Measuring Upward Mobility*

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Abstract. We develop a measure of upward mobility that combines relative and absolute approaches to measuring mobility. The core of our approach is the *Growth Progressivity Axiom*: transfers of instantaneous growth rates from relatively rich to poor individuals increases upward mobility. This axiom, along with other auxiliary restrictions, identifies a one-parameter family of upward mobility measures, linear in individual growth rates, with geometrically declining weights on baseline incomes. Our axiomatized measure does not rely on panel data, thereby expanding our analytical scope to data-poor settings.

1. Introduction

Social mobility refers to the ease of transition between socioeconomic categories. To the extent that those categories (e.g. income or wealth) are vertically ranked, mobility is linked to *directed* movement, upward or downward. So economic growth is related to mobility in this sense, without necessarily being identical to it. Moreover, mobility is higher if the relatively worse-off enjoy greater upward movement. So economic *equality* is connected to mobility in this sense, without necessarily being identical to it.

These observations connecting mobility, growth and equality lead to a view that mobility is related to *pro poor growth*, a concept that aggregates growth across individuals or groups, but weighted by their economic characteristics. (See Section 2 for relevant literature.) We follow this line of thinking and call our notion *upward* mobility. It is to be contrasted with the idea of mobility as "pure movement," which allows mobility to increase with sheer volatility across categories, with no accounting for any ranking on those categories.

Our goal is to construct the discrete-time measure in Theorem 3. But our methodology builds on economic trajectories in continuous time. We begin with the notion of "instantaneous mobility," defined on a vector of individual observations, each consisting of a baseline value of some endowment (that we shall call *income*) and an instantaneous

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growth rate of that endowment. We impose two axioms: *Growth Progressivity* and *Growth Alignment*. The former states that a transfer of growth rates from richer to poorer individuals increases upward mobility. (The rates are instantaneous, so no income crossings occur when initial incomes are distinct, but of course they could occur over an interval.) By exploiting properties of multiaffine functions, we show that Growth Progressivity implies the linearity of upward mobility in individual growth rates, with weights that decline in baseline incomes (Theorem 1). This central connection between Growth Progressivity and the *linearity* of the measure in growth rates is both of intrinsic interest and crucial to empirical implementation, as we shall see.

Growth Alignment states that mobility increases when all individuals experience higher growth. Along with some standard auxiliary restrictions, these two axioms identify a one-parameter family of upward mobility measures: a weighted sum of instantaneous growth rates for each individual, with weights that geometrically decline in income and are indexed by a single parameter to mark the speed of that decline (Theorem 2).

Now we move to domains that are typically available to the researcher: income information that is given over discrete points in time. To begin this transition, we consider income *trajectories* over some interval as the relevant elements of our domain. This is a more complex space than the one generated by instantaneous incomes and growth rates. Specifically, Growth Progressivity cannot be applied to these trajectories based on overall growth rates over the interval, for the simple reason that the trajectories might cross. We approach this extension by imposing two conditions. First, we ask that discrete upward mobility for a collection of income trajectories should be fully pinned down by the collection of all instantaneous upward mobilities at every instant of time, precipitated by those trajectories. We call this property *reducibility*. Our second condition asks that our overall measure of upward mobility defined on income trajectories should be *additive* over time: mobility between time t and t', followed by mobility between time t' and t'', should sum up to total upward mobility between t and t''. Theorem 3 combines reducibility, additivity and our earlier axioms to generate the main measure of mobility that we take to the data. This convenient discrete-time formula also has a welfarist interpretation as the annualized growth of Atkinson's equivalent income.

A central implication of our exercise — one that we develop in detail in Section 5 — is that our derived upward mobility measure can be constructed without the need for panel data. This conclusion questions the need for estimating transition probabilities, which is typically accomplished with much difficulty, as the data is often proprietary; see Chetty et al. (2017) and Acciari et al. (forthcoming). Such exercises are near-impossible to conduct in the majority of countries. Our implication is strong enough that we ask for the

reader's patience in postponing a final judgment on this assertion until Sections 3 and 4 are absorbed. If convincing, our measure significantly expands mobility measurement to a large set of countries.

Finally, Section 7 contains an initial empirical exploration. We compare our measure to perhaps the most popular measure of directional mobility, which *does* rely on a panel. This is the share of families whose absolute fortune improved across generations. Chetty, Grusky, Hell, Hendren, Manduca and Narang (2017) is the leading study that uses this measure. They estimate the fraction of children who earn more than their parents for US birth cohorts from 1940 to 1984 and document a secular decline in this fraction. We take our measure to the same data without relying on the transition matrix of Chetty et al. (2017), which is estimated from a proprietary panel of tax records. Chetty et al. (2017) need the transition matrix because their measure is panel-reliant. Ours is not, but tracks their measure very closely; see Figure 3.[2] With this empirical "proof of concept" in hand, we apply our measure to Brazil, India and France over 1970–2015, using *repeated cross-sectional data* from the World Inequality Lab. The exercise reveals new trends for these countries that may be of independent interest.

2. Related Literature

Different approaches have been taken to the measurement of mobility reflecting the variety of opinion on just what the term means (Fields 2010).

A large literature views mobility as non-directional, building on the idea that transition probabilities on intertemporal population distributions across categories are all we need, without distinguishing between gains and losses; see Prais (1955), Atkinson (1981), Bartholomew (1982), Conlisk (1974), Dardanoni (1993), Hart (1976), or Shorrocks (1978). Here, mobility is the literal converse of immobility. In Shorrocks (1978), for instance, any increase in an off-diagonal entry of the transition matrix across incomes or income increases mobility. Other measures include the income elasticity of progeny income with respect to parental income (Solon 1999, Jäntti and Jenkins 2015), and the rank-rank correlation or slope (Dahl and DeLeire 2008, Chetty, Hendren, Kline and Saez 2014a).

In the extreme form of this approach, mobility *is* movement. Good examples are the relative mobility measures of King (1983) and Chakravarty (1984) that measure the rerankings in the distribution, also called "exchange mobility" (see Dardanoni 1993 and Markandya 1982).

[2] This exercise is in the spirit of Berman (2021), who also attempts to circumvent the panel structure, though from an empirical perspective. See Section 7 for more discussion.

However, this view divests itself of the ethical connotations that swirl around the concept of mobility (Fields and Ok 1999a; Jäntti and Jenkins 2015). We all agree that gains are better than losses. When empirical studies emphasize a specific part of the transition matrix, say, transfers from bottom to upper ranks, they implicitly provide this welcome sense of direction (see, e.g., Bhattacharya and Mazumder 2011, Chetty, Hendren, Kline and Saez 2014a and Berman 2021). Contributions that embrace overall growth include Fields and Ok (1996), Mitra and Ok (1998) and Cowell and Flachaire (2018), but the results in these papers use a measure that does not depends on the direction of change in incomes. In this sense, these are also measures of "movement" rather than "upward mobility."

A different approach aims to measure mobility as an cross-person equalizer of income or income compared to initial distribution (see Chakravarty, Dutta and Weymark 1985, Maasoumi and Zandvakili 1986, or Fields 2010). In these measures too, *overall* growth is typically normalized away, so the absolute aspect of overall income growth is removed. However, we connect closely in spirit with these authors, in that we derive their approach from more primitive axioms, without assuming it.

Two well-known measures of mobility are both directional and absolute. Fields and Ok (1999b) propose "directional mobility measures" that sum individual growth rates. More recently, the absolute mobility measure in Chetty, Grusky, Hell, Hendren, Manduca and Narang (2017) records the fraction of children who earn more than their parents. We discuss both measures in more detail in Section 6.7. For now, we note that both measures throw away information about who gains and who loses. In contrast, as already discussed, the relative mobility literature is sensitive to such matters.

In our twin emphases placed on both absolute and relative growth, we are closest to a literature on *pro-poor growth*. Chenery, Jolly, Ahluwalia, Bell and Duloy (1974) introduced an index of economic performance as a weighted sum of group growth rates. This led to a literature on pro-poor growth using a variety of weights decreasing with income (see Dardanoni 1993, Essama-Nssah 2005, and Ravallion and Chen 2003) and proposing graphical tools to represent growth along the income distribution such as the growth incidence curve (Grimm 2007, Bourguignon 2011, Ravallion and Chen 2003, Dhongde and Silber 2016, Palmisano and de Gaer 2016, Creedy and Gemmell 2018, Palmisano 2018). These authors observe that the difference between "anonymous" and "non-anonymous" growth incidence curves correspond to pure exchange mobility. They also connect pro-poor growth to the literature on convergence (see in particular O'Neill and Kerm 2008, Wodon and Yitzhaki 2005, Bourguignon 2011 and Dhongde and Silber 2016). Using non-anonymous growth incidence curves, indices of directional mobility have been proposed

that place more weight on the growth rates of lower-ranked individuals (Jenkins and Van Kerm 2016, Palmisano and de Gaer 2016 and Berman and Bourguignon 2022).

Our analysis builds on these insights. As noted in the Introduction, we impose Growth Progressivity and Growth Alignment as axioms, and show that these restrictions (along with some other mild restrictions) precipitate a measure that is linear in growth rates, with weights that decline geometrically in income. Our exercise therefore establishes the foundational principles of such measures. But more than that, it leads to a discrete-time measure which applies to non-panel data. Taken together, the axioms imply that mobility for the sake of pure movement, or exchange mobility, is entirely eliminated from our measure of *upward* mobility. The development of this idea is a central theme of the paper.

3. Prelude: Instantaneous Upward Mobility

We first develop a "instantaneous" upward mobility kernel that is:

1. *Directional*: it rewards growth, and punishes decay;

2. *Progressive*: it rewards "growth transfers" from higher to lower incomes.

Section 4 extends the kernel to a measure defined on income *trajectories*. It is directional, progressive and

3. *Panel-independent*: it can be deployed on repeated cross-sections of data.

We refer to our central variable of interest as *income*. Conceptually, what we have in mind is some measure of permanent income or wealth, a variable which serves as a sufficient statistic for well-being at any moment in time. Alas, such a statistic is often not observed: a problem common to other measurement exercises pertaining to inequality, poverty, GDP and so on. See Section 5 for more discussion. For the formal analysis, however, all that matters is that we have some variable for which continuous changes can be envisaged.

3.1. Axiomatic Development. Each person i is linked to a pair $z_i = (y_i, g_i)$, where $y_i > 0$ is baseline income and g_i is the instantaneous growth rate of that income. Denote by $\mathbf{z} = \{z_i\}$ the population collection of incomes and growth rates, including repetitions. An *upward mobility kernel* is a continuous function $M(\mathbf{z})$, defined over all finite populations, anonymous to permutations of indices within $\mathbf{z}$. We place the following axioms on M.

1. **Zero Growth Anchoring.** If under both $\mathbf{z}$ and $\mathbf{z}'$, *every* individual has a zero growth rate, then $M(\mathbf{z}) = M(\mathbf{z}')$. Normalize this common value to zero.

Our fundamental axiom that connects growth to mobility is

2. **Growth Progressivity.** For any $\mathbf{z}$, i and j with $y_i < y_j$, and for $\epsilon > 0$, form $\mathbf{z}'$ by altering g_i to $g_i + \epsilon$ and g_j to $g_j - \epsilon$. Then instantaneous mobility goes up: $M(\mathbf{z}') > M(\mathbf{z})$.

Growth Progressivity "rewards" a transfer of *growth points* from rich to poor. Note that because growth rates are instantaneous, there is no immediate "income crossing" when baseline incomes are distinct. We will address income crossings in Section 4.

Despite its resemblance to the well-known Transfers Principle for inequality comparisons (Fields and Fei 1978), Growth Progressivity is a distinct concept that involves transfers of growth *rates*, not incomes. This is a central demarcation we draw between inequality, which is a static concept, and upward mobility, which is dynamic. For instance, a two-person society with starting incomes of (5000, 10000) and corresponding growth rates (8%, 8%) is more upwardly mobile than one with (6%, 10%). Or indeed, that a society with growth rates (2%, -2%) is more upwardly mobile than one with (0%, 0%). It is easy to check that Growth Progressivity implies the Transfers Principle, but also applies when the pie is changing, and in particular implies that not all aggregate growth is welcome.

We also impose an auxiliary restriction that gives M some cardinal meaning. For any pair of situations $\mathbf{z}$ and $\mathbf{z}'$, let $\mathbf{z} \oplus \mathbf{z}'$ denoted the merged or concatenated situation which simply takes the union of all income-growth pairs over both situations. For instance, $\mathbf{z} \oplus \mathbf{z}$ means that $\mathbf{z}$ has been duplicated. We place a restriction on "locally merged" situations that have identical sets of incomes and growth rates except for just one individual k:

3. **Local Merge.** Suppose $\mathbf{z}$, $\mathbf{z}'$ and $\mathbf{z}''$ are identical except for possibly just one index k, for which $g'_k = g_k - \epsilon$ and $g''_k = g_k + \epsilon$ for some $\epsilon > 0$. Then $M(\mathbf{z}' \oplus \mathbf{z}'') \neq M(\mathbf{z} \oplus \mathbf{z})$ whenever $M(\mathbf{z}'') - M(\mathbf{z}) \neq M(\mathbf{z}) - M(\mathbf{z}')$.

