

# Money Creation in Decentralized Finance: A Dynamic Model of Stablecoin and Crypto Shadow Banking\*

Ye Li<sup>†</sup>      Simon Mayer<sup>‡</sup>

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## Abstract

Stablecoins are at the center of debate surrounding decentralized finance. We develop a dynamic model to analyze the instability mechanism of stablecoins, the complex incentives of stablecoin issuers, and regulatory proposals. The model rationalizes a variety of stablecoin management strategies commonly observed in practice and characterizes an instability trap: Stability lasts for long time, but once debasement happens, price volatility persists. Capital requirement improves price stability but fails to eliminate debasement. Restricting the riskiness of reserve assets can surprisingly destabilize price. Finally, data privacy regulation has an unintended benefit of reducing the price volatility of stablecoins issued by data-driven platforms.

**Keywords:** Stablecoin, instability, regulation, decentralized finance

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<sup>†</sup>The Ohio State University Fisher College of Business. E-mail: li.8935@osu.edu

<sup>‡</sup>University of Chicago, Booth School of Business. E-mail: simon.mayer@chicagobooth.edu.

# 1 Introduction

More than a decade ago, Bitcoin heralded a new era of digital payments and decentralized finance (Brunnermeier, James, and Landau, 2019; Duffie, 2019). The substantial volatility of first-generation cryptocurrencies limits their utility as a means of payment.<sup>1</sup> Stablecoins aim to maintain a stable price against fiat currencies and meet the rising demand for blockchain-based safe assets in the fast growing DeFi (Decentralized Finance) sector. The market value of stablecoins more than tripled from January to November 2021, and the boom attracted enormous attention from policy makers. On November 1, 2021, U.S. President’s Working Group on Financial Markets, joined by the Federal Deposit Insurance Corporation (FDIC) and the Office of the Comptroller of the Currency (OCC), released a report on the recent developments of stablecoins (U.S. Department of the Treasury, 2021). U.S. Secretary of the Treasury Janet Yellen emphasized the potential of stablecoins as beneficial payments options and risks due to the lack of legal oversight. In response, U.S. Senate held a hearing on the risks of stablecoins on December 14, 2021.<sup>2</sup>

In this paper, we aim to provide clarity on the instability mechanism of stablecoins, evaluate regulatory proposals, and analyze the complex incentives of stablecoin issuers with a particular focus on large digital platforms. We formalize the operation of a stablecoin issuer in a dynamic continuous-time model and specify the stablecoin demand following the literature on agents’ transactional demand for low-risk assets (Gorton and Pennacchi, 1990; Moreira and Savov, 2017). The equilibrium features a rich set of dynamic strategies of stablecoin management commonly observed in practice (Bullmann, Klemm, and Pinna, 2019), such as open market operations, requirement of users’ collateral, user fees or subsidies, targeted price band, and the issuances of governance tokens (or “secondary units”) that function as equity shares of the stablecoin issuer.<sup>3</sup>

A key feature of our model is that the stablecoin issuer’s reserves are risky, capturing that the issuer is exposed to operational shocks or that reserve assets are risky which is in line the practice. For example, Tether, the issuer of the largest stablecoin by market capitalization (USDT), holds a significant share of reserves in commercial papers of unknown quality.<sup>4</sup> In equilibrium, negative

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<sup>1</sup>Flaws in the protocol design of early cryptocurrencies limit payment scalability (Hinzen, John, and Saleh, 2019).

<sup>2</sup>For the full hearing, “Stablecoins: How Do They Work, How Are They Used, and What Are Their Risks?”, please refer to the website of U.S. Senate Committee on Banking, Housing, and Urban Affairs ([www.banking.senate.gov](http://www.banking.senate.gov)).

<sup>3</sup>An alternative to collateralization is to use algorithmic supply rules to stabilize price but success has been limited.

<sup>4</sup>According to De and Hochstein (2021), USDT is backed by dollar cash, cash equivalents, and commercial papers (75.85%), secured loans (12.55%), corporate bonds, funds, and precious metals (9.96%), and other investments including cryptocurrencies (1.64%). The default of China Evergrande Group on its commercial papers disrupted cryptocurrency markets precisely due to the concern over Tether’s exposure to the Chinese real estate sector (See Lewitinn (2021)). Stablecoins backed by safe assets are rare and simply a form of narrow banking (Pennacchi, 2012).

shocks to the reserves may trigger instability and debasement that resemble the recent episodes, such as the 8% debasement of USDT in April 2017 and the 3.5% debasement of USDC (USD Coin) in February 2020. The stablecoin issuer debases its stablecoins and allows the price to float to maintain sufficient collateralization to avoid liquidation, for instance, triggered by a run.

However, the debasement induces an amplification mechanism that generates a bimodal distribution of states. In states of high reserves, the issuer maintains a fixed exchange rate, so the stablecoin demand is strong and transaction volume is high. Through open market operations and fees on stablecoin users, the issuer generates revenues that further grow its reserves. In states of low reserves, the issuer off-loads risk to users through debasement, which depresses the stablecoin demand and thus reduces the issuer’s revenues. The issuer can only rebuild its reserves slowly and thus falls into an instability trap. Stability can last for a long time, but once debasement happens following negative shocks to the stablecoin issuer’s reserves, volatility persists. Such ergodic instability resembles that in [Brunnermeier and Sannikov \(2014\)](#).<sup>5</sup>

We show that capital requirement reduces the volatility of stablecoin price and demonstrate how the optimal capital requirement varies with key parameters that drive the stablecoin demand or the issuer’s trade-offs. Notably, regulations restricting the riskiness of the reserve assets have different effects than capital requirement. In particular, our analysis reveals a volatility paradox: Forcing the stablecoin issuer to hold reserves in low-risk assets can make the stablecoin more volatile because the issuer endogenously responds to reduce its reserves.

Moreover, we extend our model by allowing the stablecoin issuer to profit from users’ transaction data. When data becomes a productive capital, its marginal  $q$  distorts the stablecoin issuer’s decisions of reserve management. The issuer aggressively draws down its reserves to subsidize users’ data-generating activities rather than preserving reserves for stablecoin management. The data acquisition incentive leads to a more volatile price of the stablecoin. While data privacy and stablecoins have been treated as two separate areas, both under heated debate among regulators, our analysis reveals an unintended consequence of privacy regulation on stablecoins: Limiting platforms’ usage of user-generated data reduces the price volatility of their stablecoins.

The stablecoin issuer’s reserve management is reminiscent of dynamic corporate cash management ([Riddick and Whited, 2009](#); [Bolton, Chen, and Wang, 2011](#); [Décamps, Mariotti, Rochet, and](#)

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<sup>5</sup>The banking model of [Klimenko, Pfeil, Rochet, and Nicolo \(2016\)](#) generates ergodic instability under regulations unlike the laissez-faire economy in [Brunnermeier and Sannikov \(2014\)](#) and our paper. The commonality is that the financial-slack variable, which drives the equilibrium dynamics, can be trapped in a certain region by a large probability over the long run. The continuous-time approach allows a complete characterization of equilibrium dynamics. Ergodic instability is often ignored by the traditional approach of log-linearization near the steady state.

Villeneuve, 2011; Hugonnier, Malamud, and Morellec, 2015; He and Kondor, 2016; Gao, Whited, and Zhang, 2020), but different from a traditional firm, the stablecoin issuer can depreciate its liabilities (outstanding stablecoins) through debasement, akin to a country monetizing debts through inflation. Stablecoins share similarities with contingent convertible bonds (CoCos) that share risk between equity investors and debt holders (Pennacchi, 2010; Glasserman and Nouri, 2012, 2016; Chen, Glasserman, Nouri, and Pelger, 2017; Pennacchi and Tchisty, 2018, 2019). But unlike CoCos, risk sharing is done through debasement rather than converting stablecoins into the issuer’s equity, and debasement is under the discretion of the issuer rather than pre-specified trigger events.<sup>6</sup>

Next, we provide more details on the model setup and our main results. In a continuous-time economy, a digital platform issues stablecoins (“tokens”) to a unit mass of representative users. Our setup applies to both centralized stablecoin issuers, such as Tether or Circle, and decentralized autonomous organizations (DAO), such as MakerDAO, that are governed by prescribed internet protocols.<sup>7</sup> A user’s token holdings deliver a flow utility that captures the transactional benefits. Following Moreira and Savov (2017), we assume that users’ demand for tokens declines in the volatility of token price. Such safety preference is motivated by the link between information sensitivity and asset illiquidity.<sup>8</sup>

Users can trade tokens for numeraire goods (“dollars”) at an endogenous price (the exchange rate) without frictions. The issuer (i.e., the platform) can trade tokens against its reserves, directly influencing the token price. Thus, the token price is at any time optimal from the issuer’s perspective. On the issuer’s balance sheet, the liability side has tokens outstanding and equity. On the asset side, the platform holds reserves that earn an interest rate, grow with token issuance and user fees, and load on Brownian shocks. The shocks capture operational risk and unexpected fluctuations of reserve value. Importantly, our model also allows for double-collateralization that is behind some stablecoin initiatives in practice (e.g., DAI). Under double-collateralization, users are required to post collateral to back their stablecoin holdings subject to margin requirement and the issuer’s reserves are drawn upon to cover shortfalls in users’ collateral value. Then, the reserve shocks originate from the fluctuation of users’ collateral value, and the size of the shock can be controlled through the margin requirement on the users.

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<sup>6</sup>Our model is also related to the continuous-time models of exchange rate determination in small-open economies (e.g., Penati and Pennacchi, 1989) but differs in our endogenous process of money supply.

<sup>7</sup>This is in line with the current focus of policy makers on a technology-neutral approach that emphasizes economic insights over technological aspects of implementation ECB Crypto-Assets Task Force (2019).

<sup>8</sup>To be liquid and circulate as a transaction medium, a security must be designed in a way that deters private information acquisition (e.g., via a safe payoff) and thus avoids asymmetric information between trade counterparties (Gorton and Pennacchi, 1990; DeMarzo and Duffie, 1999; Dang, Gorton, Holmström, and Ordoñez, 2014).

The dollar value of excess reserves (i.e., the platform’s equity on the balance sheet or the difference between reserves and the dollar value of stablecoins outstanding) is the state variable in the issuer’s dynamic optimization program.<sup>9</sup> The value function that solves the Hamilton-Jacobi-Bellman (HJB) equation delivers a state-contingent valuation of the stablecoin issuer’s equity shares (i.e., what practitioners call the governance tokens). The platform pays out dividends to its shareholders when it holds sufficient reserves as risk buffer, which implies an endogenous upper bound on excess reserves. The issuer accumulates reserves through the interests earned on reserves, fees charged to users, and trading profits from open market operations.

When excess reserves turn negative, the stablecoin becomes under-collateralized and therefore faces the risk of liquidation triggered by a run (i.e., all users sell or redeem their tokens at once), a concern that features prominently in policy debates.<sup>10</sup> To capture the potential hazards and costs of such a run on the stablecoin, we consider the platform is liquidated (e.g., due to a run) once excess reserves turn negative. As such, the platform maintains over-collateralization to stave off the risk of liquidation; notably, as we show, the equilibrium dynamics are similar in a model variant that allows for under-collateralization, so our framework can also be used to study under-collateralized and partially collateralized algorithmic stablecoins (e.g., Fei USD).<sup>11</sup>

In spite of over-collateralization, the stablecoin issuer cannot always sustain one-to-one convertibility between tokens and dollars. To avoid costly liquidation, the platform opts for debasement and token price becomes volatile whenever its equity (excess reserves) fall below a threshold. Debasement and volatility trigger a vicious cycle as the depressed token demand leads to a reduction in fee revenues, which causes a slow recovery of equity (excess reserves) and persistent volatility. However, debasement is a valuable option, as it allows the platform to share risk with users. When negative shocks decrease reserves, debasement causes token liabilities to shrink and stabilizes equity. Above the debasement threshold, the platform sustains one-to-one convertibility and token price is stable. Then a strong token demand allows the platform to collect revenues to grow reserves, which

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<sup>9</sup>A stablecoin issuer has stablecoin liabilities and equity and is different from a money market fund (only equity).

