The Financial (In)Stability Real Interest Rate, R**

Ozge Akinci, Gianluca Benigno, Marco Del Negro and Albert Queralto

February 23, 2023

Abstract

We introduce the concept of financial stability real interest rate using a macroeconomic banking model with an occasionally binding financing constraint as in Gertler and Kiyotaki (2010). The financial stability interest rate, r**, is the threshold interest rate that triggers the constraint being binding. We discuss r** and its dynamics, and show that persistently low real rates induce an increase in financial vulnerabilities and a consequent decline in the level of r**. We also provide a measure of r** for the US economy and discuss its evolution over the past 50 years, highlighting that during periods of financial stress that are associated with a decline in r** , the real rate tracks r** —a feature of monetary policy known as "Greenspan's put".

1 Introduction

One of the key aspects that has characterized the global economy and in particular advanced economies in the last two decades is the secular decline in real interest rates. The decline in global real interest rates has largely occurred in a context of relatively low and stable inflation suggesting that the drop in observed real interest rates reflects a fall in what researchers refer to as the "natural real interest rate," also known as r* (see, for example, Holston et al., 2017, and Del Negro et al., 2019). The concept of natural real interest rate dates to Wicksell (1898) and it is usually defined as the "real rate consistent with real GDP equals to its potential in the absence of shocks to demand. In turn potential GDP is defined to be the level of output consistent with stable price inflation absent transitory supply shocks" (see Laubach and Williams (2003)). In short, the concept of natural real interest rate is associated with the notion of macroeconomic stability.

In this paper we propose a complementary concept that we call the "financial stability real interest rate, r**." The core idea relies on determining the underlying level of real interest rate that might generate financial instability dynamics. Both conceptually and observationally r** differs from the "natural real interest rate" and from the observed real interest rate reflecting a tension in terms of macroeconomic stabilization versus financial stability objectives.

First, we discuss r** from a conceptual standpoint. To define the financial stability real interest rate one first needs to develop a concept of financial stability. To this end, we consider an environment in which some agents in the economy face a credit constraint that gives rise to debt-deflation or asset fire-sale dynamics. Importantly, the credit constraint is occasionally binding. This implies that the economy is characterized by two states: when the constraint is not binding the economy is in a normal state or tranquil period; when the constraint binds the economy is in a crisis mode and a financial instability dynamic arises. The financial stability real interest rate is the interest rate that, for a given state of the economy (for example for a given amount of private debt), would be consistent with the constraint being just binding.

Just like the natural rate of interest provides a benchmark for monetary policy in terms of macroeconomic stability, r** is meant to provide a benchmark for financial stability: if the real rate in the economy is at or above r**, the tightness of financial conditions may generate financial instability. Like the natural interest rate, the financial (in)stability real interest rate is state dependent: it evolves with the conditions of the economy, and in particular with the degree of imbalances in the financial system.

The notion of a financial (in)stability real interest rate broadly applies to any model where the economy fluctuates between normal state and a crisis state (see for example Mendoza (2010), Benigno et al. (2013), and Akinci and Chahrour (2018) in the context of the sudden stop literature). For concreteness, in this paper we use a particular model to illustrate how r** is constructed. The specific approach that we will follow in developing the concept of the financial stability real interest rate builds upon the banking framework developed by Gertler and Karadi (2011) and Gertler and Kiyotaki (2015). One of the virtues of using the Gertler-Kiyotaki framework is that it allows to relate the concept of financial stability real interest rate also to key variables for financial intermediaries such as the net worth or the asset/liability ratio.

In this framework, financial intermediaries channel funds from households to firms. The key imperfection is that banks have a limit in their ability to raise funds because of a moral hazard problem. It is assumed that after raising funds and buying assets at the beginning of the period, and then the banker decides whether to operate honestly or divert assets for personal use. This moral hazard problem gives rise to an incentive compatibility constraint that creates a link between the value of the bank and the value of the assets that can be diverted.

In their seminal work, Gertler and Karadi (2011) and Gertler and Kiyotaki (2015) always assume that the constraint is binding. The key departure that we would like to consider, as in Akinci and Queralto (2022), is to allow for the constraint to be occasionally binding so that the economy can display both a tranquil and a crisis state. This departure requires using a global solution method for solving the model and takes into account the non-linearity generated by the occasionally binding constraint.

As a first pass we focus on a version of the model in which there is no need to determine nominal variables as contracts are expressed in real terms. Therefore by construction, the real rate in this economy coincides with the natural rate of interest r*, that is, the underlying interest rate consistent with macroeconomic stability. In order to further simplify the exposition, in this paper we illustrate the mechanism in a situation where the real interest rate is exogenous as, for example, in a small open economy. We leave the discussion of the rich interactions between monetary policy and the financial (in)stability real interest rate to further research.

Within our modeling approach, we characterize some key properties of our conceptualization. Given the non-linearity built in our approach, the response of the economy to a shock differs depending on the underlying state. In particular the level and the evolution of the financial stability real rate r** depends on the economy being in a high or low fragility state. For example, when the economy is in a high fragility state (high leverage), a negative shock that triggers a binding credit constraint is associated with a spike in credit spreads and a relatively low level of the financial stability real interest rate, r**. Interestingly, during the period of financial stress r** stands below the natural real interest rate. This suggests that, under these circumstances, a policy rate that tracks the natural real interest rate leads to financial instability. Moreover, prolonged period of low real interest rate leads eventually to an increase in leverage of the banking sector and a lower level of the financial stability real interest rate. Low for long (in terms of real interest rates) tends then to reduce the policy space as the gap between the natural and the financial stability real interest rate shrinks.

We then provide an empirical measure of r** for the US economy and discuss its evolution over the past 50 years. We construct our measure by building on the properties of our model economy to identify in the data episodes of financial instability. First, periods of financial stress coincides with very volatile credit spreads. Such volatility is a consequence of the non linearity of the model: when financing constraints are binding, the financial accelerator mechanism amplifies the impact of shocks to the economy and to credit spreads in particular. Second, the level of credit spreads is connected with the tightness of the financial constraint, and therefore with the gap between r** and the real rate r. Again, this relationship is non linear as it changes depending on the constraint being binding or not: during periods of financial stress, credit spreads predict the latent r**-r gap very well. The relationship becomes much looser during tranquil financial periods, where movements in spreads are much noisier proxies for the r**-r gap. Our empirical measure of r** shows that in post-1970s US data as r** falls during periods of financial stress the real rate tends to track r** —a feature of monetary policy known as "Greenspan's put." This has been a feature of all financial stress episodes in the US, with the only exception being the later part of the Great Financial Crisis, when the nominal interest was stuck at the zero lower bound. In general we note that financial stress episodes are associated with periods in which the real interest rate is above our measure of r**.

The next section describes the model, section 3 discusses our calibration strategy and section 4 presents the quantitative properties of r**. In section 5 we construct the empirical measure of r**. Section 6 concludes.

2 Model

We propose a framework that builds upon the banking model developed in Gertler and Kiyotaki (2010). In this setting, banks make risky loans to nonfinancial firms and collect deposits from domestic households. In addition, in our setup banks may also hold a perfectly safe asset, supplied by the foreign sector (or, under an equivalent formulation, by the government sector).

Because of an agency problem, banks may be constrained in their access to external funds. A key feature of our analysis is to allow for this constraint to be occasionally binding, as in Akinci and Queralto (2022). In normal, or "tranquil," times, banks' constraints do not bind: credit spreads are small and the economy's behavior is similar to a frictionless neoclassical framework. When the constraint binds the economy enters into financial stress mode: credit spreads become large and volatile, and investment and credit drop, consistent with the evidence.

A second crucial feature of our model that differs from existing literature is that the degree of agency frictions facing a given bank depends on the composition of its assets: when bankers' assets are heavily tilted toward the safe assets, agency frictions are less severe than if they hold a large share of risky assets.

As we mentioned above for simplicity at this stage we consider a real model in which there is no nominal determination.

2.1 Households

Each household is composed of a constant fraction $(1-f)$ of workers and a fraction f of bankers. Workers supply labor to the firms and return their wages to the household. Each banker manages a financial intermediary ("bank") and similarly transfers any net earnings back to the household. Within the family there is perfect consumption insurance.

Households do not hold capital directly. Rather, they deposit funds in banks. The deposits held by each household are in intermediaries other than the one owned by the household. Bank deposits are riskless one-period securities. Consumption, C_t, deposits, D_t, and labor supply, L_t, are given by maximizing the discounted expected future flow of utility

$$\mathbb{E}_t \sum_{i=0}^{\infty} \beta^i U(C_{t+i}, L_{t+i}),$$

subject to the budget constraint

$$C_t + D_t \leq W_t L_t + R^d_{t-1} D_{t-1} + \Pi_t$$

for all t. The symbol $\mathbb{E}_t$ denotes the mathematical expectation operator conditional on information available at time t, and $\beta \in (0,1)$ represents a subjective discount factor. The variable W_t is the real wage, R^d_t is the (gross) real interest rate received from holding one-period deposits, and Π_t is total profits distributed to households from their ownership of both banks and firms.

2.2 Banks

Banks are owned by the households and operated by the bankers within them. In addition to its own equity capital, a bank can obtain external funds from domestic households, d_t. In each period the bank uses its net worth n_t and deposits d_t to purchase securities issued by nonfinancial firms, s_t, at price Q_t, as well as safe assets b_t. In turn, nonfinancial firms use the proceeds to finance purchases of physical capital.

2.2.1 Agency friction and incentive constraint

We follow Gertler and Kiyotaki (2010) in assuming that banks are "specialists" who are efficient at evaluating and monitoring nonfinancial firms and at enforcing contractual obligations with these borrowers. For this reason firms rely solely on banks to obtain funds and there are no contracting frictions between banks and firms. However, as in Gertler and Kiyotaki (2010), we introduce an agency problem whereby the banker managing the bank may decide to default on its obligations and instead transfer a fraction of assets to the households, in which case it is forced into bankruptcy and its creditors can recover the remaining funds. In recognition of this possibility, creditors potentially limit the funds they lend to banks. As a consequence, banks may or may not be credit constrained, depending on whether or not they are perceived to have incentives to disregard their contractual obligations.

More specifically, after having borrowed external funds but before repaying its creditors, the banker may decide to default on its obligations and divert fraction $\Theta(x_t)$ of his or her asset portfolio, where

$$x_t \equiv \frac{b_t}{Q_t s_t + b_t} \tag{1}$$

is the share of safe assets in the banker's portfolio. In this case, the bank is forced into bankruptcy and its creditors recover the remaining funds. To ensure that the bank does not divert funds, the incentive constraint must hold:

$$V_t \geq \Theta(x_t)(Q_t s_t + b_t), \tag{2}$$

where V_t stands for the continuation value of the bank. This constraint requires that the banker's continuation value be higher than the value of the diverted funds.

We assume that the function $\Theta(\cdot)$ satisfies $\Theta'(x_t) < 0$: as the banker's portfolio becomes more risky, the agency friction worsens. The rationale for this assumption is that risky loans are more opaque and hard to monitor relative to safe assets, which leads creditors to turn more cautious when the banker's portfolio becomes riskier. We also assume that $\Theta''(x_t) > 0$, which embeds a form of diminishing returns to holding safe assets: when x_t is very low, further diminishing it worsens the friction more than if x_t is high. The motivation for this assumption is that if x_t is already very low, further tilting the balance sheet toward risky assets entails entering segments of the asset market that are particularly sensitive to agency and information frictions.

2.2.2 The banker's problem

The bank pays dividends only when it exits. If the exit shock realizes, the banker exits at the *beginning* of $t+1$, and simply waits for its asset holdings to mature and then pays the net proceeds to the household. The objective of the bank is to maximize expected terminal payouts to the household. In addition, we assume that the banker (or the household he or she is a member of) also derives direct utility from the banker's holdings of the safe asset b_t. Motivating this assumption is the well-documented safety and liquidity properties of safe assets like U.S. Treasuries (Krishnamurthy and Vissing-Jorgensen, 2012; Del Negro et al., 2017b).

Formally, the bank chooses state-contingent sequences $\{s_t, b_t, d_t\}_{t=0}^{\infty}$ to maximize

$$V_t = \mathbb{E}_t \sum_{i=0}^{\infty} \sigma^i \Big\{ \Lambda_{t,t+1+i} \Big[(1-\sigma)(R_{K,t+1+i} Q_{t+i} s_{t+i} + R_{t+i} b_{t+i} - R^d_{t+i} d_{t+i}) \Big] + \Lambda_{t,t+i} \zeta_{t+i} b_{t+i} \Big\}, \tag{3}$$

where the (time-varying) exogenous variable ζ_{t+i} governs the direct utility derived from safe asset holdings. We assume ζ_t follows an *iid* process with mean $\overline{\zeta} > 0$. In the quantitative part of the paper, this variable plays the role of capturing movements in credit spreads that are due to "noise" (liquidity preferences) rather to a changed assessment in the creditworthiness of the financial system. Maximization is subject to a sequence of budget constraints

$$Q_t s_t + b_t + R^d_{t-1} d_{t-1} \leq R_{K,t} Q_{t-1} s_{t-1} + R_{t-1} b_{t-1} + d_t \tag{4}$$

and subject to the incentive constraint given in equation (2). Here R_t is the return on the safe asset and $\Lambda_{t,t+1}$ is the household's stochastic discount factor, given by the marginal rate of substitution between consumption at dates $t+1$ and t. The budget constraint (4) states that the bank's expenditures (consisting of asset purchases, $Q_t s_t + b_t$, and repayment of deposit financing, $R^d_{t-1} d_{t-1}$) cannot exceed its revenues, stemming from payments of previous-period asset holdings, $R_{K,t} Q_{t-1} s_{t-1} + R_{t-1} b_{t-1}$ and deposits d_t.

