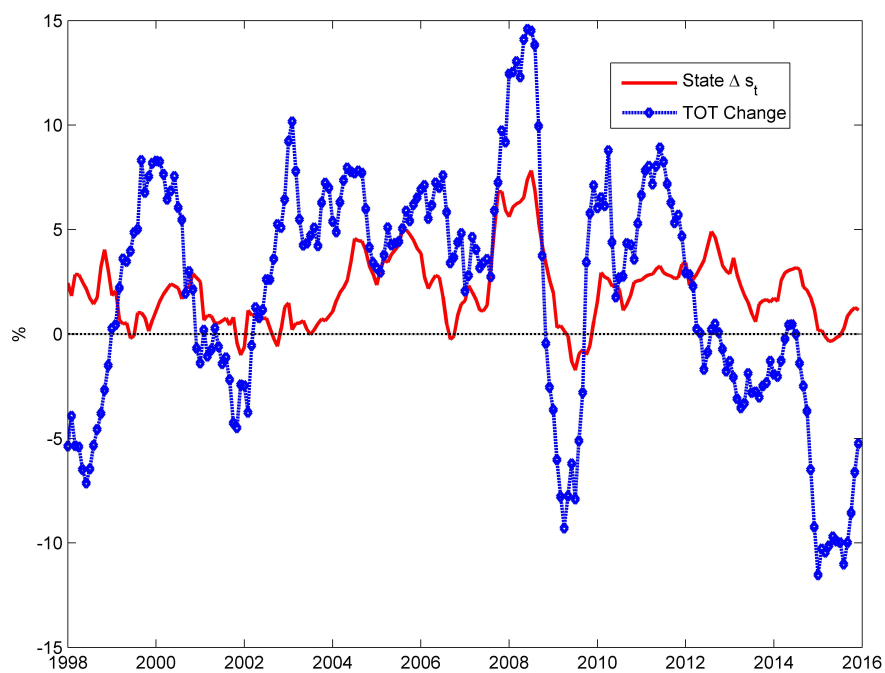
(b) State variable R_t vs. interbank call rate (monthly rate)



(c) State variable Δe_t vs. percentage changes in nominal exchange rate



(d) State variable Δs_t vs. percentage changes in terms-of-trade



(e) State variable Q_t vs. actual GDP growth rate