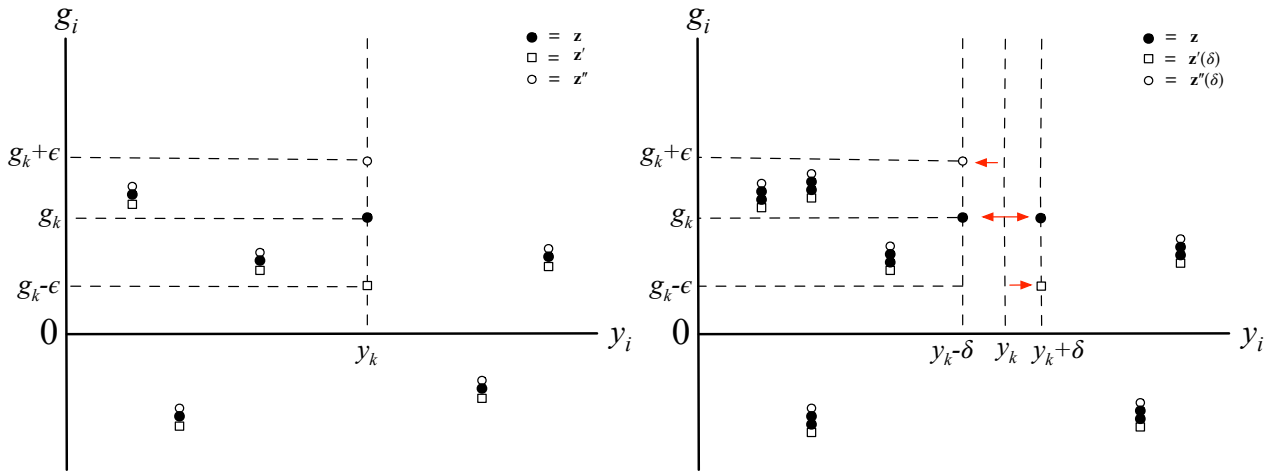
This axiom demands that if "average mobility" is altered by moving one person's growth rate up while her clone's growth rate is moved down, then mobility is also altered when both persons coexist and experience these same changes "at the same time."

Theorem 1. *Zero Growth Anchoring, Growth Progressivity, and Local Merge hold if and only if*

$$M(\mathbf{z}) = \sum_{i=1}^{n} \phi_i(\mathbf{y}) g_i \tag{1}$$

for some continuous collection $\{\phi_i\}$, *with* $\phi_i(\mathbf{y}) = \phi_j({}^i\mathbf{y}_j)$ *whenever i and j are permuted in* $\mathbf{y}$ *to get* $\tilde{\mathbf{y}}$, *and* $\phi_i(\mathbf{y}) > \phi_j(\mathbf{y})$ *when* $y_i < y_j$.

While Theorem 1 is proved in the Appendix, we illustrate here the power of the Growth Progressivity axiom. It is centrally responsible for precipitating the additivity and linearity



(a) Growth perturbation **(b)** Income perturbation

Figure 1. Illustration of proof that M is multiaffine. Part (a) shows two situations (filled and hollow dots) which are identical except for the growth rates at y_k, which are different (larger dots). Part (b) perturbs the incomes in both situation by generating incomes $y_k - \delta$ and $y_k + \delta$ instead of y_k. The perturbation δ is to be thought of as small.

of our measure in individual growth rates.[3] Without linearity, the measure must exhibit different local sensitivities to the growth rate g_i for some i. We are then able to exploit this variation in local sensitivities to construct a society in which a transfer of growth rates from one "near-clone" of i to another violates the Growth Progressivity Axiom. But these observation is both imprecise and incomplete, and the main argument runs in two central steps. The first of these is the assertion that $M(\mathbf{y}, \mathbf{g})$ must be *multiaffine* in $\mathbf{g}$; i.e., for every k, $m(g_k) \equiv M(g_k|\mathbf{y}, \mathbf{g}_{-k})$ is affine in g_k:

$$m(g_k) = Ag_k + B$$

for constants A and B that could depend on $(\mathbf{y}, \mathbf{g}_{-k})$. Because m is continuous, it is enough to show that for every $\epsilon > 0$,

$$m(g_k) = \frac{1}{2}\left[m(g_k - \epsilon) + m(g_k + \epsilon)\right]. \tag{2}$$

Suppose the claim is false, so that (2) fails for some g_k and $\epsilon > 0$. Let $\mathbf{z} = (\mathbf{y}, \mathbf{g}_{-k}, g_k)$. The filled dots in Part (a) of Figure 1 depicts this situation. It also shows two other situations, $\mathbf{z}'$ (represented by the squares) and $\mathbf{z}''$ (represented by the hollow dots). The proximity of dots and squares away from y_k is meant to imply that these three situations are identical in incomes and growths, except at y_k, where z' exhibits a lower growth rate than $\mathbf{z}$ (by ϵ)

[3]The other axioms also play their part. Axiom 1 removes any intercept term that depends on incomes. Axiom 3 takes us across populations of varying size. Without it, Growth Progressivity would still limit the curvature of the measure, but allow for some nonlinearity depending on cross-individual income gaps.

and $\mathbf{z}''$ a higher growth rate (also by ϵ). Because (2) fails, we have

$$m(g_k + \epsilon) - m(g_k) \neq m(g_k) - m(g_k - \epsilon),$$

but using the definition of m, this means $M(\mathbf{z}'') - M(\mathbf{z}) \neq M(\mathbf{z}) - M(\mathbf{z}')$, or:

$$M(\mathbf{z}' \oplus \mathbf{z}'') \neq M(\mathbf{z} \oplus \mathbf{z}) \tag{3}$$

by the Local Merge Axiom. Suppose that "<" holds in (3); the opposite ">" has a parallel argument. Part (b) of Figure 1 perturbs $\mathbf{z}$, $\mathbf{z}'$ and $\mathbf{z}''$ to separate y_k into $y_k - \delta$ and $y_k + \delta$, as shown, with the perturbed $\mathbf{z}'(\delta)$ having $y_k + \delta$ and the perturbed $\mathbf{z}''(\delta)$ having $y_k - \delta$. Part (b) also perturbs $\mathbf{z}$ in two ways: $\mathbf{z}^-(\delta)$ replaces y_k by $y_k - \delta$ while $\mathbf{z}^+(\delta)$ replaces y_k by $y_k + \delta$. Let $\mathbf{z}(\delta) \equiv \mathbf{z}^+(\delta) \oplus \mathbf{z}^-(\delta)$. Using the continuity of M and "<" in (3) and the fact that $\mathbf{z}'(\delta) \oplus \mathbf{z}''(\delta) \to \mathbf{z}' \oplus \mathbf{z}''$ and $\mathbf{z}(\delta) \to \mathbf{z} \oplus \mathbf{z}$, we must conclude that for $\delta > 0$ and small,

$$M(\mathbf{z}'(\delta) \oplus \mathbf{z}''(\delta)) < M(\mathbf{z}^+(\delta) \oplus \mathbf{z}^-(\delta)).$$

But this contradicts Growth Progressivity, for $\mathbf{z}'(\delta) \oplus \mathbf{z}''(\delta)$ can be achieved from $\mathbf{z}^+(\delta) \oplus \mathbf{z}^-(\delta)$ by transferring a growth rate of $\epsilon > 0$ from $y_k + \delta$ to $y_k - \delta$.

A parallel argument applies when ">" holds in (3), by perturbing $\mathbf{z}''(\delta)$ to the higher income $y_k + \delta$ and $\mathbf{z}'(\delta)$ to the lower income $y_k - \delta$.

So M is multiaffine, and therefore it is expressible as follows: for every $\mathbf{y} \gg 0$, there is a collection $\phi_S(\mathbf{y})$ for every nonempty subset S of $\{1, \ldots, n\}$, such that

$$M(\mathbf{z}) = \sum_S \phi_S(\mathbf{y}) \left[\prod_{j \in S} g_j \right]. \tag{4}$$

where n is the population, the sum ranges over all nonempty[4] index subsets S of $\{1, \ldots n\}$, and the $\phi_S(\mathbf{y})$ are income-vector-dependent coefficients (see, e.g., Gallier 1999, Chapter 4.5). We argue that all nontrivial product terms *must have zero coefficients*. Otherwise, for some $\mathbf{g}$, it is possible to obtain a contradiction by transferring growth rates from relatively poor to relatively rich and increasing M, so violating the Growth Progressivity Axiom. See Appendix for details. The only terms that can have zero coefficients are the linear terms in (4). The fact that the smaller index terms among $\{\phi_i(\mathbf{y})\}$ have larger values than the larger-index terms is also an immediate consequence of Growth Progressivity. That outlines our proof of Theorem 1.

3.2. A One-Parameter Family for Instantaneous Upward Mobility. Expression (1) is a key implication of the Growth Progressivity Axiom, and illustrates the power of that Axiom. To highlight this, we have (so far) placed no restrictions on how the upward mobility

[4]The empty product can be excluded by the Zero Growth Anchoring Axiom.

kernel changes with growth, or on baseline weights apart from the property derived in Theorem 1. We now proceed to impose further axioms that bring the weights $\phi_i(\mathbf{y})$ into sharper focus.

4. **Income Neutrality.** Given $\mathbf{z} = (\mathbf{y}, \mathbf{g})$, form $\mathbf{z}' = (\lambda\mathbf{y}, \mathbf{g})$ by scaling all baseline incomes by the same positive constant λ. Then $M(\mathbf{z}) = M(\mathbf{z}')$.

5. **Growth Alignment.** For any $\mathbf{y}$, if $\mathbf{g} > \mathbf{g}'$, then $M(\mathbf{y}, \mathbf{g}) > M(\mathbf{y}, \mathbf{g}')$. And if $\mathbf{g} = (g, g, \ldots, g)$, then for every $\mathbf{y}$ and $\mathbf{y}'$, $M(\mathbf{y}, \mathbf{g}) = M(\mathbf{y}', \mathbf{g})$.

6. **Binary Growth Tradeoffs.** For any ij, any (y_i, y_j), and any two growth pairs (g_i, g_j) and (g_i', g_j'), the comparison of $\mathbf{z} = ((y_i, g_i), (y_j, g_j), (\mathbf{y}_{-ij}, \mathbf{g}_{-ij}))$ and $\mathbf{z}' = ((y_i, g_i'), (y_j, g_j'), (\mathbf{y}_{-ij}, \mathbf{g}_{-ij}))$ is insensitive to the value of $(\mathbf{y}_{-ij}, \mathbf{g}_{-ij})$.

Axiom 4 asserts that only *relative* baseline incomes matter. Axiom 5 (partially) aligns upward mobility with growth, If *all* income levels grow faster, then mobility is deemed to be higher. We believe both these axioms to be innocuous, though see Section 3.3.

Axiom 6 declares that any tradeoffs across a pair of growth rates depends only on the characteristics of just *that* pair. This is in the spirit of "independence of irrelevant alternatives." There are well-known misgivings about that axiom (see the critical assessment in Pearce 2021). One qualification concerns the choice of domain for this Axiom. Under absolute upward mobility, individual growth rates matter *per se*, so that the domain of Axiom 6 is reasonable. But if the context is one of relative upward mobility, then the Axiom more properly applies to the *excess* (positive or negative) of individual growth against overall growth. See the discussion in Section 3.3.

Theorem 2. *Axioms 1-6 hold if and only if for every population of size $n \geq 3$,*

$$M_\alpha(\mathbf{z}) = \frac{\sum_{i=1}^n y_i^{-\alpha} g_i}{\sum_{i=1}^n y_i^{-\alpha}}, \text{ for some } \alpha > 0. \tag{5}$$

In effect, four axioms characterize (5): Growth Progressivity, Income Neutrality, Growth Alignment, and Binary Growth Tradeoffs. Local Merge and Zero Growth Anchoring are automatically implied. The family of instantaneous measures characterized here will form the nucleus of our main analysis, to be developed in Section 4.

While the Appendix provides a self-contained proof of Theorem 2 that does not invoke Theorem 1, it is useful to see the marginal roles played by the new axioms in precipitating it. When $n \geq 3$, Binary Growth Tradeoffs along with anonymity allow us to write

$$\phi_i(\mathbf{y}) = \psi(y_i)h(\mathbf{y})$$

for functions ψ and h. By Growth Alignment, both functions are strictly positive-valued, and we can normalize $M(\mathbf{z}) = g$ when all growth rates equal g. Using (1), $\sum_i \psi(y_i)h(\mathbf{y}) = 1$. Substituting this information in (1),

$$M(\mathbf{z}) = \frac{\sum_i \psi_i(y_i)g_i}{\sum_i \psi_i(y_i)}.$$

The Appendix shows that Income Neutrality must then imply $\psi(y) = y^{-\alpha}$, where $\alpha > 0$ by Growth Progressivity. That establishes (5), and Theorem 2.