The bank's balance sheet identity,

$$Q_t s_t + b_t \equiv n_t + d_t, \tag{5}$$

which is equivalent to a definition of net worth n_t—stating that the bank's assets are funded by the sum of net worth and deposits—can be combined with (4) to yield the law of motion of the bank's net worth:

$$n_t = (R_{K,t} - R^d_{t-1}) Q_{t-1} s_{t-1} + (R_{t-1} - R^d_{t-1}) b_{t-1} + R^d_{t-1} n_{t-1}. \tag{6}$$

We use the method of undetermined coefficients to solve the banker's problem. We guess that the value function satisfies $V_t(n_t) = \alpha_t n_t$, where α_t is a coefficient to be determined. Define $\Omega_{t+1} \equiv 1 - \sigma + \sigma \alpha_{t+1}$, and let

$$\mu_t \equiv \mathbb{E}_t[\Lambda_{t+1} \Omega_{t+1} (R_{K,t+1} - R^d_t)], \tag{7}$$

$$\mu_{B,t} \equiv \mathbb{E}_t[\Lambda_{t+1} \Omega_{t+1}](R_t - R^d_t), \tag{8}$$

$$\nu_t \equiv \mathbb{E}_t[\Lambda_{t+1} \Omega_{t+1}] R^d_t. \tag{9}$$

Note that α_{t+1}, capturing the value to the bank of an extra unit of net worth the following period (in case the banker does not exit), acts by "augmenting" the banker's stochastic discount factor (SDF) so that their effective SDF is given by $\Lambda_{t,t+1}(1 - \sigma + \sigma \alpha_{t+1}) = \Lambda_{t,t+1} \Omega_{t+1}$.

With these definitions, the problem simplifies to

$$\alpha_t n_t = \max_{s_t, b_t} \quad \mu_t Q_t s_t + \mu_{B,t} b_t + \nu_t n_t + \zeta_t b_t \tag{10}$$

subject to the incentive constraint

$$\alpha_t n_t \geq \Theta(x_t)(Q_t s_t + b_t). \tag{11}$$

We define the banker's *leverage ratio* ϕ_t as the ratio of total assets to net worth:

$$\phi_t \equiv \frac{Q_t K_t + B_t}{N_t}. \tag{12}$$

Given this definition, n_t drops out of (10)-(11), and the problem can be equivalently written as

$$\alpha_t = \max_{x_t, \phi_t} \quad [\mu_t(1 - x_t) + (\mu_{B,t} + \zeta_t)x_t]\phi_t + \nu_t \tag{13}$$

subject to

$$[\mu_t(1 - x_t) + (\mu_{B,t} + \zeta_t)x_t]\phi_t + \nu_t \geq \Theta(x_t)\,\phi_t. \tag{14}$$

Taking first-order conditions with respect to x_t of the corresponding Lagrangian, we obtain the following condition:

$$\begin{aligned} \mu_{B,t} - \mu_t &= \mathbb{E}_t[\Lambda_{t+1}\Omega_{t+1}(R_{K,t+1} - R_t)] \\ &= \zeta_t + \frac{\overline{\lambda}_t}{1 + \overline{\lambda}_t}[-\Theta'(x_t)], \end{aligned} \tag{15}$$

where $\overline{\lambda}_t \geq 0$ denotes the Lagrange multiplier on the incentive constraint. Equation (15) states that positive discounted excess returns on risky relative to safe assets are positively linked to both the marginal utility derived from the safe asset (ζ_t) and to the tightness of the incentive constraint (recall that $\Theta'(x_t) < 0$).

Differentiating the Lagrangian with respect to ϕ_t, we obtain

$$\frac{\overline{\lambda}_t}{(1 + \overline{\lambda}_t)}\Theta(x_t) = \mu_t(1 - x_t) + (\mu_{B,t} + \zeta_t)x_t \equiv \overline{\mu}_t, \tag{16}$$

which links the Lagrange multiplier $\overline{\lambda}_t$ positively to the "total" excess returns on banks' assets (inclusive of the preference shock ζ_t), which we define as $\overline{\mu}_t$.

The solution for overall banker leverage ϕ_t is as follows. If $\overline{\mu}_t = 0$, the constraint is not binding, and the banker is indifferent as to its leverage choice. If $\overline{\mu}_t > 0$, the banker leverages up as much as allowed by the incentive constraint. Rearranging (14), maximum leverage, denoted $\overline{\phi}_t$, is given by

$$\overline{\phi}_t = \frac{\nu_t}{\Theta(x_t) - \overline{\mu}_t}. \tag{17}$$

Observe that $\overline{\phi}_t$ is decreasing in $\Theta(x_t)$, and therefore falls as the banking sector's porftolio shifts toward risky assets (i.e. as x_t falls).

Since the bankers problem is linear, we can easily aggregate across banks. For surviving banks, the evolution of net worth is given by (6). We assume entering bankers receive a small exogenous equity endowment, given by fraction ξ/f of the value of the aggregate capital stock. Thus the law of motion of aggregate net worth is

$$N_t = \sigma\big[\, \left(R_{K,t} - R^d_{t-1}\right) \underbrace{Q_{t-1}K_{t-1}}_{=Q_{t-1}S_{t-1}} + (R_{t-1} - R^d_{t-1})B_{t-1} + R^d_{t-1}N_{t-1}\big] + (1-\sigma)\xi Q_{t-1}K_{t-1}. \quad (18)$$

where we have used the market-clearing condition $K_t = S_t$.[1]

2.2.3 Credit spreads and the financial constraint

The model highlights how the behavior of credit spreads depends on both the tightness of the financial constraint and the preference shock ζ_t. We define the credit spread as the (annualized) expected return on nonfinancial firms' securities, $\mathbb{E}_t(R_{K,t+1})$, minus the rate on the safe asset, R_t. When the constraint is not binding, banks can fully arbitrage away excess returns, and as a result the spread depends only on ζ_t. From (15),

$$\mathbb{E}_t\left[\boldsymbol{\Omega}_{t+1}(R_{Kt+1} - R_t)\right] = \zeta_t, \quad (19)$$

where $\boldsymbol{\Omega}_{t+1} \equiv \Lambda_{t,t+1}\Omega_{t+1}$ is the banker's "total" SDF. If the economy is far away from the constraint, the credit spread $\mathbb{E}_t(R_{K,t+1}) - R_t$ will only reflect fluctuations in ζ_t, and therefore will tend to be low on average and relatively stable. The model then implies a behavior of investment during tranquil periods similar to standard (frictionless) models, with $\mathbb{E}_t(R_{K,t+1})$ approximately tracking R_t, which helps determine the response of investment to movements of the real rate: a higher R_t, for example, raises the required expected return on investment, triggering a fall in Q_t and I_t.

By contrast, when the constraint binds, banks' lending is constrained by their net worth, and therefore banks cannot fully arbitrage away the returns between risky and safe assets. As a consequence, we have

$$\mathbb{E}_t\left[\boldsymbol{\Omega}_{t+1}(R_{Kt+1} - R_t)\right] > \zeta_t,$$

and the credit spread will tend to be large and volatile. (Note that the assumption that $\Theta' < 0$ is crucial for this result). In this regime, investment behavior is heavily influenced by financial accelerator and fire-sale dynamics: a lower asset price Q_t erodes net worth and tightens the constraint further, which pushes investment down, triggering another round of decline in Q_t. Along the way, credit spreads skyrocket.

2.3 Nonfinancial Firms

There are two categories of nonfinancial firms: final goods firms and capital producers. In turn, within final goods firms we also distinguish between "capital leasing" firms and final goods producers, in order to clarify the role of bank credit used to finance capital goods purchases.

[1] We use N_t, S_t, B_t to refer to the aggregate counterparts of n_t, s_t, b_t.

2.3.1 Final Goods Firms

We assume that there are two types of final goods firms: capital leasing firms and final goods producers. The first type of firm purchases capital goods from capital good producers, stores them for one period, and then rents them to final goods firms. The second type uses physical capital (rented from capital leasing firms) and labor to produce final output. Importantly, capital leasing firms have to rely on banks to obtain funding to finance purchases of capital, as explained below. In addition, final goods producers need to rely on banks to finance working capital.

In period $t-1$, a representative capital leasing firm purchases K_{t-1} units of physical capital at price Q_{t-1}. It finances these purchases by issuing S_{t-1} securities to banks which pay a state-contingent return $R_{K,t}$ in period t. At the beginning of period t, the firm rents out this capital to final goods firms at price Z_t, and then sells the undepreciated capital $(1-\delta)K_{t-1}$ in the market at price Q_t. The payoff to the firm per unit of physical capital purchased is thus $[Z_t + (1-\delta)Q_t]$. Given frictionless contracting between firms and banks, it follows that the return on the securities issued by the firm is given by the following (note that this equation implies that capital leasing firms make zero profits state-by-state): $R_{K,t} = \frac{Z_t+(1-\delta)Q_t}{Q_{t-1}}$.

In the aggregate, the law of motion for capital is given by

$$K_t = \Gamma(I_t) + (1-\delta)K_{t-1} \tag{20}$$

Final goods firms produce output Y_t using capital and labor: $Y_t = A_t F(K_{t-1}, L_t)$, where A_t is a TFP shock. We assume a working capital requirement, following Neumeyer and Perri (2005), whereby firms need to borrow a fraction Υ of the wage bill before production takes place. These loans are obtained from bankers at the beginning of the period, and pay gross return $R_{W,t} = R_t^d + \frac{\mu_t}{\mathbb{E}_t[\mathbf{\Omega_{t+1}}]}$.

The first-order conditions for labor and for physical capital are as follows:

$$A_t F_1(K_t, L_t) = Z_t \tag{21}$$

$$A_t F_2(K_t, L_t) = W_t \left[1 + \Upsilon(R_{W,t} - 1)\right] \tag{22}$$

2.3.2 Capital Goods Producers

Capital producers, owned by households, produce new investment goods using final output, and they sell those goods to firms at the price Q_t. The quantity of newly produced capital, $\Gamma(I_t)$, is an increasing and concave function of investment expenditure to capture convex adjustment costs.

The objective of the capital producer is then to choose $\{I_t\}$ to maximize profits distributed to households:

$$\max \quad Q_t \Gamma(I_t) - I_t \tag{23}$$

The resulting first-order condition yields a positive relation between Q_t and I_t:

$$Q_t = \left[\Gamma'(I_t)\right]^{-1} \tag{24}$$

2.4 Interest rate determination

We assume that the safe rate, R_t, evolves (mostly) exogenously. The goal is to capture fluctuations in the natural real interest rate, without taking a stance on their causes. Accordingly, R_t satisfies

$$R_t = \overline{R} + \mathcal{R}_t + f(x_t - \overline{x}), \tag{25}$$

where $\overline{R}$ and $\overline{x}$ are parameters, and $\mathcal{R}_t$ follows the stochastic process

$$\log(\mathcal{R}_t) = \rho_R \log(\mathcal{R}_{t-1}) + \epsilon_{R,t},$$

with $\epsilon_{R,t} \sim N(0, \sigma_R)$. The (endogenous) term $f(x_t - \overline{x})$ is a small portfolio cost we introduce for technical reasons, as it helps ensure stationarity of safe asset holdings B_t (Schmitt-Grohe and Uribe (2003)).

2.5 Resource Constraint, Market Clearing, and Equilibrium

The resource constraint and the balance of payments equations, respectively, are given by:

$$Y_t = C_t + I_t + \mathbf{T}_t \tag{26}$$

$$T_t = B_t - R_{t-1}B_{t-1} \tag{27}$$

where $\mathbf{T}$ stands for net exports (or transfers (taxes) under an equivalent formulation where safe assets are provided by the government sector). An equilibrium is defined as stochastic sequences for the eight quantities $Y_t, C_t, I_t, \mathbf{T}_t, B_t, L_t, K_t, N_t$, five prices $R_{K,t}, Q_t, R_t, R_t^d, W_t$, and six banking sector coefficients $\mu_t, \mu_{B,t}, \nu_t, \alpha_t, \phi_t, x_t$ such that households, banks, and firms solve their optimization problems, and all markets (for short-term debt, securities, new capital goods, final goods, and labor) clear, given exogenous stochastic sequences for A_t, ζ_t, and $\mathcal{R}_t$.

