Rational PhD Glut*

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Abstract

There exists a growing glut of PhDs in some academic fields. In this study, we develop an occupational choice model with rational PhDs to explain such a phenomenon and provide quantitative and welfare analysis. We distinguish two stages of market demand for PhDs. The first stage is the professors' demand for doctoral students in knowledge production. The second stage is the market demand for professors in higher education. Students will choose an academic career if their expected lifetime income, i.e., the training wage in the first stage plus the discounted professor wage, meets the participation constraint. By calibrating the model to Taiwanese data as an example, we find some deviations of the number of doctoral students from the social optima in all fields.

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1. Introduction

There exists a growing glut of PhDs in some academic fields. A survey of Sauermann and Roach (2012) indicates that most PhD students in science fields prefer academic careers in research and teaching. However, Cyranoski, Gilbert, Ledford, Nayar, and Yahia (2011) show that in 2006 only 15% of science PhDs in the US found a tenure-track position within six years after receiving their degrees. Comparing annual PhD graduates in science between 1998 and 2008, Cyranoski, et al. (2011) also find that the total number has increased by nearly 40% among OECD countries. Therefore, given such a low rate of finding academic positions, there exists a growing glut of PhDs in science fields. This seems to contradict economic equilibrium hypotheses such as the engineer-scientist model of Arrow and Capron (1959) which suggests that any oversupply in the labor market should decrease overtime. Nevertheless, as Ehrenberg (1992) points out, due to limited data there is little economic research on the occupational choices of doctorates.

In this paper, we develop an occupational choice model to justify the aforementioned phenomenon. The model is based on the overlapping generations (OLG) structure of Samuelson (1958) in which doctoral students overlap professors. Doctoral students here are individuals in the training stage and thus include PhD students and postdocs. They are assumed to have two sources of lifetime income in their academic career. One is the student wage received while being a teaching or research assistant. The other is the expected wage from being a professor, but students have some risks when finding academic positions. Thus, given an uncertain lifetime income and a low probability of finding academic positions, students would choose to stay in an academic career only in cases of having a satisfactory wage level during the training stage. That is, their expected lifetime income from the student wage plus the discounted professor wage should meet the participation constraint of an academic career. This decomposition of lifetime income provides a plausible explanation for the PhD glut in academia.

In particular, we distinguish the PhD glut problem and the labor market oversupply by introducing two stages of market demand. The first stage is the professors' demand for doctoral students in knowledge production. The second stage is the market demand for professors as providers of higher education. The labor supply of PhDs is only determined in the first stage, i.e., the number of students who decide to pursue doctoral degrees. As long as the expected lifetime income meets the participation constraint, we could have more PhD graduates than market vacancies for professors in the second stage. In this aspect of sequential demand, the market in the first stage actually does not have an oversupply problem since the number of doctoral students just equals the quantity demanded in knowledge production in which professors are employers. However, a PhD glut problem could still exist because the equilibrium number of doctoral students can be larger than the socially optimal level.

Previous studies of occupational choice focus on the process of expectation formation. Freeman (1975a, 1975b, 1976) has related studies on professional markets with cobweb-type models of adaptive expectation. In a cobweb model, the inelasticity of labor supply is due to a lag in response to demand shocks because individuals take the current wage as a prediction of future wages. On the other hand, Siow (1984) proposes a forward-looking model in which individuals have rational expectations in assessing future wages and response to demand shocks is rapid. Ryoo and Rosen (2004) develop a framework to model different processes of expectation formation and find that the rational expectation process is well-suited to the data in the US engineer market. Nevertheless, Manski (1993, 2004) argues that observed data may be consistent with some alternative specifications of expectation formation and then suggests to measuring subjective beliefs directly. According to surveys of students, Betts (1996), Jensen (2010), and Zafar (2011) find that individuals commonly have biased beliefs about current wages; but in an experimental environment Wiswall and Zafar (2015a, 2015b) demonstrate that students rationally revised their beliefs in response to new information.

In our model, doctoral students also have rational expectations in assessing future market conditions; however, their occupational choices could be inelastic over time. The most important information in higher educational market is the number of students. As Zarkin (1985) points out, the demand for school teachers is determined by the number of children who are already born. Hence, the demand for professors can be explicitly estimated by the number of existing students prior to college, which should be known to doctoral students in determining their occupational choices. However, given negative information about demand shocks on future enrollment reduction, the number of doctoral students could still be the same in the first stage and result in a glut in the second stage. This is because the effect of demand shocks can be compensated by adjusting current training wages. In other words, professors would provide higher incentives to attract students in knowledge production. Indeed, our analysis shows that professors will do so if the doctoral students' productivity is high. The two-stage demand is how rational PhDs could respond slowly to demand shocks, as a result which contradicts most rational models on occupational choice.

In a series of papers debating the PhD glut problem in the US biomedical sciences, there are some policy recommendations to the National Institutes of Health (NIH), such as reducing the number of new entrants for PhDs, increasing the financial compensation for postdocs (Alberts, Kirschner, Tilghman, and Varmus, 2014, 2015), and diversifying PhD training for nonacademic research careers (Daniels, 2015). Nevertheless, Kelly and Marians (2014) argue that the NIH may not have sufficient data to determine the optimal level of new entrants and this should be optimized by market mechanisms. In fact, Germain (2015) and Sauermann and Roach (2016) find that PhD students and postdocs have recognized future career risks but there still exists persistent labor market imbalances in science fields.

Our model can clarify the aforementioned debates and provide quantitative analysis for the optimal level of new entrants. In equilibrium, rational students, through market mechanisms, still exhibit a persistent glut because of the compensation from professors, which is consistent with the observations of Germain (2015) and Sauermann and Roach (2016). To provide quantitative analysis, we calibrate the model to data in Taiwan during the period from 2004-2013 as an example. The Taiwanese data has several advantages in practice. First, the wages of professors in Taiwan are publicly available and hence there is no concern about biased-wage beliefs such as in Betts (1996), Jensen (2010), and Zafar (2011). The uncertainty of lifetime income is mainly from the probability of finding an academic position. Second, the supply of local PhDs can be explicitly determined since most do not search for academic positions outside of Taiwan. Following Bound, Braga, Golden, and Turner (2013), we also count the PhDs with doctoral degrees from abroad and then estimate market vacancies and actual demand for local PhDs by estimating the proportions of newly-hired professors who receive local PhD degrees every year.

We also highlight the distinction between fields and calibrate for the following five fields: engineering, science, medical sciences, humanities, and social sciences. The result suggests that PhD students in engineering, science, and medical sciences have higher academic productivities than those in humanities and social sciences. In particular, the academic productivity of PhD students in medical sciences is almost twice as high as that in social sciences. This implies that professors in medical sciences strongly need PhD students as inputs in knowledge production, such as research assistants conducting experiments in laboratories. In fact, our data shows that the proportion of students studying PhD programs in medical sciences is also higher than that in social sciences.

In addition, using the calibrated result as a benchmark, we conduct policy experiments by setting various population growth rates and wage premiums for PhD workers. The wage premiums for PhD workers in industry are defined as the additional wages for workers with PhD degrees compared to workers without PhD degrees. We highlight these two variables because a lower birth rate is a global issue and wage premiums are related to the policy suggestion of Daniels (2015) in diversifying PhD training for nonacademic needs. The result shows that, when population declines, a lower education demand will lead to a smaller proportion of students entering PhD programs. However, the magnitude is quite different between fields and it mainly depends on the field wage premiums for PhD workers. For example, the wage premium in social sciences is highest and this provides more incentive for students in this field to study a PhD. As a result, given a negative population growth rate, social sciences has the lowest decline in PhD entrants and hence has a lower probability for their PhD graduates to find an academic position. In contrast, PhD graduates in other fields have higher probabilities to find an academic position under the same circumstance.

Finally, we define the socially optimal proportion of doctoral students by solving the cost minimization problem in which discounted-lifetime working wages is the opportunity cost for doctoral students. The wage premium for PhD workers, the population growth rate, the academic productivity of PhD students, and the discount rate are determinant factors in the social optima. We then use a calibrated parameter to measure the differences between the actual proportions of doctoral students and their socially optimal levels in different fields. The result shows that PhD entrants are higher than their social optima in all fields. In other words, PhD gluts actually exist. The policy suggestion of Alberts, et al. (2014) for reducing PhD entrants toward the optimal level is reasonable in this sense. However, the optimal level is determined by the aforementioned factors, which are different between fields. For example, social sciences has the largest deviation from social optima because of having the lowest population growth rate and student's academic productivity, while medical sciences has the smallest deviation. Such a distinction should be taken into account when making policy suggestions for solving the PhD glut problem.