3.3. Instantaneous Relative Mobility. Summarizing, we see that our measure has both absolute and relative features, the former embodied in Growth Alignment and the latter in Growth Progressivity. Our preference is to retain both these aspects. But we might also want to "net out" aggregate growth and view what remains as a purely relative measure of mobility. We call this the *relative upward mobility kernel*, and we can define it by

$$R_\alpha(\mathbf{z}) = \frac{\sum_{i=1}^n y_i^{-\alpha} g_i}{\sum_{i=1}^n y_i^{-\alpha}} - g = \frac{\sum_{i=1}^n y_i^{-\alpha} e_i}{\sum_{i=1}^n y_i^{-\alpha}}, \tag{6}$$

where g is the overall rate of growth and $e_i \equiv g_i - g$ is the individual *excess* growth rate. Noting that $g = (\sum_j y_j g_j)/(\sum_j y_j)$, this can be rewritten as

$$R_\alpha(\mathbf{z}) = \sum_{i=1}^n \phi_i^*(\mathbf{y})g_i, \text{ where } \phi_i^*(\mathbf{y}) = \frac{y_i^{-\alpha}}{\sum_{j=1}^n y_i^{-\alpha}} - \frac{y_i}{\sum_{j=1}^n y_i}. \tag{7}$$

Because $\phi_i^*(\mathbf{y}) > \phi_j^*(\mathbf{y})$ whenever $y_i < y_j$, the relative upward mobility kernel satisfies Growth Progressivity (it satisfies Local Merge and Zero Growth Anchoring as well). It is therefore one of the measures accommodated in the characterization of Theorem 1. But growth is not the variable of central interest in the relative mobility kernel. Rather, it is *excess growth* over and above the overall aggregate growth rate. From an axiomatic perspective, we must change our domain to pairs of the form $(\mathbf{y}, \mathbf{e})$: $\mathbf{y}$ is a vector of baseline incomes, and $\mathbf{e}$ is the vector of excess growth rates $e_i = g_i - g$. Axioms 2–5 apply without any change, in the sense that they can be shown to be equivalent across the two domains. But Axiom 6, on Binary Growth Tradeoffs, does change its meaning: the independence condition on binary tradeoffs across g_i and g_j is not the same as the independence condition on binary tradeoffs across e_i and e_j. Finally, because overall growth has no meaning, Axiom 7 is eliminated, while Axiom 1 is replaced by

1′. **Zero Excess Growth Anchoring.** If in two situations $\mathbf{z}$ and $\mathbf{z}'$, *every* individual has the same growth rate, then $M(\mathbf{z}) = M(\mathbf{z}')$; normalize this common value to zero.

These reconfigured axioms fully characterize the relative upward mobility kernel in (6). We omit the proof, which follows that of Theorem 2.

4. Upward Mobility Over Time Intervals

Growth data typically come *across* intervals of time, and not *at* instants of time. The latter generate our mobility kernels, and fully allow Growth Progressivity to be applied. After all, the ranking of two distinct incomes cannot be reversed in an instant, no matter how disparate the growth rates. But over intervals of time, income crossings can and do occur. Individual 1 might initially be poorer than 2, and then richer. Growth progressivity cannot, therefore, be unambiguously applied "in favor of" individual 1: we do not want to ascribe a higher weight to the income growth of individual 1 over the entire interval, just because she was initially poorer.

To address this issue, suppose we could fully observe all income trajectories over some given time interval. We break up the trajectories into smaller sub-trajectories — in the limit, into instants of time — placing more weight on individual 1's growth rate only while she is poorer than 2. Overall upward mobility will then be derived from all the instantaneous measures of upward mobility over the time interval.

Of course, we typically do not get fully observe income trajectories, and we will certainly address this core issue. But to begin with, assume that income trajectories are observed over entire intervals. We will view upward mobility as a functional defined on such trajectories. Specifically, for every starting date s and ending date t, denote a trajectory by $\mathbf{y}(s,t) = \{y_i(s,t)\}$, where i indexes individuals. Upward mobility is then a mapping $\mathbf{y}(s,t) \mapsto \mu(\mathbf{y}(s,t))$. We presume translation invariance in calendar time as a self-evident restriction: for all pairs of trajectories $(\mathbf{y}(s,t), \tilde{\mathbf{y}}(0,t-s))$ with $\tilde{\mathbf{y}}(\tau) = \mathbf{y}(s+\tau)$ for $\tau \in [0, t-s]$,

$$\mu(\mathbf{y}(s,t)) = \mu(\tilde{\mathbf{y}}(0,t-s)). \tag{8}$$

Assume that income paths are strictly positive and continuously differentiable everywhere. Then instantaneous growth rates are well-defined and continuous everywhere (but see Section 6.4). We now impose two conditions on the upward mobility measure.

4.1. Reducibility. As motivation, return to our individuals 1 and 2, with incomes $\{y_1(0), y_2(0)\}$ at date 0 and $\{y_1(T), y_2(T)\}$ at date T. Recall that 1 is initially poorer, $y_1(0) < y_2(0)$, but eventually $y_1(T) > y_2(T)$. See Figure 2a. Growth Progressivity cannot be indiscriminately applied over the entire interval. The higher growth experienced by individual 1 should not necessarily be viewed as conferring greater upward mobility just because 1 was poorer than 2 to start with. After all, 1 is richer than 2 in the second phase of the process.

As mentioned earlier, a natural resolution is to examine the trajectories in "pieces". If 1 is initially poorer, let Growth Progressivity act in favor of 1, but ask that Growth Progressivity

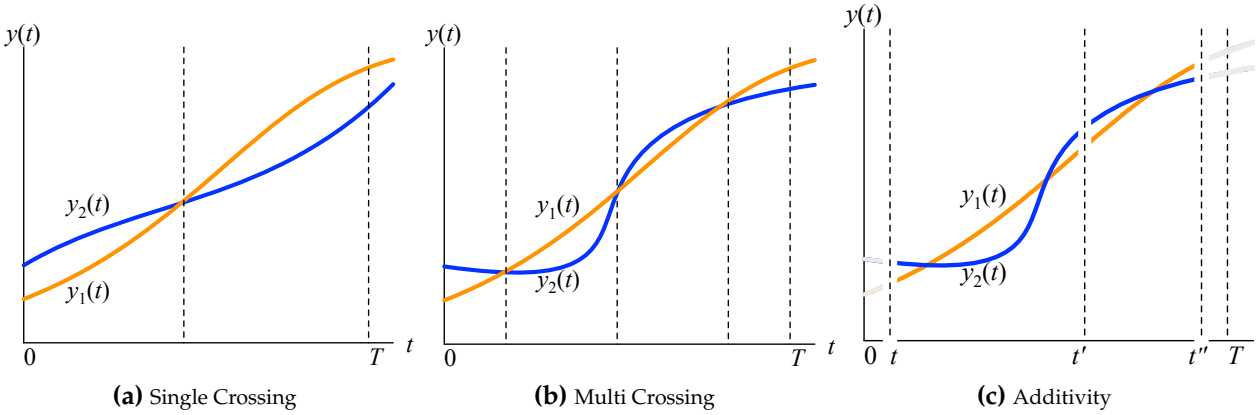


Figure 2. Possible Paths for Discrete Data.

act in favor of 2 once a crossing occurs. (Such switches could occur on multiple occasions, as in Figure 2b.) That leads to the notion of *reducibility*: regard our measure as being fully determined by the upward mobility kernels at every instant of time during the interval. That is, if μ measures upward mobility over $[s, t]$, reducibility asks that μ be expressible as

$$\mu(\mathbf{y}(s,t)) = \Psi(\{M(\mathbf{z}(\tau))\}_s^t) \tag{9}$$

for some pointwise, nondecreasing "aggregator" Ψ, where M is our instantaneous mobility kernel, and $\{\mathbf{z}(\tau)\}$ is the collection of income and growth rate pairs induced by the right-hand derivatives of $\mathbf{y}(s,t)$. We normalize Ψ by asking that if instantaneous upward mobility is constant at any $m \in \mathbb{R}$ over the entire time interval, then upward mobility over that interval also equals m.

4.2. Additivity. Our second condition concerns the time separability of our upward mobility measure. Divide a time interval into two subintervals of equal lengths, and suppose that measured mobility is M_1 and M_2 in each interval. We would then like to say that overall mobility across the entire interval is just $(M_1 + M_2)/2$, just as we would average logarithmic growth. Adjusting in the obvious way for intervals of unequal size — as in Figure 2(c), for instance — say that an upward mobility measure is *additive* if for every collection of income trajectories of the form $\mathbf{y}(s,t)$ and every intermediate date $u \in (s,t)$,

$$(u-s)\mu(\mathbf{y}(s,u)) + (t-u)\mu(\mathbf{y}(u,t)) = (t-s)\mu(\mathbf{y}(s,t)). \tag{10}$$

Observe that additivity only makes sense when the trajectories are differentiable, so that mobility kernels are defined at every moment in time. For instance, a piecewise-flat trajectory with jumps will not satisfy additivity, unless those instantaneous jumps are fully accounted for. In Section 6.4 we show how to accommodate those discontinuities.

It should be noted that additivity, or reducibility, or indeed the joint imposition of the two, are compatible with a variety of mobility measures, including those that simply describe "pure movement." But in conjunction with our earlier axioms for instantaneous upward mobility, the allowable class of measures has a particularly narrow form.

4.3. Discrete Upward Mobility.

Theorem 3. *Axioms 1–6, reducibility (9), and additivity (10) hold if and only if over any collection* $\mathbf{y}(s,t)$ *of continuous and right-differentiable income trajectories on* $[s,t]$, $\mu(\mathbf{y}(s,t))$ *has a representation of the form:*

$$\mu_\alpha(\mathbf{y}(s,t)) = \frac{1}{t-s}\ln\left[\frac{\sum_{i=1}^n y_i^{-\alpha}(t)}{\sum_{i=1}^n y_i^{-\alpha}(s)}\right]^{-\frac{1}{\alpha}} \text{ for some } \alpha > 0. \tag{11}$$

We mention the main lines of the argument here. It is not hard to see that (11) satisfies all the conditions of the Theorem, so we focus on "only if." By Theorem 2 — and therefore Axioms 1–6 — the upward mobility kernel at any date is $M_\alpha(\mathbf{z}) = \sum_{i=1}^n \left[y_i^{-\alpha}/\sum_{j=1}^n y_j^{-\alpha}\right] g_i$ for some $\alpha > 0$, where α is independent of calendar time by the translation invariance condition (8). Therefore, by reducibility,

$$\mu_\alpha(\mathbf{y}(s,t)) = \Psi\left(\left\{\sum_{i=1}^n \frac{y_i(\tau)^{-\alpha} g_i(\tau)}{\sum_{i=1}^n y_i(\tau)^{-\alpha}}\right\}_s^t\right), \tag{12}$$

and by the additivity of μ, (12) further simplifies to

$$\mu_\alpha(\mathbf{y}(s,t)) = \int_s^t h\left(\sum_{i=1}^n \frac{y_i(\tau)^{-\alpha} g_i(\tau)}{\sum_{i=1}^n y_i^{-\alpha}}\right) d\tau \tag{13}$$

for some function h. (See Steps 1 and 2 of the formal proof.) The normalization on the aggregator Ψ tells us that $\mu(\mathbf{y}(s,t)) = m$ if the upward mobility kernel is constant at m over $[s,t]$. Applying this restriction to (13), we must conclude that for every such $m \in R$,

$$(t-s)h(m) = m, \tag{14}$$

and substituting (14) into (13), and integrating (see Step 3 of the formal proof for details), we obtain (11), which is the form in which we wish to take our mobility measure to the data. Notice that (11) divides by the normalization term $t-s$, and so picks out "average mobility" over the period, expressible as, say, an annual percentage rate.

Theorem 3 also applies to the relative mobility measure introduced in Section 3.3. Impose reducibility just as we did for absolute mobility, but using relative mobility kernels. Then Theorem 3 asks us to integrate the kernel described in Section 3.3 over a income trajectory to generate a corresponding *discrete* relative mobility measure; call it ρ. That measure

is independent of the particular trajectory, for the same reason that the discrete upward mobility measure and the overall growth rate both are. Letting $\bar{y}$ denote per-capita income, we have:

$$\rho_\alpha(\mathbf{y}(s,t)) = \mu_\alpha(\mathbf{y}(s,t)) - \frac{1}{t-s}\Big[\ln(\bar{y}(t)) - \ln(\bar{y}(s))\Big] = \frac{1}{t-s}\left\{\ln\left[\sum_i \left(\frac{y_i(t)}{\bar{y}(t)}\right)^{-\alpha}\right]^{-\frac{1}{\alpha}} - \ln\left[\sum_i \left(\frac{y_i(s)}{\bar{y}(s)}\right)^{-\alpha}\right]^{-\frac{1}{\alpha}}\right\}. \quad (15)$$

5. Panel Independence

Theorem 3 has the implication that our derived mobility measure does not need panel data for its implementation. Even though both reducibility and additivity rely conceptually on the full observability of income trajectories, that observability is discarded in the sequel. Equation (11) makes it clear that only information on starting and terminal incomes is needed for our upward mobility computation. That is, while i is an index that sums weighted incomes in both the numerator and denominator of the expression in (11), there is no presumption that i stands for the same person at the beginning and the end of the interval. Because the identity connection between starting and terminal incomes is thereby broken, we see that reducibility already begins to de-emphasize the need for panel data. Additivity and our derived linearity of the upward mobility kernels do the rest.[5]

Panel data with good-quality income data is a scarce commodity in many countries and sometimes requires access to proprietary data (Chetty et al. 2017 and Acciari et al. forthcoming),[6] so as to estimate a transition probability matrix over incomes. The panel-independence property questions the very need for estimating that matrix.

Panel-independence might appear counterintuitive. After all, one could respond that mobility is essentially a dynamic construct for dynasties or lineages. So those dynamics should be reflected in any measure of it. We agree with the first statement, but not the second. It is certainly true that to assess the fortunes of a family over time, *that* family must be tracked. (For instance, from the perspective of the "mobility of an individual dynasty," it does indeed matter whether the trajectories in question are given by Figure 2a or by Figure 2b.) But to assess upward mobility overall, it is not an individual family that the researcher is after, but the contributions of all families to upward mobility at every point of time. A family is given more social weight at any instant that it is poorer than another, but those weights are reversed as soon as their rankings are reversed.