2.6 Constructing R**

The model has three endogenous state variables, in addition to three exogenous states associated with the shock processes. The endogenous state variables are the beginning-of-period value of the capital stock, K_{t-1}, of bankers' holdings of safe assets, B_{t-1}, and of bankers' aggregate deposits issued to households, $D_{t-1} = Q_{t-1}K_{t-1} + B_{t-1} - N_{t-1}$. Thus the period-$t$ state vector is

$$\mathbf{S}_t \equiv \{K_{t-1}, B_{t-1}, D_{t-1}, A_t, \zeta_t, \mathcal{R}_t\} \tag{28}$$

The financial stability interest rate, r**, is defined as the threshold real rate above which financial instability arises; i.e., the real interest rate that makes the financial constraint *just* bind. Accordingly, we compute r** by moving the underlying real interest rate in the economy while keeping all other states variables of the economy unchanged.

Specifically, if the economy is in the unconstrained region, we increase the last element of $\mathbf{S}_t$ up until the point at which the constraint just starts to bind (holding constant the other elements of $\mathbf{S}_t$). We then call r** the hypothetical value of the interest rate that makes the constraint just bind. Conversely, if the economy is in the constrained regime, we lower $\mathcal{R}_t$ up until the point where the constraint ceases to bind.

As such, r** can be viewed as a *threshold*: real interest rates below r** ensure that the economy remains in the "financial stability" regime. Note also that the financial stability rate *gap* $\mathbf{r}^{**} - \mathbf{r}$ (an object we will make reference to throughout) generally depends on the evolution of the other state variables, such as leverage and the share of safe assets in banks' portfolio (we analyze the dynamic implications on r** and on the r** gap of changes in the state of the economy in Section 4.2).

3 Functional Forms and Parameter Values

In this section we describe, in turn, the functional forms and the parameter values used in the model simulations.

3.1 Functional Forms

The functional forms of preferences, production function, and investment adjustment cost are the following:

$$U(C_t, L_t) = \frac{\left(C_t - \chi \frac{L_t^{1+\epsilon}}{1+\epsilon}\right)^{1-\gamma} - 1}{1-\gamma} \tag{29}$$

$$F(K_t, H_t) = A_t (K_{t-1})^{\eta} L_t^{1-\eta} \tag{30}$$

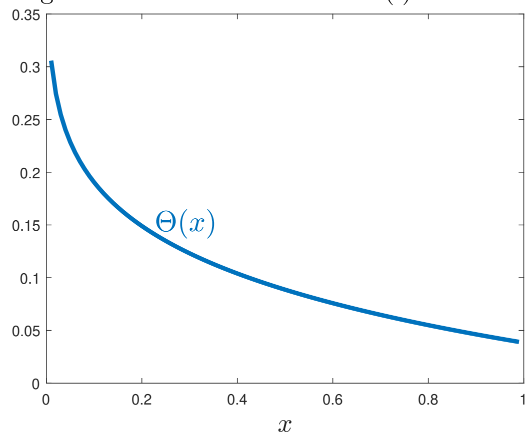
$$\Gamma\left(I_t\right) = a_1 \left(I_t\right)^{1-\vartheta} + a_2 \tag{31}$$

$$\Theta(x_t) = \theta \left(1 - \frac{\lambda}{\kappa} {x_t}^{\kappa}\right) \tag{32}$$

The utility function, equation (29), is defined as in Greenwood et al. (1988), which implies non-separability between consumption and leisure. This assumption eliminates the wealth effect on labor supply by making the marginal rate of substitution between consumption and labor independent of consumption. The parameter γ is the coefficient of relative risk aversion, and ϵ determines the wage elasticity of labor supply, given by $1/\epsilon$. The production function, equation (30), takes the Cobb-Douglas form. The coefficient η is the elasticity of output with respect to capital. Equation (31) defines the the investment technology, with the ϑ corresponding to the elasticity of the price of capital with respect to investment.

Equation (32) defines the relationship between the ratio of safe-to-risky assets, x_t, and the degree of financial frictions, $\Theta(x)$, in the economy, as described before. Our calibration assumes Θ is quite convex, meaning that it is relatively flat around high levels of the safe asset ratio, x, but becomes very steep as the safe asset ratio, x, falls toward low values. Figure 1 illustrates how $\Theta(x)$ changes as a function of x.

Figure 1: Functional Form of $\Theta(\cdot)$ Function



We calibrate preference, production, and financial sector parameters to standard values when possible, and report them in Table 1. We set the discount factor, β, to 0.995, which implies an annual real neutral rate of interest rate of 2%. The following four parameters are standard values in business cycle literature: The risk aversion parameter, γ, the capital share, η, and the depreciation of capital, δ, are set to 2, 0.33, and 0.025, respectively.

We set the Frisch labor supply elasticity (given by $1/\epsilon$) to 4, a value at the higher end of a wide range of values used in the literature. As in Gertler and Kiyotaki (2010), this relatively high value represents an attempt to compensate for the absence of frictions such as nominal wage and price rigidities, which are typically included in quantitative DSGE models. While our framework excludes these frictions to preserve simplicity, they likely have a role in accounting for employment and output volatility, so we partly compensate for their absence by setting a relatively high elasticity of labor supply.

We follow Gertler et al. (2019) in choosing the parameters governing the investment technology. More specifically, we set ϑ, which corresponds to the elasticity of the price of capital with respect to investment rate, equal to 0.25, a value within the range of estimates from panel data. We then choose a_1 and a_2 to hit two targets: first, a ratio of quarterly investment to the capital stock of 2 percent and, second, a value of the price of capital Q equal to unity in the risk-adjusted steady state.

The reference safe asset ratio, $\overline{x}$ is set to 0.2. This value roughly corresponds to the average holdings of safe assets by U.S. depository institutions (relative to total assets), defining safe assets as the sum of cash, reserves, federal funds, and Treasury and agency-backed securities. We then let $\overline{R}$ adjust such that x equals the target $\overline{x}$ in the steady state.

We then need to assign values to the five parameters relating to financial intermediaries: the survival rate of bankers, σ, the transfer to entering bankers, ξ, and the parameters governing the $\Theta(\cdot)$ function: θ, λ, and κ. We calibrate σ to 0.925, implying that bankers survive for about 3.5 years on average. This value of banks' survival rate is within the range of values found in the literature. The start-up transfer rate ξ, which ensures that entering bankers have some funds to

Table 1: Calibrated Model Parameters

Parameter	Symbol	Value	Source/Target
Conventional			
Discount factor	β	0.995	Interest rate 2%, ann.)
Risk aversion	γ	2	Standard RBC value
Capital share	η	0.33	Standard RBC value
Capital depreciation	δ	0.025	Standard RBC value
Elasticity of R to x	φ	0.005	Standard RBC value
Reference safe asset ratio	$\overline{x}$	0.2	
Labor disutility	χ	2.5	Steady state labor of 33%
Inverse Frisch elast.	ϵ	1/4	Gertler and Kiyotaki (2010)
Elasticity of Q w.r.t. I	ϑ	0.25	Gertler, Kiyotaki, Prestipino (2019)
Investment technology	a_1	1.1261	$Q = 1$
Investment technology	a_2	-0.1696	$\Gamma(I) = I$
Financial Intermediaries			
Survival rate	σ	0.925	Exp. survival of 3.5 yrs
Transfer rate	ξ	0.20	{ Frequency of crises around 3%,
Fraction divertable	θ	0.69	Leverage of 6}
Elasticity of $\Theta(x)$ w.r.t. x	κ	0.124	
	λ	0.117	
Shock Processes			
Persistence of interest rate	ρ_R	0.95	
SD of interest rate innov. (%)	σ_R	0.06	
Persistence of TFP	ρ_A	0.90	
SD of TFP innov. (%)	σ_A	0.44	
Steady State level of liquidity shock	$\bar{\zeta}$	0.00125	
SD of liquidity shock innov. (%)	σ_ζ	0.0313	

start operations, is set to target a leverage ratio of around 6 in the risk-adjusted steady state. This target is an estimate of the leverage ratio of the aggregate financial sector (broadly defined). We then set the three parameters governing the asset diversion function to hit three targets: a frequency of severe financial crises of 3 percent annually, an asset diversion fraction that is nearly zero as x approaches unity, and a Θ function that is very flat at high values of x (see Figure 1). The second target is based on the presumption that a portfolio composed of purely safe assets is nearly impossible to divert. The third target captures that notion that when the banker's asset portfolio is already very safe, there are almost no gains (in terms of reduced agency frictions) of marginally making it safer.

Finally, as we have direct observations on real interest rates, we fix the persistence and standard deviation of innovation for the interest rate shocks, ρ_R and σ_R, to the real interest rate from U.S. data. We then choose the standard deviation of the TFP shock so that the model matches the standard deviation of output growth in the United States, equal to about 2 percent annually since the mid 80s. The mean of process for the liquidity shock, $\overline{\zeta}$, is calibrated to 0.00125 to deliver a steady state liquidity premium of 50 basis points, and we set its standard deviation to deliver a small volatility of ζ_t (equal to one-fourth of its mean).

4 Model Results

We now turn to the key quantitative properties of the model, and then discuss the drivers and dynamics of the financial stability rate, r**.

4.1 Quantitative Properties of the Model

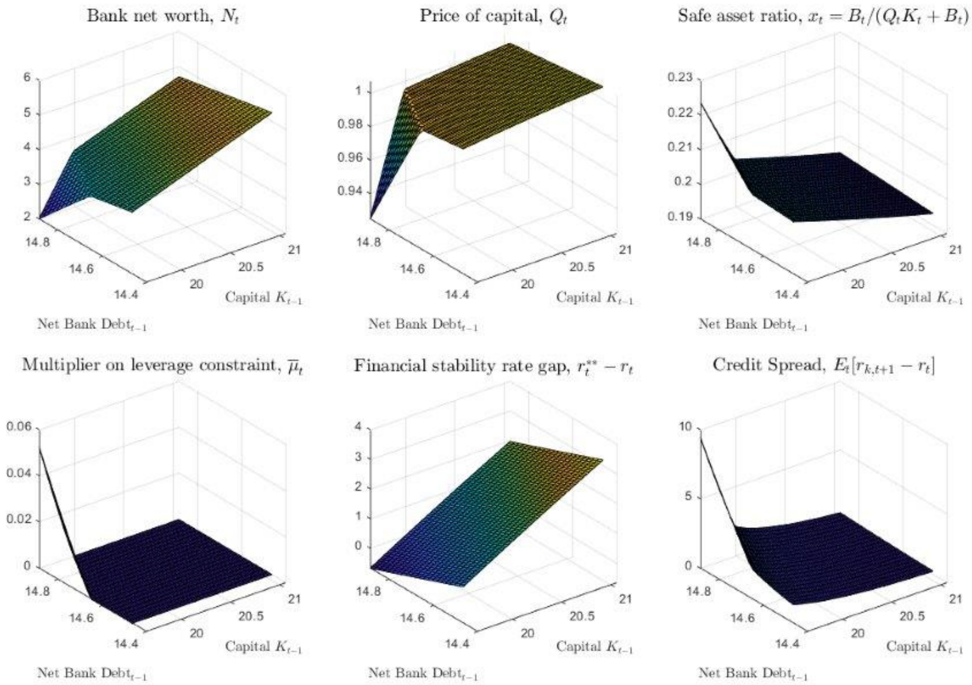
In this section we show that while the model is simple, it is quantitatively realistic, especially in capturing the dynamics surrounding financial crises. Quantitative realism is important as it enables us to use it for constructing an empirical measure of the financial stability rate in the data, which is one of the main objectives of our paper.

The model economy displays nonlinearity and state-dependence, which is induced by the financial constraint, as well as amplification via the financial accelerator mechanism that occurs when the constraint binds. In order to illustrate both of these features of the model, we first show banks' behavior as a function of some of the endogenous states in our calibrated economy. Figure 2 displays the three-dimensional policy functions with aggregate banking sector debt aggregate capital as arguments. It is apparent that when the financing constraints become binding, as reflected by positive values of the Lagrange multiplier on the constraint $\overline{\mu}$, the responses of banks' net worth, asset prices, and the holdings of safe assets to a given change in the states are much larger compared to the region in which the constraint is slack (i.e. $\overline{\mu} = 0$). The constrained region is not only characterized by very low values of banker's capital or by very high values of banking sector debt, but also by a combination of relatively low values the former and relatively high values of the latter. The threshold of banking sector debt for which the constraint becomes binding, and hence the level of r**, is a function of the level of banker's assets. Note that interestingly, while all the other charts are very non linear, the r** chart looks very linear. This is because the power of changes in the real interest rate affecting the financing conditions varies with the extent to which the economy is constrained: When the economy is deep in the constrained region, the financial accelerator becomes very powerful, thus it benefits tremendously from a rate cut.