The rest of the paper is organized as follows. Section 2 presents the model to illustrate how students choose a career according to their expected lifetime income, and then solves the model equilibrium in steady state. Section 3 provides quantitative analysis and policy experiments based on Taiwanese data. Section 4 presents welfare analysis. Section 5 concludes the paper and discusses possible extensions.

2. Model

This is an occupational choice model for academic careers, which applies an OLG structure in which doctoral students and professors are overlapped with two stages of market demand. In the first stage, professors hire doctoral students as an input in knowledge production. For doctoral students, our definition includes PhD students and postdocs as well. As Cantwell and Lee (2010) emphasize, postdocs are not faculty because their duty is to do research. A survey of Nerad and Cerny (1999) indicates that the main reasons for choosing postdocs are training and as a necessary step to tenured positions. In addition, Stephan and Ma (2005) find that the financial benefits of pension and health care have significantly positive contribution to the increased frequency and duration of postdocs in the US. Therefore, in our model, both PhD students and postdocs are defined generally as doctoral students in the training stage and the financial benefits for them play a crucial role as an equilibrium price. After the training stage, there exists a second-stage demand for professors as providers of knowledge in higher education and in this stage doctoral students will become professors or otherwise become workers.

2.1 Occupational states

Time is infinite, $t = 0,1,2,...$. At generation t, there are n_t agents living for two periods, a young and an old period. Let $g > 0$ be the gross rate of population growth and then we have $n_{t+1} = gn_t$. For every generation t, $y_t(i) \in \{0,1\}$ and $o_t(i) \in \{0,1\}$ are respectively the occupational states of agent i when young and old for $i = 1,2,...,n_t$. A young agent can choose to be a worker as $y_t(i) = 0$ or to be a doctoral student as $y_t(i) = 1$. A young worker $y_t(i) = 0$ will surely become an old worker $o_t(i) = 0$ in the next period. A doctoral student $y_t(i) = 1$ will become a professor $o_t(i) = 1$ with a probability of $p_{t+1} \in [0,1]$ and become an old worker $o_t(i) = 0$ with a probability of $(1 - p_{t+1})$. That is, if doctoral students cannot become professors, they will switch to being workers in the next period.

Let $w_t > 0$ and $E_t[w_{t+1}] > 0$ respectively be the current and expected wage of a worker. The expected lifetime income for a worker of generation t is

$$W_t \equiv w_t + \beta E_t[w_{t+1}], \tag{1}$$

where $\beta \in (0,1)$ is the discount factor. On the other hand, the expected lifetime income for a doctoral student of generation t is

$$D_t \equiv s_t + \beta p_{t+1} E_t[d_{t+1}] + \beta(1 - p_{t+1}) E_t[w_{t+1}], \tag{2}$$

where $s_t \in (0, w_t)$ is the current wage of a doctoral student as a teaching assistant or research assistant; $E_t[d_{t+1}] > 0$ is the expected wage as a professor in the next period. A young agent will choose to be a doctoral student if $D_t \geq W_t$, which is the participation constraint of an academic career.

2.2 Knowledge production

Following Levin and Stephan (1991), the income of professors is generated by knowledge production. In most fields, Wuchty, Jones, and Uzzi (2007) show that the team production of knowledge dominates solo work. In addition, Petersen, Riccaboni, Stanley, and Pammolli (2012) demonstrate decreasing marginal return with team size and emphasize that team productivity is related to group efficiency in knowledge production. Thus, we highlight the collaboration between professors and doctoral students and their productivity in the process of knowledge production.

Let μ_t be the proportion of young agents of generation t who choose to be doctoral students, i.e., $\mu_t = \Sigma_{i=1}^{n_t} y_t(i) / n_t$. Thus, at time t, there are $n_{t-1}\mu_{t-1}p_t$ professors and $n_t\mu_t$ doctoral students. We define the knowledge production function of professors and doctoral students as

$$X_t = A n_t^{1-\alpha}(n_{t-1}\mu_{t-1}p_t + \lambda n_t \mu_t)^{\alpha}, \tag{3}$$

where X_t is the output; $A > 0$ and $\alpha \in (0,1)$ are respectively parameters for total factor productivity and marginal return; $\lambda \in (0,1)$ is the doctoral student productivity. The output function implies that professors and students are imperfectly substitutable and one professor is equal to $1/\lambda$ doctoral students. Hence a larger λ means a relatively higher productivity of doctoral students. Moreover, the output is increasing with the size of the younger generation, which reflects the spillover tendency of knowledge as seen in Jaffe (1989) and Audretsch and Feldman (1996).

For every generation t, there are two stages of market demand for PhDs in knowledge production. In the first stage, professors ($n_{t-1}\mu_{t-1}p_t$) from the last generation hire doctoral students ($n_t\mu_t$) to produce knowledge. In the next period, the market has a second-stage demand for new professors ($n_t\mu_t p_{t+1}$) as providers of knowledge. We assume that every young agent needs to have (consume) knowledge of e. When knowledge is produced, professors then provide (sell) knowledge to young agents and the total need of knowledge e constitutes the second-stage demand for professors.

In the first stage, professors as employers try to maximize their benefit after paying wages to doctoral students. Note that (3) is homothetic. By defining $x_t = X_t / n_t$, we have

$$x_t = A(\frac{\mu_{t-1} p_t}{g} + \lambda \mu_t)^\alpha . \tag{4}$$

Since all doctoral students earn $s_t n_t \mu_t$ by being teaching or research assistants, the wage for a professor is thus

$$d_t = \frac{n_t x_t - s_t n_t \mu_t}{n_{t-1} \mu_{t-1} p_t} , \tag{5}$$

for $t = 0,1,2,\ldots$. Note that $n_t \mu_t$, the number of students who choose to be doctoral students, is not directly controlled by professors; however, professors can set the enrollment limit to maximize (5). Let m_t be the enrollment capacity, i.e., the proportion of young agents n_t who can be enrolled to be doctoral students. By maximizing (5), we can derive the demand function for doctoral students by substituting $\mu_t = m_t$ into (5) and then obtain the first order condition of

$$m_t = \lambda^{\frac{\alpha}{1-\alpha}} \left(\frac{\alpha A}{s_t} \right)^{\frac{1}{1-\alpha}} - \frac{\mu_{t-1} p_t}{\lambda g} . \tag{6}$$

Equation (6) is the first-stage demand for doctoral students. However, if s_t is too low, professors might not have enough students, i.e., $\mu_t < m_t$. Therefore, the supply of doctoral students μ_t in this stage should also be considered when determining the equilibrium path.

2.3 Equilibrium

In this subsection, we solve the model equilibrium in steady state. That is, given parameters, the proportion of doctoral students, the probability of being professors, and their wages converge to the equilibrium values over time. Recall the participation constraint that a young agent will choose to be a doctoral student if $D_t \geq W_t$. Thus, $D_t = W_t$ determines the supply function of doctoral students as follows:

$$s_t = w_t + \beta p_{t+1} E_t [w_{t+1} - d_{t+1}] . \tag{7}$$

In equilibrium, demand equals supply and then $\mu_t = m_t$ for $t = 0,1,2,\ldots$. Substituting (7) into (6) yields the equilibrium condition as

$$m_t = \lambda^{\frac{\alpha}{1-\alpha}} \left(\frac{\alpha A}{w_t + \beta p_{t+1} E_t [w_{t+1} - d_{t+1}]} \right)^{\frac{1}{1-\alpha}} - \frac{m_{t-1} p_t}{\lambda g} . \tag{8}$$

where d_{t+1} contains m_t, m_{t+1}, and s_{t+1}. Substituting $\mu_t = m_t$ and $\mu_{t+1} = m_{t+1}$ into (5) gives

$$d_{t+1} = \frac{g}{m_t p_{t+1}} \left(A \left(\frac{m_t p_{t+1}}{g} + \lambda m_{t+1} \right)^{\alpha} - s_{t+1} m_{t+1} \right). \tag{9}$$

Hence (8) is the condition for the equilibrium path of $\{m_t; t = 1,2,3,...\}$. Moreover, we have $E_t[s_{t+1}] = E_t[w_{t+1} + \beta p_{t+2} E_{t+1}[w_{t+2} - d_{t+2}]]$ in (8). Because all agents of generation t make decisions according to information in period t, we have $E_t[w_{t+2}] = E_t[E_{t+1}[w_{t+2}]] = E_t[w_{t+1}]$ by the law of iterated expectations. In general, for all young agents of generation t, we define $E_t[w_{t+1+j}] = E_t[E_{t+j}[w_{t+1+j}]] = E_t[w_{t+1}] = \overline{w}$ for $j = 1,2,3,...$.