<sup>10</sup>In December 2020, three U.S. house representatives proposed the [Stablecoin Tethering and Bank Licensing Enforcement \(STABLE\) Act](#) that emphasized full collateralization. On June 16, 2021, a bank run happened to IRON, a partially collateralized token soft pegged to the U.S. dollar. This was the first large-scale bank run in the cryptocurrency market, and major cryptocurrency investors were calling for regulators’ attention ([Tiwari, 2021](#)). The policy literature emphasizes the fragility of stablecoins due to bank runs ([Brainard, 2019](#); [G7 Working Group on Stablecoins, 2019](#); [ECB Crypto-Assets Task Force, 2019](#); [Massad, 2021](#); [Gorton and Zhang, 2021](#)). [Routledge and Zetlin-Jones \(2021\)](#) study speculative attacks on under-collateralized stablecoins and coordination failure.

<sup>11</sup>Algorithmic stablecoins, such as Terra USD or Fei USD, are often backed by reserves consisting of their own governance tokens and many (but not all) algorithmic stablecoin are only partially collateralized. Through the lens of our model, algorithmic stablecoins differ from reserve-backed stablecoins like USDC mostly in that they hold less reserves and riskier reserves.

further strengthens the peg to dollar. This virtuous cycle implies persistent expansion of platform reserves until it reaches the optimal payout boundary. The stationary distribution of equity (excess reserves) is thus bimodal with two peaks near zero and the payout boundary, respectively.

Importantly, we show that token price debasement occurs even when the platform can replenish its reserves by issuing new equity subject to issuance costs similar to [Riddick and Whited \(2009\)](#): Following negative shocks, the platform first debases its token and only relies on equity financing as a last resort when the equity (excess reserves) fall to zero.

We evaluate three types of stablecoin regulations. The first is a standard capital requirement that stipulates the minimal degree of excess reserves. It reduces token price volatility and increases welfare but debasement still happens in equilibrium. As long as the threat of liquidation (or costly equity issuance) exists, whether it is due to reserve depletion or the violation of regulation, it is optimal for the stablecoin issuer and users to share risk through debasement. The second type of regulation, which enforces a fixed token price, only hurts welfare by destroying the economic surplus from risk sharing. In practice, it is difficult to commit against debasement, but even if such commitment is possible, our results show that it would not be optimal.<sup>12</sup> Intuitively, while commitment to price stability reduces price volatility in good times, it exacerbates the risk of stablecoin failure (for instance, due to a run), which is costly.

The third type of regulation restricts the riskiness of reserve assets. A volatility paradox emerges: When reserve assets are riskier, the platform hoards more reserves and token price actually becomes more stable. Forcing the platform to hold low-risk reserve assets may destabilize token price as the platform will respond by holding less reserves. As long as the regulation cannot completely eliminate risk, the volatility paradox is a robust equilibrium feature. In terms of welfare, the regulatory outcome depends on the risk-return trade-off (i.e., whether riskier reserves deliver higher expected returns). In sum, capital requirement is the most effective in generating unambiguous welfare gains and stabilizing token price, but still fails to entirely eliminate debasement.

Stablecoins became the subject of heated debate after Facebook and its partners announced their own stablecoin, Libra (now “Diem”), in June 2019.<sup>13</sup> More recently, the payment provider

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<sup>12</sup>In other words, the system does not feature dynamic inconsistency in the issuer’s choice of debasement. Admittedly, our model may underestimate the value of a perfectly stable token. For example, debasement invites speculation that can in turn amplify price fluctuation and triggers a vicious cycle ([Scheinkman and Xiong, 2003](#); [Kondor, 2009](#); [Mayer, 2020](#)).

<sup>13</sup>The announcement triggered a globally-coordinated response under the umbrella of the G7. From then on, the G20, the Financial Stability Board (FSB), and central banks around the world have also embarked on efforts to address the potential risks while harnessing the potential of technological innovation. In 2022, Meta has sold the stablecoin project to Silvergate, a crypto-focused bank ([Yahoo Finance, 2022](#)).

PayPal announced to venture into stablecoin issuance too.<sup>14</sup> The enormous amount of transaction data brought by a payment system lures digital platforms to develop their own stablecoins. Following Parlour, Rajan, and Zhu (2020), we extend our model to incorporate data as by-product of transactions.<sup>15</sup> Data helps the platform to improve its productivity in locking in users’ attention and stimulating user activities (for example, through targeted content delivery). When data becomes a productive capital, its marginal q distorts the platform’s stablecoin management. Specifically, the platform faces a new trade-off between data acquisition and reserves preservation. The former requires lower fees to stimulate user activities while the latter calls for higher fees to grow reserves. We show that when data becomes more productive, the platform cuts fees aggressively and, as a result, token price becomes more volatile and debasement becomes more likely.

Therefore, a paradox exists: Stablecoins built primarily for data acquisition become increasingly unstable when data becomes more valuable. As for any digital applications, stablecoins processing digital payment also raise data privacy concerns (Chen, Huang, Ouyang, and Xiong (2021)). Our analysis reveals an unintended benefit of data privacy regulation that limits platforms’ use of user-generated data (Liu, Sockin, and Xiong (2020)). By making data less productive, privacy regulation tilts the platforms’ incentive towards reserve preservation rather than data acquisition and therefore stabilizes the platforms’ stablecoins.

## 2 Background: Crypto Shadow Banking in Decentralized Finance

Blockchain technology supports peer-to-peer transfer of assets on distributed ledgers, potentially eliminating the need to transact through intermediaries (Raskin and Yermack, 2016; Abadi and Brunnermeier, 2019; Brainard, 2019). Decentralization avoids sizable intermediation costs (Philippon, 2015). Depending on the blockchain protocols, decentralization can enhance operational resilience by eliminating single point of failure while still achieve scalability (John, Rivera, and Saleh, 2020).<sup>16</sup> In addition, decentralization through tokenization has the potential to resolve conflicts

<sup>14</sup>Recent news articles on [Coindesk.com](#) and [Bloomberg](#) discuss PayPal’s effort to integrate cryptocurrency payments as well as to develop their own stablecoin. Another example is JPM Coin, a blockchain-based digital coin for fast payment settlement that is being developed by JP Morgan Chase and was announced in February 2019.

<sup>15</sup>In the broader literature on the economics of data, [Veldkamp \(2005\)](#), [Ordoñez \(2013\)](#), [Fajgelbaum, Schaal, and Taschereau-Dumouchel \(2017\)](#), and [Jones and Tonetti \(2020\)](#) model data by-product of economic activities.

<sup>16</sup>Decentralized ledger technology is nascent and faces many challenges. Settlement finality can be compromised when the nodes of a distributed network disagree ([Biais, Bisiere, Bouvard, and Casamatta, 2019](#); [Ebrahimi, Routledge, and Zetlin-Jones, 2020](#)). Law of one price fails in segmented markets ([Makarov and Schoar, 2020](#)). Proof-of-work protocols face limits on adoption [Hinzen, John, and Saleh \(2019\)](#), system security risks ([Budish, 2018](#); [Pagnotta, 2021](#)), and requires energy consumption that crowds out other users ([Benetton, Compiani, and Morse, 2021](#)). Researchers are active in studying alternative protocols, such as proof-of-stake (e.g., [Saleh, 2020](#); [Fanti, Kogan, and Viswanath, 2019](#);

of interest between platforms and their users (Sockin and Xiong (2022)). Decentralized finance (“DeFi”) offers blockchain-based alternatives to traditional financial services, such as banking, brokerage, and exchanges (Lehar and Parlour, 2021). It also uses smart contracts with coded enforcement via programmable money (Tinn, 2017; Cong and He, 2019; Goldstein, Gupta, and Sverchkov, 2019), a concept independent from blockchain (Halaburda, 2018).

This emerging financial architecture requires blockchain-based currencies. A viable means of payment should maintain a stable value at least within the settlement period (i.e., time needed for generating decentralized consensus on transactions (Chiu and Koepl, 2017)). However, most cryptocurrencies are highly volatile (Hu, Parlour, and Rajan, 2019; Stulz, 2019; Liu and Tsyvinski, 2020). They are platform-specific currencies (Catalini and Gans, 2018; Sockin and Xiong, 2018; Li and Mann, 2020; Bakos and Halaburda, 2019; Gryglewicz, Mayer, and Morellec, 2020; Cong, Li, and Wang, 2021; Danos, Marcassa, Oliva, and Prat, 2021) whose values are unbacked and fluctuate with the supply and demand dynamics native to the hosting platforms (Cong, Li, and Wang, 2019).<sup>17</sup>

Stablecoins are advertised as blockchain-based copies of fiat currencies. The total market value is \$130 billion dollars as of November 2021 (up from \$ 28 billion in January). Stablecoins are heavily used in DeFi activities (Saengchote, 2021; Werner, Perez, Gudgeon, Klages-Mundt, Harz, and Knottenbelt, 2021), and in May 2021 alone, \$766 billion worth of stablecoins were transferred.<sup>18</sup> The issuer can be a corporate entity or a consortium (e.g., a consortium led by Facebook, the developer of Diem).<sup>19</sup> It can also be a decentralized autonomous organization (DAO), an internet protocol whose rules may be updated upon users’ consensus on the blockchain (e.g., MakerDAO, the issuer of DAI).<sup>20</sup> A stablecoin is backed by the issuer’s reserve assets. Notably, this applies to a large extent also to so-called algorithmic stablecoins that tend to be backed by less and/or riskier reserves.<sup>21</sup> The price stability is sustained by the issuer conducting open market operations

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John, Rivera, and Saleh, 2021). The cost of decentralization also depends on the market structure of decentralized ledger keepers (Huberman, Leshno, and Moallemi, 2019; Pagnotta and Buraschi, 2018; Easley, O’Hara, and Basu, 2019; Cong, He, and Li, 2020; John, Rivera, and Saleh, 2020; Lehar and Parlour, 2020; Prat and Walter, 2021).

<sup>17</sup>Unbacked cryptocurrencies are exposed to platform-specific risks (Liu, Sheng, and Wang, 2020; Shams, 2020), issuers’ moral hazard (Chod and Lyandres, 2019; Davydiuk, Gupta, and Rosen, 2019; Gan, Tsoukalas, and Netessine, 2021; Garratt and Van Oordt, 2019), and self-fulfilling and speculative beliefs (Garratt and Wallace, 2018; Benetton and Compiani, 2020). Their returns exhibit a factor structure like other risky assets (Liu, Tsyvinski, and Wu, 2019).

<sup>18</sup>See Rajpal and Marshall (2021), Op-ed: Stablecoin is the future of virtual payments. How wise regulation can foster its growth, *CNBC* July 13, 2021.

<sup>19</sup>Central banks digital currencies are alternatives to privately issued stablecoins (Bech and Garratt, 2017).

<sup>20</sup>It is technologically feasible to hard-code certain aspects of a protocol. Kim and Zetlin-Jones (2019) propose an ethical framework for developers to determine which aspects should be immutable and which should not.

<sup>21</sup>Often, algorithmic stablecoin, like Terra USD or Fei USD, are backed by reserves consisting of their own governance tokens. In our framework, such algorithmic stablecoins differ from reserve-backed stablecoins like USDC mostly because they hold less reserves and/or riskier reserves.



(i.e., trading reserves against stablecoins) and meeting redemption requests (Bullmann, Klemm, and Pinna, 2019). The blockchain-based distributed ledger records the ownership and transfer of stablecoins but verifying reserves still relies on traditional auditing (Calle and Zalles, 2019).