It is worth emphasizing that the model features a form of precautionary behavior in bankers' choice of the safe asset ratio x_t (shown in the right column on the top row). When the economy is far from the constrained region (where the bank capital is high and the net bank debt is low), the safe asset ratio of the banker is quite small. Interestingly, as the banker is approaching the constrained region (either via lower capital or higher bank debt), even before the constraint starts to bind, banks start to accumulate safe assets and de-lever on risky capital (so that x rises) in an attempt to avoid the crisis. Nonetheless, crises occasionally happen in the model, as either the precautionary behavior arrives too late or is too little to avoid it. A similar behavior occurs within the constrained region, with x_t now increasing more steeply as the economy enters the constrained region. It is because, now the value to the banks of relaxing the constraint rises sharply, inducing them to tilt their balance sheet toward safe assets. One can also see from the figure that credit spreads start to rise as the economy moves towards the binding region, and they eventually rise much more steeply along with sharply deteriorating equity values and falling asset prices when the crisis happens.

We next evaluate our model's quantitative performance in matching the following three facts

Figure 2: Equilibrium objects as a function of states



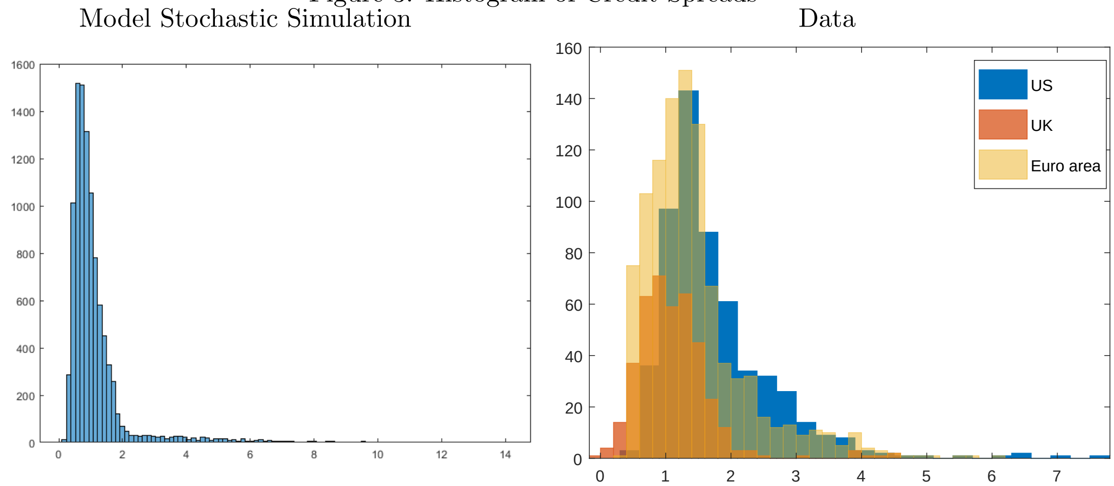
Note: Model endogenous variables as a function of two of the model state variables: The beginning-of-period value of the capital stock, K_{t-1}, and of bankers' net indebtedness (defined as total deposits minus excess returns on the safe asset). All other states kept at risk-adjusted-steady-state value.

associated with the relationship between financial stress episodes and the real economy (see Akinci and Queralto (2022) for a more extensive characterization of these empirical regularities associated with crises in the data). First, we show that our calibrated model can account for the fact that credit spreads display occasional spikes. Second, we demonstrate that the model captures the asymmetric relationship between credit spreads and economic activity. Lastly, we show that the average financial crisis in the model is consistent with the evolution of real and financial variables around actual crises episodes.

Figure 3 shows histograms of credit spreads, both from model-simulated observations (left panel) and from actual data (right panel). As shown in the panel on the right, credit spreads display occasional large spikes in the data: Spreads hover around 100 basis points a large fraction of the time, while they infrequently take values as large as 700 basis points. The panel on the left shows a histogram of credit spreads obtained from model stochastic simulations. As the figure makes clear, the model delivers a right skewed distribution of credit spreads, as observed in the data.

Our model economy displays strong nonlinearities, consistent with the evidence from the macro-finance literature (see, for example Merton (2009), Kenny and Morgan (2011), Hubrich et al. (2013), He and Krishnamurthy (2019), or more recently Adrian et al. (2019)). Figure 4 illustrates the asymmetric relation between credit spreads and economic activity predicted by the model: when financial stresses is relatively elevated, higher spreads tend to be more strongly associated with

Figure 3: Histogram of Credit Spreads

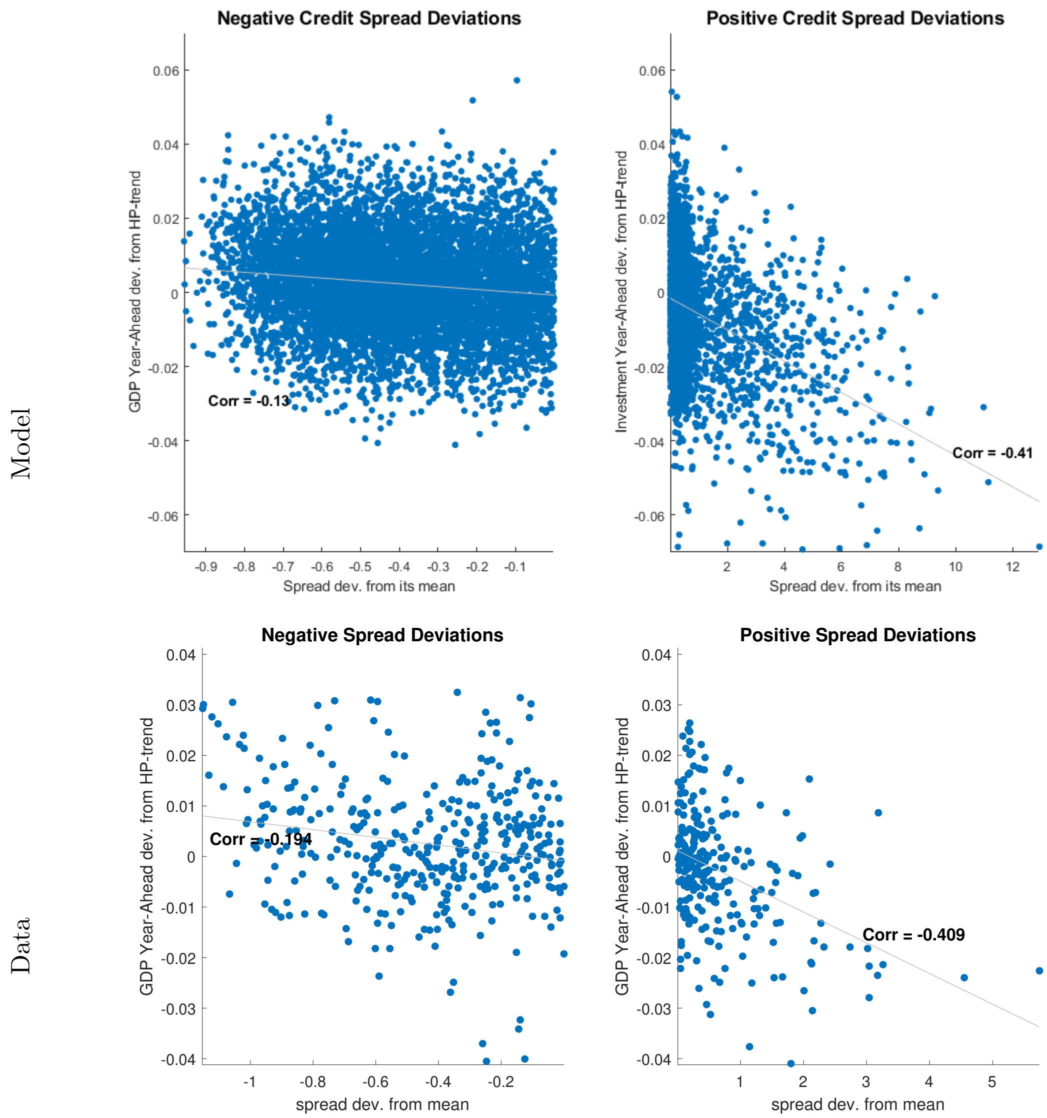


Note: Credit spreads stand for corporate bond spreads for non-financial firms. Euro area includes Germany, France, Spain, and Italy. Spreads are calculated as the average spreads between the yield of private-sector bonds in Italy, Spain, Germany and France relative to the yield on German government securities, in the UK relative to UK government securities, and in the US relative to US government securities, of matched maturities. Data sources: Gilchrist and Mojon (2014), Bank of England, Gilchrist and Zakrajsek (2012).

weaker real activity than in more tranquil times. In particular, when spreads are above their sample mean there exists a correlation between credit spreads and real economic activity (calculated as year-ahead deviation of real output from its HP trend) of about -0.41, compared with -0.13 obtained when spreads are below the sample mean. These patterns are consistent with the empirical results shown in the lower panel of the same figure. Key to explaining the model's ability to generate this asymmetry is the occasionally binding financial constraint: a binding constraint is associated with financial stress and higher spreads, since banks are prevented from arbitraging away the difference between risky and safe returns, and at the same time with an amplified response of real activity to shocks via the financial accelerator.

Figure 5 illustrates further the nonlinearity and state-dependence induced by the financial constraint, as well as the amplification via the financial accelerator mechanism that occurs when the constraint binds. The figure shows the responses of credit spreads, investment, and output to a combination of positive real interest rates and negative TFP shocks. The blue solid line displays the reaction of model variables to the shocks when they arrive in a "tranquil" period. The effects of the shocks are not large enough to push the economy into the constrained region. As a consequence, the shocks has only modest effects on output, investment and credit spreads. We next perform a similar experiment, i.e. we hit the model with the exact same shocks at $t = 1$, but we now assume that these shocks arrive when the economy is in a vulnerable state—with bank leverage very close to the maximum allowed by the constraint. The red dashed line in Figure 5 shows the dynamic effects of these shocks when the economy starts from this high-fragility state (in deviation from the path that the economy would have followed absent the shock). The decline in bank net worth (not shown) is now large enough to bring banks up against their constraints. As a consequence, the spread jumps

Figure 4: Credit Spreads and Output



Note: The upper (lower) left (right) panel shows the relationship between year-ahead real GDP, expressed as a deviation from its HP trend, and the negative (positive) deviations of the credit spread from its mean in the model (data). The lower panel shows Data sources: Haver Analytics, Gilchrist and Mojon (2014), Bank of England, Gilchrist and Zakrajsek (2012), authors' calculations.

by about 400 basis points annually. The sharp decline in net worth is explained by the financial accelerator mechanism that operates when the constraint binds: falling net worth leads investment to drop, which drives asset prices down, leading net worth to drop further. There is also a severe drop in output, of about 4.5 percent –several times larger than the decline of 1 percent that occurs when the economy is not in a fragile state.

The first panel of the figure shows the dynamic evolution of financial stability interest rate gap,

$r^{**} - r$, when the shock hits starting at vulnerable (tranquil) period as shown by the red dashed (blue solid) lines. The economy's initial states are different in these two cases, thus the model-implied financial stability rate differs markedly at these initial points. In tranquil periods the economy is generally farther away from the constraint, causing the financial stability rate gap $r^{**} - r$ to take much higher values compared with values it takes in vulnerable periods (around 1.5 percent compared to just 0.25 percent). Note that the constraint is not binding in either cases at the initial points, as a result the real interest rate gap, $r^{**} - r$ is still positive before the shock hits.

The shock that hits in tranquil period leads the financial stability gap to fall from 1.5 percent to around .75 percent, implying that $r**$ is still above the underlying real interest rate in the model economy. When the same shock arrives in vulnerable period, on the other hand, the constraint binds and r^{**} falls below the real interest rate of 2 percent. It is because the real interest rates consistent with financial stability has to be much smaller than the underlying real interest rate to be able to alleviate financial instability pressures generated by the binding financing constraints by boosting asset prices and bank equity valuations.