We focus on the equilibrium quantities and prices of the market. In Proposition 1, we first describe the equilibrium path of m_t, the proportion of young agents who choose to be doctoral students in the first stage, and equilibrium path of p_t, the probability of being a professor in the second stage. The equilibrium prices are discussed later in Proposition 3.

Proposition 1. *In steady state, we have* $\{m_t, p_t\} = \{m_{t+1}, p_{t+1}\} = \{m^*, p^*\}$ *such that*

$$m^* = \left(\frac{g}{p^* + \lambda g} \right) \left(\frac{A\left(\lambda\alpha(1-\beta g) + \beta(p^* + \lambda g)\right)}{\overline{w}\left(1 + \beta p^*\right)} \right)^{\frac{1}{1-\alpha}},$$

$$p^* = \frac{A\lambda\left(\alpha\left(1-\beta g\right) + \beta g\right) - \overline{w}a}{\beta\left(\overline{w}a - A\right)},$$

where $a = (e / A)^{(1-\alpha)/\alpha}$.

Proof. See Appendix 1.

In Proposition 1, we are interested in the marginal effect of student productivity (λ) and population growth (g) on m^* and p^*. Note that in steady state p^* should be in the range of $[0,1]$, this also restricts the valid values of parameters. First, if $A > \overline{w}^{\alpha} e^{1-\alpha}$, we must have $\beta\left(\overline{w}a - A\right) < 0$. The denominator of p^* in Proposition 1 is thus negative and the numerator of it should also be negative. Specifically, λ and g should be sufficiently small such that $\beta\left(\overline{w}a - A\right) \le A\lambda\left(\alpha\left(1-\beta g\right) + \beta g\right) - \overline{w}a \le 0$ to maintain $p^* \in [0,1]$. In these cases, we would have $dp^* / d\lambda < 0$ and $dp^* / dg < 0$ because of $\alpha\beta < \beta$. Second, if $A < \overline{w}^{\alpha} e^{1-\alpha}$, the denominator of p^* in Proposition 1 is positive. However, if λ or g is sufficiently small such that $\lambda\beta g \le 1$, we will have $\lambda\left(\alpha\left(1-\beta g\right) + \beta g\right) \le 1$; in such cases, the numerator is negative and hence we cannot have $p^* \in [0,1]$. We conclude this result in Proposition 2.

Proposition 2. *If* λ *or* g *is sufficiently small, we have* $p^* \in [0,1]$ *and* p^* *is a decreasing function of* λ *and* g.

In other words, an increase in doctoral student productivity or population growth rate will lower the probability of becoming a professor. A reasonable conjecture is that higher values of λ or g will induce more students to choose academic careers. For example, given a higher value of λ, professors may raise the training wage to attract more students for optimal knowledge production. However, the implication of changing λ and g on m^* in Proposition 1 is complicated because it depends on the relative values of parameters. In order to demonstrate such a conjecture, we solve for the equilibrium wages of students and professors in Proposition 3.

Proposition 3*. In steady state, we have* $\{s_t, d_t\} = \{s_{t+1}, d_{t+1}\} = \{s^*, d^*\}$ *such that*

$$s^* = \overline{w}(1+\beta p^*) + \left(\overline{w}(1+\beta p^*) - A\left(\lambda + \frac{p^*}{g}\right)^{\alpha} (m^*)^{\alpha-1}\right)\left(\frac{\beta g}{1-\beta g}\right),$$

$$d^* = \frac{gA}{p^*}\left(\lambda + \frac{p^*}{g}\right)^{\alpha} (m^*)^{\alpha-1} - \frac{gs^*}{p^*},$$

where m^* *and* p^* *are derived from Proposition 1.*

Proof. See Appendix 2.

Proposition 3 shows that s^* and d^* are functions of m^* and it is also difficult to observe the marginal effect of λ and g on s^*, d^*, and m^* directly. We thus compute the equilibrium value of m^*, p^*, s^*, and d^* on various λ given a parameter set of $(g, \alpha, \beta, A, w, e) = (1, 0.85, 0.75, 2, 2, 0.1)$ in Figure 1. As we can see, the number of doctoral students m^* is increasing as doctoral student productivity λ increases. There are two reasons for this increasing trend. First, professors will provide a higher wage s^* to students if students' productivity increases. Second, the professor wage d^* increases dramatically when λ increases. These reasons are also shown in Figure 1. Thus, a higher current wage plus a larger future wage is attractive to students in choosing an academic career, so m^* is increasing in λ. Consequently, there are too many PhDs in the second-stage market and the probability of finding an academic position p^* decreases when λ increases in Figure 1, which has been proved in Proposition 2.

In addition, we are also interested in how the population growth rate affects the equilibrium results. Figure 2 illustrates the equilibrium value of m^*, p^*, s^*, and d^* on various g, given a parameter set of $(\lambda, \alpha, \beta, A, w, e) = (0.5, 0.85, 0.75, 2, 2, 0.1)$. We can observe similar patterns in that m^* is an increasing function of g and p^* is a decreasing function of g. However, the incentive for attracting more students to choose an academic career is not due to higher training wages as doctoral students but due to higher expected wages as professors. Figure 2 shows that s^* does not change when g increases. In contrast, d^* is increasing in g. That is, a larger population growth rate implies a higher demand for professors and hence a greater wage for professors in the second stage. Therefore, there are more students choosing an academic career in the first stage and this results in a lower p^* in the second stage.

2.4 Wage Premium

In this subsection, we extend the baseline model to incorporate the contribution of PhD training for nonacademic research careers. This extension allows us to analyze the policy recommendation of diversifying PhD training for nonacademic jobs suggested, for example, in Daniels (2015). Specifically, we introduce an exogenous parameter $\kappa \geq 1$, which is the wage premium of a PhD worker. That is, the wage of a PhD worker on a nonacademic job in period t is κw_t. Thus, the expected lifetime income for a doctoral student of generation t is

$$D_t = s_t + \beta\left(p_{t+1}d_{t+1} + \left(1 - p_{t+1}\right)\kappa w_{t+1}\right),\ \kappa \geq 1. \tag{10}$$

Following the analysis in section 2.3, we focus on the equilibrium in steady state, where $w_t = \overline{w}$, $m_t = m^*$, and $p_t = p^*$ are constant over time. The supply of doctoral students is decided by the optimal occupation choice of young agents, and the demand for doctoral students is decided by the optimal choices of professors. The equilibrium is reached when the demand for doctoral students equals the supply of doctoral students in each period.

As long as there are young agents optimally choosing to be doctoral students, that is, as long as $0 < m^* < 1$, young agents should be indifferent between a worker and a doctoral student. That is, $D_t = W_t$. Therefore, the equilibrium salary of the doctoral students, s^*, should satisfy the following equation.

$$s^* = \overline{w} + \beta\left[\left(1 - \kappa(1 - p^*)\right)\overline{w} - p^* d^*\right]. \tag{11}$$

Comparing equation (7) with equation (11), we find that for the same probability of becoming a professor and the same income for a professor, young agents are willing to accept a lower salary to become doctoral students when κ is larger. This is because young agents expect to get a higher wage after PhD training if they work in a nonacademic job. This effect will have a tendency to increase the supply of PhDs and drive down s^*. On the other hand, a lower salary would encourage professors to hire more doctoral students and has a positive effect on d^*, the wage of professors. However, because the demand for college and master-level education remains unchanged, a higher inflow of doctoral students will lead to a lower probability for PhD graduates to find academic jobs and a lower wage in the next period. The decrease of the probability to become professors and the decrease of the wage for professors would raise the required salary of doctoral students. Since the decrease of p^* and d^* are driven by the inflow of doctoral students, the supply of PhDs must go up and s^* must go down.

Following a similar analysis as in Section 2.3, we summarize the equilibrium in steady state when there exists a wage premium for PhD workers in the following proposition. We state the result below and leave the proof in Appendix 3.