[5]So Growth Progressivity along with the other conditions is also instrumental in precipitating panel-independence, because it implies the linearity of instantaneous upward mobility in growth rates.

[6]Chetty et al. (2017) use their transition matrix to estimate the fraction of children who fare better than their parents. This leads to a measure that we will discuss in Sections 6.7 and 7.

One qualification to this argument is that the choice of whatever it is that we're measuring the mobility *of* should be settled at the outset. We can only say that there is no justification for history-dependence in a variable if that variable is a reasonable sufficient statistic for a family's overall economic position at the present time. The choice of variable — wealth, income or permanent income — should therefore be made with care, subject to data limitations of course. This cautionary note is common to other measurements — of GNP, inequality, poverty. It is expected that the researcher will be reasonably comfortable with her chosen variable before applying the measure to it.

Additionally, beyond the question of choosing a proper individual- or household-level variable, an individual's current socioeconomic position might also be driven by stigma or status for some identifiable *social group* to which that individual belongs. We return to this theme in Section 6.6.

6. Discussion

6.1. Upward Mobility and Growth. Our measure connects upward mobility to *pro-poor growth* (Chenery, Jolly, Ahluwalia, Bell and Duloy 1974, Dardanoni 1993, Ravallion and Chen 2003, Essama-Nssah 2005, Jenkins and Van Kerm 2006, 2011, Palmisano and de Gaer 2016 and Berman and Bourguignon 2022). Theorem 2 declares that the weights on different incomes must be powers of the inverses of those incomes. For instance, $\alpha = 0.5$ doubles the weight on someone earning \$40,000 relative to someone earning \$160,000. If $\alpha \simeq 0$ then our upward mobility measure (1) converges to the sum of the log growth rates of individual income, as in Fields and Ok (1999b),[7] and as $\alpha \to \infty$, it becomes Rawlsian.

Using (20), the distinction between upward mobility and aggregate growth becomes very sharp. The former tolerates a sacrifice of aggregate growth provided the relatively poor grow faster. The logarithm of aggregate growth over the same period is given by

$$\text{Log Growth} = \mu_{-1}(\mathbf{y}(s,t)) = \frac{1}{t-s} \ln \left[\frac{\sum_{j=1}^{m} n_j(t) y_j(t)}{\sum_{j=1}^{m} n_j(s) y_j} \right], \tag{16}$$

which is formally in the class of our measures as shown, but is emphatically excluded because the implicit value of α under the growth measure is -1. In other words, (16) is "fully" separated from our class of measures, and does not even sit on the boundary of that class as $\alpha \to 0$. Nevertheless, this exercise shows that our measure can be viewed as a "growth rate equivalent." Indeed, when all growth rates are the same, our measure *is* the (logarithm of) growth. Otherwise it corrects for the progressivity of that growth.

[7] This can be seen by applying L'Hospital's Rule to $-\ln\left(\frac{\sum_i y_i(t)^{-\alpha}}{\sum_i y_i(s)^{-\alpha}}\right)/\alpha(t-s)$, as $\alpha \to 0$.

6.2. Upward Mobility and Inequality. Upward mobility rewards greater equalization of "terminal" incomes, but only in the equalization implicit in the *change* of incomes, as a consequence of rewarding differential growth for the relatively poor. That comes from Growth Progressivity. Therefore upward mobility is not a measure of equality. For instance, (11) is insensitive to inequality in baseline incomes. As already noted, if all incomes grow at the same rate, our measure returns the same answer irrespective of the initial distribution of income. It also values growth all around relative to zero-growth situations, even if that growth is disequalizing.

6.3. Upward Mobility as Change in Welfare. Consider the Atkinson welfare function given by

$$a_\alpha(\mathbf{y}) = \left(\frac{1}{n}\sum_i y_i^{-\alpha}\right)^{-\frac{1}{\alpha}}, \tag{17}$$

but where we restrict α to be strictly positive. We can think of $a_\alpha(\mathbf{y})$ as the *Atkinson equivalent income* of an income vector $\mathbf{y}$, when the welfare (or curvature) parameter is α. It is then trivial to see that our instantaneous mobility kernel is precisely the instantaneous rate of of growth of Atkinson equivalent income; that is,

$$M_\alpha(\mathbf{z}) = \sum_{i=1}^{n} \frac{\partial a_\alpha(\mathbf{y})}{\partial y_i}\frac{1}{y_i(t)}\frac{dy_i(t)}{dt} \tag{18}$$

along any differentiable trajectory of incomes. And if we then turn to the discrete measure, we have the expected implication that for any $s < t$ and differentiable trajectory $\mathbf{y}(s,t)$,

$$\ln a_\alpha(\mathbf{y}(t)) - \ln a_\alpha(\mathbf{y}(s)) = (t-s)\mu(\mathbf{y}(s,t)). \tag{19}$$

Equation (19) has the interpretation that upward mobility over the period s to t can be viewed as the "average percentage change" in Atkinsonian welfare (or Atkinson equivalent income) over that period.

A similar interpretation applies to the relative mobility index. The expression in (15) can be rewritten as

$$\rho_\alpha(\mathbf{y}(s,t)) = \frac{1}{t-s}\Big[\ln\left(W_\alpha(\mathbf{y}(t)\right) - \ln\left(W_\alpha(\mathbf{y}(s)G)\right)\Big],$$

where $G = \bar{y}(t)/\bar{y}(s)$ is the overall growth factor. That is, ρ_α can be seen as a net adjustment in welfare experienced in from going from the initial distribution to final distribution, relative to going from the initial distribution to a hypothetical distribution $\mathbf{y}(s)G$, obtained by giving everyone the average growth rate experienced by the economy. This view of relative mobility as a net change in social welfare over and above balanced growth underlies the ethical measures of mobility of Chakravarty et al. (1985), though in our case it has emerged endogenously from more primitive axioms.

It is worth noting, however, that our interpretation of mobility as change in Atkinson equivalent income is *restricted* to coefficients of inequality aversion that exceed one, or to welfare curvatures exceeding that of the logarithmic function. This is an implication of growth progressivity, which imposes a strong preference for equalization. So only a subclass of the Atkinson family is relevant for our analysis.

6.4. Trajectories With Jumps. The trajectories that connect initial and final income vectors were assumed to be continuously differentiable in time. This is done so that growth rates are everywhere well-defined (and continuous) so that we can apply our instantaneous measure. But discontinuous events might well occur, such as an inheritance, a sudden loss of job, or a promotion. If incomes are stationary except at these crucial events, instantaneous mobility would be zero at every date except at the isolated jumps. But of course, overall mobility is not zero.

It is, however, not difficult to modify the analysis so that they apply to paths with simple jump discontinuities at finitely many dates. Every such continuous trajectory, differentiable or not, can be approximated by continuously differentiable trajectories, and they all generate the same answer as in (11). The reason is that (11) is independent of intermediate trajectories as long as they are continuously differentiable.

6.5. Population Shares. Income-growth observations could be repeated, so there is no need for population weighting in (11). But this presumes that the population is constant. Typically, data on income- or income-specific growth rates are provided by categories (say m quantiles), and are available as $\{y_i(\tau), n_i(\tau)\}_{i=1}^m$, where $n_i(\tau)$ is the population share in quantile i at date τ. To incorporate the data in this form, with varying populations, we need to take a stand on what happens when a new individual is added to or removed from the set of observations.

To do this, we need an obvious population neutrality principle, analogous to income neutrality (Axiom 4): when populations at any instant are replicated by some positive integer, keeping their distributions over (y, g) unchanged, the mobility kernel is unchanged.

Now proceed as follows. With finite populations, changes will happen at discrete instants in time. Suppose that between s and t there are M consecutive time intervals $I_1, \ldots I_M$ such that in interval m, the population is constant at n_m. Define n^* to be the lowest common multiple of $(n_1, \ldots, n_m)$, and scale population in each interval I_m from n_m to n^*. Mobility kernels are unchanged at every instant that they are defined, by population neutrality. Moreover, population will now remain stationary at n^* over the entire interval.

All that remains is to connect the trajectories so that each of the n^* individuals have incomes that are fully defined on $[s, t]$. Panel independence ensures that it won't matter how we connect the different trajectories. Of course, any connection rule will generally entail jump discontinuities in the trajectories, but these can be taken care of exactly as we did in Section 6.4. With this procedure in hand, and explicitly keeping track of identical observations at each income our mobility measure becomes:

$$\mu_\alpha(\mathbf{y}(s,t)) = \frac{1}{t-s} \ln \left[\frac{\sum_{j=1}^{m_t} n_j(t) y_j^{-\alpha}(t)}{\sum_{j=1}^{m_s} n_j(s) y_j^{-\alpha}(s)} \right]^{-\frac{1}{\alpha}}, \tag{20}$$

where n_j is the share of the population earning income y_j. Note that the number of distinct incomes at each date do not have to be the same; it is possible that $m_s \neq m_t$ in (20).

6.6. Mobility and Social Groupings. We return to a discussion initiated in Section 5. Suppose that we do have an individual- or household-level variable that serves as a good proxy for that individual or household's overall economic standing. That does not take care of other social variables that might confer additional status or stigma. The fact that person B is currently richer than person A might not detract from the reality that B belongs to an under-served social group, perhaps demarcated by ethnicity, race, gender or religion. If such groupings are salient, our measures of upward mobility in (11) and (15) may need to incorporate this fact. We indicate one such approach here.

Suppose that there are K social groups in society. Each person belongs to one such group. Returning temporarily to the case of instantaneous mobility, the data in hand is now a collection $\{\mathbf{z}_k\}_{k \in K}$, where for each k, $\mathbf{z}_k = (z_i)_{i \in k}$ with $z_i = (y_i, g_i)$ as before for every $i \in k$. Recall from Section 6.3 that instantaneous mobility within a (homogeneous) group can be thought of as the growth of Atkinson's equivalent income within that group; see equation (**??**). For an individual, that measure reduces precisely to the instantaneous growth rate of her income. So we could proceed as follows: consider a group of individuals, each indexed by their baseline Atkinson equivalent income, and transfer population-weighted growth points (of the growth of Atkinson income) from relatively rich to relatively poor, again measured by Atkinson income. Then mobility should go up for society as a whole.

This is certainly overkill when proceeding from individual to group: nothing is gained by replacing income with Atkinson income for an individual! But it suggests a way to recursively proceed from individual to group, and then onwards from group to society. For each k, calculate mobility $M_\alpha(\mathbf{z}_k)$. Now re-apply the mobility axioms to groups as units, with Growth Progressivity applying to population-weighted "mobility point transfers" from relatively rich to relatively poor groups, as measured by group-level Atkinson

equivalent income. We then obtain the following extension of our baseline measure:

$$M_{\alpha\beta}(\mathbf{z}) = \sum_k \frac{n_k a_k^{-\beta}}{\sum_q n_q a_q^{-\beta}} M_\alpha(\mathbf{z}_k) = \sum_k \frac{n_k a_k^{-\beta}}{\sum_q n_q a_q^{-\beta}} \left[\frac{\sum_{i=1}^{n_k} y_i^{-\alpha} g_i}{\sum_{j=1}^{n_k} y_j^{-\alpha}} \right]$$
$$= \sum_k \frac{n_k^{(\alpha-\beta)/\alpha} \left(\sum_{i=1}^{n_k} y_i^{-\alpha} \right)^{\beta/\alpha}}{\sum_q n_q^{(\alpha-\beta)/\alpha} \left(\sum_{i=1}^{n_q} y_i^{-\alpha} \right)^{\beta/\alpha}} \left[\frac{\sum_{i=1}^{n_k} y_i^{-\alpha} g_i}{\sum_{i=1}^{n_k} y_i^{-\alpha}} \right], \tag{21}$$

where $\alpha > 0$ and $\beta > 0$ are (possibly distinct) indices of progressivity, the former at the level of the individual, and the latter at the level of social groupings. Observe that if $\alpha = \beta$, then the distinction disappears and the measure reduces to our standard mobility measure.

6.7. Our Axioms and Alternative Measures. We illustrate our central axiom by using it to evaluate some alternative mobility measures. It will be easiest to do this by going back to the instantaneous upward mobility kernel. For our first comparison, it will be useful to consider an extended family of measures, which includes our measures described in (5):

$$M_\alpha^h(\mathbf{z}) = \frac{\sum_{i=1}^n y_i^{-\alpha} h(g_i)}{\sum_{i=1}^n y_i^{-\alpha}} \tag{22}$$

for some possibly non-linear h. A special case can be found by setting $\alpha = 0$ and h equal to the indicator function $I(g) = 0$ for $g < 0$, and $I(g) = 1$ for $g \geq 0$. For this setting, the general measure in (22) is easily seen to reduce to

$$M_0^I(\mathbf{z}) = \text{Population share under } \mathbf{z} \text{ for whom the future improves on the present.}$$

This measure is used in Chetty et al. (2017) and Berman (2021), and we've already encountered it in the context of our discussion on panels. Our points here are different. By nesting the measure within (22), which also includes our characterized family, we uncover two essential contrasts with our measure. First, the growth experiences of the poor are treated on par with those of the rich: M_0^I counts only the unweighted share of those families whose absolute fortunes improved ($\alpha = 0$). Second, h is a step function, while in our case it is the identity function. Both these differences can be traced back to a failure of M_0^I to satisfy the Growth Progressivity axiom.[8] The former difference is easily fixed by injecting some weighting to create a modified measure in which poorer families that improve receive larger weight than richer families that also improve. This modified measure M_α^I could, of course, approximate M_0^I as closely as we please. But the second difference cannot be easily fixed. To see this, consider the following example.