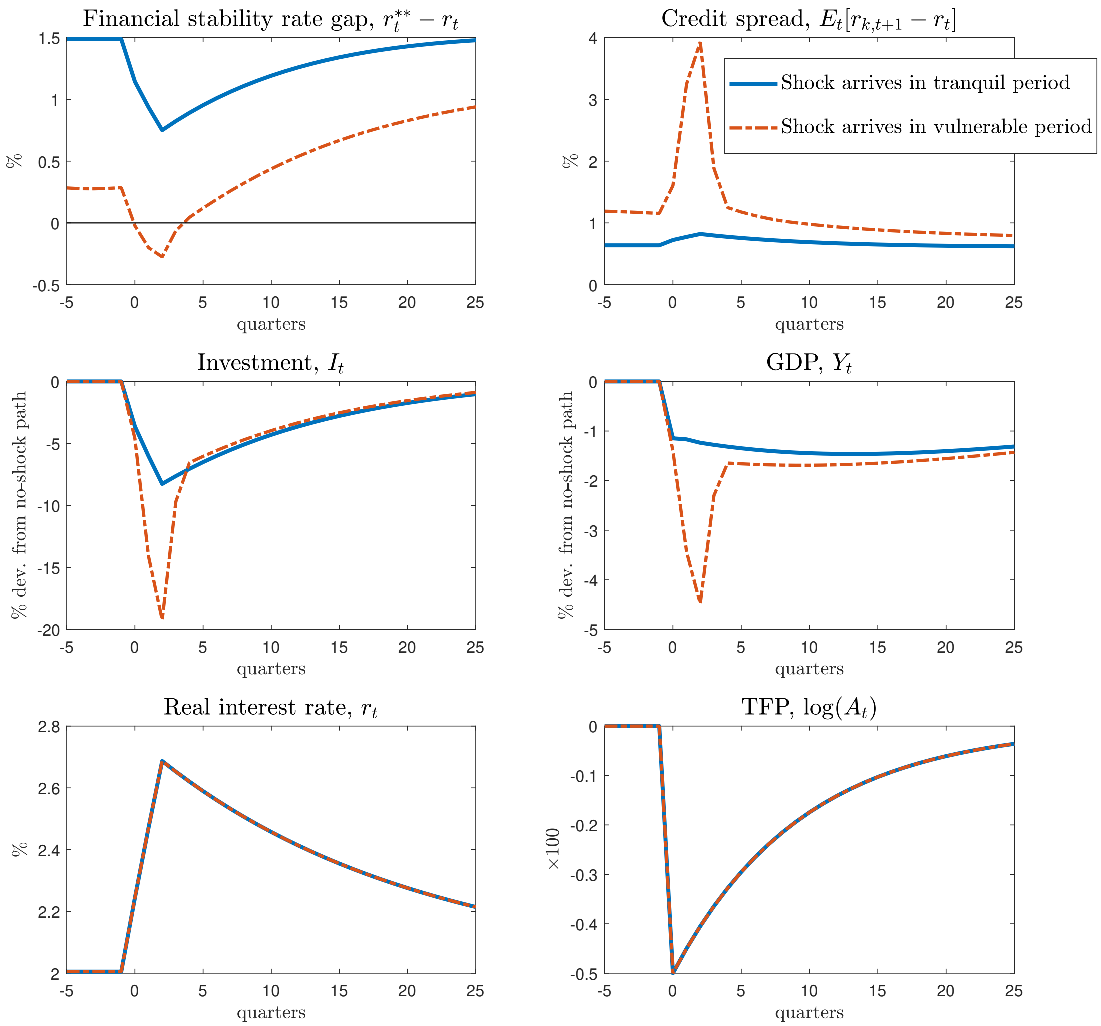
This exercise illustrates that responses of the economy to a shock can be very different depending on the underlying state. More importantly, this figure displays that both the level and the evolution of financial stability real rate, r**, could be different depending on the underlying state of the economy.

Finally, Figure 6 shows how the calibrated model can produce quantitatively realistic crisis dynamics, including the size of the increase in spreads. In the quarters leading up to the crisis, bank equity (not shown) deteriorates sharply. These equity losses eventually put banks up against their borrowing constraints, leading to a significant jump in credit spreads (first panel, first row) whenever adverse shocks hit the economy. As the financial accelerator mechanism kicks in, the increase in spreads is associated with declines in net worth, investment, and asset prices reinforcing each other. Investment at the trough is about 20 percent below trend in the simulation, close to the average drop in investment in the U.S. during the global financial crisis. Similarly, output decreases by about 4 percent below trend, and asset prices also drop sharply.

An important observation from the event study figure pertains to the behavior of the safe asset ratio, x_t, as well as the evolution of real interest rates surrounding crisis events. Leading up to the crisis, the economy features a safe asset ratio below its mean value (first row, second column). This is in part driven by below-mean values of the real interest rate leading up to the crisis (possibly due to, in a monetary model, accommodative monetary policy, which contributes to the risk-taking behavior of banks). Thus, while the crisis is ultimately triggered by exogenous forces (a sharp upward movement in the real rate, along with deteriorating TFP in the periods leading up to the crisis), the pre-conditions for crises to occur in fact reflect endogenous choices—namely, a low value of the safe asset ratio x_t.

The lower right panel of Figure 6 illustrates the behavior of r**, the benchmark rate for financial stability, during the crisis episode. As discussed, we construct r** by calculating the implied real interest rate in the economy that makes the constraint just binding. When the leverage constraint is slack, r** is the hypothetical (higher) level of the real rate at which the financial constraint would start to bind, leading to financial instability. In states of the world in which the constraint binds,

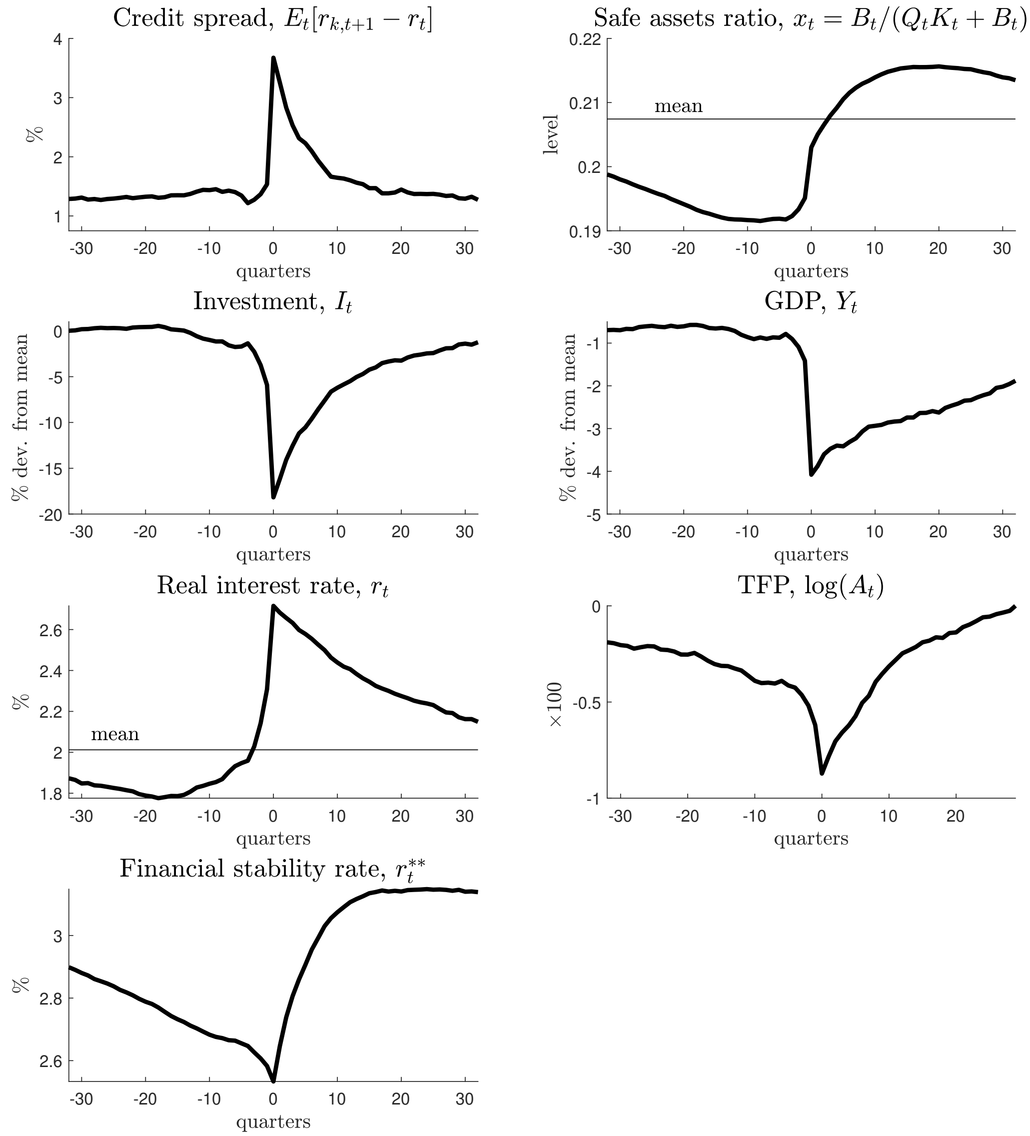
Figure 5: State Dependence



Note: The figure shows the effects of adverse interest rate and TFP shocks depending on the economy's initial state. In the blue solid lines, the economy is initially far away from the constrained region. In the red dashed lines, the economy is initially close to the constrained region.

conversely, we compute the counterfactual (lower) real interest rate that would make the constraint cease to bind. Thus, we define r** as the *threshold* rate above which financial instability arises. Figure 6 shows that in the period preceding the crisis, the financial stability rate declines, indicating that the economy is becoming more fragile. As the crisis hits, r** starts rising, partly because banks accumulate safe assets, but for a while remains below the real rate.

Figure 6: Average Financial Crisis in the Model



Note: A financial crisis event in the model is defined as an event in which banks' constraint binds for at least two consecutive quarters. We simulate the economy for a large number of periods and compute averages across identified financial crisis events.

4.2 The financial stability rate, r**: Dynamics

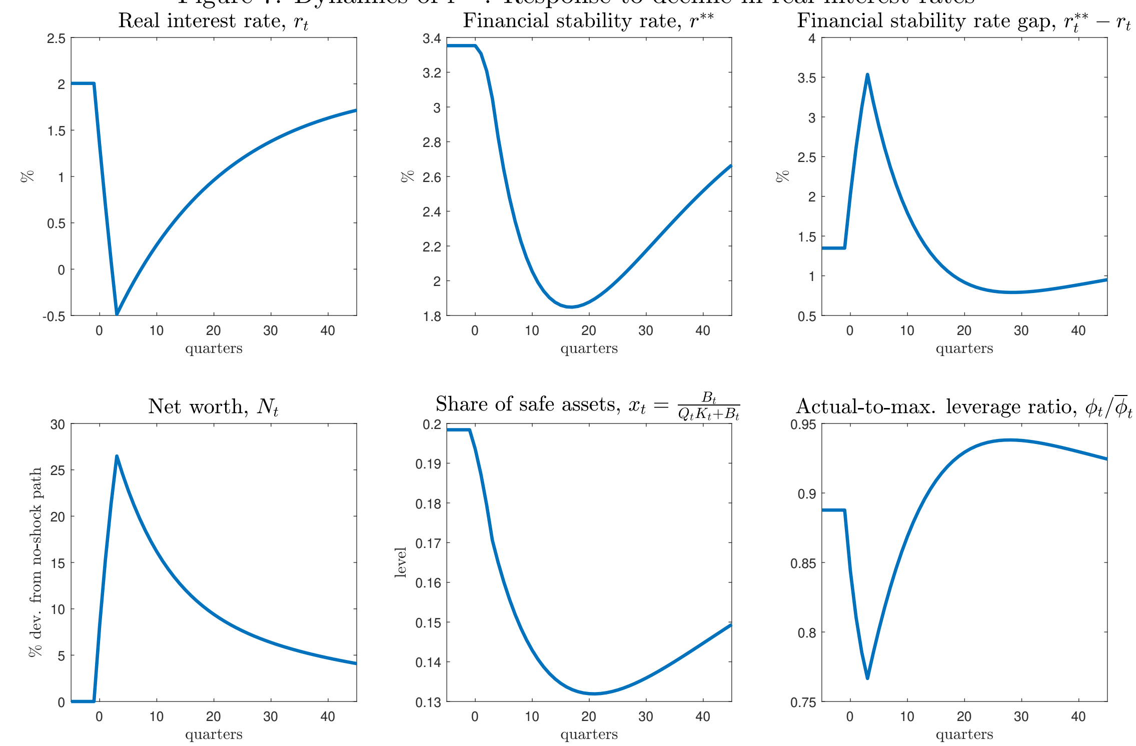
In this section we characterize the dynamic properties of the financial stability interest rate r** in the calibrated version of the model as described above. We start by showing the dynamic evolution of the endogenous variables in the model, such as banker's net worth, the share of safe assets in bank's balance sheets, the actual-to-maximum leverage ratio of the banker (i.e., the distance to the endogenous leverage constraint), and the financial stability interest rate, in response to an unexpected fall in the real rate of interest. These results are displayed in Figure 7. The figure also shows the dynamics of financial stability interest rate gap, $r^{**} - r$, defined as the difference between the financial stability rate and the underlying real rate of interest in the model economy.