Proposition 4. *The steady state can be solved by*

$$m^* = \frac{\lambda^{\frac{\alpha}{1-\alpha}} \left(\dfrac{\alpha A(1-\beta g)}{\overline{w} + \beta\left(1-\kappa(1-p^*)\right)\overline{w} - \dfrac{\beta g e}{m^*}} \right)^{\frac{1}{1-\alpha}}}{\left(1 + \dfrac{1}{\lambda}\dfrac{p^*}{g}\right)},$$

$$p^* = g\left(\left(\frac{e}{A}\right)^{\frac{1}{\alpha}} \frac{1}{m^*} - \lambda \right),$$

$$s^* = \frac{\overline{w} + \beta\left(1-\kappa(1-p^*)\right)\overline{w} - \dfrac{\beta g e}{m^*}}{1-\beta g},$$

$$d^* = \frac{g\left(e - \left(1 - \beta\left(1-\kappa(1-p^*)\right)\right)\overline{w} m^*\right)}{\left(1-g\beta\right) m^* p^*}.$$

Proof. See Appendix 3.

3. Quantitative Analysis

This section provides quantitative analysis on the issue of PhD glut. The theoretical framework constructed in Section 2 can be applied to countries with a PhD glut. For numerical purposes, we take Taiwan from 2004-2013 as an example. There are several advantages to use Taiwanese data for this calibration. First of all, Taiwan has also faced a PhD glut in the past decade. Second, wages for professors in Taiwan are observable and almost identical across schools and fields. Therefore, the uncertainty of lifetime income for pursuing a PhD degree is mainly due to the probability of finding an academic position. Third, the supply of local PhD graduates can be clearly counted because most of them do not search for academic positions abroad.

Our quantitative strategies are as follows. First, the model is calibrated to the data from Taiwan. Specifically, the calibration is conducted by field. This enables us to explore the differentiated impacts of educational policies among the different fields. The second subsection discusses calibrated results. Using the calibrated results as a benchmark, policy experiments are provided in the third subsection.

3.1 Calibration

The model in Section 2 is calibrated to the data in Taiwan from 2004-2013. In particular, the following five fields are calibrated: engineering, science, medical sciences, humanities, and social sciences. Engineering includes the fields of engineering, manufacturing, and construction; science refers to the field of science; medical sciences contains the fields of medical sciences, health, and social welfare; humanities includes humanities and arts; and social sciences refers to social sciences, business, and law. The classification of fields is based on a dataset of the Ministry of Education (MOE), Taiwan.[1] Table 1 summarizes the parameters for calibration. The second column (aggregate) of the table refers to the calibration for the aggregation of the five fields. Each calibration is solved as a steady state.

Five parameters are preset. T is model period. There are two generations in our model. The first period is the period of being a PhD student and then becoming a professor in the second period. Thus, we set T to be the number of years that is required to complete a PhD program. According to 2014 Annual Employment Survey Report of Earned Doctorates (AESED), the average time to complete a PhD program during 2001-2005 was 5.24 years, during 2006-2010 it was 5.69 years, and during 2011-2015 it was 6.22 years.[2] Therefore, T is set to be 6 years in the calibration. The annual time preference β_0 is chosen to be 0.75. In the calibration, the annual time preference is discounted by the model period so that β is equal to 0.178. A is a scaling factor and we set A to be 2.

The other two preset parameters, g and κ are computed according to the data. The gross rate of population growth g is interpreted as the gross growth rate of the sum of

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[1] The dataset is provided by the Department of Statistics, Ministry of Education (MOE), Taiwan. In the following subsections, we simply use MOE for referring to the dataset. Details about the dataset can be found in http://english.moe.gov.tw/lp.asp?ctNode=11429&CtUnit=1345&BaseDSD=16&mp=1.

[2] The survey is conducted by Science and Technology Policy Research and Information Center, National Applied Research Laboratories.

university and master graduates. To compute the gross growth rate, we first calculate the annual gross growth rate of the sum of university and master graduates from the MOE. Then the average of the annual gross growth rate from 2004 to 2013 is computed. Finally, the average of the annual gross growth rate is multiplied by six times in order to be consistent with the model period. According to Table 1, the gross rate of population growth for the aggregate data is greater than one, which implies that the amount of university and master graduates was still growing during the periods we calibrated for. With the low birth rates, the gross rate of population growth is expected to become smaller than one in the near future. We will conduct an experiment to discuss this issue in Section 3.3.

The wage premium of a PhD worker κ is interpreted to be the monthly wage ratio of a PhD graduate to a master graduate. The average monthly wages of a PhD graduate and a master graduate are obtained from the MOE's "The Big Data Analysis on Employment and Wages of 2010-2012 College Graduates" report.[3] Table 1 suggests that the wage premium exists for PhD graduate. On average the wage of a PhD graduate is about 1.35 times higher than that of a university or master graduate. Furthermore, it is interesting to find that the wage premium of PhD workers in the field of medical sciences is relatively smaller than that of other fields. This may reflect the fact in Taiwan that most doctors are university graduates. Compared with university and master graduates in other fields, doctors' incomes are relatively high, so the wage premium of PhD workers in medical sciences is not large. This implies that the marginal benefit of pursuing a PhD degree in medical sciences is limited.

There are four calibrated parameters, e, w, λ, and α. They are solved together by using equations in Proposition 4 to match four data moments, p^*, m^*, s^*, and d^*. The four data moments are described as follows.

In order to compute the probability of finding an academic position p^*, the number of domestic PhD graduates and the domestic demand for professors for each year are both required. The number of domestic PhD graduates for each year is obtained from the MOE. However, there is no direct measurement on the demand for professors faced by domestic PhD graduates. Therefore, we estimate the demand for professors faced by domestic PhD

[3] The classification of fields in the report is different from that of university and master graduates. To be comparable, we classify engineering and computer science as the field of engineering; natural science, life science, math, and statistics are to be the field of science; medical science and health are included in the field of medical science; humanities is humanities; and business and management and social and behavior science are in the field of social sciences.

graduates based on two assumptions: the retirement rate in each year is 5%, and 60% of the vacancies are occupied by PhD graduates who obtained a degree in other countries.[4] To estimate the demand for professors faced by domestic PhD graduates, we first obtain the number of professors in each year from the MOE. Deducting retirement and taking the difference between the two years, the total demand for professors in each year is obtained. With the assumption that about 60% of the vacancies are occupied by PhD graduates who obtained degrees in other countries, we compute the demand for professors faced by domestic PhD graduates. Then, to be consistent with the model period, we compute the probability of finding an academic position for each year using a six-year accumulated number of PhD graduates divided by the six-year accumulated demand for professors faced by domestic PhD graduates. Finally, the average of the probability of finding an academic position from 2004-2013 is calculated to be the target of the calibration.

To calculate the proportion of population studying PhD programs, m^*, we need the number of university and master graduates and the number of first-year PhD students. The number of university and master graduates is obtained from the MOE. However, the number of first-year PhD students is not available. Only the stock of PhD students is obtained. Therefore, we estimate by taking the difference of the stock of PhD students between the two years and adding the number of PhD graduates in the corresponding year back in. This becomes the estimated number of first-year PhD students. Then, to be consistent with the model period, the six-year accumulated estimated number of first-year PhD students is divided by the six-year accumulated number of university and master graduates to obtain the proportion of the population studying PhD programs in each year. Finally, the average of the proportion between 2004 and 2013 is computed to be our target.

The third target is the wage of a PhD student, s^*. In the data, it refers to scholarships and fellowships. Unfortunately, there is no such survey. Only the ceiling for scholarships is provided: 28000 Taiwanese dollars (TWD) per month for a PhD student and 32000 TWD per month for a PhD candidate.[5] Therefore, we assume the monthly scholarship for a PhD student is 20000 TWD and 25000 TWD per month for a PhD candidate. The monthly wage

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[4] The retirement rate 5% is assumed to match the survey data of PhD graduates finding academic positions in AESED during 2008-2015. It is close to the turnover rate of 7%-10% for full-time associate and full professors in the US, as reported in Ehrenberg (1992). The assumption that PhD graduates who obtained a degree in other countries is 60% also based on the AESED survey during 2008-2015.

[5] This is the standard of scholarship offered by the Ministry of Science and Technology, Taiwan.

for a first-year postdoc is 56650 TWD and a second year postdoc is 58710 TWD. We further assume that a young agent has to be a PhD student for two years, a PhD candidate for four years, and a postdoc for two years. Adjusting for the model period, the wage of a PhD student is about 2.428 million TWD. Thus, we set 2.428 to be our target in the calibration.