[8]Strictly speaking, Growth Alignment is also not satisfied, as the measure only seeks to know if future prospects improved or deteriorated without asking by how much, but this is a minor issue which can easily be rectified by any increasing approximation h of I.

Suppose that there are two income or lifetime income groups of equal size, at levels \$19,000 and \$20,000. This is a decaying society: assume that their children have \$18,700 and \$19,700 respectively. Call this situation $\mathbf{z}$; then $M_0^I(\mathbf{z}) = 0$; no one earns more than their parents. Now suppose that we alter the situation so that the children of the first group have still lower income; say \$18.000. Give that extra loss to the children of the second group, so that they now earn \$20,400. Call this situation $\mathbf{z}'$. We believe it would be hard to argue that upward mobility is higher under $\mathbf{z}'$ compared to $\mathbf{z}$, but $M_0^I(\mathbf{z}') = 0.5 > M_0^I(\mathbf{z})$.

Or consider a growing society with two equally-sized groups at incomes \$10,000 and \$20,000, and with growth rates equal to 1% for each group. Then we see that $M_0^I(10000, 20000, 1, 1) = 1$: all individuals improve. If we transfer 2 percentage points of growth from the rich group to the poor, Growth Progressivity states that upward mobility must go up. But upward mobility as measured by M_0^I declines: $M_0^I(10000, 20000, 1, 1) = 1 > M_0^I(10000, 20000, 3, -1) = 0.5$. This sort of example can be constructed for *any* nonlinear h function. Indeed, that is why the linearity of mobility in individual growth rates is implied by Theorem 1.

To this one might respond that the fault lies in our axioms and not the measure M_0^I. We disagree. The new situation in the second example has poorer families actually *catching up* with their richer counterparts. Upward mobility rewards — and in our opinion *should* reward — this narrowing of inequalities.[9] It is the fact that M_0^I actually falls instead that is problematic. In this case it comes from a psychological anchor built into the zero-improvement threshold. Cross that threshold, and policymakers are presumably delighted. Fail to cross it, and they are not. This knife-edge preoccupation with the zero threshold is, we feel, not warranted, especially in a world where granular data is increasingly available, and indeed, already available to some of the authors who have used M_0^I.[10]

Fields and Ok (1999b) also provide an axiomatic derivation for a mobility measure that (a) rewards growth and (b) is sensitive to inequality. Without going into detail about the setting or the axioms, we simply record the measure that they obtain:[11]

$$M^{FO}(\mathbf{z}) = \frac{1}{n}\sum_{i=1}^{n} \ln(1 + g_i) = \frac{1}{n}\sum_{i=1}^{n} \left[\ln(w_i') - \ln(y_i)\right],$$

[9] This parallel between pro-poor growth and convergence has been emphasized by O'Neill and Kerm (2008), Wodon and Yitzhaki (2005), Bourguignon (2011) and Dhongde and Silber (2016), among others.

[10] Our points are echoed in a different context in the critique of the head count measure of poverty, in which a disequalizing transfer from the poor to the less poor could result in a fall in the head count, which is an unsatisfactory property. See Sen (1976) and Foster, Greer and Thorbecke (1984).

[11] They call this a "directional measure" to emphasize that "upward" changes in income are preferred to downward changes; as already discussed, their other measures do not have this property.

where y_i and w_i' are initial and final incomes over two periods. While Fields and Ok use different axioms in a different setting, we can use our axioms to evaluate this measure. It rises with higher growth rates for all individuals, so that Growth Alignment is satisfied. Income Neutrality and Binary Growth Tradeoffs are easily seen to be satisfied as well. We must conclude, therefore, that their measure cannot satisfy Growth Progressivity.

The following example shows this directly. Suppose that there are just two individuals, with incomes 100 and 200, and suppose that both grow at 10%. Compare this with another situation in which the poor person grows at 15%, while the rich person grows at 5%. Growth is now pro-poor, and our mobility measure goes up. The measure M^{FO}, however, comes *down*. That is not to say that the measure we propose is necessarily "better" — though our support for Growth Progressivity suggests that we believe it is — but to observe that the researcher has an explicit axiomatic (and therefore intuitive) basis on which to compare the measures.

7. Upward Mobility in the Data

A central feature of our measure, discussed in Section 4.3, is that it does not rely on panel data for its implementation. In this section, we apply our measure of upward mobility to the United States, Brazil, India and France using repeated cross-sectional data from the World Inequality Lab (World Inequality Database 2021, see Appendix .1 for more on the data). The exercise that follows demonstrates the applicability of our measure, and also contributes to a growing literature comparing upward mobility across regions (among others Ayala and Sastre 2002, Fields and Ok 1999a, Jenkins and Van Kerm 2011, Chetty et al. 2014a) with the added advantage that we are using a measure of mobility with explicit conceptual foundations.

7.1. An Initial Comparison with Existing Empirical Studies. Perhaps the most popular measure of directional mobility (deployed empirically) is the share of families whose absolute fortune has improved across generations.[12] In Section 6.7, we discussed how our measure of upward mobility differs from this "absolute mobility" measure. In this section, we are interested in comparing how these measures behave empirically. As pointed out by Deutscher and Mazumder (2020), in practice the trends exhibited by various mobility measures do tend to be similar, differing mainly depending on whether they are directional or not. This is quite apart from the conceptual considerations highlighted in this paper.

[12]This measure is more often used in the context of parents and children to measure inter-generational mobility but a similar measure can also be used to measure absolute intra-generational mobility over time.

In well-known work, Chetty et al. (2017) estimated this absolute mobility measure — the fraction of children who earn more than their parents — for US birth cohorts from 1940 to 1984 and documented its decline. They combined the estimated transition matrix of the parent-child income distribution from a unique panel of tax records (Chetty et al. 2014b), with estimates of the marginal income distributions by generation using the CPS and decennial Census data. Chetty et al. (2017)'s estimates of absolute mobility are plotted in both panels of Figure 3.

Their exercise exploits the panel structure of the data, of course. Yet Berman (2021) shows that *in practice*, these estimates of absolute mobility depend largely on the *marginal* income distributions, and relatively little on the estimated transition matrix. From this finding, he concludes that it is possible to approximate Chetty et al. (2017)'s measure of absolute mobility on non-panel data by using available transition matrices estimated for other countries or periods. The World Inequality Database 2021 provides yearly percentile distributions of income for the adult US population:

$$\mathbf{y}^c(\tau) \equiv \{y_1^c(\tau), y_2^c(\tau), ...y_{100}^c(\tau)\} \text{ for } c = \text{US and year } \tau \in [1940, 1984].$$

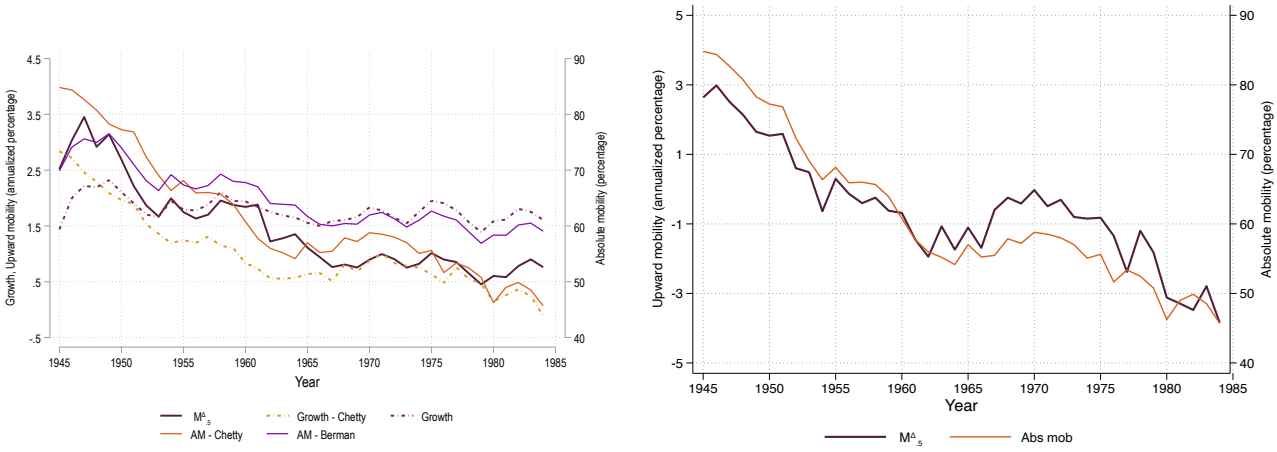
Using these, Berman (2021) estimates the mean and variance of each marginal distribution which, under a log-normality restriction, suffices to characterize the entire marginals at 30-year intervals with starting year ranging from 1945 to 1985. Applying his empirical approximation, he then obtains estimates of absolute income mobility. Figure 3(a) plots our replication of these estimates using Berman's approach.[13]

Next, using the same data, we calculate our measure of upward mobility $M_\alpha^\Delta(\mathbf{y}(t), \mathbf{y}(t+30))$, *which needs no approximation for non-panels*, over the same 30-year intervals for $\alpha = 0.5$ and add it to Figure 3(a). And finally, Figure 3(a) also contains estimates of annual growth (measured as the difference in natural logarithm of the per capita income[14]) averaged over thirty-year intervals: one series from the dataset in Chetty et al. (2017) and the other from World Inequality Database (2021). All estimates of mobility and growth are tagged by their *starting* year.

We see that despite the difference in data and approaches, all three measures capture the overall large decline in mobility in the generations that followed World War II. It appears that the panel-dependent Chetty *et al.* estimates, the (empirically motivated) panel-independent Berman variant, and our (conceptually) panel-free measure move closely with one another. Some differences do arise, and they appear to stem largely from differences in the growth patterns recorded by the two datasets. Figure 3(b) compares our

[13] We thank Yonathan Berman for sharing his code with us. Our estimates and Berman (2021) differ slightly due to updates to the WID database.

[14] That is, annual log growth equals $[\ln(\bar{y}(t+30)) - \ln(\bar{y}(t))]/30$.



(a) WID data

(b) Chetty *et al.* data

Figure 3. Mobility Trends in the United States. This figure displays trends in mobility over thirty-year intervals for the United States, indexed by starting years. Panel (a) builds on percentile income data from World Inequality Database (2021). We use $M^{\Delta}_{0.5}$ for the upward mobility measure. We also display the Chetty et al. (2017) measure and replicate Berman (2021) using the World Inequality Database (2021) data. These values of absolute mobility are recorded on the right vertical axes. Upward mobility and growth rates are displayed on the left vertical axes. Panel (b) displays $M^{\Delta}_{0.5}$ applied to the decile distribution from Chetty et al. (2017) along with the Chetty et al. (2017) measure. Sources: Berman (2021), Chetty et al. (2017) and World Inequality Database (2021).

measure and that of Chetty et al. (2017), using their dataset rather than World Inequality Database 2021.[15] Figure 3b plots upward mobility computed on the decile data and reproduces Chetty et al. (2017)'s absolute mobility as shown in Panel (a). This exercise confirms even more strongly that the two measures are very closely aligned in their ordinal movements.

We used $\alpha = 0.5$ as a benchmark for this exercise,[16] but may be interested in the robustness of our findings to different values of the pro-poor factor. Figure 6 in Appendix .2 shows that very similar patterns of decline in upward mobility are observed for various values of the pro-poorness factor α ranging from 0.1 to 5, as well as $\alpha \simeq 0$, which corresponds to Fields and Ok (1999b). At the same time, increasing α predictably puts more weight on growth at the lowest quantiles. See Appendix .2 for more discussion.

[15] Chetty et al. (2017)'s sample has some negative and zero income entries among the poorest percentiles, while we know that all individuals must receive something. Our measure tend to be sensitive to the imputation assumptions for these low values, especially for higher values of α. Our favorite solution to this issue is to measure upward mobility on decile data. See the appendix for a discussion on the topic.

[16] Choosing a pro-poorness factor is a question of judgment. A pro-poor factor of $\alpha = 0.5$ doubles the weight in the instantaneous mobility measure on someone earning \$40,000 relative to someone earning \$160,000, while a pro-poor factor of $\alpha = 1$ doubles the weight on someone earning \$40,000 relative to someone earning \$80,000.

Finally, notice that these measures do not merely track overall growth — something that will become even more apparent in the next section. There is a good reason for this. Figure 5 in Appendix .2 shows how starting in the early 1950s, the upper income quintile has experienced higher than average 30-year growth while the bottom two quintiles of the distribution have seen their real growth almost vanish. These trends are reflected in mobility.

7.2. Upward Mobility in Brazil, India and France. Encouraged by the comparison above, and certainly given our theoretical remarks on panel data, we now study upward mobility in settings where panel data are not available. This can bring developing countries into focus.