Stablecoins can potentially be the link between DeFi and the real economy. In a statement in November 2021, U.S. Treasury Secretary Janet Yellen commented on stablecoins: “Stablecoins that are well-designed and subject to appropriate oversight have the potential to support beneficial payments options.”<sup>22</sup> The volatility of the first-generation cryptocurrencies, such as Bitcoin and Ether, limits their adoption in real-world transactions. Stablecoins, designed to have stable exchange rates with respect to the reference fiat currencies, have the potential to mediate blockchain-based transactions of goods, services, and real assets.

Stablecoins also play important roles in the cryptocurrency community. Traders’ activities heavily involve rebalancing between stablecoins and more volatile cryptocurrencies. Cryptocurrency has become an emerging asset class with the total market capitalization around \$1.5 trillion dollars (with roughly \$700 billion in Bitcoin).<sup>23</sup> It is estimated that 50 to 60% of Bitcoin trading volume is against USDT, the stablecoin issued by Tether (J.P. Morgan Global Research, 2021).

In spite of the importance of stablecoins, there does not exist clear legal and regulatory frameworks. Unlike depository institutions, a stablecoin issuer does not have any obligation to guarantee the quality of reserve assets or maintain a fixed token price. Many are concerned that a major stablecoin “breaks the buck” may trigger financial turmoil beyond the cryptocurrency community (Massad, 2021; Kozhan and Viswanath-Natraj, 2021). The creation of stablecoins is essentially a new form of shadow banking—unregulated safety transformation—with its distinct features.<sup>24</sup>

The reserve assets are risky. Major stablecoin issuers hold commercial papers without disclosure on the identities of commercial paper issuers. The default of China Evergrande Group disrupted cryptocurrency markets precisely due to the concern over Tether’s exposure to the Chinese real estate sector.<sup>25</sup> Tether is the issuer of USDT, the largest stablecoin by market capitalization.<sup>26</sup> Other stablecoins, including algorithmic ones, are often back by cryptoassets.

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<sup>22</sup>See Livni and Lipton (2021), Regulators Ask Congress to Create New Rules for Cryptocurrencies, *The New York Times* November 1, 2021.

<sup>23</sup>Nearly half of millennial millionaires have at least 25% of their wealth in cryptocurrencies (CNBC Survey).

<sup>24</sup>A stable value is essential for a transaction medium because it reduces asymmetric information between transaction counterparties (Gorton and Pennacchi, 1990; DeMarzo and Duffie, 1999; Dang, Gorton, Holmström, and Ordoñez, 2014). Without informational frictions, stability may not be necessary (Schilling and Uhlig, 2019).

<sup>25</sup>See Lewitinn (2021), Tether, Bitcoin and Chinese Commercial Paper at Scale: A look at the leading stablecoin’s market cap and China’s real estate industry., *CoinDesk Insights* November 12, 2021.

<sup>26</sup>According to De and Hochstein (2021), USDT is backed by dollar cash, cash equivalents, and commercial papers (75.85%), secured loans (12.55%), corporate bonds, funds, and precious metals (9.96%), and other investments including digital tokens (1.64%).

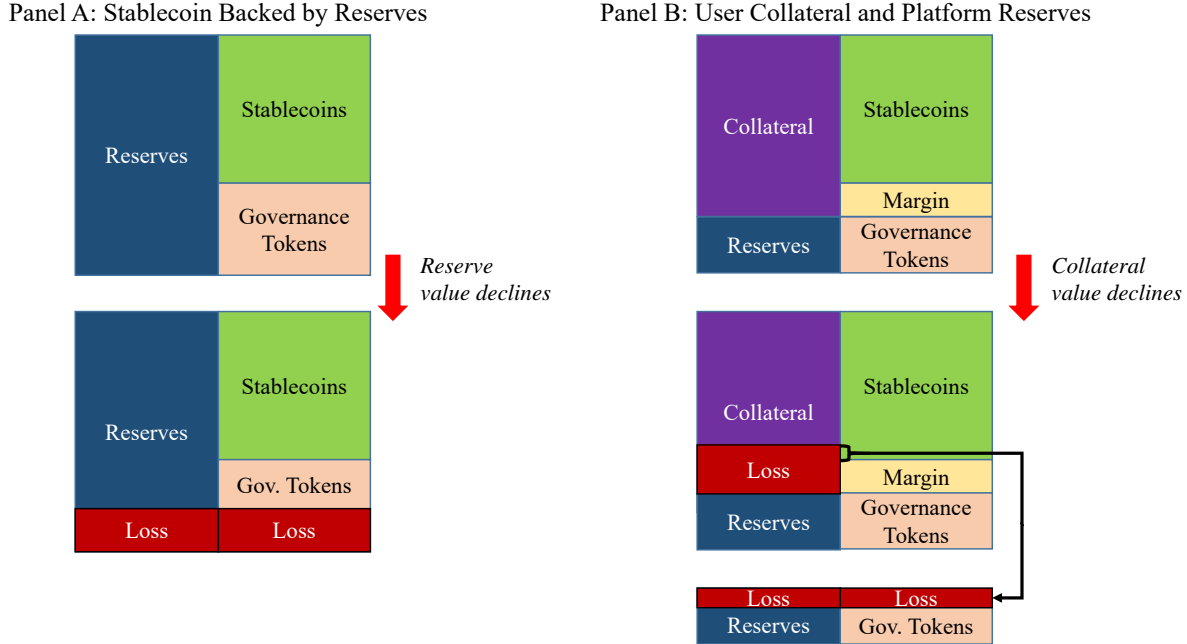


Figure 1: **Crypto Shadow Banking.** This illustrates the two structures of stablecoins. In Panel A, a platform issues stablecoins backed by its reserves. The excess reserves belong to the holders of governance tokens who have the control right (i.e., the control over platform policies). When reserves are invested in risky assets, a potential loss is absorbed by the equity position. As long as the stablecoins are over-collateralized, their value is intact. In Panel B, stablecoins are backed by both the user’s collateral and the platform’s reserves. When the collateral value declines and the user fails to meet the margin requirement, the platform liquidates the collateral and uses the proceeds (and its own reserves) to buy back stablecoins in the secondary market.

Panel A of Figure 1 illustrates stablecoin creation with over-collateralization, a common practice among stablecoin issuers (Bullmann, Klemm, and Pinna, 2019). The issuer’s excess reserves buffers the fluctuation of reserve value. The equity shares are called governance tokens (or “secondary units”) that carry the rights to vote on changes of protocols (i.e., control rights) and pay out cash flows generated by fees charged on the stablecoin users. Governance tokens can be issued to replenish reserves, just as traditional corporations can raise cash by issuing equity. A stablecoin issuer essentially takes a leveraged bet on the value of reserve assets. The issuer can increase its leverage by issuing new stablecoins to finance the purchase of reserve assets, just as banks finance their lending and security trading with newly issued deposits (i.e., inside money creation (Tobin, 1963; Bianchi and Bigio, 2014; Piazzesi and Schneider, 2016; Faure and Gersbach, 2017; Donaldson, Piacentino, and Thakor, 2018; Parlour, Rajan, and Walden, 2020)). Unlike banks that commit to redeem deposits at par, the stablecoin issuer can debase the stablecoins.

Panel B of Figure 1 illustrates a more complex structure that is similar to the one adopted

by MakerDAO, the issuer of DAI and an early decentralized autonomous organizations.<sup>27</sup> A user pledges her holdings of cryptocurrencies and other assets as collateral for newly created stablecoins, subject to a haircut (margin requirement). The user may transfer the stablecoins, which then circulate in the market, but she must maintain the margin requirement. If the collateral value declines and the user cannot maintain the margin, she loses her collateral to the stablecoin issuer, who then liquidates the collateral and uses the proceeds to buy back (and burn) the stablecoins created for this user.<sup>28</sup> If the liquidation of collateral does not generate sufficient proceeds, the stablecoin issuer’s reserves supplement the expense of stablecoin buyback.<sup>29</sup> In the example of DAI, the reserves are called the System Surplus Buffer which finances stablecoin buybacks and repurchases of the governance token MKR.

The structure in Panel B of Figure 1 resembles shadow banking: A bank sets up a conduit (special purpose vehicle) that tranches risky investments into debt and equity and extends a guarantee to the debt investors (Acharya, Schnabl, and Suarez, 2013). The stablecoin is like the debt (senior) tranche of the conduit. The stablecoin issuer and the user correspond, respectively, to the bank and the conduit. The issuer’s commitment to buy back stablecoins potentially with her own reserves is analogous to the bank’s guarantee. Despite double collateralization, the stablecoin may still break the buck, consistent the findings of Kozhan and Viswanath-Natraj (2021) on a positive relationship between collateral risk and the price volatility of stablecoin DAI.<sup>30</sup>

We set up our model in the next section following the structure in Panel A of Figure 1 and present the solution in Section 4. Section 5 provides an analysis of several regulatory proposals. In Section 6, we extend our model to incorporate double collateralization in Panel B of Figure 1 and analyze the optimal margin requirement. Section 7 focuses on stablecoins issued by large digital platforms that have strong user network effects and can profit from users’ data.

### 3 A Model of Stablecoins

Consider a continuous-time economy where a continuum of representative agents (“users”) of unit measure demand stablecoins (“tokens”) that are issued by a platform. The generic consumption

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<sup>27</sup>Decentralized autonomous organizations (DAOs) are organizations represented by rules encoded as computer programs and controlled by the organization members through various voting mechanisms on blockchains.

<sup>28</sup>Burning is to send the stablecoins to an irretrievable digital address

<sup>29</sup>While the repurchase (and burn) of stablecoins is recorded on the blockchain, the liquidation of non-cryptocurrency collateral and reserves happens off-chain and still requires the traditional financial and legal systems.

<sup>30</sup>The capital structure of a stablecoin issuer (stablecoin liabilities and equity) is different from that of a money market fund (full equity). Money market funds have different fragility mechanisms (Kacperczyk and Schnabl, 2013; Parlatore, 2016; Schmidt, Timmermann, and Wermers, 2016; La Spada, 2018; Li, Li, Macchiavelli, and Zhou, 2021).

goods (“dollars”) are the numeraire in this economy, and we take as exogenous a prevailing interest rate  $r$ . Let  $P_t$  denote the token price in units of dollars. Users trade tokens with the platform and amongst themselves at the market price  $P_t$  per unit without frictions. The platform can influence the token price  $P_t$  by trading tokens against its reserves in the market. Notice that when users sell tokens to the platform, they essentially redeem stablecoins at (per-unit) value  $P_t$ , so our model describes both stablecoins that allow for redemption and that do not. In equilibrium, the dollar price of token has a law of motion which the atomic users take as given:

$$\frac{dP_t}{P_t} = \mu_t^P dt + \sigma_t^P dZ_t, \quad (1)$$

where the standard Brownian shock,  $dZ_t$ , will be introduced below as a shock to the platform’s reserves. We will show how  $\mu_t^P$  and  $\sigma_t^P$  in equilibrium depend on the platform’s optimal strategies. Next, we first introduce users and then set up the platform’s problem.

**Users.** There is a unit mass of risk-neutral users with time discount rate  $r$ . We use  $u_{i,t}$  ( $i \in [0, 1]$ ) to denote the dollar value of user  $i$ ’s holdings of tokens, so user  $i$  holds  $k_{i,t} = u_{i,t}/P_t$  units of tokens. The aggregate dollar value of token holdings is  $N_t \equiv \int_{i \in [0, 1]} u_{i,t} dt$ . Users hold stablecoins for transactions and as a store of value. We model the transactional benefit of holding stablecoins in a reduced form. Specifically, user  $i$  derives a flow utility (convenience yield) from token holdings:

$$\frac{1}{\xi} u_{i,t}^\xi A^{1-\xi} dt - \eta u_{i,t} |\sigma_t^P| dt, \quad (2)$$

where  $\xi \in (0, 1)$ ,  $A > 0$ , and  $\eta > 0$ . We model the utility from holding means of payment following the classic models of monetary economics (e.g., [Baumol, 1952](#); [Tobin, 1956](#); [Feenstra, 1986](#); [Freeman and Kydland, 2000](#)) and related empirical studies (e.g., [Poterba and Rotemberg, 1986](#); [Lucas and Nicolini, 2015](#); [Nagel, 2016](#)). In this literature, agents derive utility from the real value of holdings, i.e.,  $u_{i,t}$ .<sup>31</sup> The quality of the payment system is captured by parameter  $A$  which we will endogenize in Section 7. We define transactional utility from an ex ante perspective and the first term in (2) can be viewed as the *expected* transaction benefits in  $dt$ . We do not model the ex post circulation of tokens in line with the aforementioned literature on money-in-utility and cash-in-advance constraint.