At time $t = 0$, before the shock hits, the economy is assumed to be at the risk-adjusted state state, which features a real rate of interest of 2 percent and a financial stability rate of about 3.4 percent (shown in the first two panels of the figure, respectively). In the experiment, the real rate then falls at $t = 0$ and stays low for an extended period before gradually returning to its steady-state. In the model, persistent reductions in real rates (which in a monetary model would be associated with accommodative monetary policy) lead to an improvement in financial conditions in the near term (price of capital, Q, rises sharply on impact, not shown) and banks' balance sheets improve significantly (lower left panel). The fall in real rates also triggers a "reach for yield" behavior by bankers: banks shift their portfolios from safe assets (B falls) towards riskier capital (K increases). This type of behavior arises naturally in the model: in the face of persistently lower return on safe assets, agents respond by saving less in safe assets and more in risky assets (note that a return on capital, R_k, falls, causing bankers to increase their investment in risky capital over time). Thus, the ratio of safe-to-risky assets, x_t, declines persistently. As discussed earlier, the degree of agency friction facing bankers depends on the asset composition of banks' balance sheet: frictions are more severe when balance sheets have a greater share of risky assets as opposed to safe assets (i.e. when x_t is low). Importantly, as one moves toward the left of the $\Theta(\cdot)$ function shown earlier, the degree of agency frictions rises much more sharply for a given change in the safe asset ratio (a feature we can capture thanks to our nonlinear solution method). Thus, a large and persistent downward move in the real rate, by inducing a sizable and persistent reduction in x_t, has the by-product of substantially worsening the degree of agency frictions. The lower x_t then puts downward pressure on the maximum leverage ratio $\overline{\phi}_t$. As a result, the ratio of actual-to-maximum leverage, which initially falls due to the positive effects of lower rates on asset prices and net worth, begins rising gradually after about a year; after roughly three years it has surpassed its initial point, leaving the economy more vulnerable than in its initial (pre-shock) state.

The dynamics of the financial stability interest rate gap follow closely those of the banker's actual-to-maximum leverage ratio. On impact, as the underlying states of the economy remain unchanged, the financial stability interest rates does not move, and so the financial stability rate gap ($r^{**} - r$) rises sharply as r declines. Over time, as the banker shifts their portfolio towards riskier assets from safe government bonds, banker's leverage gets closer to the endogenous leverage constraint, causing the financial stability interest rate and the financial stability rate gap to fall significantly. As a result, the model implies that persistent declines in real interest rates today cause the real rate consistent with financial stability to be low in the future, implying that sudden increases in the real rate may cause turmoil in financial markets. Brunnermeier (2016) calls this phenomenon "financial dominance.". While the model we are currently working with features flexible prices, this result suggests that the extension with nominal rigidities may feature interesting trade-offs between macroeconomic and financial stability.

To further illustrate the dynamic relationship between the real interest rate and the financial stability rate gap, Figure 8 shows the model-implied cross-correlogram between the real interest rate,

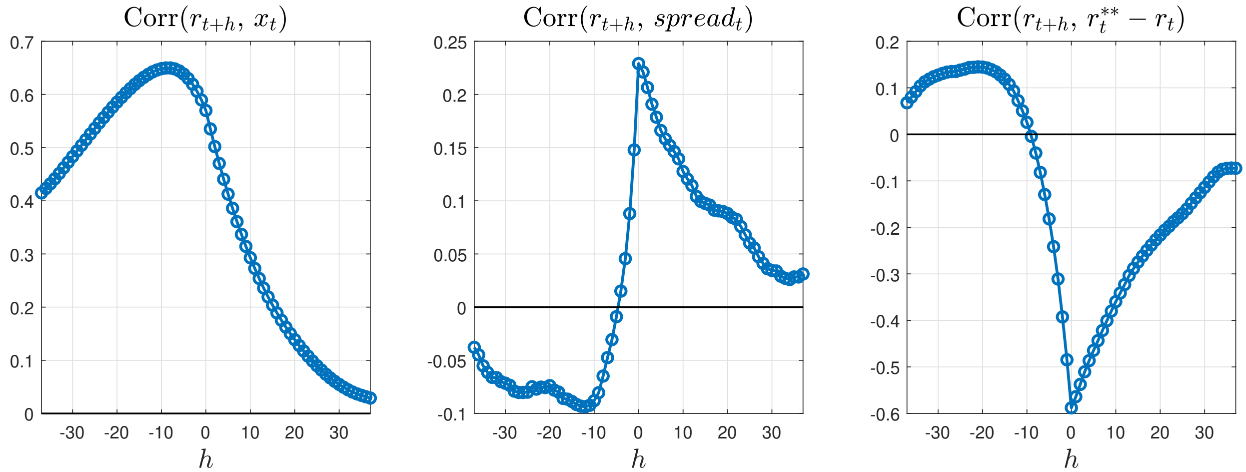
Figure 7: Dynamics of r**: Response to decline in real interest rates



Note: Responses to a one standard deviation shock to the real interest rate, r_t, at time 0. The economy is at the risk-adjusted steady in the initial period. Variables indicated % dev. computed as percent deviations relative to their risk-adjusted steady state values.

the ratio of safe-to-risky assets in banks' portfolio (x_t), credit spreads, and the financial stability interest rate gap, $(r^{**} - r)$. The first panel shows the correlation between the real interest rate r_{t+h} and the period-t safe asset ratio x_t, for a range of values of h. In response to a fall in real interest rates, the share of safe assets plummets, as discussed before, giving rise to a positive correlation between x and the real rate. Lower safe asset ratio, in turn, gives rise to an increase in financial frictions faced by the bank, contributing financial vulnerabilities to build up. This phenomena has also been reflected in the cross-correlogram between the real rates and the credit spreads (middle panel). While the contemporaneous correlation between real rates and credit spreads is positive, the correlation between current credit spreads and the lagged interest rate is *negative*: *low* levels of interest rates today are associated with greater *future* financial stress. The reason, again, is that low interest rates are associated with a lower ratio of safe-to-risky assets in the banking sector. This buildup of risky lending moves the economy closer to the financial stress region, as it raises the extent of financial frictions (measured by $\Theta(x_t)$). Accordingly, the model implies a positive association between the past values of real interest rate and the the financial stability rate gap. In other word, current low real interest rate forecasts future low real interest rate gaps. This positive association between the past values of real interest rates and the financial stability rate gap, however, is quite small. This is largely because in the model, as in reality, crises are eventually triggered by

Figure 8: Real Rate, Share of Safe Assets, and Financial stability rate gap, Lead-Lag Correlations



several other factors including a deteriorating TFP levels so that the relationship between low real interest rates today and future crisis is far from mechanical and is difficult to use in trying to predict the timing of crises.

5 Measuring r**

The previous sections defined r** and discussed its properties. This section provides a measure r** for the US economy and discusses its evolution over the past 50 years.

Within the context of the model measuring r** is straightforward: as shown above there is a mapping between the model's state variables (debt and quality adjusted assets) and the financial instability real interest rate on which we elaborate in the previous sections. In principle one could measure these very same variables in the data and use the same mapping to derive r**. In practice this approach may not be very promising however, because it is hard to construct empirical counterparts for these state variables.

A possibly more promising avenue which we pursue here is to identify a variable that is easy to measure in the data and that is tightly associated with r**. In the reminder of this section we argue that this variable is credit spreads. Given the model's nonlinearities however, the relationship between any observable, and specifically spreads, and r** is likely to be different depending on whether the constraint is binding. For this reason it is important to first identify periods where the economy is likely to be under financial stress, in the sense that the intermediaries' leverage constraints are binding. As it turns out, credit spreads are very helpful for this task as well. In the first part of the section we will argue that the *volatility* of spreads helps identify episodes of financial stress. In the second part of the section, we show that the *level* of spreads is tightly associated with r**, and more precisely with the gap between r** and the real rate r, especially during episodes of financial stress. Finally, we will present estimates of r** in the data and discuss some specific historical episodes, such as the Great Recession.

5.1 Identifying Financial Stress Episodes

As discussed in the section 2 there is a tight relationship in our non linear model between credit spreads and financial constraints. When financial constraints are not binding, intermediaries can arbitrage between riskless and risky assets, thereby keeping spreads very tight.[2] When the constraint is binding however this arbitrage is neither possible—because of the binding constraint—nor desirable as risky assets become a very poor hedge for intermediaries, so spreads open up. Moreover, as discussed before, financial accelerator dynamics kick in this non linear model, with the result that the economy becomes very sensitive to shocks and spreads turn very volatile.

Figure 9 illustrates these dynamics using data simulated from the model. The top panel displays the financial stability rate along with the real interest rate. The second panel shows the financial stability rate gap. The third and the last panels display the credit spreads and the value of the Lagrange multiplier on the leverage constraint, $\overline{\mu}$, respectively. Two observations are worth highlighting from the figure: First, the financial stability interest rate is slower moving object compared with the real interest rate. By construction, it remains above the prevailing rate in the economy times at which the constraint does not bind (which is the majority of time in the model economy), and falls below the real rate when the constraint binds (or, when $\overline{\mu}$ becomes positive). Second, whenever $\overline{\mu}$ is positive spreads are more volatile.

Of course μ is not directly observable in the data. We therefore want to construct a heuristic rule for identifying financial stress episodes that works correctly the model and that can be applied to the data. The above observations lead us to construct such a rule as follows. Call "spread jumps" changes in spread $\Delta spread_t$ that are above some quantile q of the distribution, i.e., $|\Delta spread_t| > q$. We then define a financial stress region as a sequence of jumps no more than two quarters/six months apart, beginning with an upward jump and ending with a downward jump. One can think of this heuristic approach as an alternative to estimating a regime switching model where the spread data is divided into high and low volatility regions. The requirement that jumps are no more than two quarters apart is dictated by the desire to avoid including in our definition non constrained regions in which sporadic increases/decreases in spreads, which in the model are driven by liquidity shocks, take place.

Figure 9 also shows how well the rule works in the model: the shaded areas are the financial stress episodes as identified by the rule when we pick $q = q_{85}$, that is, the 85^{th} quantile of the distribution. The red and green dashed lines mark the beginning and the end of true financial stress episodes. The red and green crosses mark spread jumps satisfying $|\Delta spread_t| > q_{85}$ (red are positive, green are negative). The figure shows that the episodes defined as crisis by the heuristic rule are indeed $\mu > 0$ periods, that is, the rule rarely entails false positives (we verified that this is the case in a much longer simulation). This occurs both because the initial rise in spread is not large enough to satisfy the $|\Delta spread_t| > q_{85}$ requirement or more often because more than two quarters pass between

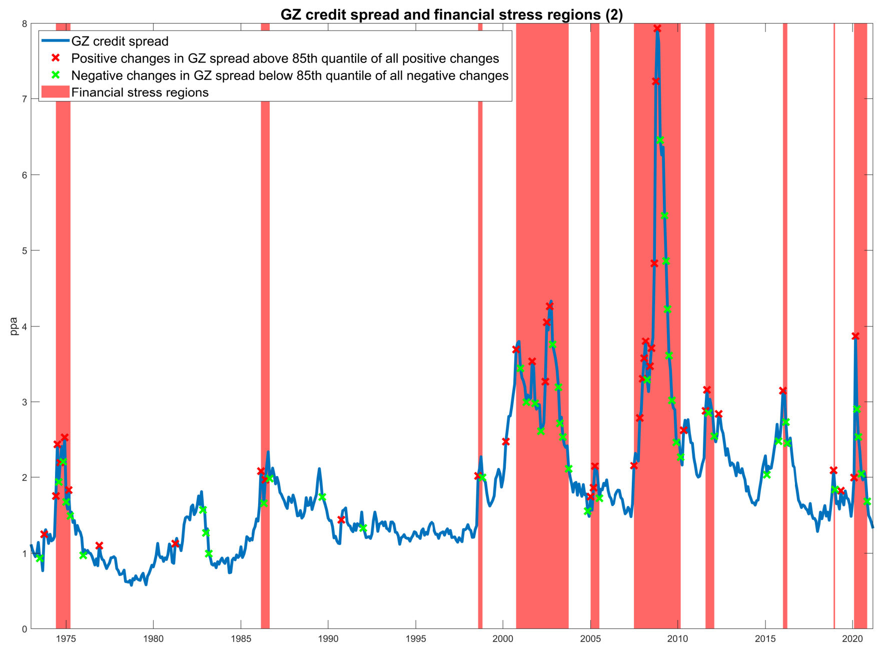
[2] As highlighted before, the model features exogenous liquidity premia that keep spreads positive even when financing constraints are not biding.

Figure 9: Financial Stability Rate Gap, Spreads, and Financial Constraints, Model

jumps in spreads. If course, lowering the required quantile to, say, q_{80} would reduce false negatives but would also introduce some false positives. We want to avoid doing so for reasons discussed in the next section.

Figure 10 shows the result of the heuristic rule when applied to Gilchrist and Zakrajsek (2012)'s GZ spread in the period for which this spread is available. The shaded areas are all arguably periods

Figure 10: Credit Spreads and Financial Stress Episodes, Data



associated with some degree of financial stress, from the LTCM crisis in the late 1990s to the period following 9/11/2001 to the Great Recession and its aftermath.