Specifically, we apply our measures to study ten-year upward mobility in Brazil, India and France using decile data from the World Inequality Database (2021).[17] We apply our discrete measure (11) for the benchmark value of $\alpha = 0.5$ to measure upward mobility $M^{\Delta}_{0.5}(\mathbf{y}(t), \mathbf{y}(t+10))$ over ten-year intervals, for each of our three countries and for all t ranging from 1980 to 2010.[18] (Figure 8b in Appendix .3 shows robustness to different values of α.) For this exercise, we also bring on board our measure of relative upward mobility, which nets out growth. Recall that upward mobility is akin to an equivalent growth rate and hence it can take positive or negative values and can be expressed as an annual percentage, just as growth rates are. The relative mobility kernel shows the departure of the measured upward mobility from the per capita income growth (measured as the difference in the natural logarithm of the per capita income).

Figure 4 plots these measures, with — it is fair to say — striking effect.

After the debt crisis of 1980, Brazil entered a long decade of stagnation. In Figure 4, we see that ten-year upward mobility fluctuated between −1% and 1% over the period, but also that upward mobility and growth co-moved closely. The relative mobility kernel is therefore null over the period. Figure 8a in the Appendix confirms that all quintiles experienced the same ten-year growth throughout the 1980s. By the mid-1990s, however, Brazil had been transformed by trade liberalization, a series of privatizations and several pro-business policies. Growth reappeared between 1997 and 2007, but the quintiles diverged significantly in their growth experiences. The second to the fourth quintile did sustain positive growth, but incomes of the lowest quintile essentially decayed for most of these years except for the mid 90s. Finally, income growth in the top quintile significantly surpassed those in the other quintiles between 1999 and 2003. This is mirrored in a

[17] Panel data do exist for France (European Community Household Panel ECHP) and India (India Human Development Survey IHDS), though not for all years.

[18] The first year for which the data are available for all three countries is 1980.

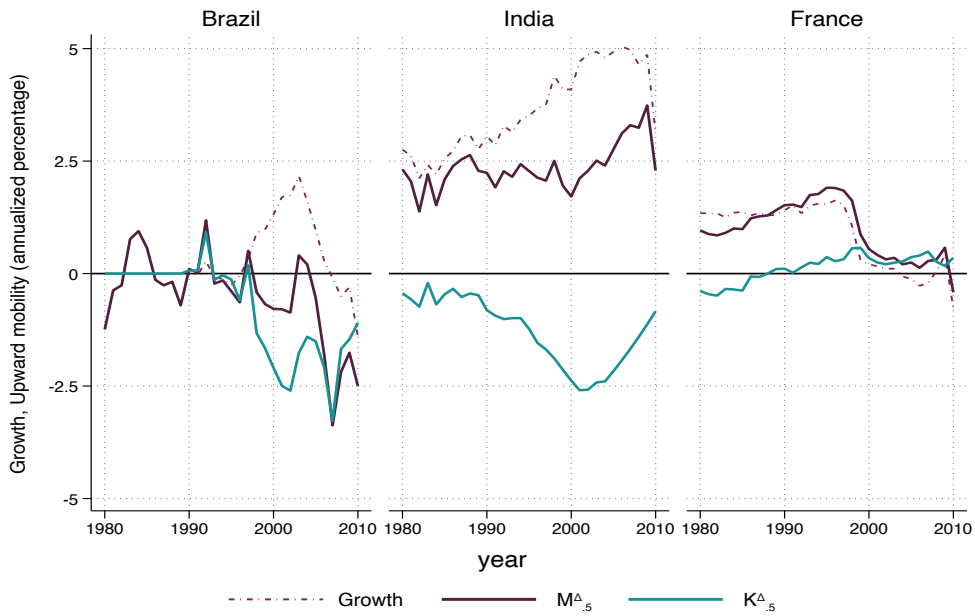


Figure 4. Upward Mobility in Brazil, India and France. These diagrams show growth (approximated by the difference in the natural logarithm of the per capita income), upward mobility and the relative mobility kernel for $\alpha = 0.5$. The numbers capture annualized averages over ten-year intervals, and are indexed by starting years.

dramatic drop-off in upward mobility even as growth rose, with an attendant and even more severe plunge in the relative mobility kernel. Indeed, upward mobility is negative between 1993 and 2003. The implementations of strong social programs in 2003 may have helped to partially reverse the trend then. In 2007, Brazil's growth was negative and upward mobility was at its lowest at -3.35%.

The Indian story is equally dramatic, albeit on a different growth scale. Unlike Brazil, the overall period was one of steady economic growth. Following market deregulation in the early 1990s, India's per-capita growth rate experienced a steady acceleration, from 2.75% to an impressive 5% in 2005. But upward mobility, already short of growth after 1980, increasingly departs from it after 1990. This is reflected in the relative mobility kernel which trends sharply downward into the 2000s (note: our mobility estimates are indexed by *starting* years), though a later recovery is visible. The overall picture is consistent with a post-1990s reform regime that is unambiguously pro-business. Separately, Figure 8a in Appendix .3 makes it abundantly clear that this acceleration of growth is purely concentrated in the top quintile. Our findings are in line with the inequality estimates of Chancel and Piketty (2019), who showed that the share of income of the top 1% rose from 6% in 1980 to over 22% in 2005. In the late 2000s, India suffered from the severe contraction in global trade when the financial meltdown morphed into a worldwide

economic downturn. This shock particularly affected the top quintile (Figure 8a, which explains the upward trend in the relative upward mobility kernel.

Finally, France paints a very different picture. Despite growth stagnating at about 1.5 % until 1997, upward mobility has risen to about 2% in 1995. Figure 8a in Appendix .3 reveals how the growth has been systematically higher among the lowest quintiles. As a result, upward mobility *exceeded* income growth from 1988 onwards and the relative mobility kernel displays positive values. In fact, even though the great recession made resulted in negative growth rates, we see that upward mobility remained positive at around 0.5% between 2000 and 2009, which is in striking contrast to the experiences of India and Brazil.

7.3. Summary. These vignettes are not a substitute for a detailed study of mobility trends. But they serve as proof of concept for a new measure of upward mobility. We have argued that our measure is *conceptually* free from the need for panel data. If this argument is found convincing, it has significant implications for the *empirical* study of mobility in many countries for which panel data are unavailable. Indeed, for many countries in which such data are available, as in the case of India and Brazil, they are present in very limited settings. The current measure therefore greatly expands the scope for analyzing mobility in these societies.

At the same time, we do not rest our case on the theory alone. We have shown that empirically, our measure moves very closely with leading studies in the field which uses a different measure that *does* rely on a panel for its application. This empirical conformity gives us additional confidence that our analysis elsewhere can be viewed as useful.

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Appendix A: Proofs

Proof of Theorem 1. Certainly, (1), along with the restrictions on $\{\phi_i\}$, satisfies Axioms 1–3. We therefore establish the converse.

Step 1. Suppose that $\mathbf{z}^*$ has i and j with $y_i = y_j$. For $\epsilon > 0$, define $\mathbf{z}^{**}$ identical to the old situation except that $g_i' = g_i - \epsilon$ and $g_j' = g_j + \epsilon$. Then $M(\mathbf{z}^*) = M(\mathbf{z}^{**})$.

Proof. For $\delta > 0$ but small, define $\mathbf{z}^*(\delta)$ and $\mathbf{z}^{**}(\delta)$ as follows: each has the same set of incomes and the same growth rates for every individual as $\mathbf{z}^*$ and $\mathbf{z}^{**}$ respectively, except that y_i and y_j are replaced by $y_i - \delta$ and $y_j + \delta$. By Growth Progressivity, $M(\mathbf{z}^*(\delta)) > M(\mathbf{z}^{**}(\delta))$ for every $\delta > 0$. Passing to the limit as $\delta \to 0$ and using the continuity of M, we have $\lim_{\delta\to 0} M(\mathbf{z}^*(\delta)) = M(\mathbf{z}^*)$ and $\lim_{\delta\to 0} M(\mathbf{z}^{**}(\delta)) = M(\mathbf{z}^{**})$, so that

$$M(\mathbf{z}^*) \geq M(\mathbf{z}^{**}). \tag{23}$$

Next, define a new situation $\mathbf{z}'^{**}(\delta)$ which is exactly like $\mathbf{z}^{**}(\delta)$ except that the growth rates are flipped: income $y_i - \delta$ now has the growth rate $g_i + \epsilon$, while income $y_j + \delta$ has the growth rate $g_j - \epsilon$. Applying Growth Progressivity again, we now have $M(\mathbf{z}^*(\delta)) < M(\mathbf{z}'^{**}(\delta))$ for every $\delta > 0$. Passing to the limit as $\delta \to 0$ just as we did before, we must now conclude that

$$M(\mathbf{z}^*) \leq M(\mathbf{z}^{**}). \tag{24}$$

Combining (23) and (24), we obtain Step 1.

Step 2. $M(\mathbf{y}, \mathbf{g})$ is multiaffine in $\mathbf{g}$; i.e., for every k, $M(\mathbf{y}, \mathbf{g}_{-k}, g_k)$ is affine in g_k:

$$M(\mathbf{y}, \mathbf{g}_{-k}, g_k) = A(\mathbf{y}, \mathbf{g}_{-k})g_k + B(\mathbf{y}, \mathbf{g}_{-k}) \tag{25}$$

for two functions A and B.

Proof. Because M is continuous, it is enough to show that for every $\epsilon > 0$,

$$M(\mathbf{y}, \mathbf{g}_{-k}, g_k) = \frac{1}{2}[M(\mathbf{y}, \mathbf{g}_{-k}, g_k - \epsilon) + M(\mathbf{y}, \mathbf{g}_{-k}, g_k + \epsilon)]. \tag{26}$$

Suppose that (26) fails for some g_k and $\epsilon > 0$. Let $\mathbf{z} = (\mathbf{y}, \mathbf{g}_{-k}, g_k)$. Define two situations $\mathbf{z}'$ and $\mathbf{z}''$, both identical to $\mathbf{z}$ for all income-growth pairs other than at y_k, where under $\mathbf{z}'$, $g_k' = g_k - \epsilon$, whereas under $\mathbf{z}''$ it is $g_k'' = g_k + \epsilon$. Because (26) fails, it is easy to see that

$$M(\mathbf{z}'') - M(\mathbf{z}) \neq M(\mathbf{z}) - M(\mathbf{z}'),$$

or that

$$M(\mathbf{z}' \oplus \mathbf{z}'') \neq M(\mathbf{z} \oplus \mathbf{z}) \tag{27}$$

by Local Merge. But that contradicts Step 1 with $\mathbf{z}^* = \mathbf{z} \oplus \mathbf{z}$ and $\mathbf{z}^{**} = \mathbf{z}' \oplus \mathbf{z}''$.

A well-known consequence of multiaffine real-valued functions (see, e.g., Gallier 1999, Chapter 4.5) is that M has the following representation: for every $\mathbf{y} \gg 0$, there is $\phi_S(\mathbf{y})$ for every nonempty $S \subseteq \{1, \ldots, n\}$, such that

$$M(\mathbf{z}) = \sum_S \phi_S(\mathbf{y}) \left[\prod_{j \in S} g_j \right]. \tag{28}$$

(The empty product can be excluded by the Zero Growth Anchoring Axiom.)

Step 3. $\phi_S(\mathbf{y}) = 0$ for any S with $|S| \geq 2$.

Proof. Suppose the assertion is false. Then there are indices i and j and $S \subseteq \{1, \ldots, n\}$ such that $\{ij\} \subset S$ and $\phi_S(\mathbf{y}) \neq 0$. Fix any numbers $\{\bar{g}_k\}$, for $k \neq i, j$, such that

$$\zeta \equiv \sum_{T: i,j \in T} \phi_T(\mathbf{y}) \left[\prod_{k \neq i,j} \bar{g}_k \right] \neq 0. \tag{29}$$

Also define

$$\beta \equiv \sum_{T: i \in T, j \notin T} \phi_T(\mathbf{y}) \left[\prod_{k \in T - i} \bar{g}_k \right], \quad \gamma \equiv \sum_{T: i \notin T, j \in T} \phi_T(\mathbf{y}) \left[\prod_{k \in T - j} \bar{g}_k \right], \text{ and } \delta \equiv \sum_{T: i \notin T, j \notin T} \phi_T(\mathbf{y}) \left[\prod_{k \in T} \bar{g}_k \right], \tag{30}$$

where the numbers are to be interpreted as zero in case any of the above products are empty. For any $G > 0$, consider any growth vector $\mathbf{g}$ such that $g_k = \bar{g}_k$ for all $k \neq i, j$, and such that $g_i + g_j = G$. (We will be placing more restrictions on g_i, g_j and G below.) Then, combining (29) and (30), it is easy to see that

$$M(\mathbf{y}, \mathbf{g}) = \zeta g_i g_j + \beta g_i + \gamma g_j + \delta.$$

Differentiating with respect to g_i and g_j and using $g_j = G - g_i$, we see that

$$\frac{\partial M(\mathbf{y}, \mathbf{g})}{\partial g_i} - \frac{\partial M(\mathbf{y}, \mathbf{g})}{\partial g_j} = \zeta G - 2\zeta g_i + \beta - \gamma.$$

In what follows, recall from (29) that $\zeta \neq 0$. Now we consider the following cases. First, if $y_i = y_j$, we know from Step 1 that the above derivative should be zero, but that clearly cannot hold for arbitrary values of G and g_i, both of which we are absolutely free to choose. Second, if $y_i < y_j$, we know from Growth Progressivity that the above derivative should be positive. Again note that we are free to choose G and g_i. If $\zeta > 0$, choose $G > 0$ and large and g_i smaller than G but close to it; then the above derivative must be negative, a contradiction. Finally, if $\zeta < 0$, again choose $G > 0$ and large, but choose g_i to be small; then the above derivative must be negative, a contradiction. (The case $y_i > y_j$ similarly generates a contradiction if $\zeta \neq 0$.)