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<sup>31</sup>We refer readers to the textbook treatments (e.g., [Galí, 2015](#); [Ljungqvist and Sargent, 2004](#); [Walsh, 2003](#)). For the nominal value (i.e.,  $k_{i,t}$ ) to affect agents’ decisions, additional frictions, such as nominal illusion (e.g., [Shafir, Diamond, and Tversky, 1997](#)) or sticky prices (e.g., [Christiano, Eichenbaum, and Evans, 2005](#)), have to be introduced.

The user's preference for stability is captured by the parameter  $\eta (> 0)$ , and is defined on the absolute value of  $\sigma_t^P$  to capture the fact that users are averse to token price fluctuation no matter whether the price moves with ( $\sigma_t^P > 0$ ) or against ( $\sigma_t^P < 0$ ) the platform's reserve shock  $dZ_t$ . We motivate such preference for stability following [Moreira and Savov \(2017\)](#): to be liquid and circulate as a transaction medium, a security must be designed in a way that deters private information acquisition and thus avoids asymmetric information between trade counterparties ([Gorton and Pennacchi, 1990](#); [DeMarzo and Duffie, 1999](#); [Dang, Gorton, Holmström, and Ordoñez, 2014](#)).<sup>32</sup> One may interpret the flow utility given by (2) as the convenience yield relative to its reference fiat currency due to functionalities such as relaxing collateral constraints in smart contracts.

User  $i$  pays a proportional fee on her token holdings,  $u_{i,t}f_t dt$ , where  $f_t$  is set by the platform. When  $f_t$  is negative, users earn subsidies on their token holdings. Note that as long as the money (token) velocity is constant within the small time interval ( $dt$ ), transaction volume is proportional to token holdings,  $N_t$ . Therefore, an alternative interpretation of  $f_t$  is transaction fee. There exists a technical upper bound on the volume of transactions that the platform can handle per unit of time ([Hinzen, John, and Saleh, 2019](#)). Without loss of generality, we model the bound as follows:

$$N_t \leq \bar{N}. \quad (3)$$

Let  $R_{i,t}$  denote user  $i$ 's (undiscounted) *cumulative* payoff from platform activities. The instantaneous payoff depends on user  $i$ 's choice of  $u_{i,t} \geq 0$  and is given by

$$dR_{it} \equiv \left( \frac{1}{\xi} u_{i,t}^\xi A^{1-\xi} - \eta u_{i,t} |\sigma_t^P| \right) dt + u_{i,t} \left( \frac{dP_t}{P_t} - r dt - f_t dt \right), \quad (4)$$

where the first term is the flow utility (2) and the second term includes the return from token price change net of forgone interests  $r$  and fees  $f_t$ . A representative user  $i$  chooses  $u_{i,t} \geq 0$  to maximize

$$\max_{u_{i,t} \geq 0} \mathbb{E}_t [dR_{it}] = \max_{u_{i,t}} \left[ \frac{1}{\xi} u_{i,t}^\xi A^{1-\xi} dt + u_{i,t} \left( \mu_t^P - r - f_t - \eta |\sigma_t^P| \right) dt \right]. \quad (5)$$

Note that  $r$  is essentially the users' (opportunity) cost of capital as in [Jorgenson \(1963\)](#).<sup>33</sup>

<sup>32</sup>The disutility from token volatility can also be motivated by risk-averse preference or users' aversion to exchange-rate shocks that cause losses of net worth when assets and liabilities are denominated in different currencies (tokens and dollars) ([Doepke and Schneider, 2017](#); [Gopinath, Boz, Casas, Díez, Gourinchas, and Plagborg-Møller, 2020](#)).

<sup>33</sup>For agents with infinite intertemporal elasticity of substitution, we do not need to explicitly model the dynamic consumption-savings trade-off and portfolio allocation of savings because the marginal investment in any asset (including tokens) should deliver an expected return  $r$ , which is also equal to the marginal return on total savings. Therefore, a user simply maximizes the excess return on token holdings when choosing  $u_{i,t}$ . As will be shown later,

**The Platform.** Let  $S_t$  denote the total units of tokens outstanding (the token supply). The token market clearing condition is given by

$$S_t = \int_{i \in [0,1]} \frac{u_{i,t}}{P_t} di, \quad (6)$$

or equivalently, in the numeraire (dollar) value:

$$S_t P_t = N_t = \int_{i \in [0,1]} u_{i,t} di. \quad (7)$$

The platform chooses the fees and controls the token price,  $P_t$ , by adjusting the token supply. This is akin to central banks using open market operations to intervene in the foreign exchange markets (e.g., [Calvo and Reinhart, 2002](#)). When the platform issues more tokens ( $dS_t > 0$ ), it collects dollar revenues as users buy tokens with dollars. When the platform repurchases and burns tokens from users ( $dS_t < 0$ ), it spends dollars to buy tokens from users.<sup>34</sup>

Let  $M_t$  denote the dollar value of the platform's reserve assets;  $M_t$  is publicly observable and cannot turn negative. For simplicity, we do not microfound the composition of reserve assets and specify the following law of motion<sup>35</sup>

$$dM_t = rM_t dt + (P_t + dP_t)dS_t + N_t f_t dt + N_t \sigma dZ_t - dDiv_t. \quad (8)$$

The first term is the interests earned on the reserves balance as  $r$  is the prevailing interest rate. The second term is the revenues (losses) from issuing (buying back) tokens the secondary market. From  $t$  to  $t+dt$ , the quantity adjustment  $dS_t$  is multiplied by the end-of-period price  $P_{t+dt} = P_t + dP_t$ . The third term is the fee. In the fourth term,  $Z_t$  is a standard Brownian motion, and its increment,  $dZ_t$ , captures the shocks to the reserve holdings, which can stem from unexpected operating expenses or risks in the reserve assets. This shock is the only source of uncertainty in the model, and it scales with dollar market capitalization of tokens,  $S_t P_t = N_t$ .<sup>36</sup> Let  $Div_t$  denote the *cumulative* dividend process. The platform's reserves decrease when the platform pays its owners dividends,  $dDiv_t$ . In the baseline, we assume  $dDiv_t \geq 0$ , reflecting that platform shareholders have limited liability and

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the first order condition of the problem given by (5) equates the marginal benefit of  $u_{i,t}$  to  $r$ .

<sup>34</sup>In practice, token burning is to send tokens to irretrievable digital addresses.

<sup>35</sup>In [Appendix B.3](#), we consider a more general law of motion of  $M_t$ . [Section 5.3](#) studies the implications of regulating the riskiness of stablecoin reserve assets.

<sup>36</sup>The assumption that reserve shocks scale with  $N_t$  is inconsequential for our main findings. [Appendix B.3](#) considers an alternative specification for the dynamics  $dM_t$  in which reserve shocks scale with the level of reserves  $M_t$  rather than with  $N_t$ . We show that the results remain qualitatively unchanged under this alternative specification.

it is not possible to issue new equity. In Section 4.3, we extend our model to incorporate (costly) equity issuance (i.e.,  $dDiv_t < 0$  (Bolton et al., 2011)).

The platform maximizes the expected discounted value of dividend payouts to its owners:

$$V_0 \equiv \max_{\{f_t, dS_t, dDiv_t\}} \mathbb{E} \left[ \int_0^\infty e^{-\rho t} dDiv_t \right] \quad \text{subject to (8) and } dDiv_t \geq 0. \quad (9)$$

We assume that the platform’s shareholders apply a discount rate  $\rho$  which exceeds the interest rate  $r$ , that is,  $\rho > r$ . The assumption that shareholders apply a higher discount rate than the interest rate on reserves is standard in dynamic liquidity management models, (e.g., Bolton et al. (2011); Décamps et al. (2011)) and made to rule out a degenerate solution in which the firm forever accumulates financial slack and never pays out dividends.<sup>37</sup>

**Liquidation and runs.** A prominent concern among policy makers is that, similar to traditional banks, stablecoins are prone to runs when they are undercollateralized in that the reserve value lies below the dollar value of outstanding stablecoins (i.e.,  $M_t < S_t P_t$ ).<sup>38</sup> Such a run, in which stablecoin holders would rush to sell (or to redeem) their stablecoin holdings at price  $P_t$  before reserves are depleted, would lead to a drastic failure of the stablecoin. To capture the potential hazards and costs of such a run scenario in a tractable way, we assume that the platform is liquidated and token price and platform owners’ value falls permanently to zero when the platform’s equity (i.e., the excess reserves,  $M_t - S_t P_t$ ) becomes negative. The interpretation is that users sell their stablecoins and a run occurs the first time the stablecoin becomes under-collateralized and excess reserves  $M_t - S_t P_t$  fall below a critical threshold  $\underline{C}$  which we set to zero.<sup>39</sup> As such, the platform maintains over-collateralization (i.e.,  $M_t \geq S_t P_t$ ) to prevent liquidation and a run. Section 4.2 provides some additional discussion about the instability mechanism of the stablecoin and its relation to runs.

While we do not explicitly model users’ decisions to run, we note that such a threshold strategy, in which there is a run once fundamentals (in our case, excess reserves) fall below a certain threshold,

<sup>37</sup>The wedge,  $\rho - r$ , can be microfounded with an exogenous Poisson-arriving liquidation with intensity  $\rho - r$ . The literature on agency cost of cash holdings also provides a rationale for why the return on liquidity holdings is below shareholders’ discount rate (Nikolov and Whited, 2014; Nikolov, Schmid, and Steri, 2019).

<sup>38</sup>In December 2020, the U.S. house representatives proposed the [Stablecoin Tethering and Bank Licensing Enforcement \(STABLE\) Act](#) that emphasized full collateralization. On June 16, 2021, a bank run happened to IRON, a partially collateralized token. This was the first large-scale run in the cryptocurrency market (Tiwari, 2021). The policy literature emphasizes the fragility of stablecoins due to bank runs (Brainard, 2019; G7 Working Group on Stablecoins, 2019; ECB Crypto-Assets Task Force, 2019; Massad, 2021; Gorton and Zhang, 2021).

<sup>39</sup>In this analogy, because all model quantities evolve continuously, the run then occurs precisely when  $M_t = S_t P_t$  in which case the platform uses its reserves to buy back  $S_t$  tokens from the users at price  $P_t$ . That is, users effectively redeem their tokens with the platform at price  $P_t$ .

is typically the outcome of a global games approach analyzing bank runs (Goldstein and Pauzner (2005) or currency attacks (Morris and Shin, 1998; Goldstein, Ozdenoren, and Yuan, 2011)).<sup>40</sup> Importantly, our results would be qualitatively similar if we considered a negative liquidation threshold,  $\underline{C} < 0$ .<sup>41</sup> In Appendix B.6, we present a model variant that allows under-collateralization (i.e., negative equity and  $\underline{C} < 0$ ) and show that, indeed, the dynamics are similar.

## 4 Equilibrium

In this section, we characterize the analytical properties of the dynamic equilibrium and, to sharpen the economic intuition, we also provide graphical illustrations based on the numerical solutions.