The wage of a professor with the same rank in Taiwan (d^*) is almost homogenous, regardless of the field. Seniority results in a slight difference. The average annual salary of an assistant professor in a public university in Taiwan is about 1.05 million TWD; 1.18 million TWD for an associate professor; and 1.36 million TWD for a full professor.[6] To compute the lifetime salary of a professor, we assume a young agent works for 30 years after PhD graduation: 6 years at the rank of assistant professor, 12 years at the rank of associate professor, and 12 years at the rank of full professor. Then, the average annual salary is weighted by the years of each rank and adjusted by the model period to obtain the wage of a professor in the model, 7.364 million TWD. This is our fourth target in the calibration.

3.2 The Benchmark

Table 1 reports the calibrated results for each field. These will be our benchmark economies when policy experiments are conducted. The calibrated result of the aggregate implies that the ratio of lifetime wage in academia to that in industry is around 1.5.[7] For comparison, the average monthly salary in Taiwan from 2004-2013 was about 44138 TWD; the weighted average monthly salary for a PhD student was 31920 TWD; and the weighted average monthly salary for a professor was 90917 TWD.[8] Therefore the data implies that the ratio of average monthly wage in academia to that in industries is about 1.2. The performance of our benchmark economy is quite reasonable.

The calibrated results capture an intuitive difference between fields. The courses of

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[6] Source: The salary table for National Chung Hsing University, which is the same as all universities in Taiwan.

[7] It is computed by $(s^* + d^*)/(w + \kappa w)$ using the calibrated values.

[8] The average monthly salary in Taiwan from 2004-2013 is from the Yearbook of Earnings and Productivity Statistics, published by the Directorate General of Budget, Accounting and Statistics, Taiwan. The weighted monthly wage of a PhD student is computed by assuming 2 years of being a PhD student, 4 years of being a PhD candidate and 2 years of postdoc. The weighted monthly wage of a professor is calculated with the assumption that 6 years are necessary for the rank of an assistant level, 12 years for the associate level and 12 years for the full professor. The annual bonus in academia in Taiwan is homogenous at 1.5 months, while that in industry varies. Therefore, here we simply use monthly wage to compute the ratio of wage in academia to that in industries.

humanities and social sciences are always provided by giving lectures. However, in addition to lectures, doing experiments in laboratories is important in the fields of engineering, science, and medical sciences. Therefore, the calibrated results in Table 1 suggest a higher average demand for college and master education (e) for the field of engineering, science, and medical sciences, while it is lower in humanities and social sciences. This difference is also reflected in the academic productivity of a PhD student, λ. PhD students in engineering, science, and medical sciences are able to help professors to provide education supply by conducting experiments in laboratories. In contrast, PhD students in humanities and social sciences are not easily able to directly help professors in lectures. Therefore, the academic productivity of PhD students in engineering, science, and medical sciences is higher. The academic productivity of PhD students in medical sciences is almost twice as high as that in social sciences.

3.3 Policy Experiments

Two policy experiments are provided in this sub-section. First, because of the low birth rate trend, it is expected that the population of university and master graduates will be shrinking in the near future. Therefore, we conduct experiments on g to explore its influence on the PhD market. Second, a common suggestion about the PhD glut is that universities should provide more training to PhD students so that they will more easily find work in industries after graduation. We perform experiments on wage premiums for PhD workers to evaluate this policy.

3.3.1 Slower Population Growth

Lower birth rate is a global issue. It happens not only in developed countries but also in developing countries. A current lower birth rate will result in a smaller population in the next two decades, and thereby less demand for university and master education. In equilibrium, the education supply must be lower. Therefore, people argue that the trend of having a lower birth rate will worsen the PhD glut problem and lead to serious unemployment for professors. This subsection studies the issue of lower birth rate and its impacts on the PhD market.

Two scenarios are conducted to evaluate the issue of lower birth rate. First, we consider a case where there is no population growth. In other words, the gross growth rate of a population (university and master graduates) is equal to one. Then, we further explore a scenario with a shrinkage of population. The results are both summarized in Table 2. The percentage changes relative to the benchmark are reported in parentheses.

The results of the two scenarios are consistent. We find that, because of the slower population growth, education demand is lower. Because agents have perfect foresight, lower education demand lowers the incentive to study a PhD program. Therefore, the proportion of the population studying PhD programs declines. In addition, the wages of professors also decrease because they depend on the number of students studying PhD programs. Since the proportion of the population studying PhD programs declines, professors have to provide higher wages to PhD students in order to attract more PhD students. Finally, the smaller proportion of the population studying PhD programs increases the probability of finding an academic position for a PhD graduate in equilibrium.

It is interesting to find that the response to slower population growth in the field of social sciences is different from that of other fields. Because of a higher wage premium for PhD workers, the decline in the proportion of the population studying PhD programs in social sciences is relatively smaller when slower population growth occurs. Hence, different from the other fields, the probability of finding an academic position decreases in social sciences.

A common concern about the lower birth rate is that it will become more difficult for PhD graduates to find a position in academia. Our quantitative analysis suggests that the above argument is true only in the field of social sciences. For other fields with relatively lower wage premiums for PhD workers, because of perfect foresight, agents take the issue of lower birth rate into consideration when they decide to study PhD programs. Therefore, the probability of finding an academic position goes up instead. The assumption of perfect foresight in our model is supported by Sauermann and Roach (2016). They find that, in contrast to the common argument, junior scientists actually have quite accurate estimations of career prospects in academia.

3.3.2 Higher Wage Premium for PhD Worker

A common suggestion for a solution to the PhD glut is that universities should provide training for PhD students to fulfill the requirements of industrial jobs. If the training for PhD students to satisfy the requirements of industrial jobs is successful, industries will be willing to hire PhD workers with higher wages. Therefore, the policy refers to an increase in wage premium for PhD workers in our experiment. This subsection performs two scenarios (1% and 5% increases in wage premium for PhD workers) to evaluate the effects of this policy on the PhD market. Table 3 provides the results for the two scenarios.

The two experiments suggest that an increase in the wage premium for PhD workers encourages a proportion of the population to study PhD programs. This is because the population expects that their future wages after PhD graduation will go up. More PhD students result in a lower probability of finding an academic position. Besides, there are more PhD students, and thus professors can hire PhD students with a lower wage. Although the wage of a PhD student declines, the number of PhD students increases. Therefore, the part of academic production that belongs to PhD students goes up, while the part that goes to all professors declines. The wage of professors decreases as well. It is interesting to note that the response of professors' wages in medical sciences is different from that of other fields. This is mainly due to the fact that the increase in the part of academic production that belongs to PhD students in medical sciences is relatively small. Together with a smaller increase in m^*, the wage of professors in medical sciences goes up.

The policy aims to solve the problem of PhD glut. Our result suggests that providing training for PhD students to fulfill the requirements of industrial jobs could lead to more PhD students and lower probabilities of finding a position in academia. However, here we should be aware of the possible difference between an increase in wage premium for PhD workers and the training for PhD students to fulfill the requirements of industrial jobs. We also ignore the possible cost that may exist when the training is provided. If the cost is paid by students or by the academic production, the policy still motivates more PhD students but the magnitude could be smaller.

3.4 Discussion

Our framework assumes that a wage premium exists for PhD workers. Even if they may not be able to stay in academia, PhD graduates still benefit from a PhD degree by having higher wages than non-PhD workers when they work in industries. This assumption provides an incentive to study PhD programs. Therefore, this subsection investigates the importance of the existence of a wage premium for PhD workers.

To explore the role of a wage premium for PhD workers on the PhD market, we conduct a counterfactual experiment by setting the wage premium at one. Other variables remain unchanged. The results are reported in Table 4.

We find that the existence of a wage premium for PhD workers encourages the studying of PhD programs. Specifically, the magnitude is larger in engineering and science.

In engineering and science, the existence of a wage premium enlarges the proportion of the population studying PhD programs by 1.5 and 1.2 percentage points, respectively. In contrast, it only increases the proportion by 0.4 percentage points in humanities and social sciences. This suggests that a wage premium for PhD workers plays an important role in the decision to study PhD programs, especially for the fields where the percentage of PhD graduates who work in industries is high. Without a wage premium for PhD workers, we might underestimate the proportion of the population studying PhD programs and overestimate the probability of finding an academic position.

4. Welfare Analysis

In this section, we first define the socially optimal proportion of doctoral students by solving the cost minimization problem to meet the education demand. Then we use the calibrated parameter to determine whether the actual proportion of doctoral students is above or below its socially optimal value and compare different fields to see which field has the most severe PhD glut problem.