It follows that $\zeta = 0$, which means that $\phi_S(\mathbf{y}) \neq 0$ *only* for sets S that are singletons. But that establishes (1). The continuity of each ϕ_i follows from that of M. By anonymity, it must be that $\phi_i(\mathbf{y})g_i = \phi_j(\mathbf{y})g_j$ when $(y_i, g_i) = (y_j, g_j)$, which implies that $\phi_i(\mathbf{y}) = \phi_j(\mathbf{y})$ whenever $y_i = y_j$. And if $y_i < y_j$, Growth Progressivity implies that $\phi_i(\mathbf{y}) > \phi_j(\mathbf{y})$. □

Proof of Theorem 2. The representation (5) clearly satisfies Axioms 1–6. We prove the converse in several steps. (As discussed in the main text, we will not need to invoke Axiom 3 or Local Merge, while Axiom 1 — Zero Growth Anchoring — is obviously implied by the stronger Growth Alignment axiom.)

Step 1. Under $n \geq 3$, Growth Alignment, Binary Growth Tradeoffs, and the anonymity and continuity of M, Theorem 2 in Chatterjee ⓡ Ray ⓡ Sen (2021) applies, and implies that M can be written as

$$M(\mathbf{z}) = f\left(\sum_{i=1}^{n} h(y_i, g_i), \mathbf{y}\right), \tag{31}$$

where h is strictly increasing in g_i for each y_i, and $x \mapsto f(x, \mathbf{y})$ is strictly increasing over x in the range of $\sum_i h(y_i, g_i)$ as we range over all $(\mathbf{y}, \mathbf{g})$.[19]

Step 2. By Growth Alignment, whenever $g_1 = \cdots = g_n = g$, then

$$f\left(\sum_{i=1}^{n} h(y_i, g), \mathbf{y}\right) = \lambda(g) \text{ for every income vector } \mathbf{y} \tag{32}$$

for some increasing, continuous λ, where we can normalize $\lambda(0) = 0$.

Step 3. For any $\mathbf{z} = (\mathbf{y}, \mathbf{g})$, define $g = g(\mathbf{z})$ by the equality

$$\sum_{i=1}^{n} h(y_i, g) \equiv \sum_{i=1}^{n} h(y_i, g_i). \tag{33}$$

Such a g is always well-defined for every $\mathbf{z}$. To see this, observe that h is increasing in its second argument by Step 1, so

$$\sum_{i=1}^{n} h(y_i, \min_j g_j) \leq \sum_{i=1}^{n} h(y_i, g_i) \leq \sum_{i=1}^{n} h(y_i, \max_j g_j).$$

So there is $g \in [\min_j g_j, \max_j g_j]$ such that (33) holds. Because h is strictly increasing in its second argument, this value of g is uniquely pinned down.

Combining (32) in Step 2 with (33) in Step 3, we conclude that for all $\mathbf{z}$,

$$M(\mathbf{z}) = f\left(\sum_{i=1}^{n} h(y_i, g(\mathbf{z})), \mathbf{y}\right) = \lambda(g(\mathbf{z})), \tag{34}$$

[19] Chatterjee ⓡ Ray ⓡ Sen (2021) build on Debreu (1960), Gorman (1968) and Wakker (1988).

Step 4. We claim that $h(w, g)$ is affine in g;[20] i.e., there exist $\psi(w)$ and $\mu(w)$ such that

$$h(w, g) = \psi(w)g + \mu(w). \tag{35}$$

Suppose not; then there is $w > 0$, $g \in \mathbb{R}$ and $\epsilon > 0$ such that

$$h(w, g + \epsilon) - h(w, g) \neq h(w, g) - h(w, g - \epsilon). \tag{36}$$

Consider any pair $\mathbf{z}^*$ and $\mathbf{z}^{**}$ such that $\mathbf{y} = \mathbf{y}'$, and for two indices i and j, $y_i = y'_i = y_j = w'_j = w$, while $g_i = g_j = g$, $g'_i = g_i - \epsilon$, $g'_j = g_j + \epsilon$, and $\mathbf{g}_{-ij} = \mathbf{g}'_{-ij}$. Observe that

$$\sum_{i=1}^{n} h(y_i, g_i) - \sum_{i=1}^{n} h(w'_i, g'_i) = [h(w, g + \epsilon) - h(w, g)] - [h(w, g) - h(w, g - \epsilon)] \neq 0,$$

and so, because f is strictly increasing in its first argument, it must be that

$$M(\mathbf{z}^*) - M(\mathbf{z}^{**}) = f\left(\sum_{i=1}^{n} h(y_i, g_i), \mathbf{y}\right) - f\left(\sum_{i=1}^{n} h(w'_i, g'_i), \mathbf{y}\right) \neq 0. \tag{37}$$

But this contradicts Step 1 in the proof of Theorem 1 (that Step only uses Growth Progressivity).

Step 5. Applying (35) to (33), we obtain

$$\left[\sum_{i=1}^{n} \psi(y_i)\right] \lambda(g(\mathbf{z})) = \sum_{i=1}^{n} \psi(y_i) g_i + \sum_{i=1}^{n} \mu(y_i).$$

Because $h(w, g)$ is strictly increasing in g, we have $\psi(w) > 0$ (and therefore $\sum_{i=1}^{n} \psi(y_i) > 0$ as well). So the above equality and (34) allow us to conclude that

$$M(\mathbf{z}) = \lambda(g(\mathbf{z})) = \frac{\sum_{i=1}^{n} \psi(y_i) g_i}{\sum_{i=1}^{n} \psi(y_i)} + \frac{\sum_{i=1}^{n} \mu(y_i)}{\sum_{i=1}^{n} \psi(y_i)}. \tag{38}$$

By applying (38) to the case in which all growth rates are 0, and applying Growth Alignment, we must conclude that the second term on the right hand side is a constant, independent of $\mathbf{y}$. We can therefore normalize it to 0, and so $M(\mathbf{z})$ can be written as

$$M(\mathbf{z}) = \lambda(g(\mathbf{z})) = \frac{\sum_{i=1}^{n} \psi(y_i) g_i}{\sum_{i=1}^{n} \psi(y_i)}, \tag{39}$$

where by Growth Alignment and Growth Progressivity, $\psi(w)$ is a positive-valued, continuous function decreasing in w.

[20] Unlike in the proof of Theorem 1, Local Merge will not be required here.

Step 6. We claim, finally, that $\psi(w)$ is proportional to $w^{-\alpha}$ for some $\alpha > 0$. To prove this, we first show that for every strictly positive (y_1, y_2, λ),

$$\frac{\psi(y_1)}{\psi(y_2)} = \frac{\psi(\lambda y_1)}{\psi(\lambda y_2)}. \tag{40}$$

Suppose that this is false for some (y_1, y_2, λ). Without loss, suppose that $y_1 < y_2$ and that ">" holds in (40).[21] Pick values g_1, g_1', g_2, g_2' such that $g_1 > g_1'$ and $g_2' > g_2$, and such that

$$\frac{\psi(y_1)}{\psi(y_2)} > \frac{g_2' - g_2}{g_1 - g_1'} > \frac{\psi(\lambda y_1)}{\psi(\lambda y_2)}. \tag{41}$$

Now consider two situations $\mathbf{z}$ and $\mathbf{z}'$. Under $\mathbf{z}$, the values for persons 1 and 2 are (y_1, g_1) and (y_2, g_2), while under $\mathbf{z}'$, the corresponding values are (y_1, g_1') and (y_2, g_2'). Otherwise, the two situations are identical. Manipulating the left inequality in (41), we must conclude that

$$\psi(y_1)g_1 + \psi(y_2)g_2 > \psi(y_1)g_1' + \psi(y_2)g_2',$$

and consequently, that $M(\mathbf{z}) > M(\mathbf{z}')$. Now scale every income in $\mathbf{y}$ and $\mathbf{y}'$ by the common factor λ in (40) and call the new situations $\mathbf{z}_\lambda = (\mathbf{y}_\lambda, \mathbf{g}_\lambda)$ and $\mathbf{z}_\lambda' = (\mathbf{y}_\lambda', \mathbf{g}_\lambda')$. Manipulating the right inequality in (41), we must conclude that

$$\psi(\lambda y_1)g_1 + \psi(\lambda y_2)g_2 < \psi(\lambda y_1)g_1' + \psi(\lambda y_2)g_2',$$

so that now we have $M(\mathbf{z}_\lambda') > M(\mathbf{z}_\lambda)$. But this reversal contradicts Income Neutrality. Therefore (40) must be true.

By defining $w = y_1$, $w' = y_2/y_1$, and $\lambda = 1/y$, we see from (40) that ψ satisfies the fundamental Cauchy equation

$$\psi(w)\psi(w') = \psi(ww')\psi(1) \tag{42}$$

for every $(w, w') \gg 0$. The class of solutions to (42) (that also satisfy continuity and $\psi(w) > 0$ for $w > 0$) must be proportional to $\psi(p) = p^{-\alpha}$ for some constant α (see Aczél 1966, p.41, Theorem 3). Invoking Growth Progressivity, it is obvious that α must be positive.

Combining Step 6 with (39), we obtain

$$M(\mathbf{y}, \mathbf{g}) = \frac{\sum_{i=1}^n y_i^{-\alpha} g_i}{\sum_{i=1}^n y_i^{-\alpha}},$$

thereby completing the proof. □

[21] Asking for $y_1 < y_2$ is without loss. Suppose, however, that "<" holds in (40). Then simply rename λy_1 to w_1', λy_2 to w_2' and set $\lambda' = 1/\lambda$. Then the assertion in the main text holds: $w_1' < y_2'$ and ">" holds in (40).

Proof of Theorem 3. By Theorem 2 and reducibility, we know that (12) holds; that is:

$$\mu_\alpha(\mathbf{y}(s,t)) = \Psi\left(\{M_\alpha(\tau)\}_s^t\right), \tag{43}$$

for every (s,t) with $s < t$, where we recall that the kernel M_α is given by

$$M_\alpha(\tau) = \sum_{i=1}^n \frac{y_i(\tau)^{-\alpha} g_i(\tau)}{\sum_{i=1}^n y_i(\tau)^{-\alpha}},$$

and is everywhere continuous, by our assumptions on $\mathbf{y}(s,t)$. We now proceed in steps.

Step 1. For any $u \geq 0$, the limit expression

$$\lim_{v \to u} \Psi\left(\{M_\alpha(\tau)\}_u^v\right), \text{ is well-defined and equals } M_\alpha(u). \tag{44}$$

Proof. Fixing u and picking any $v > u$ (the case $v < u$ is symmetric), define $\bar{m}(v)$ and $\underline{m}(v)$ to be, respectively, the supremum and infimum of $M_\alpha(\tau)$ for $\tau \in [u,v)$. Let C^v be the function on $[u,v)$ that takes constant value $\bar{m}(v)$, and c^v the function on $[u,v)$ that takes constant value $\underline{m}(v)$. Then, because Ψ is nondecreasing and normalized,

$$\underline{m}(v) = \Psi(\{c^v\}_u^v) \leq \Psi\left(\{M_\alpha(\tau)\}_u^v\right) \leq \Psi(\{C^v\}_u^v) = \bar{m}(v). \tag{45}$$

Because $M_\alpha(\tau)$ is continuous, both $\underline{m}(v)$ and $\bar{m}(v)$ converge to $M_\alpha(u)$ as $v \downarrow u$. Using this information in (45), we must conclude that (44) holds.

Step 2. for any s and t with $s < t$, and every strictly positive and continuously differentiable trajectory $\mathbf{y}(s,t)$,

$$\mu(\mathbf{y}(s,t)) = \frac{1}{t-s}\int_s^t M_\alpha(u)du = \frac{1}{t-s}\int_s^t \sum_{i=1}^n \frac{y_i(u)^{-\alpha} g_i(u)}{\sum_{i=1}^n y_i(u)^{-\alpha}} du. \tag{46}$$

To prove (38), fix $0 \leq s < t$ and some strictly positive and continuously differentiable trajectory $\mathbf{y}(s,t)$. For any $s \leq u < t$, define

$$L(u) \equiv (t-u)\mu_\alpha(\mathbf{y}(u,t)). \tag{47}$$

By additivity, we know that for any $0 \leq u < v < t$, $(v-u)\mu_\alpha(\mathbf{y}(u,v)) + (t-v)\mu_\alpha(\mathbf{y}(v,t)) = (t-u)\mu_\alpha(\mathbf{y}(u,t))$. Equivalently, using (43) and (47),

$$(v-u)\Psi\left(\{M_\alpha(\tau)\}_u^v\right) + L(v) = L(u),$$

so that

$$\frac{L(v)-L(u)}{v-u} = -\Psi\left(\{M_\alpha(\tau)\}_u^v\right). \tag{48}$$

Using Step 1, we must conclude that L is differentiable at u. Using (44) in (48), we have

$$L'(u) = -M_\alpha(u) \text{ for all } s \leq u < t. \tag{49}$$

Integrating the formula in (49) over all u between s and t, we must conclude that

$$(t-s)\Psi\left(\{M_\alpha(\tau)\}_s^t\right) = L(s) = L(t) - \int_s^t L'(u)du = L(t) + \int_s^t M_\alpha(u)du = \int_s^t M_\alpha(u)du,$$

where the very last equality uses the fact that $L(t) = 0$ (use Step 1 and the definition of $L(u)$). Therefore

$$\mu_\alpha(\mathbf{y}(s,t)) = \frac{1}{t-s}\int_s^t M_\alpha(u)du = \frac{1}{t-s}\int_s^t \sum_{i=1}^n \frac{y_i(u)^{-\alpha}g_i(u)}{\sum_{i=1}^n y_i(u)^{-\alpha}}du,$$

which establishes (38).