### 4.1 Managing Stablecoin: Optimal Strategies

**User Optimization.** A representative user  $i$  solves the problem in (5) with the following first-order condition for  $u_{i,t}$

$$u_{i,t}^{\xi-1} A^{1-\xi} + \mu_t^P - f_t - \eta|\sigma_t^P| = r, \quad (10)$$

so total utility from the marginal token holdings is equal to  $r$ , the user's discount rate and the prevailing interest rate which is users' (opportunity) cost of capital. In equilibrium,  $N_t = u_{i,t}$  under user homogeneity, which, through (10), implies

$$N_t = \frac{A}{(r + f_t - \mu_t^P + \eta|\sigma_t^P|)^{\frac{1}{1-\xi}}}. \quad (11)$$

Token demand decreases in the fees,  $f_t$ , and depends on the token price dynamics, which the platform controls. This is the solution within the system throughput (i.e.,  $N_t < \bar{N}$ ); otherwise, we have  $u_{i,t} = N_t = \bar{N}$ . Note that  $N_t$  quantifies token usage and can be interpreted as transaction volume.

**Platform Optimization.** To solve for the platform's optimal strategies, we first note that, given the token price dynamics (i.e.,  $\mu_t^P$  and  $\sigma_t^P$ ), the platform can directly set  $N_t$  through the fees  $f_t$ .

<sup>40</sup>Alternatively, one could model the risk of a run by considering that at time  $t$ , run occurs with probability  $\delta_t dt$ , whereby  $\delta_t$  is a function of the stablecoin's collateralization level, i.e.,  $\delta_t = \delta(M_t - S_t P_t)$ . Then,  $\delta(x) = 0$  for  $x \geq 0$  as there are no runs when the stablecoin is over-collateralized, and  $\delta'(x) > 0$  to capture that the risk of a run rises as the platform becomes increasingly under-collateralized.

<sup>41</sup>In fact, our findings carry through as long as there is some lower (liquidation) bound on excess reserves.



Rearranging (11), we can back out the fees implied by the platform's choice of  $N_t$ :

$$f_t = \left( \frac{A}{N_t} \right)^{1-\xi} - r + \mu_t^P - \eta|\sigma_t^P|. \quad (12)$$

Using (12), we substitute out  $f_t$  in the law of motion of reserves (8) and obtain

$$dM_t - (P_t + dP_t)dS_t = rM_t dt + N_t^\xi A^{1-\xi} dt - rN_t dt + N_t (\mu_t^P - \eta|\sigma_t^P|) dt + N_t \sigma dZ_t - dDiv_t. \quad (13)$$

Next, we show the state variable for the platform's dynamic optimization is the *excess reserves*,

$$C_t \equiv M_t - S_t P_t. \quad (14)$$

To derive the law of motion of  $C_t$ , we first note that

$$\begin{aligned} dC_t &= dM_t - d(S_t P_t) = dM_t - (P_t + dP_t)dS_t - S_t dP_t \\ &= dM_t - (P_t + dP_t)dS_t - N_t (\mu_t^P dt + \sigma_t^P dZ_t). \end{aligned} \quad (15)$$

The second equality uses  $d(S_t P_t) = dS_t P_t + S_t dP_t + dS_t dP_t$  (by Itô's lemma) and the last equality uses (1) and  $N_t = S_t P_t$ . From a balance-sheet perspective, the reserves,  $M_t$ , are the platform's assets and the outstanding tokens,  $S_t P_t$ , are the liabilities. The excess reserves constitute the (book) equity. Thus, equation (15) is essentially the differential form of the balance-sheet identity. Using (13) to substitute out  $dM_t - (P_t + dP_t)dS_t$  in (15), we obtain the following law of motion:

$$dC_t = \left( rC_t + N_t^\xi A^{1-\xi} - N_t \eta |\sigma_t^P| \right) dt + N_t (\sigma - \sigma_t^P) dZ_t - dDiv_t, \quad (16)$$

with drift  $\mu_{C,t} \equiv rC_t + N_t^\xi A^{1-\xi} - N_t \eta |\sigma_t^P|$  and diffusion  $\sigma_{C,t} \equiv N_t (\sigma - \sigma_t^P)$ . Note that  $N_t \mu_t^P$  disappears. As shown in (13), the platform receives more fee revenues (see (12)) when users expect tokens to appreciate ( $N_t \mu_t^P$ ), but such revenues do not increase the platform's excess reserves as they are cancelled out by the appreciation of token liabilities. After netting out the two forces, the drift term,  $rC_t + N_t^\xi A^{1-\xi} - N_t \eta |\sigma_t^P|$ , is the expected appreciation of the platform's excess reserves.

The platform controls the law of motion of  $C_t$  through dividend payouts,  $dDiv_t$ , aggregate token demand,  $N_t$  (or equivalently, fees  $f_t$ ), and token price volatility  $\sigma_t^P$ . We will show that once we solve for these optimal control variables, the equilibrium processes of token supply,  $S_t$ , and token price,  $P_t$ , can be obtained. We characterize a Markov equilibrium with the platform's excess reserves,

$C_t$ , as the only state variable. We solve for the platform's control variables,  $dDiv_t$ ,  $\sigma_t^P$ , and  $N_t$ , as functions of  $C_t$ , and thereby, show that (16) is an autonomous law of motion of  $C_t$ .

The platform owners' value function at time  $t$  is given by

$$V_t = V(C_t) = \max_{\{N, \sigma^P, Div\}} \mathbb{E} \left[ \int_{s=t}^{\infty} e^{-\rho(s-t)} dDiv_s \right]. \quad (17)$$

The platform pays dividends when the marginal value of excess reserves is equal to one, i.e., one dollar has the same value either held within the platform or paid out,

$$V'(\bar{C}) = 1. \quad (18)$$

The optimality of payouts at  $\bar{C}$  also requires the following super-contact condition (Dumas, 1991),

$$V''(\bar{C}) = 0. \quad (19)$$

The next proposition states that the value function is concave. The declining marginal value of excess reserves implies that  $\bar{C}$  in (18) is an endogenous upper bound of the state variable  $C_t$ . At any  $C_t \in (0, \bar{C})$ , the platform does not pay dividends to its owners because the marginal value of excess reserves,  $V'(\bar{C})$ , is greater than one, i.e., the owners' value of dividend.

**Proposition 1 (Value Function and Optimal Payout).** *There exists  $\bar{C} > 0$  such that  $C_t \leq \bar{C}$ . For  $C_t < \bar{C}$ , the value function is strictly concave, and  $V'(C_t) > 1$ . At  $C_t = \bar{C}$ ,  $V'(\bar{C}) = 1$  and the platform pays dividends when  $dC_t > 0$  so that dividend payments cause  $C_t$  to reflect at  $\bar{C}$ .<sup>42</sup>*

Before characterizing the solution as  $C_t$  approaches zero (the lower boundary), we note that

$$\max_{\{N \in [0, \bar{N}]\}} \left\{ N^\xi A^{1-\xi} - \eta N \sigma \right\} > 0, \quad (20)$$

because  $N^\xi A^{1-\xi} - \eta N \sigma = 0$  at  $N = 0$  and, under  $\xi < 1$  as previous stated, the first derivative goes to infinity as  $N$  approaches zero, (i.e.,  $\lim_{N \rightarrow 0} \xi \left(\frac{A}{N}\right)^{1-\xi} - \eta \sigma = +\infty$ ).

As  $C_t$  approaches zero, the platform can only avoid liquidation by reducing the diffusion of  $C_t$  (i.e., the shock exposure  $\sigma_C(C_t) = N_t(\sigma - \sigma_t^P)$ ) to zero, which requires

$$\lim_{C \rightarrow 0^+} \sigma^P(C) = \sigma, \quad (21)$$

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<sup>42</sup>When  $dC_t > 0$  at  $C_t = \bar{C}$ , the dividend amount is equal to  $dC_t$  (i.e., exactly the amount needed to avoid  $C_t > \bar{C}$ ).

and by keeping the drift of  $C_t$  (i.e.,  $\mu_C(C_t) = rC_t + N_t^\xi A^{1-\xi} - N_t \eta |\sigma_t^P|$ ) non-negative, which is already guaranteed by (20) under (21). It is optimal to do so and avoid liquidation because, as we show below, the value of platform as an ongoing concern is positive. In the region  $C \in (0, \bar{C})$ , we obtain the following Hamilton-Jacobi-Bellman (HJB) equation (with time subscripts suppressed):

$$\rho V(C) = \max_{\{N \in [0, \bar{N}], \sigma^P\}} \left\{ V'(C) \left( rC + N^\xi A^{1-\xi} - \eta N |\sigma^P| \right) + \frac{1}{2} V''(C) N^2 (\sigma - \sigma^P)^2 \right\}. \quad (22)$$

Setting  $\sigma^P = \sigma$  is always feasible in the HJB equation, which implies:

$$\begin{aligned} V(C) &\geq \frac{V'(C)}{\rho} \left( rC + \max_{\{N \in [0, \bar{N}]\}} \{N^\xi A^{1-\xi} - \eta N \sigma\} \right) \\ &\geq \frac{1}{\rho} \left( \max_{\{N \in [0, \bar{N}]\}} \{N^\xi A^{1-\xi} - \eta N \sigma\} \right) > 0, \end{aligned} \quad (23)$$

where the second inequality uses  $C \geq 0$  and  $V'(C) \geq 1$  and the last inequality follows (20). By the continuity of the value function  $V(C)$ , a strictly positive lower bound of  $V(C)$  on  $(0, \bar{C})$  implies that  $\lim_{C \rightarrow 0} V(C) > 0$ . In sum, we have shown that it is optimal for the platform to implement (21) and thereby avoid liquidation because the value as an ongoing concern is positive as  $C_t$  approaches zero. Finally, (21) implies that, when taking the right-limit on both sides of (22), we obtain

$$\lim_{C \rightarrow 0^+} \frac{V(C)}{V'(C)} = \frac{1}{\rho} \max_{\{N \in [0, \bar{N}]\}} \{N^\xi A^{1-\xi} - \eta N \sigma\} = \frac{1}{\rho} \{ \underline{N}^\xi A^{1-\xi} - \eta \underline{N} \sigma \} > 0, \quad (24)$$

where the second equality follows from plugging in the optimal  $N_t$  given by

$$\underline{N} \equiv \lim_{C \rightarrow 0^+} N(C) = \arg \max_{N \in [0, \bar{N}]} \{N^\xi A^{1-\xi} - \eta N \sigma\} = A \left( \frac{\xi}{\eta \sigma} \right)^{\frac{1}{1-\xi}} \wedge \bar{N} > 0. \quad (25)$$

The condition (24) serves as another boundary condition for the HJB equation.

As an interim summary, the next proposition summarizes the value function solution as solution to an ordinary differential equation (ODE) problem with an endogenous boundary  $\bar{C}$ . Figure 2 plots the numerical solution of value function (Panel A) and the decreasing marginal value of excess reserves with the red dotted line marking the payout boundary  $\bar{C}$ .