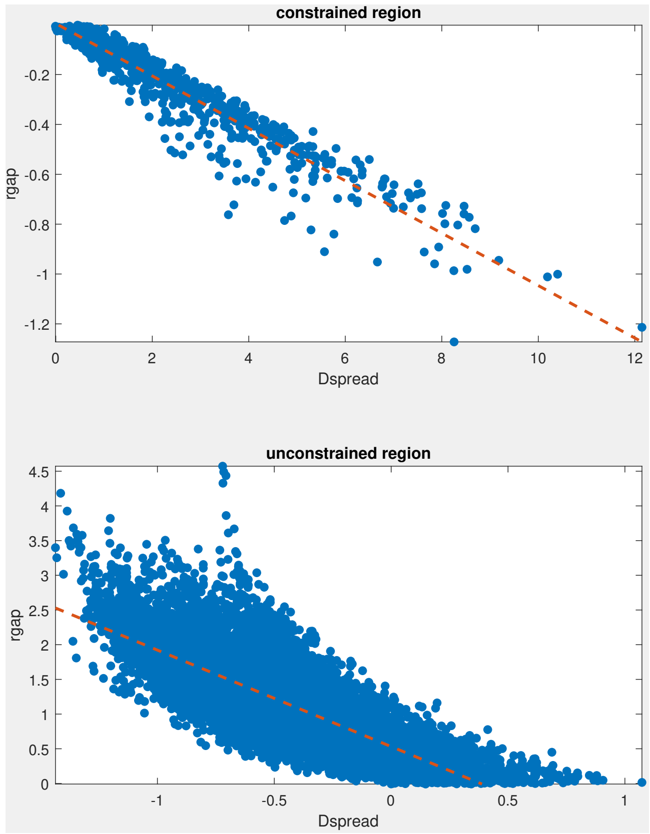
5.2 Credit Spreads and r**

Figure 9 show that the level of spreads is very correlated with the leverage multiplier, μ, *when the constraint is binding*, but of course not correlated at all (since $\mu = 0$) when the constraint does not bind. In the constrained region, the level of the multiplier μ is likely to be correlated with the (negative) gap between r** and the current level of the real rate: how much r would need to fall from its current level in order to improve the intermediaries' balance sheet (via its effect on asset prices) depends on how binding the constraint is. If the constraint is just binding, a small cut in the real rate may suffice. If the economy is in the throngs of a financial crisis, a larger cut in real rates may be needed to restore the health of the financial system.

These considerations lead us to run two separate regressions of the r**-r gap on the level of spreads, one for financial stress periods and one for "normal" periods. Ultimately, we want to use the estimated relationship on the data and map the observed level of spreads onto a measure for r**-r, and then use the level of the real rate to infer r** itself.

When considering US data, this approach runs into the following problem: if in the simple model we built spreads are stationary, there is ample evidence that in US data they are not (e.g., Del Negro et al., 2017a). Looking at Figure 10 it is apparent that the peak of spreads during the LTCM crisis in the late 1990s amounts to a relatively low level of spreads in the 2010s, for instance. For this

Figure 11: Spreads and and r**-r in the model



reason we amend the above strategy as follows: instead of using the level of spreads, we use (both in the model and in the data) the level of spreads relative to what they were in the period right before the economy first entered the current regime. To the extent that right before entering the constrained regime (or right after exiting the constrained regime) the constraint is "close to" being binding, then in that period the gap between r** and r is close to zero, and therefore normalizing spreads using their initial level is harmless in the model, and beneficial in the data as it effectively removes the trend.

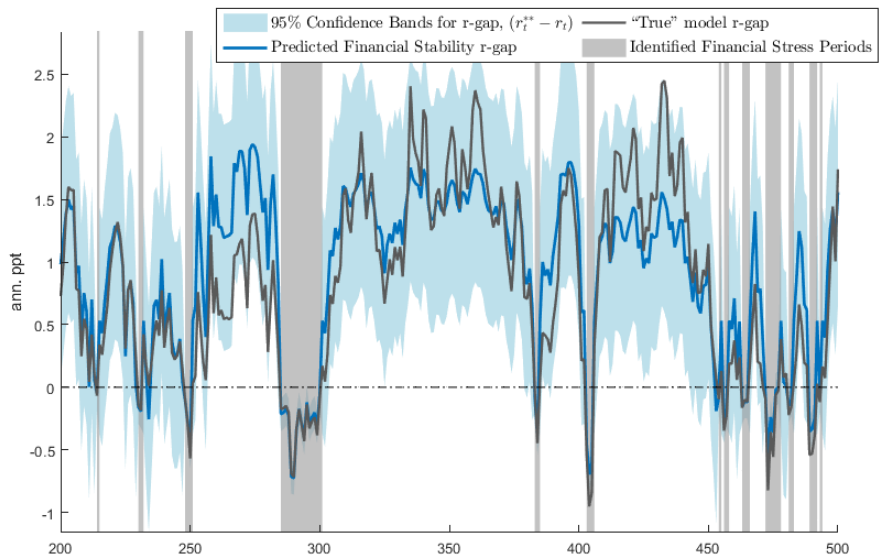
In sum, we run the separate regressions using model-generated data for the *financially constrained* regime ($\mu > 0$)

$$(r_t^{**} - r_t) = \alpha_c + \beta_c Dspread_t + \epsilon_t,\ Var(\epsilon_t) = \sigma_c^2;\quad \hat{\beta}_c = -0.11,\ \hat{\sigma}_c^2 = 0.0666 \tag{33}$$

and *unconstrained* ($\mu = 0$) regime

$$(r_t^{**} - r_t) = \alpha_u + \beta_u Dspread_t + \epsilon_t,\ Var(\epsilon_t) = \sigma_u^2;\quad \hat{\beta}_u = -1.40,\ \hat{\sigma}_u^2 = 0.4525 \tag{34}$$

Figure 12: True vs Predicted r-gap, $r_t^{**} - r_t$, **Model**

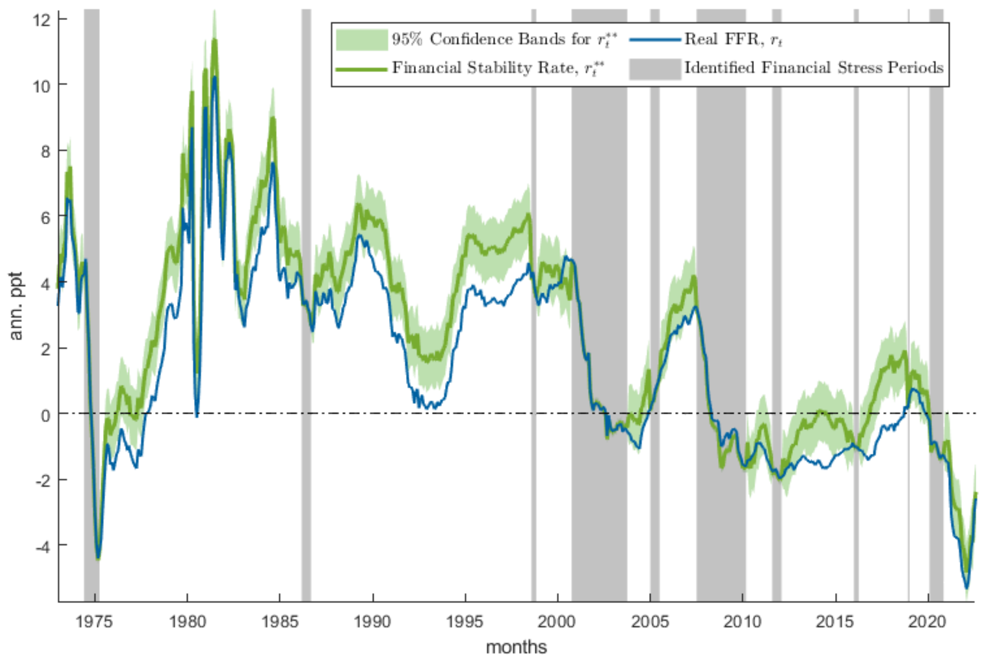


where $Dspread_t = (spread_t - spread_{\underline{\tau}})$, with $\underline{\tau}$ being the period before/after the economy enters/exits a financial stress episode. The estimates coefficients are displayed next to each regression and the fitted regression line is shown in Figure 11 together with the simulated data.

The results of the regressions confirm the intuition outlined above. In periods of financial stress there is a very tight relationship between credit spreads and the tightness of financial constraints, implying that the r**-r gap is well predicted by the level of spreads. Conversely, in unconstrained periods the relationship is much looser, as indicated by the higher standard deviations of the regression errors. The slope is negative in both regions—when spreads are low/high, r** is well above/below r and hence the gap is high/low—but the slope is very different. It is much lower in the constrained region indicating that in this region an increase in the tightness of the constraint (and therefore a decrease in the r**-r gap) leads to large increases in credit spreads. Since such spreads are in the right hand side in the above regression, this translates into a low slope. Vice versa, in the unconstrained region the slope is more negative: when the economy is close to the constraint (that is, the r**-r gap is positive but close to zero), in this non linear model intermediaries incorporate the risk that the constraint may become binding in pricing assets, leading to an increase in spreads. But this increase is still relatively mild, so that when spreads are on the right hand side of the regression the slope is more negative.

Figure 12 shows how well the fitted regressions do in capturing the r**-r gap in the model. Specifically, the black solid lines display the true gap, the blue line is the fitted gap, and the shaded areas are the 95 percent coverage intervals implied by the estimated $\hat{\sigma}_c$ and $\hat{\sigma}_u$. Shaded gray areas identify financial stress episodes. The figure shows that at least in the model spreads work very well

Figure 13: Financial stability rate, r_t^{**}, vs real FFR, r_t, Data



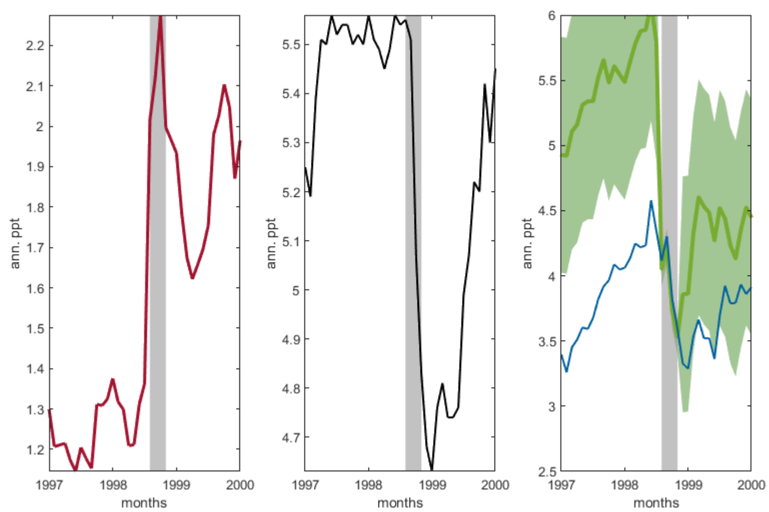
in essentially nailing the r**-r gap during the periods of financial stress. The distance between true and fitted values is generally small in these regions, and almost always within the relatively narrow bands. Outside of financial stress periods the fit becomes much poorer. This ignorance is reflected however in the wider bands, so that at least in those simulated data it is never the case that the true value of the gap falls outside the bands.

5.3 r** in the Data

The bottom line of the previous two sections is that the volatility of credit spreads helps to identify regions of financial distress, and that especially in these regions the level of spreads can quite accurately pin down the gap between r** and r, at least in the model. In this section we will make use of these results to provide an estimate of the time series of r** for US data over the past fifty years, and argue that this estimate is sensible. We will also show that the popular notion of a "Greenspan's put", namely that the central bank cuts rates whenever financial intermediaries become constrained, seems to be supported by the data: when financial constraints become binding and r** falls, the real rate soon follows it down so to close the gap between the two and ameliorate impact of the constraint.

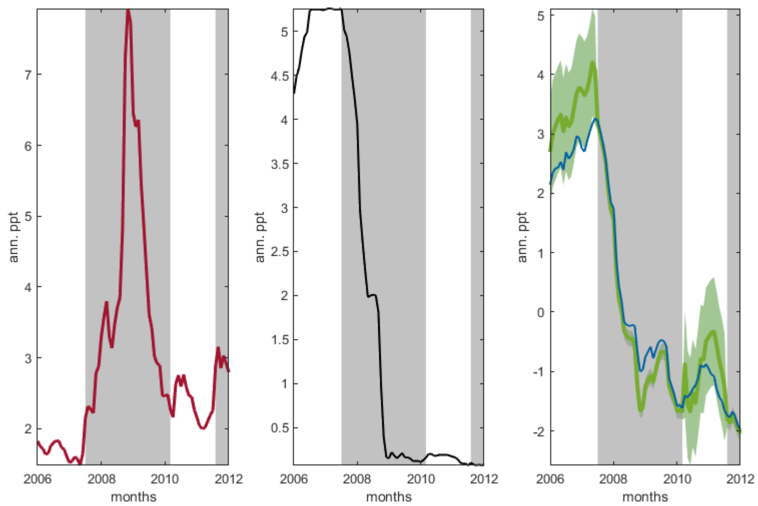
The blue line in Figure 13 shows the real rate, as measured by the ex-post real federal funds rate. The green line shows the point estimate of r** implied by the regressions (33) and (34), with the green shaded areas being the 95 percent coverage intervals. Vertical shaded gray areas identify financial stress episodes as in Figure 10. By construction, r** is below r during periods of financial stress, and above it otherwise, although the uncertainty is often large enough that that the 95 intervals include r. Broadly speaking, it appears that during the first part of the Great Moderation period, in the mid to late 80s and the 90s, r** is significantly above r except for short-lived episodes of stress such as the LTCM crisis. In the 2000s and right after the Great Recession the gap between r** and r is close to zero, meaning that the constraints is close to being binding, even in periods that are not classified as financial stress episodes. In the mid to late 2010s r** is generally well above r, except again for a couple of very short-lived periods of stress, until the Covid pandemic hits the economy in March 2020. We also note that in most financial stress episode r** is rarely if ever significantly below r for extended periods of time, with the Great Recession being the only exception, when monetary policy was constrained by the zero lower bound.