4.1 Socially Optimal Proportion of Doctoral Students

Suppose the proportion of doctoral students is decided by a social planner who aims to meet the education demand. If the social planner assigns a young agent to be a worker, the agent's lifetime income will be $\overline{w}(1+\beta)$ in the steady state, which is assumed to be his or her productivity in a nonacademic career. If the young agent is assigned to be a doctoral student, then he or she cannot work in a nonacademic job unless he or she does not become a professor in the next period. Taking into account the wage premium of a PhD worker, the net costs of assigning a young agent to be a doctoral student, measured by the productivity loss in nonacademic jobs, is $C=\overline{w}[(1+\beta-\kappa\beta(1-p)]$. We define the socially optimal proportion of doctoral students as the proportion that minimizes such opportunity costs.

Definition. *The socially optimal proportion of doctoral students,* $\hat{m}^*$*, is the solution to the following cost minimization problem.*

$$\min_{m} \quad C=\overline{w}\Big[(1+\beta)-\kappa\beta(1-p)\Big],$$

where

$$A\left(\frac{mp}{g}+\lambda m\right)^{\alpha}=e\,.$$

We find that the socially optimal proportion of doctoral students depends on the wage premium of PhD workers κ, the gross rate of population growth g, the academic productivity of PhD students λ, and the discount rate β. Intuitively, the socially optimal proportion of doctoral students should be low when κ, g, and λ are low. We summarize those conditions in the proposition below and leave the proof in Appendix 4.

Proposition 5. *If* $\kappa(1+g\lambda) < 1 + 1/\beta$, *then the socially optimal* m *is*

$$\hat{m}^* = m_{\min} = \left(\frac{e}{A}\right)^{\frac{1}{\alpha}} \frac{1}{\lambda + \frac{1}{g}}.$$

If $\kappa(1+g\lambda) > 1 + 1/\beta$, *then*

$$\hat{m}^* = m_{\max} = \left(\frac{e}{A}\right)^{\frac{1}{\alpha}} \frac{1}{\lambda}.$$

Proof. See Appendix 4.

4.2 Socially Optimal Level in the Quantitative Analysis

Using the parameters of our benchmark economy and Proposition 5 for the definition of socially optimal $\hat{m}^*$, we are able to provide quantitative analysis on the socially optimal $\hat{m}^*$ and compute the PhD glut in each field. They are summarized in Table 5.

First, we find that the optimal $\hat{m}^*$ for the society in all fields is $m_{\min}$. Second, the proportions of the population studying PhD programs in the calibration are all larger than those of the social optima. In other words, the PhD glut problem exists in every field. Third, the differences between fields also exist. For example, in the field of engineering the m in the calibration is 2.69 percentage points larger than that for the socially optimal level, while it is only 0.89 percentage points larger in the field of social sciences. However, in terms of percentage change relative to the socially optimal $\hat{m}^*$, we find that social sciences has the largest PhD glut. This is mainly because in social sciences the average quantity demanded of university and master education (e) and the gross rate of population growth (g) are both relatively small; therefore the socially optimal m for social sciences is small.

Agents in our model have perfect foresight. The proportion of the population studying PhD programs in equilibrium is determined by the demand and supply in the PhD market. However, our welfare analysis shows that the m^* in equilibrium is still higher than the socially optimal level in all fields. In other words, PhD gluts actually exist. From this point of view, the common suggestion for reducing m toward an optimal level is reasonable.

Furthermore, the optimal m in our welfare analysis is determined by parameters, such as population growth, academic productivity of PhD students and average quantity demanded of college and master education. These parameter values are different between fields, thereby the socially optimal m varies and the gap between equilibrium and socially optimal m are different. For example, the gap is smaller in the field of medical sciences. Hence, the different characteristics of these fields should be taken into account when policies for solving the PhD glut are formed.

5. Concluding remarks

In this study, we provide a simple model and quantitative analysis to explain the PhD glut problem in academia. Agents in our model are rational but their labor supply could be inelastic over time. In fact, we distinguish the PhD glut and the market oversupply by introducing two stages of market demand. The first stage is the professors' demand for doctoral students in knowledge production. The second stage is the market demand for professors as providers of higher education. We argue that professors actually provide high enough compensation for PhD students in deciding their academic occupation choice. The market does not have an oversupply problem since the number of doctoral students equals the quantity demanded in the first stage. However, in equilibrium there are more PhD entrants than the socially optimal levels in all academic fields, i.e., the PhD glut problem.

There are other factors such as gender differences and non-financial benefits in determining doctoral students' occupational choices. For example, female and male students may have different time preferences and hence exhibit various responses to market shocks. The model also does not consider professors' flexibility in choosing research topics and doing research in the late morning, which should be considered as non-financial benefits compared to working wages. Finally, data from other countries may provide different quantitative results. Those extensions are left for future research.

Appendix 1. Equilibrium in steady state

In the steady state, we have $\{m_t, p_t\} = \{m_{t+1}, p_{t+1}\} = \{m^*, p^*\}$. Substituting $m_{t-1} = m_t = m^*$ and $p_t = p_{t+1} = p^*$ into (8) gives

$$(1+\frac{p^*}{\lambda g})m^* = \left(\frac{\alpha A}{s_t}\right)^{\frac{1}{1-\alpha}} = \lambda^{\frac{\alpha}{1-\alpha}}\left(\frac{\alpha A}{w_t + \beta p^* E_t[w_{t+1} - d_{t+1}]}\right)^{\frac{1}{1-\alpha}}. \tag{A1}$$

Note that $\beta E_t[w_{t+1} - d_{t+1}] = \beta\overline{w} - \beta E_t[d_{t+1}]$ and

$$E_t[d_{t+1}] = \frac{gA}{p^*}\left(\lambda + \frac{p^*}{g}\right)^{\alpha}(m^*)^{\alpha-1} - \frac{gE_t[s_{t+1}]}{p^*}. \tag{A2}$$

from (9). In the steady state, we also have $w_t = E_t[w_{t+1}] = \overline{w}$. Thus (A1) can be rewritten as

$$(1+\frac{p^*}{\lambda g})m^* = \lambda^{\frac{\alpha}{1-\alpha}}\left(\frac{\alpha A}{\overline{w} + \beta p^*\overline{w} - \beta g A\left(\lambda + \frac{p^*}{g}\right)^{\alpha}(m^*)^{\alpha-1} + \beta g E_t[s_{t+1}]}\right)^{\frac{1}{1-\alpha}}, \tag{A3}$$

where $E_t[s_{t+1}] = \overline{w} + \beta p^*\overline{w} - \beta g A(\lambda + (p^*/g))^{\alpha}(m^*)^{\alpha-1} + \beta g E_t[s_{t+2}]$. Substituting the sequence of $E_t[s_{t+j}]$, $j = 1, 2, 3, \ldots$, into (A3) yields

$$(1+\frac{p^*}{\lambda g})m^* = \lambda^{\frac{\alpha}{1-\alpha}}\left(\frac{\alpha A}{\overline{w}(1+\beta p^*) + \left(\overline{w}(1+\beta p^*) - A\left(\lambda + \frac{p^*}{g}\right)^{\alpha}(m^*)^{\alpha-1}\right)\left(\frac{\beta g}{1-\beta g}\right)}\right)^{\frac{1}{1-\alpha}}. \tag{A4}$$

Solving (A4) obtains the equilibrium proportion as

$$m^* = \left(\frac{g}{p^* + \lambda g}\right)\left(\frac{A\left(\lambda\alpha(1-\beta g) + \beta(p^* + \lambda g)\right)}{\overline{w}\left(1+\beta p^*\right)}\right)^{\frac{1}{1-\alpha}}. \tag{A5}$$

Since all young agents need knowledge of e provided from knowledge production, we shall have $x_t = e$ in equilibrium. Substituting m^*, p^*, and $x_t = e$ into (4), we have

$$e = A\left(\left(\left(\frac{g}{p^* + \lambda g}\right)\left(\frac{A\left(\lambda\alpha(1-\beta g) + \beta(p^* + \lambda g)\right)}{\overline{w}\left(1+\beta p^*\right)}\right)^{\frac{1}{1-\alpha}}\right)\left(\frac{p^*}{g} + \lambda\right)\right)^{\alpha}. \tag{A6}$$

Solving (A6) yields the equilibrium p^* of the second-stage demand for professors as

$$p^{*} = \frac{A\lambda\left(\alpha\left(1-\beta g\right)+\beta g\right)-\overline{w}a}{\beta\left(\overline{w}a-A\right)} \tag{A7}$$

where $a = (e \,/\, A)^{(1-\alpha)/\alpha}$.