**Proposition 2 (Value Function).** *The value function,  $V(C)$ , and the boundary  $\bar{C}$  solve the ODE (22) on  $(0, \bar{C})$  subject to the boundary conditions (18), (19), and (24). As  $C \geq 0$ , reserves  $M = C + SP$  are positive in any state  $C \geq 0$ .*

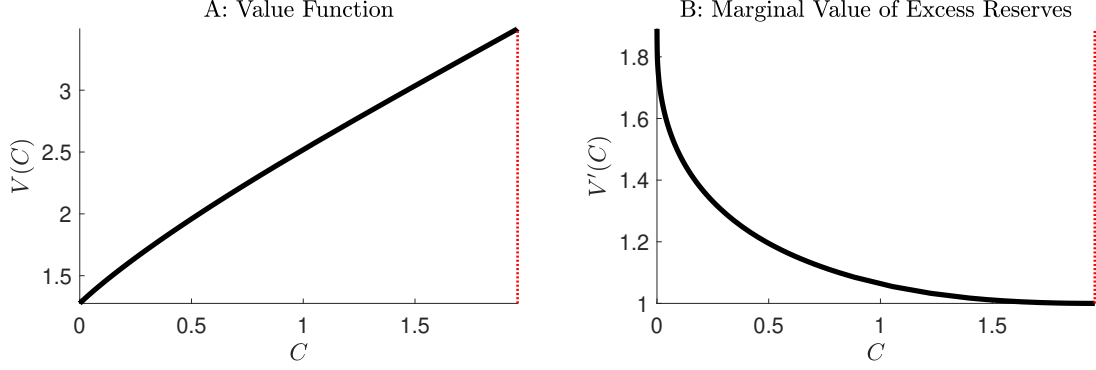


Figure 2: **Value Function.** This illustrates the level and first derivative of the platform’s value function. The red dotted lines in both panels mark  $\bar{C}$  (defined in Proposition 1). The parameters are  $r = 0.05$ ,  $\rho = 0.06$ ,  $\sigma = 0.1$ ,  $\bar{N} = 5$ ,  $\eta = 0.15$ ,  $\xi = 0.5$ , and  $A = 0.0025$ .

Next, we fully characterize the platform’s optimal choices of  $\sigma_t^P$  and  $N_t$  as functions of the state variable,  $C_t$  (via the derivatives of  $V(C)$ ). First, we define the platform’s *effective risk aversion*:

$$\gamma(C) \equiv -\frac{V''(C)}{V'(C)}. \quad (26)$$

This definition is analogous to the classic measure of absolute risk aversion of consumers (Arrow, 1965; Pratt, 1964). From Proposition 1,  $\gamma(C) \geq 0$  and, in  $(0, \bar{C})$ ,  $\gamma(C) > 0$ . The next proposition states the monotonicity of  $\gamma(C)$  in  $C$  and summarizes the optimal  $\sigma_t^P = \sigma^P(C_t)$  and  $N = N(C_t)$ .

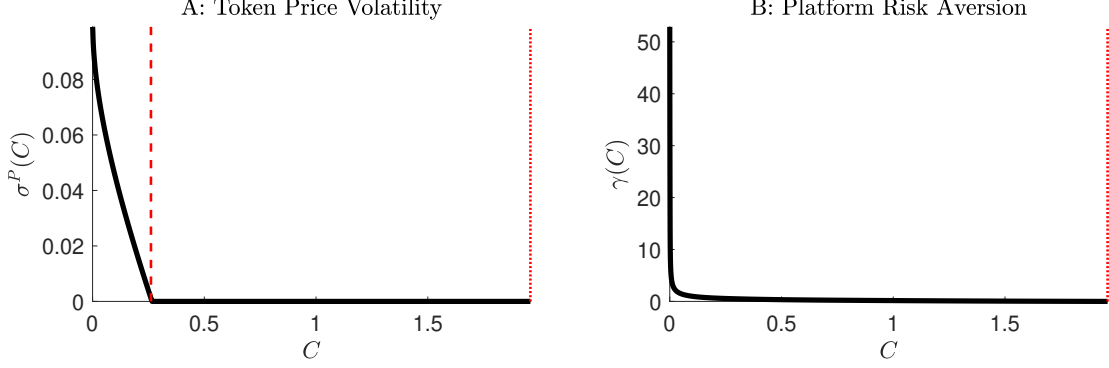
**Proposition 3 (Risk Aversion, Token Volatility, and Token Usage).** *The platform’s effective risk aversion,  $\gamma(C)$ , strictly decreases in the level of excess reserve holdings,  $C$ . There exists  $\tilde{C} \in (0, \bar{C})$  such that, at  $C \in (0, \tilde{C})$ ,  $N(C) = \underline{N}$  and  $\sigma^P(C)$  strictly decreases in  $C$ , given by,*

$$\sigma^P(C) = \sigma - \frac{\eta}{\gamma(C)\underline{N}} \in (0, \sigma), \quad (27)$$

and at  $C \in [\tilde{C}, \bar{C}]$ ,  $\sigma^P(C) = 0$  and  $N(C)$  increases in  $C$ , given by

$$N(C) = \min \left\{ \left( \frac{\xi A^{1-\xi}}{\gamma(C)\sigma^2} \right)^{\frac{1}{2-\xi}}, \bar{N} \right\}. \quad (28)$$

When the platform’s reserves are low, i.e.,  $C \in (0, \tilde{C})$ , it is the ratio of users’ risk aversion to the platform’s risk aversion that determines token volatility. Equation (27) shows that, in this region, when the platform accumulates more reserves and becomes less risk-averse, it absorbs risk



**Figure 3: Token Volatility and Platform Risk Aversion.** This figure plots token return volatility  $\sigma^P(C)$  in Panel A and the effective risk aversion  $\gamma(C)$  in Panel B. In Panel A, the red dashed line marks  $\tilde{C}$  (in Proposition 3). The red dotted lines in both panels mark  $\bar{C}$  (in Proposition 1). The parameterization follows Figure 2.

from users by tuning down  $\sigma_t^P$ , and when the platform exhausts its reserves, it off-loads the risk in its dollar revenues to users.<sup>43</sup> The platform and its users engage actively in risk-sharing when  $C \in (0, \tilde{C})$ . This is illustrated by the numerical solution in Panel A of Figure 3 with  $\tilde{C}$  marked by the dashed line. In Panel B, we show that the platform’s risk aversion declines in  $C$ . In this region of low reserves, transaction volume is pinned to the lowest level given by  $\underline{N}$  in (25).

Once the platform’s reserves surpass the critical threshold  $\tilde{C}$ , its risk aversion becomes sufficiently low and it optimally absorbs all the risk in its dollar revenues, setting  $\sigma^P(C)$  to zero which also implies that in this region  $\mu^P(C) = 0$ .<sup>44</sup> As a result, transaction volume starts to rise above the “hibernation level”,  $\underline{N}$ , as illustrated by Panel A of Figure 4. Therefore, reserves are absolutely essential for stimulating economic activities on a stablecoin platform.

Interestingly, even though the platform shelters its users from risk at any  $C > \tilde{C}$ , its risk aversion still shows up in  $N_t$  given by (28). As shown in (12), the choice of  $N_t$  is implemented through fees. Therefore, the intuition can be more easily explained when we substitute (28), the optimal  $N_t$ , and the optimal  $\sigma_t^P = 0$  (as well as  $\mu_t^P = 0$ ) into (12) to solve  $f_t$ : when  $\left(\frac{\xi A^{1-\xi}}{\gamma(C)\sigma^2}\right)^{\frac{1}{2-\xi}} < \bar{N}$ ,

$$f(C) = \left(\frac{A\gamma(C)\sigma^2}{\xi}\right)^{\frac{1-\xi}{2-\xi}} - r, \quad (29)$$

i.e., the platform charges higher fees to build up its reserves when its precautionary savings motive is strong ( $\gamma(C)$  is higher); when  $\left(\frac{\xi A^{1-\xi}}{\gamma(C)\sigma^2}\right)^{\frac{1}{2-\xi}} \geq \bar{N}$ , i.e.,  $C$  is sufficiently high such that  $\gamma(C)$  falls

<sup>43</sup>Equation (27) implies that the condition (24) is equivalent to  $\gamma(C)$  (or  $-V''(C)$ ) approaching infinity in the limit.

<sup>44</sup>This result arises because we express the equilibrium token price as a function of  $C$ , in that  $P_t = P(C_t)$ . Thus, token volatility and token returns can be expressed as functions of  $C$  too, in that  $\sigma_t^P = \sigma^P(C_t)$  and  $\mu_t^P = \mu^P(C_t)$ . Since  $\sigma^P(C) = 0$  for  $C > \tilde{C}$ ,  $P'(C)$  must be zero by Itô’s lemma (i.e.,  $P(C)$  is constant), implying  $\mu^P(C) = 0$ .

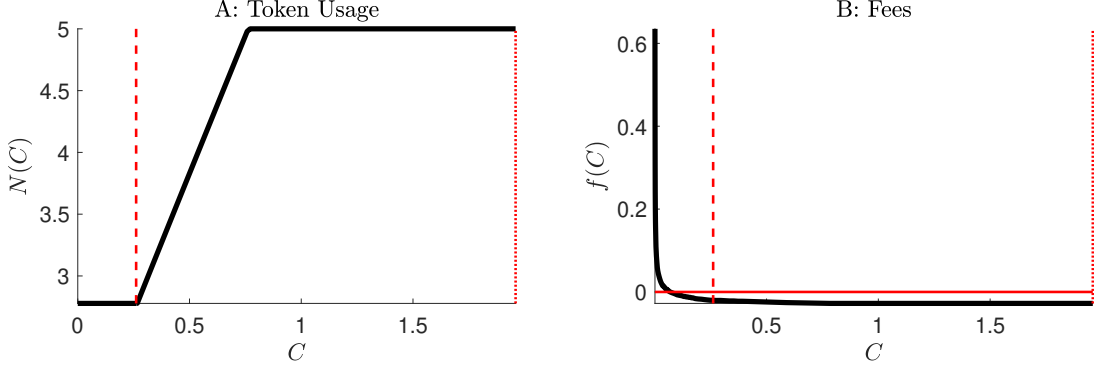


Figure 4: **Token Usage and Fees.** This figure plots token usage  $N(C)$  in Panel A and user fees  $f(C)$  in Panel B. The red dotted line marks  $\bar{C}$  (in Proposition 1). In both panels, the red dashed line marks  $\tilde{C}$  (in Proposition 3). In Panel B, the red solid line marks zero. The parameterization follows Figure 2.

below  $\frac{\xi A^{1-\xi}}{\sigma^2 \bar{N}^{2-\xi}}$ , the platform de-links the fees from its risk aversion,

$$f(C) = \left(\frac{A}{\bar{N}}\right)^{1-\xi} - r. \quad (30)$$

The platform faces a risk-return trade-off. The fees serve as a compensation for risk exposure but discourages users from participation. So when the platform’s risk aversion rises, it charges users more per dollar of transaction at the expense of a smaller volume. When the platform’s risk aversion declines, the fees decline while the total total usage increases. Once reserves are sufficiently high such that  $\gamma(C) \leq \frac{\xi A^{1-\xi}}{\sigma^2 \bar{N}^{2-\xi}}$ , the fees no longer decline with the platform’s risk aversion, as the platform has maxed out its transaction capacity, i.e.,  $N_t = \bar{N}$ , and it becomes impossible to further stimulate user participation. Likewise, when the platform’s reserves are below  $\tilde{C}$  and  $\sigma^P(C) > 0$ ,  $N_t = \underline{N}$ , and the fees are given by

$$f(C) = \left(\frac{A}{\underline{N}}\right)^{1-\xi} + \mu^P(C) - \eta \sigma^P(C) - r. \quad (31)$$

Even though the platform’s risk aversion is high, it no longer sacrifices transaction volume for higher fees because user participation has already fallen to a very low level. And, while user fees are high, expected token returns  $\mu^P(C)$  are high too, counteracting fees. The fact that one can earn positive expected returns through buying the stablecoin after debasement ceteris paribus boosts the demand for the stablecoin, driving up its price, and mirrors the stabilizing role of “arbitrageurs” for stablecoins (Lyons and Viswanath-Natraj (2020)).

Panel B of Figure 4 plots the numerical solution of optimal fees that decrease in excess reserves.

Depending on the parameters, fees can actually turn into user subsidies (i.e., fall below zero) when excess reserves are sufficiently high.<sup>45</sup> The next corollary summarizes the results on fees.