Step 3. To finally establish (11), consider any collection of positive-valued, continuously differentiable income trajectories $\mathbf{y}(s,t)$ that connect $\mathbf{y}(s)$ and $\mathbf{y}(t)$. That generates an implied trajectory of instantaneous growth rates $\mathbf{g}(\tau) \equiv \{g_1(\tau), g_2(\tau), \ldots, g_n(\tau)\}$ for every $\tau \in [s,t]$ with the properties that for every $i = 1, \ldots, n$,

$$\frac{d\ln y_i(\tau)}{d\tau} = g_i(\tau) \text{ and } \int_s^t g_i(\tau)d\tau = \ln y_i(t) - \ln y_i(s).$$

Using all this information in (13) along with (14), we have:

$$\begin{aligned}
\mu_\alpha(\mathbf{y}(s,t)) &= \frac{1}{t-s}\int_s^t \frac{\sum_i y_i(\tau)^{-\alpha}g_i(\tau)}{\sum_i y_i(\tau)^{-\alpha}}d\tau \\
&= \frac{1}{t-s}\int_s^t \left[\frac{\sum_i y_i(s)^{-\alpha}\exp\left\{-\alpha\int_s^\tau g_i(x)dx\right\}g_i(\tau)}{\sum_i y_i(s)^{-\alpha}\exp\left\{-\alpha\int_s^\tau g_i(x)dx\right\}}\right]d\tau \\
&= -\frac{1}{\alpha(t-s)}\left[\ln\left(\sum_i y_i(s)^{-\alpha}\exp\left\{-\alpha\int_s^\tau g_i(x)dx\right\}\right)\right]_{\tau=s}^{\tau=t} \\
&= -\frac{1}{\alpha(t-s)}\ln\left[\frac{\sum_i y_i(s)^{-\alpha}\exp\left\{-\alpha\int_s^t g_i(x)dx\right\}}{\sum_i y_i(s)^{-\alpha}}\right] \\
&= \frac{1}{t-s}\ln\left[\frac{\sum_i y_i(t)^{-\alpha}}{\sum_i y_i(s)^{-\alpha}}\right]^{-\frac{1}{\alpha}},
\end{aligned}$$

which yields (11) as desired. □

Appendix B: Upward Mobility using the WID

.1. Data. We use the World Inequality Database 2021. It combines fiscal, survey and national accounts data. In countries with small informal sectors and high-quality tax microdata, that tax data is the main source. Surveys and imputation methods are used to

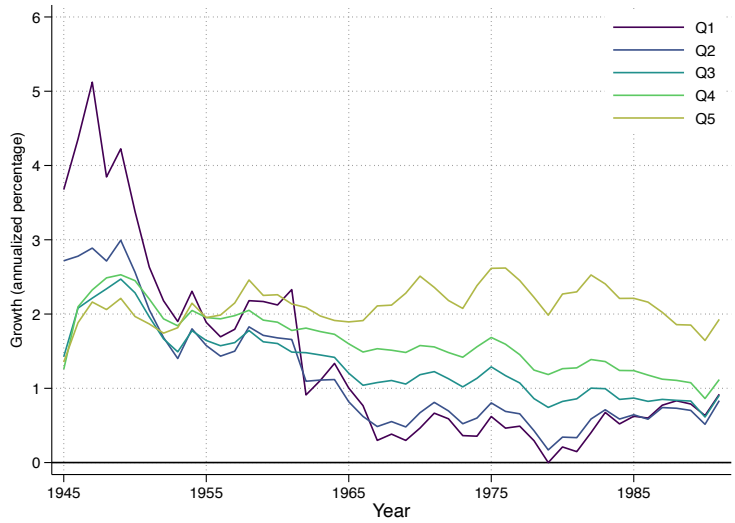


Figure 5. Growth Incidence Curves for the United States. These diagrams show the annualized growth rate of each income quintile for a 30-year interval, and are indexed by starting years.

make minor adjustments in order to account for non-filers and certain tax-exempt incomes. In contrast, income surveys are the main sources for most emerging economies, and tax datasets are only used to correct the top of the income distribution. Income surveys come mainly from the World Bank (via PovcalNet). A detailed description of the methodology is available on the WID website.

The income data are pre-tax total incomes, computed using the equal-split assumption (that is, if the tax unit has more than one income-contributing individual contributing, the assumption is that everyone contributes in equal part to the total income of that tax unit). All incomes are expressed in PPP and in real terms, with a base year of 2021.

.2. *Upward Mobility in the United States: 30 Year Intervals.*

Growth Incidence Curves. The upward mobility measures do not merely track overall growth, and there is a good reason for this. Figure 5 shows how starting in the early 1950s, the upper income quintile has experienced higher than average 30-year growth while the bottom two quintiles of the distribution have seen their real growth almost vanish.

Sensitivity to α. Chetty et al. (2017)'s sample has negative and zero income entries among the poorest percentiles. As observed in Section 6.1, our measure converges to the growth rate of the lowest percentile as $\alpha \to \infty$. Negative or zero values are therefore problematic, especially for α, and our measure could be sensitive to imputation assumptions. We

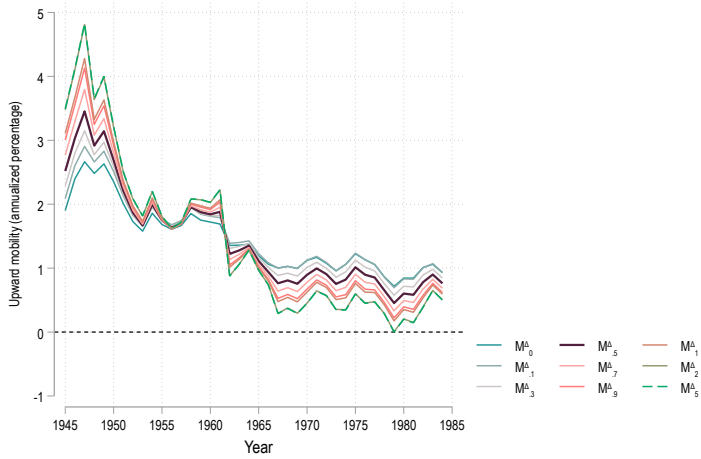


Figure 6. Upward Mobility in the US over 30-year Intervals. This figure displays upward mobility for different values of α, indexed by starting years.

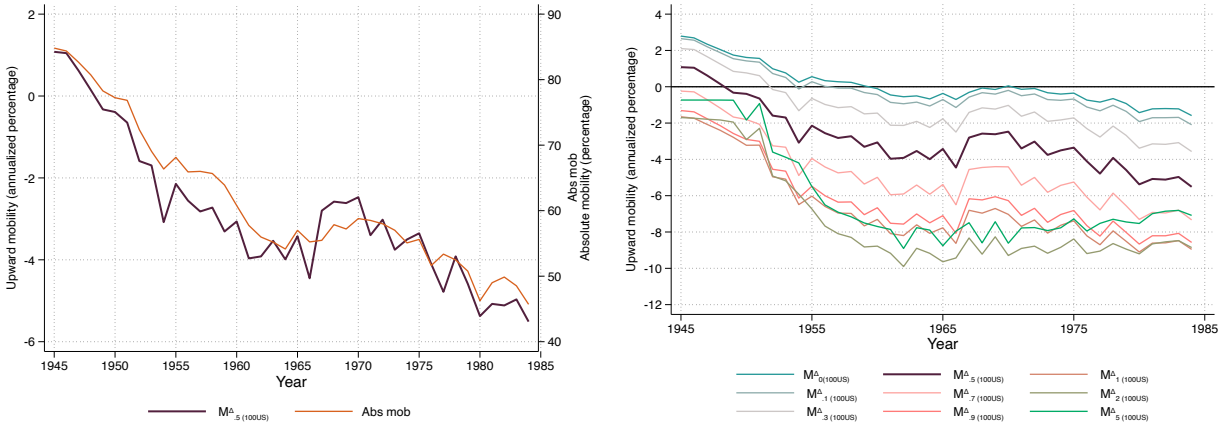
therefore measure upward mobility on the Chetty et al. (2017) data aggregated into deciles. Figure 6 plots 30-year upward mobility in the US for a range of values of α ranging from 0 (Fields and Ok 1999b) to 5. We see that the exact value of α does not affect the pattern to any large degree.

Censoring Low Incomes in Chetty et al. (2017)'s Sample. Aggregating the data into deciles is our preferred approach to deal with low income values. Alternatively, we could "censor" Chetty et al. (2017)'s sample and set all income values to some minimum; e.g., $100. However, the level of upward mobility can be sensitive to these imputations, and we avoid them.

.3. *Brazil, India and France.*

Growth Incidence Curves. Panel (a) of Figure 8 shows the ten-year growth rates by quintile for each country.

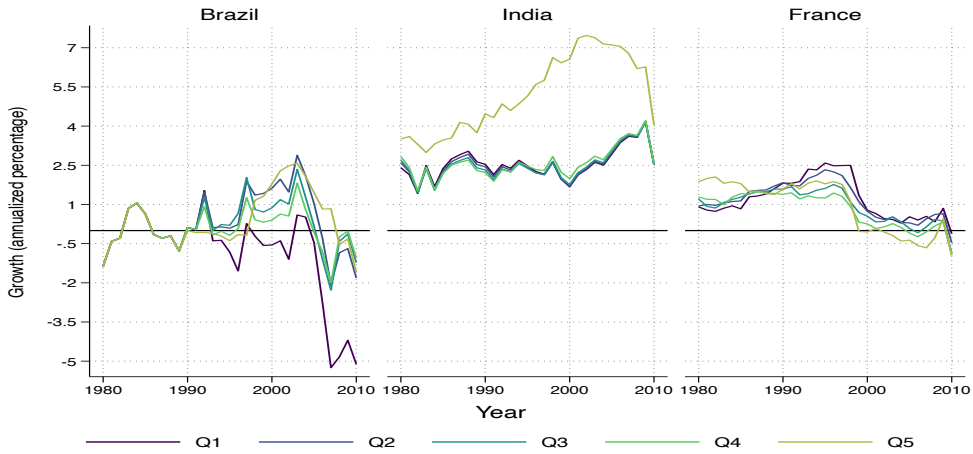
Sensitivity to α As we did for the US data, we explore the robustness to α. Panel (b) of Figure 8 plots 10-year upward mobility in Brazil, India and France for values of α ranging from 0 to 5. We see that the measure is not very sensitive to the exact value of α for India where similar growth was experienced by the bottom four quintiles (see Panel (a)). In Brazil and France, where the growth patterns among the lower quintiles are more differentiated, increasing α affects upward mobility in a predictable manner: it lowers it in Brazil where the bottom quintile fared relatively poorly and increases it in France when the bottom quintiles outperform the others.



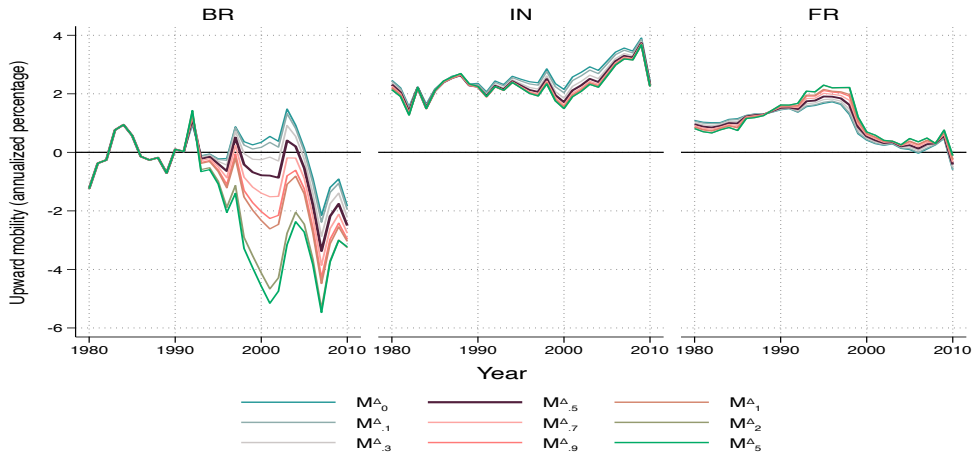
(a) Absolute and Upward Mobility, US

(b) Different values of α

Figure 7. Chetty *et al* data censored at \$100. This figure displays trends in mobility over 30-year intervals for the United States, indexed by starting years. Panel (a) builds on Chetty et al. (2017), censored at \$100 to remove negative and zero incomes, and displays $M^{\Delta}_{0.5}$ along with the Chetty et al. (2017) measure. Panel (b) display upward mobility for a range of values of α.



(a) Growth Incidence



(b) Upward Mobility

Figure 8. Growth Incidence and Upward Mobility over 10-year Intervals, Various α. Panel (a) shows annualized growth rates for each quintile over ten-year intervals, indexed by starting years. Panel (b) displays trends in upward mobility for different values of α, indexed by starting years.