Appendix 2. Equilibrium wages

From (A1), we know that $\overline{w}(1+\beta p^{*})+\left(\overline{w}(1+\beta p^{*})-A(\lambda+p^{*}/g)^{\alpha}(m^{*})^{\alpha-1}\right)\beta g/(1-\beta g)$ in (A4) is equal to s^{*} in the steady state. Hence, substituting m^{*} and p^{*} of Proposition 1 into s^{*}, we obtain the equilibrium wage of students as

$$s^{*}=\overline{w}(1+\beta p^{*})+\left(\overline{w}(1+\beta p^{*})-A\left(\lambda+\frac{p^{*}}{g}\right)^{\alpha}(m^{*})^{\alpha-1}\right)\left(\frac{\beta g}{1-\beta g}\right). \tag{A8}$$

Similarly, in the steady state, substituting m^{*}, $E_{t}[s_{t+1}]=s^{*}$, and $E_{t}[d_{t+1}]=d^{*}$ into (A2) yields the equilibrium wage of professors as

$$d^{*}=\frac{gA}{p^{*}}\left(\lambda+\frac{p^{*}}{g}\right)^{\alpha}(m^{*})^{\alpha-1}-\frac{gs^{*}}{p^{*}}. \tag{A9}$$

Appendix 3. Equilibrium with wage premium

For $\kappa \geq 1$, equation (7) needs to be rewritten as

$$s_t = w_t + \beta\Big(\big(1-\kappa(1-p_{t+1})\big)w_{t+1} - p_{t+1}d_{t+1}\Big). \tag{A10}$$

Note that in the steady state, we have $s_t = s^*$, $p_t = p^*$, $d_t = d^*$, $m_t = \mu_t = m^*$, and $x_t = e$. Therefore, from equations (4), (5), (6), and (A3), we have

$$e = A\left(\frac{m^* p^*}{g} + \lambda m^*\right)^{\alpha}, \tag{A11}$$

$$d^* = \frac{g(e - s^* m^*)}{m^* p^*}, \tag{A12}$$

$$m^*\left(1 + \frac{1}{\lambda}\left(\frac{p^*}{g}\right)\right) = \lambda^{\frac{\alpha}{1-\alpha}}\left(\frac{\alpha A}{s^*}\right)^{\frac{1}{1-\alpha}}, \tag{A13}$$

and

$$s^* = \overline{w} + \beta\Big[\big(1-\kappa(1-p^*)\big)\overline{w} - p^* d^*\Big]. \tag{A14}$$

Substituting (A12) into (A14), we have

$$s^* = \frac{\overline{w} + \beta\big(1-\kappa(1-p^*)\big)\overline{w} - \dfrac{\beta g e}{m^*}}{1-\beta g}. \tag{A15}$$

Substituting (A15) into (A12) and (A13), gives us

$$d^* = \frac{g\Big(e - \Big(1-\beta\big(1-\kappa(1-p^*)\big)\Big)\overline{w}m^*\Big)}{\big(1-g\beta\big)m^* p^*}, \tag{A16}$$

and

$$m^* = \frac{\lambda^{\frac{\alpha}{1-\alpha}}\left(\dfrac{\alpha A(1-\beta g)}{\overline{w} + \beta\big(1-\kappa(1-p^*)\big)\overline{w} - \dfrac{\beta g e}{m^*}}\right)^{\frac{1}{1-\alpha}}}{\left(1 + \dfrac{1}{\lambda}\dfrac{p^*}{g}\right)}. \tag{A17}$$

Finally, rewrite (A11) and we have

$$p^* = g\left(\left(\frac{e}{A}\right)^{\frac{1}{\alpha}}\frac{1}{m^*} - \lambda\right). \tag{A18}$$

Appendix 4. Socially optimal proportion of doctoral students

The social cost function is

$$C = \overline{w}\left[(1+\beta) - \kappa\beta(1-p)\right], \tag{A19}$$

where $p \in [0,1]$ must satisfy

$$A\left(\frac{mp}{g} + \lambda m\right)^{\alpha} = e\,. \tag{A20}$$

Equation (A20) can be rewritten as

$$p = g\left\{\left(\frac{e}{A}\right)^{\frac{1}{\alpha}}\frac{1}{m} - \lambda\right\} \tag{A21}$$

Substituting (A21) into (A19) gives us

$$C = \overline{w}\left[(1+\beta) - \kappa\beta(1+g\lambda)\right]m + \overline{w}\kappa\beta g\left(\frac{e}{A}\right)^{\frac{1}{\alpha}}. \tag{A21}$$

Therefore, when $\kappa(1+g\lambda) > 1 + 1/\beta$, the minimal cost is attained at $m = m_{\min}$, the minimal value of m. When $\kappa(1+g\lambda) > 1 + 1/\beta$, the minimal cost is attained at $m = m_{\max}$ the maximal value of m. Finally, note that since $p \in [0,1]$, from (A21) we must have

$$m \in [m_{\min}, m_{\max}] \equiv \left[\left(\frac{e}{A}\right)^{\frac{1}{\gamma}}\frac{1}{\lambda + \frac{1}{g}},\ \left(\frac{e}{A}\right)^{\frac{1}{\gamma}}\frac{1}{\lambda}\right].$$

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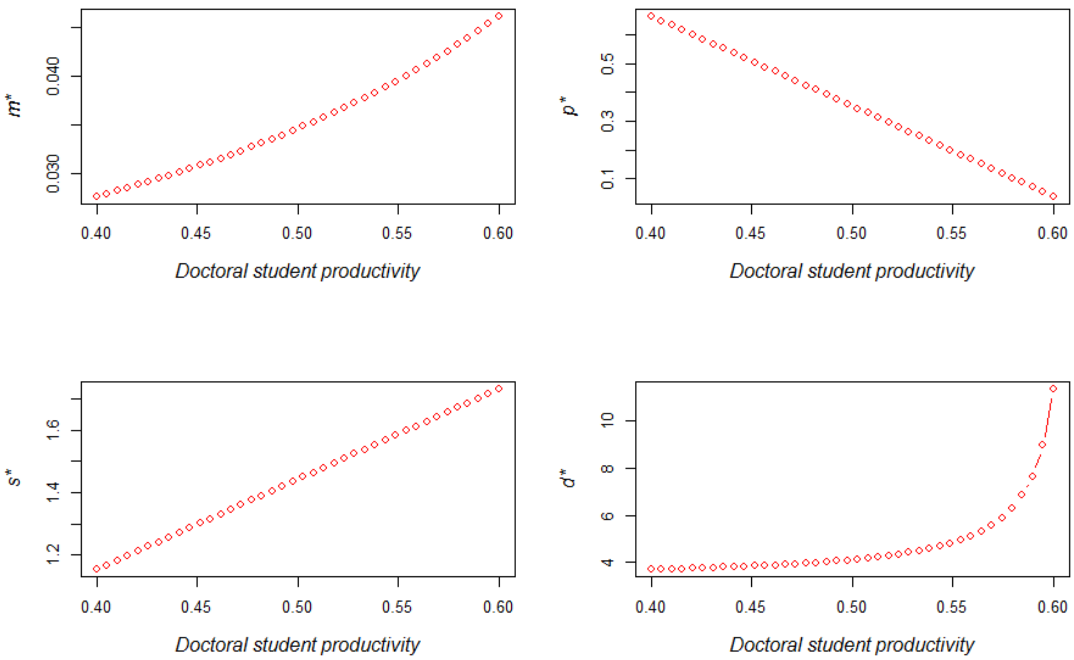