**Corollary 1 (Optimal Fees).** *Fees,  $f(C)$ , decrease in excess reserves,  $C$ . At  $C \in (0, \tilde{C})$ , where  $\tilde{C}$  is defined in Proposition 3, fees are given by (31). At  $C \in [\tilde{C}, \tilde{C}')$ , where  $\tilde{C}'$  is defined by  $\gamma(\tilde{C}') = \frac{\xi A^{1-\xi}}{\sigma^2 N^{2-\xi}}$ , fees are given by (29). At  $C \in [\tilde{C}', \bar{C})$ , where  $\bar{C}$  is defined in Proposition 1, fees are given by (30).*

When  $C$  is below  $\tilde{C}$ , an interesting implication of (31) is that the platform charges (compensates) users the expected appreciation (depreciation) of tokens over risk-free rate (i.e.,  $\mu_t^P - r$  shows up in  $f_t$ ). To fully solve the fees, we need to know both  $\gamma(C_t)$  and the function  $\mu_t^P = \mu^P(C_t)$ . In fact, the platform's choice of  $\sigma_t^P = \sigma^P(C_t)$  already pins down the function of token price,  $P_t = P(C_t)$ , so  $\mu^P(C_t)$  can be obtained from Itô's lemma. Next, we solve  $P_t = P(C_t)$  from the function  $\sigma^P(C_t)$ . By Itô's lemma,

$$\sigma^P(C) = \frac{P'(C)}{P(C)} N(C) (\sigma - \sigma^P(C)) , \quad (32)$$

where  $N(C) (\sigma - \sigma^P(C))$  is the diffusion of state variable  $C$ . Using Proposition 2, we solve the value function  $V(C)$  and obtain  $\gamma(C)$ . Using Proposition 3, we obtain the functions  $\sigma^P(C)$  and  $N(C)$ . Plugging  $\sigma^P(C)$  and  $N(C)$  into (32), we obtain a first-order ODE for the function  $P(C)$ .

To uniquely solve the function  $P(C)$ , we need to augment the ODE (32) with a boundary condition. In our model, both the platform and users are not concerned with the level of token price and only care about the expected token return,  $\mu_t^P$ , and volatility,  $\sigma_t^P$ . Therefore, we have the liberty to impose the following boundary condition:

$$P(\bar{C}) = 1 . \quad (33)$$

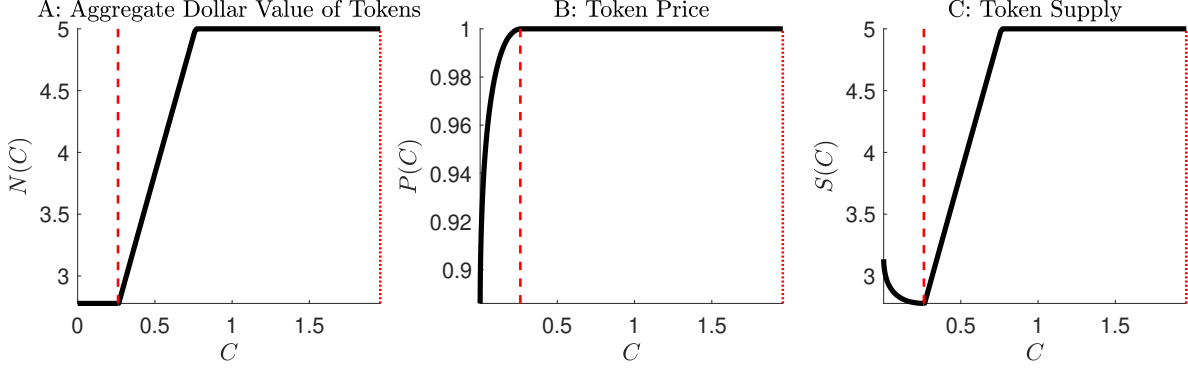
i.e., the platform sets an exchange rate of one dollar for one token when  $C_t$  reaches  $\bar{C}$ . The next corollary states the solution of token price as solution to a first-order ODE problem.

**Corollary 2 (Solving Equilibrium Token Price).** *Given the solutions of  $V(C)$  (and  $\gamma(C)$ ) from Proposition 2 and  $\sigma^P(C)$  and  $N(C)$  from Proposition 3, the equilibrium dollar price of token,  $P(C)$ , is a function of  $C$  that solves the ODE (32) under the boundary condition (33).*

Proposition 3 states that, once  $C$  crosses above the critical threshold  $\tilde{C}$ ,  $\sigma^P(C) = \mu^P(C) = 0$ ,

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<sup>45</sup>Specifically, under the particular parameterization, the condition is for fees to turn into subsidies near  $\bar{C}$  is that  $\frac{A^{1-\xi}}{N^{1-\xi}} < r$  where we use (30) and the fact that  $\mu^P(C) = 0$  for  $C \in (\tilde{C}, \bar{C})$  (to be discussed later in this section).



**Figure 5: Token Price and Quantity Dynamics.** This figure plots the aggregate dollar value of tokens  $N(C)$  in Panel A, token price  $P(C)$  in Panel B, and token supply  $S(C)$  in Panel C. The red dotted lines in all panels mark  $\bar{C}$  (defined in Proposition 1). The red dashed lines in all panels mark  $\tilde{C}$  (defined in Proposition 3). The parameterization follows Figure 2.

which, by Itô's lemma, implies that  $P'(C) = 0$ . Therefore, if the platform's reserves are sufficiently high, it optimally fixes the dollar price (or the redemption value by no arbitrage) of token at  $P(C) = 1$ . When  $C$  falls below  $\tilde{C}$ , (32) implies that  $P'(C) > 0$  (because  $\sigma^P(C) \in (0, \sigma)$  in Proposition 3) so token price comoves with the platform's excess reserves.

The endogenous transition between  $P_t = 1$  and  $P_t < 1$  happens as the platform accumulates or depletes reserves through various activities laid out in (8) (and then in (16)), including the platform's issuance of new tokens ( $dS_t > 0$ ), users' token redemption ( $dS_t < 0$ ), fee revenues, and shocks to the dollar reserves. The platform's choice of token price is optimally chosen and thus credible in the sense that it the platform does not have any incentives to deviate.

**Proposition 4 (Endogenous Token Price Regimes).** *At  $C \in [\tilde{C}, \bar{C}]$ , where  $\tilde{C}$  is defined in Proposition 3, the platform maintains token price equal to one, i.e.,  $P(C) = 1$ . At  $C \in (0, \tilde{C})$ , token price comoves with the platform's excess reserves (i.e.,  $P'(C) > 0$ ).*

Proposition 4 states that token price stays at one if and only if the platform holds a sufficiently large amount of *excess* reserves ( $C > \tilde{C}$ ). When excess reserves fall below  $\tilde{C}$ , the platform optimally debases its tokens. By allowing token price to comove with its excess reserves, the platform off-loads the risk in its dollar reserves to users and thereby prevents liquidation.

Figure 5 plots the numerical solutions of aggregate token value,  $N(C) = S(C)P(C)$  (Panel A), token price  $P(C)$  (Panel B), and the total quantity of tokens  $S(C)$  implied by  $N(C)$  and  $P(C)$  (Panel C). The dashed line marks  $\tilde{C}$ . The platform implements the optimal token price through the manipulation of token supply. When the platform has enough reserves to credibly sustain



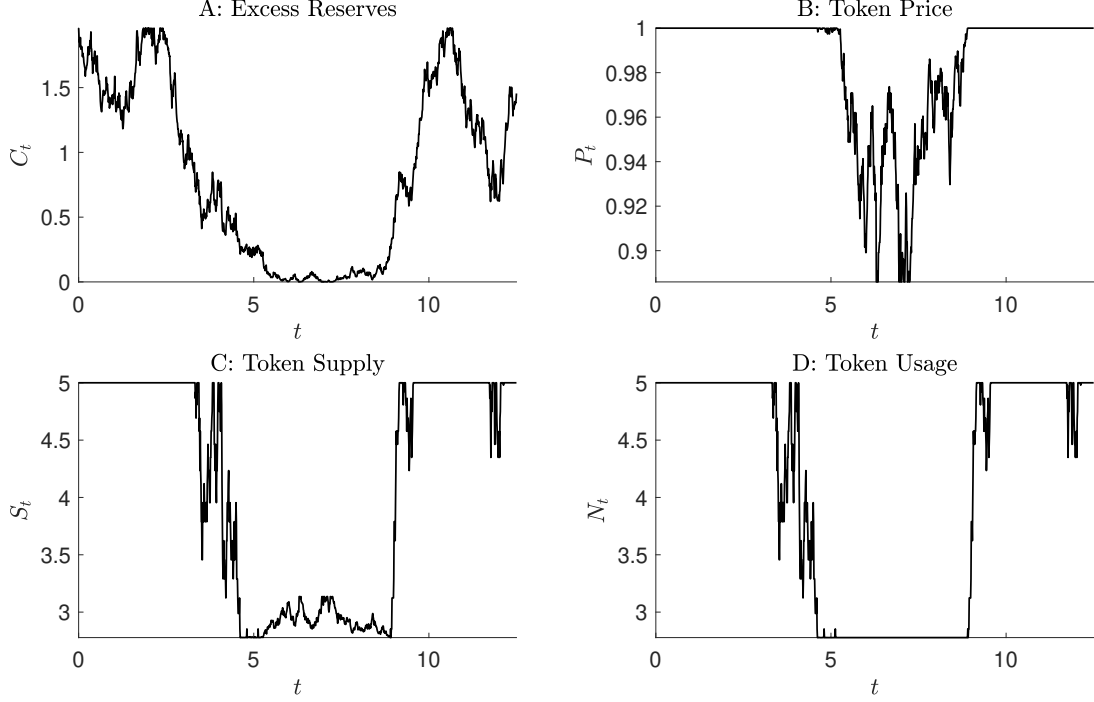


Figure 6: **The Instability Trap in Simulation.** Using the numerical solutions, we simulate a path of excess reserves (Panel A), token price (Panel B), token supply (Panel C), and token usage (Panel D). The horizontal axis records the number of years. The parameterization follows Figure 2.

$P_t = 1$  (i.e.,  $C > \tilde{C}$ ), token supply comoves with demand so that  $P(C)$  is fixed at one. Below  $\tilde{C}$ , a decrease of reserves triggers the platform to supply more tokens in exchange for dollars that replenish reserves. The users respond to token debasement by reducing their token demand to  $\underline{N}$ , which in turn reinforces the debasement by reducing the platform’s revenues from open market operations (i.e.,  $dS_t > 0$ ) and fees. The system falls into an instability trap.

In practice, stablecoin platforms often claim commitment to maintain a stable token price and substantiate their claims by holding reserves that cover token liabilities. However, such a claim is only credible (incentive-compatible) if the excess reserves are sufficiently high; otherwise, as shown in Proposition 4, it is in the platform’s interest to debase its tokens.

Using parameters in Figure 2 and numerical solutions, we simulate in Figure 6 a path of  $C_t$  (Panel A), token price  $P_t$  (Panel B), token supply  $S_t$  (Panel C), and transaction volume  $N_t$  (Panel D). The horizontal axis records the number of years. In the first three years, in spite of the volatility in  $C_t$ , the platform manages to sustain a stable token price, and with the transaction volume (or token demand) at the full capacity at  $\bar{N}$ , a fixed dollar price of token implies a fixed token supply. Following a sequence of negative shocks between the third and fourth years, the platform raises

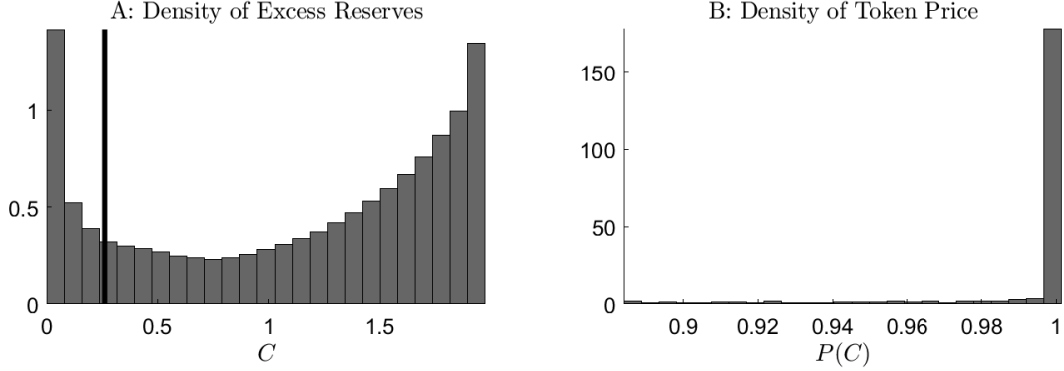


Figure 7: **Long-Run Dynamics and Stationary Density.** We plot stationary probability densities of excess reserves  $C$  (Panel A) and token value  $P(C)$  (Panel B) in numerical solutions. The solid black line in Panel A depicts  $C = \tilde{C}$ . The parameterization follows Figure 2.

fees. Users respond by reducing their token demand  $N_t$ , so the platform reduces token supply, maintaining  $P_t = 1$ . The platform optimally trades off replenishing dollars reserves by raising fees and using dollar reserves in token buy-back. As more negative shocks hit between the fourth and ninth years, the platform gives up the peg and off-loads risk to users through the fluctuation of token price. Users' token demand hits  $\underline{N}$ , and the platform starts actively expanding token supply in exchange for dollar revenues. Then following a sequence of positive shocks, the recovery started in the ninth year, and by the tenth year, the platform finally restores token price back to one.