Figure 14: Episode 1, LTCM



The bird's eye view on r** afforded by Figure 13 makes it difficult to disentangle what happens during specific episodes. For this reasons in the reminder of the section we will zoom into two such episodes. The first, shown in Figure 14, is the LTCM financial stress period in the late 1990s. Because of the currency crisis in Russia and related turmoil in emerging markets in the summer of 1998, the hedge fund LTCM ran into liquidity and solvency problems and had to be bailed out. As LTCM had large trades with a number of important counterparties, the events of 1998 put the US financial system under considerable stress. The left panel of Figure 14 shows that credit spreads jumped by about 100 basis points within two months. The right panel shows that r** (green line) falls by about 75 bps from the beginning to the end of the financial stress episode. That is exactly

Figure 15: Episode 2, Financial Crisis



by how much Greenspan cut interest rates during this period, thereby quelling the financial distress (middle panel). During the first part of the Great Recession (Figure 15) the story is quite similar. Spreads increase and, as a consequence, r** falls. Initially the real rate r follows r** downward, thereby closing the r**-r gap and limiting the effects of the financial turmoil. In mid 2008 the nominal rate hit the zero lower bound however, and as a consequence r could not fall any longer. When the Lehman crisis hit the economy, spreads increased further, r** fell and a gap between r** and r opened until late 2009 and early 2010.

6 Conclusion

In this paper, we introduce the concept of financial stability real interest rate, r**. As a vehicle to illustrate our idea, we use a macroeconomic banking model based on Gertler and Kiyotaki (2010) where the banking sector faces a constraint in terms of a limit on the amount of funds that it can raise. When the constraint binds the economy experiences financial instability with increasing credit spreads, declining asset prices and contraction in economy activity.

We show that as the banking sector becomes more leveraged, the financial stability interest rate becomes lower. This has implications for monetary policy, in that even relatively low levels of the real interest rate could trigger financial instability.

Our analysis is conducted within a simple real model where the natural real interest rate is exogenously determined. In future work we plan to explore the interaction between macroeconomic stability and financial stability within a richer framework in which monetary policy is endogenously specified.

References

Adrian, Tobias, Nina Boyarchenko, and Domenico Giannone, "Vulnerable growth," *American Economic Review*, 2019, *109* (4), 1263–89. 4.1

Akinci, Ozge and Albert Queralto, "Credit Spreads, Financial Crises, and Macroprudential Policy," *American Economic Journal: Macroeconomics*, April 2022, *14* (2), 469–507. 1, 2, 4.1

_ **and Ryan Chahrour**, "Good news is bad news: Leverage cycles and sudden stops," *Journal of International Economics*, 2018, *114* (C), 362–375. 1

Benigno, Gianluca, Huigang Chen, Christopher Otrok, Alessandro Rebucci, and Eric R Young, "Financial crises and macro-prudential policies," *Journal of International Economics*, 2013, *89* (2), 453–470. 1

Brunnermeier, Markus K., *Financial Dominance*, Rome: Banca d'Italia, 2016. 4.2

Del Negro, Marco, Domenico Giannone, Marc P. Giannoni, and Andrea Tambalotti, "Safety, Liquidity, and the Natural Rate of Interest," *Brookings Papers on Economic Activity*, 2017, *48* (Spring), 235–316. 5.2

_ **,** _ **, Marc P Giannoni, and Andrea Tambalotti**, "Global trends in interest rates," *Journal of International Economics*, 2019, *118*, 248–262. 1

Gertler, Mark and Nobuhiro Kiyotaki, "Financial intermediation and credit policy in business cycle analysis," *Handbook of Monetary Economics*, December 2010, *3* (C), 547–599. 2, 2.2.1, 3.1, 6

_ **and** _ , "Banking, Liquidity, and Bank Runs in an Infinite Horizon Economy," *American Economic Review*, 2015, *105* (7), 2011–43. 1

_ **and Peter Karadi**, "A model of unconventional monetary policy," *Journal of Monetary Economics*, 2011, *58* (1), 17 – 34. 1

_ **, Nobuhiro Kiyotaki, and Andrea Prestipino**, "A Macroeconomic Model with Financial Panics," *The Review of Economic Studies*, 05 2019, *87* (1), 240–288. 3.1

Gilchrist, Simon and Benoît Mojon, "Credit Risk in the Euro Area," Working Paper 20041, National Bureau of Economic Research April 2014. 3, 4

_ **and Egon Zakrajsek**, "Credit Spreads and Business Cycle Fluctuations," *American Economic Review*, 2012, *102* (4), 1692–1720. 3, 4, 5.1

Greenwood, Jeremy, Zvi Hercowitz, and Gregory W Huffman, "Investment, Capacity Utilization, and the Real Business Cycle," *American Economic Review*, June 1988, *78* (3), 402–17. 3.1

He, Zhiguo and Arvind Krishnamurthy, "A Macroeconomic Framework for Quantifying Systemic Risk," *American Economic Journal: Macroeconomics*, 2019, *11* (4), 1–37. 4.1

Holston, Kathryn, Thomas Laubach, and John C. Williams, "Measuring the natural rate of interest: International trends and determinants," *Journal of International Economics*, 2017, *108* (S1), 59–75. 1

Hubrich, Kirstin, Antonello d'Agostino, Marianna Cervena, Matteo Ciccarelli, Paolo Guarda, Markus Haavio, Philippe Jeanfils, Caterina Mendicino, Eva Ortega, Maria Teresa Valderrama et al., "Financial shocks and the macroeconomy: heterogeneity and non-linearities," *ECB Occasional Paper*, 2013, (143). 4.1

Judd, Kenneth L, Lilia Maliar, and Serguei Maliar, "Numerically stable and accurate stochastic simulation approaches for solving dynamic economic models," *Quantitative Economics*, 2011, *2* (2), 173–210. 6

Kenny, Geoff and Julian Morgan, "Some lessons from the financial crisis for the economic analysis," Technical Report, European Central Bank 2011. 4.1

Krishnamurthy, Arvind and Annette Vissing-Jorgensen, "The aggregate demand for treasury debt," *Journal of Political Economy*, 2012, *120* (2), 233–267. 2.2.2

Laubach, Thomas and John C. Williams, "Measuring the Natural Rate of Interest," *Review of Economics and Statistics*, November 2003, *85* (4), 1063–1070. 1

Marcet, Albert and Guido Lorenzoni, "Parameterized expectations approach; some practical issues," 1998. 6

Mendoza, Enrique G., "Sudden Stops, Financial Crises, and Leverage," *American Economic Review*, December 2010, *100* (5), 1941–66. 1

Merton, Robert M., "Observations on the Science of Finance in the Practice of Finance," 2009. 4.1

Negro, Marco Del, Domenico Giannone, Marc P. Giannoni, and Andrea Tambalotti, "Safety, liquidity, and the natural rate of interest," *Brookings Papers on Economic Activity*, 2017, *2017* (1), 235–316. 2.2.2

Neumeyer, Pablo A. and Fabrizio Perri, "Business cycles in emerging economies: the role of interest rates," *Journal of Monetary Economics*, March 2005, *52* (2), 345–380. 2.3.1

Schmitt-Grohe, Stephanie and Martin Uribe, "Closing small open economy models," *Journal of International Economics*, October 2003, *61* (1), 163–185. 2.4

Appendix: Equilibrium

Equilibrium is characterized by the following system of equations.

$$Y_t = C_t + I_t + T_t \tag{35}$$

$$T_t = B_t - R_{t-1}B_{t-1} \tag{36}$$

$$K_t = (a_1 I_t^{1-\eta} + a_2) + (1-\delta)e^{\psi_t}K_{t-1} \tag{37}$$

$$Q_t = \left[a_1(1-\eta)I_t^{-\eta}\right]^{-1} \tag{38}$$

$$\mathbb{E}_t(\Lambda_{t+1})R_t = 1 \tag{39}$$

$$\Lambda_t = \beta \frac{U_{Ct}}{U_{Ct-1}} \tag{40}$$

$$U_{C,t} = \left(C_t - \frac{\chi}{1+\epsilon}L_t^{1+\epsilon}\right)^{-\gamma} \tag{41}$$

$$R_{K,t} = \frac{\alpha \frac{Y_t}{e^{\psi_t}K_{t-1}} + (1-\delta)Q_t}{Q_{t-1}} \tag{42}$$

$$Y_t = K_{t-1}^{\alpha}L_t^{1-\alpha} \tag{43}$$

$$(1-\alpha)\frac{Y_t}{L_t} = \chi L_t^{\epsilon}\left[1 + \Upsilon\left(R_t^d + \frac{\mu_t}{\mathbb{E}_t[\Lambda_{t+1}\Omega_{t+1}]} - 1\right)\right] \tag{44}$$

$$\mu_t = \mathbb{E}_t[\Lambda_{t+1}\Omega_{t+1}(R_{K,t+1} - R_t^d)] \tag{45}$$

$$\mu_{K,t} = \mathbb{E}_t[\Lambda_{t+1}\Omega_{t+1}(R_{K,t+1} - R_t)] \tag{46}$$

$$\nu_t = \mathbb{E}_t[\Lambda_{t+1}\Omega_{t+1}]R_t^d \tag{47}$$

$$\overline{\mu}_t = \mu_t + (\zeta_t - \mu_{K,t})x_t \tag{48}$$

$$\Omega_t = 1 - \sigma + \sigma(\nu_t + \overline{\mu}_t\phi_t) \tag{49}$$

$$N_t = \sigma[(R_{K,t} - R_{t-1})Q_{t-1}K_{t-1} + (R_{B,t-1} - R_{t-1}^d)B_{t-1} + R_{t-1}^d N_{t-1}] + (1-\sigma)\xi Q_{t-1}K_{t-1} \tag{50}$$

$$\overline{\phi}_t = \frac{\nu_t}{\Theta_t - \overline{\mu}_t} \tag{51}$$

$$\Theta_t = \theta(1 - \frac{\lambda}{\kappa}x_t^{\kappa}) \tag{52}$$

$$x_t = \frac{B_t}{B_t + Q_tK_t} \tag{53}$$

$$Q_tK_t + B_t = \phi_t N_t \tag{54}$$

$$\mu_{K,t} = \zeta_t + \overline{\mu}_t \frac{\lambda x_t^{\kappa-1}}{(1 - \frac{\lambda}{\kappa}x_t^{\kappa})} \tag{55}$$

$$\overline{\mu}_t \times (\overline{\phi}_t - \phi_t) = 0 \tag{56}$$

$$R_t = \overline{R} - \varphi(e^{x_t - \overline{x}} - 1) + e^{R_t^* - 1} - 1 \tag{57}$$

$$\log(R_t^*) = \rho_r \log(R_{t-1}^*) + \sigma_r \varepsilon_{r,t} \tag{58}$$

$$\zeta_t = \overline{\zeta} + \varepsilon_{\zeta,t} \tag{59}$$

We have variables $Y_t, C_t, NX_t, B_t, K_t, I_t, Q_t, \Lambda_t, R_t, R_{B,t}, U_{C,t}, L_t, R_{K,t}, \mu_t, \mu_{K,t}, \overline{\mu}_t, \nu_t, \Omega_t, N_t, \phi_t, \overline{\phi}_t,$ $x_t, \Theta_t, R_t^*, \zeta_t$ (25 variables for 25 equations). We have $\overline{\mu}_t \geq 0$ and $(\overline{\phi}_t - \phi_t) \geq 0$. Equation (56) indicates that if $\overline{\mu}_t = 0$, we must have $\phi_t < \overline{\phi}_t$ (banks' leverage constraint does not bind); conversely, if $\overline{\mu}_t > 0$, we have $\phi_t = \overline{\phi}_t$ (the leverage constraint binds).

Our computational strategy is based on Judd et al. (2011), but relies on approximating one-step-ahead expectations rather than policy functions, as in the "parameterized expectations" approach (Marcet and Lorenzoni, 1998). We use Hermite polynomials to approximate the expectations in (35)-(59) and use stochastic simulations to iterate until convergence. Given knowledge of the expectations and of the states, it is possible to solve system (35)-(59) in closed form in the unconstrained regime, and to collapse it to just one non-linear equation in the constrained regime.