Figure 1. Examples of different doctoral student productivities

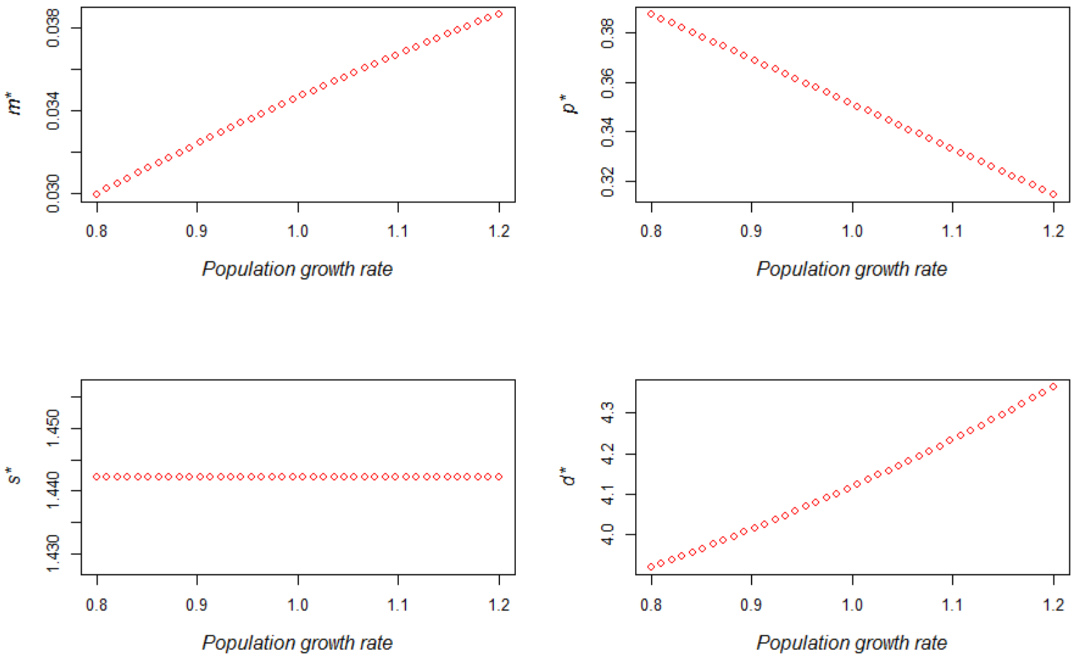

Figure 2. Examples of different population growth rates

Table 1. Parameters

	Aggregate	Engineering	Science	Medical sciences	Humanities	Social sciences	Explanation
Preset parameter							
T	6	6	6	6	6	6	model period
β_0	0.75	0.75	0.75	0.75	0.75	0.75	annual time preference
A	2	2	2	2	2	2	total factor productivity
g	1.208	1.228	1.187	1.225	1.287	1.167	gross rate of population growth
κ	1.347	1.472	1.379	1.060	1.346	1.477	wage premium of PhD worker
Calibrated parameter							
e	0.107	0.178	0.146	0.150	0.070	0.055	solved together to match data moments
w	2.728	2.813	2.744	2.580	2.748	2.823	solved together to match data moments
λ	0.685	0.716	0.734	0.883	0.620	0.476	solved together to match data moments
α	0.777	0.740	0.772	0.841	0.782	0.736	solved together to match data moments
Data moments							
p^*	0.130	0.099	0.115	0.197	0.193	0.164	probability of finding an academic position
m^*	0.031	0.052	0.043	0.044	0.019	0.015	proportion of population to study PhD programs
s^*	2.428	2.428	2.428	2.428	2.428	2.428	wage of PhD student
d^*	7.364	7.364	7.364	7.364	7.364	7.364	wage of professor

Note: e, w, λ, and α are solved together to match the data moments of p^*, m^*, s^*, and d^*.

Table 2. Slower Population Growth

	Aggregate	Engineering	Science	Medical sciences	Humanities	Social sciences
Benchmark						
p^*	0.130	0.099	0.115	0.197	0.193	0.164
m^*	0.031	0.052	0.043	0.044	0.019	0.015
s^*	2.428	2.428	2.428	2.428	2.428	2.428
d^*	7.364	7.364	7.364	7.364	7.364	7.364
No population growth (g=1)						
p^*	0.153	0.132	0.143	0.245	0.208	0.162
	(17.3%)	(33.6%)	(25.0%)	(24.3%)	(7.7%)	(-1.3%)
m^*	0.029	0.048	0.04	0.04	0.017	0.014
	(-6.8%)	(-8.1%)	(-6.8%)	(-8.8%)	(-8.9%)	(-4.5%)
s^*	2.437	2.442	2.437	2.434	2.439	2.436
	(0.4%)	(0.6%)	(0.4%)	(0.2%)	(0.4%)	(0.3%)
d^*	7.091	7.294	7.056	6.365	7.111	7.485
	(-3.7%)	(-1.0%)	(-4.2%)	(-13.6%)	(-3.4%)	(1.6%)
Negative population growth (g=0.9)						
p^*	0.161	0.142	0.155	0.262	0.210	0.159
	(23.3%)	(44.1%)	(35.3%)	(33.1%)	(8.8%)	(-3.0%)
m^*	0.028	0.046	0.038	0.038	0.017	0.014
	(-10.4%)	(-11.8%)	(-10.7%)	(-13.1%)	(-12.3%)	(-7.5%)
s^*	2.441	2.449	2.442	2.437	2.443	2.441
	(0.5%)	(0.9%)	(0.6%)	(0.4%)	(0.6%)	(0.5%)
d^*	6.95	7.196	6.889	6.029	7.014	7.542
	(-5.6%)	(-2.3%)	(-6.4%)	(-18.1%)	(-4.8%)	(2.4%)

Note: percentage changes relative to the benchmark are reported in parentheses.

Table 3. Higher Wage Premium for PhD Worker

	Aggregate	Engineering	Science	Medical sciences	Humanities	Social sciences
Benchmark						
p^*	0.130	0.099	0.115	0.197	0.193	0.164
m^*	0.031	0.052	0.043	0.044	0.019	0.015
s^*	2.428	2.428	2.428	2.428	2.428	2.428
d^*	7.364	7.364	7.364	7.364	7.364	7.364
Wage premium for PhD worker increases by 1%						
p^*	0.118	0.085	0.100	0.181	0.183	0.158
	(-9.5%)	(-14.4%)	(-13.1%)	(-7.9%)	(-5.2%)	(-3.9)
m^*	0.032	0.053	0.044	0.044	0.019	0.015
	(1.7%)	(2.0%)	(2.0%)	(1.5%)	(1.3%)	(1.2%)
s^*	2.426	2.425	2.425	2.427	2.426	2.426
	(-0.1%)	(-0.1%)	(-0.1%)	(0.0%)	(-0.1%)	(-0.1%)
d^*	7.212	7.031	7.214	7.467	7.228	7.138
	(-2.1%)	(-4.5%)	(-2.0%)	(1.4%)	(-1.8%)	(-3.1%)
Wage premium for PhD worker increases by 5%						
p^*	0.064	0.022	0.033	0.112	0.140	0.131
	(-51.2%)	(-77.7%)	(-71.2%)	(-43.4%)	(-27.8%)	(-20.1%)
m^*	0.034	0.058	0.048	0.047	0.021	0.016
	(9.7%)	(11.7%)	(11.9%)	(8.6%)	(7.4%)	(6.6%)
s^*	2.417	2.410	2.414	2.423	2.420	2.417
	(-0.5%)	(-0.7%)	(-0.6%)	(-0.2%)	(-0.3%)	(-0.4%)
d^*	5.923	0.766	5.103	8.306	6.441	5.975
	(-19.6%)	(-89.6%)	(-30.7%)	(12.8%)	(-12.5%)	(-18.9%)

Note: percentage changes relative to the benchmark are reported in parentheses.

Table 4. The Role of Wage Premium for PhD Worker

	Aggregate	Engineering	Science	Medical sciences	Humanities	Social sciences
Benchmark						
p^*	0.130	0.099	0.115	0.197	0.193	0.164
m^*	0.031	0.052	0.043	0.044	0.019	0.015
s^*	2.428	2.428	2.428	2.428	2.428	2.428
d^*	7.364	7.364	7.364	7.364	7.364	7.364
No Wage Premium for PhD Worker						
p^*	0.355	0.385	0.382	0.274	0.383	0.323
m^*	0.024	0.037	0.031	0.041	0.015	0.011
s^*	2.461	2.485	2.468	2.432	2.454	2.474
d^*	8.446	9.163	8.274	7.038	8.702	10.356

Table 5. Socially Optimal Level and PhD Glut

	Aggregate	Engineering	Science	Medical sciences	Humanities	Social sciences
$m_{\min}$	1.54%	2.48%	2.13%	2.70%	0.98%	0.57%
$m_{\max}$	3.39%	5.31%	4.58%	5.20%	2.22%	1.59%
Socially optimal $\hat{m}^*$	$m_{\min}$	$m_{\min}$	$m_{\min}$	$m_{\min}$	$m_{\min}$	$m_{\min}$
Benchmark m^*	3.13%	5.17%	4.28%	4.36%	1.92%	1.46%
PhD glut						
percentage point change	1.59%	2.69%	2.15%	1.66%	0.94%	0.89%
percentage change	103.91%	108.10%	100.51%	61.49%	95.10%	156.98%