We demonstrate the long-run dynamics of the model in Figure 7. Panel A plots the stationary probability density of excess reserves. It shows how much time over the long run the platform spends in different regions of  $C$ . The distribution is bimodal. The concentration of probability mass near  $C = 0$  is due to the fact that, when the transaction volume (or token demand) gets stuck at the hibernation level  $\underline{N}$ , the platform can only grow out of this region very slowly by accumulating reserves through fee revenues and proceeds from expanding token supply. The platform also spends a lot of time near the payout boundary  $\bar{C}$  as this is a stable region where, given a sufficiently high level of reserve buffer, shocks' impact is limited. In Panel B, we show that, even though  $P_t = 1$  seems to be the norm, the system exhibits significant risk of token debasement ( $P(C) < 1$ ), so the stationary probability density of token price has a very long left tail.

## 4.2 Discussion: Instability mechanism, liquidation, and runs

As the previous section has shown, token price volatility rises and the price falls below its peg, because the platform allows token price to float so to prevent liquidation as  $C$  approaches zero.

This instability mechanism can be interpreted as follows: The impending threat of a run, that occurs when  $C$  falls below zero and triggers liquidation, causes instability and debasement when excess reserves dwindle. Importantly, the platform is better-off debasing token price and sharing risk with users than incurring the risk of a run and liquidation.<sup>46</sup>

The extent to which users are willing to bear token price risk is governed by the parameter  $\eta$ . In the limit case  $\eta \rightarrow \infty$ , users are unwilling to bear any token price risk, so there is no price volatility (i.e.,  $\sigma^P(C) = 0$  for  $C > 0$ ) and the platform is unable to prevent liquidation (i.e., a run), leading to  $V(0) = 0$ ,  $\underline{N} = 0$ , and  $P(0) = 0$ . In a similar vein, when  $\eta$  is large, the equilibrium dynamics resemble the one of a stablecoin that maintains its peg until there is a run at  $C \rightarrow 0$ : There is no price volatility for most of the time (i.e.,  $C > 0$ ), but user token holdings and token price drop sharply once  $C$  approaches zero. As  $\lim_{\eta \rightarrow \infty} \mu_C(0) = 0$ , the state  $C = 0$  is absorbing and there is effectively no more recovery possible when  $\eta$  is large.

Next, we emphasize that our analysis and the findings remain largely unchanged when we consider a different liquidation threshold  $\underline{C}$ . Appendix B.6 presents a model variant that allows under-collateralization (i.e.,  $\underline{C} < 0$ ) and show that, indeed, the dynamics are similar. As we show, token price volatility rises and token price falls below the peg as  $C$  approaches  $\underline{C}$ . In particular, through risk-sharing with users, the platform is able to prevent liquidation.

Finally, we provide some alternative motivation for the “run” threshold  $\underline{C} = 0$ .<sup>47</sup> We argue that with publicly observable  $C_t$ , if a run occurs at finite time  $\tau$ , this run must occur in state  $C_\tau = 0$ . For this sake, consider the threshold strategy that all users run and sell all of their token holdings to the platform (i.e., redeem) once  $C_t = \underline{C} < 0$ .<sup>48</sup> At the time of the run  $\tau$  when  $C_\tau = \underline{C}$ , users sell all of their token holdings to the platform (i.e., users redeem tokens) and the platform pays in total  $M_\tau$  dollars to users. We consider two scenarios. First, all “redemptions” are executed at the same price. Since platform reserves equal  $M_\tau$  dollars, token price at liquidation reads  $P_\tau = M_\tau/S_\tau$ , contradicting  $C_\tau < 0$ .<sup>49</sup> Second, we consider “first come, first served:” That is, users can redeem at price  $P_\tau$  until the platform runs out of reserves. Then, some users end up empty-handed upon redemption. But, in this case, users facing the risk of not being able to redeem, have incentives to

<sup>46</sup>If the platform reserves were exposed to sufficiently large Poisson shocks, it likely would not be able to stave off liquidation.

<sup>47</sup>A similar line of arguments as below implies that if the platform is liquidated, then liquidation occurs at  $C \geq 0$ . Upon liquidation at time  $\tau$ , token price is zero,  $P_\tau = 0$ . As such,  $C_\tau = M_\tau - S_\tau P_\tau \geq 0$ .

<sup>48</sup>Clearly, there is some strategic complementarity in redemption decisions, supporting such an equilibrium, as in [Diamond and Dybvig \(1983\)](#): If all users sell their token holdings at time  $\tau$ , the platform is liquidated and token price becomes zero thereafter. As such, any individual user finds it optimal to sell at time  $\tau$  as well instead of waiting until after liquidation when tokens have become worthless.

<sup>49</sup>Also note that price is continuous, in that  $\lim_{t \rightarrow \tau} P_t = P_\tau$ .

front-run and to sell/redeem their tokens “just before” time  $\tau$ , i.e., “just before”  $C_t$  reaches  $\underline{C}$ , a contradiction. This unravelling continues to the point that users redeem at  $C = 0$ . As such, we conclude that if users followed the strategy to run once  $C_t = \underline{C}$ , then  $\underline{C} = 0$ .<sup>50</sup>

### 4.3 The Optimal Issuance of Equity and Governance Tokens

In this subsection, we take an excursion to analyze the issuance of platform equity (or “governance tokens”). In practice, governance tokens are popular among stablecoin platforms as means of financing. For example, MakerDAO, the issuer of DAI (one of the top five stablecoins by market value), introduced its governance tokens MKR. MKR holders vote on protocol changes and receive payout via buy-backs, just like stock market investors receive payout through share repurchases. In other instances, the stablecoin issuer raises new equity by tapping into VC financing, i.e., equity financing need not take the form of governance token issuance but can also represent traditional forms of equity financing for early-stage firms (e.g., VC financing).

So far, the platform recovers from the low- $C$  region through the accumulation of internal funds. We now allow the platform to raise dollar funds by issuing equity shares subject to a fixed financing cost,  $\chi_0$ , and a proportional cost,  $\chi_1$ , following [Bolton et al. \(2011\)](#) or [Gao, Whited, and Zhang \(2020\)](#). To characterize the optimal issuance policy, we first note that when issuing equity, the platform raises funds so that  $C$  jumps from the (lower) issuance boundary, denoted by  $\underline{C}$ , to an interior value  $\hat{C}$  where  $V'(\hat{C}) = 1 + \chi_1$ , that is when the issuance amount is  $\hat{C} - \underline{C}$ , the marginal contribution of equity issuance to platform value is equal to the marginal cost of issuing new shares. Note that  $\hat{C} \leq \bar{C}$  (the payout boundary) because  $V'(\hat{C}) = 1 + \chi_1 \geq V'(\bar{C}) = 1$  under the concavity of  $V(C)$ . Next, we show that the platform issues equity only when  $C$  falls to zero (i.e.,  $\underline{C} = 0$ ).

Consider the *change* of existing shareholders’ value after equity issuance:  $[V(\hat{C}) - (\hat{C} - \underline{C}) - \chi_0 - \chi_1(\hat{C} - \underline{C})] - V(\underline{C})$ . To obtain the post-issuance value of existing shareholders, we deduct the competitive new investors’ equity value (equal to the funds they invested),  $(\bar{C} - \underline{C})$ , and deduct the fixed and proportional issuance costs from the total platform value post-issuance,  $V(\hat{C})$ . To calculate the change, we subtract  $V(\underline{C})$ , the value without issuance. Taking derivative with respect to  $\underline{C}$ , we obtain  $1 + \chi_1 - V'(\underline{C}) < 0$  for  $\underline{C} < \hat{C}$  because  $V'(\underline{C}) > V'(\hat{C}) = 1 + \chi_1$  under the concavity

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<sup>50</sup>Conversely, when  $C_t$  is perfectly observable, any equilibrium with a run in state  $\hat{C} < 0$  must involve randomization, i.e., there is a run in state  $\hat{C}$  with endogenous intensity  $\delta$ . Alternatively, following [Morris and Shin \(1998\)](#), [Goldstein and Pauzner \(2005\)](#) or [He and Xiong \(2012\)](#), one could adopt a global games approach and assume that users observe only noisy signal about the state  $C_t$ : This set of assumptions may possibly support a negative run threshold. The study of equilibria with randomization or a global games based run mechanism are beyond the scope of the paper.

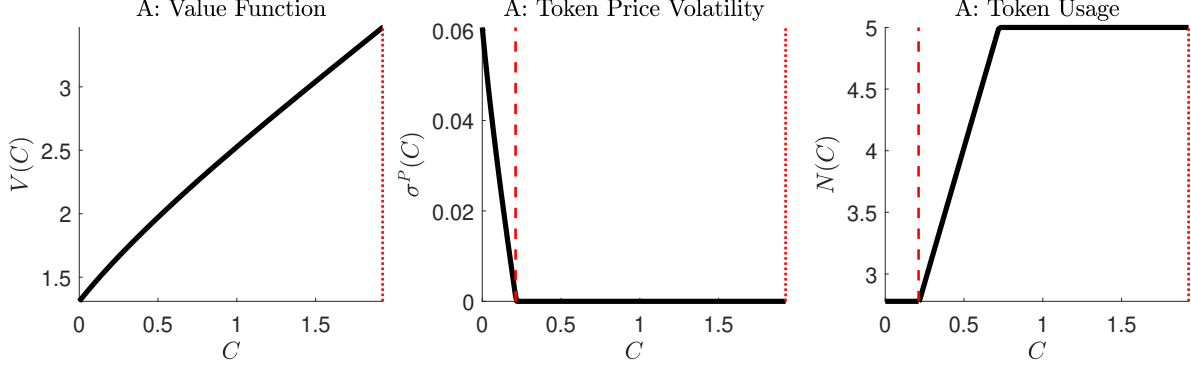


Figure 8: **Solution with Costly Equity Issuance.** In this figure, we plot the valuation function (Panel A), token price volatility (Panel B), and token demand or users' transaction volume (Panel C). The parameterization follows Figure 2, and we set  $\chi_0 = 1$  and  $\chi_1 = 0.1$ . The curves start at  $C = 0$ , the equity issuance boundary.

of  $V(C)$ .<sup>51</sup> Therefore, the platform prefers  $\underline{C}$  to be as low as possible and optimally sets it to zero.

Finally, as in the baseline model, the platform can avoid liquidation by off-loading risk to users, as shown in (21), and obtain the value given by (24). Therefore, as  $C$  approaches zero, the platform only opts for recapitalization at  $C = 0$  if recapitalization generates a higher value than liquidation. Accordingly, the lower boundary condition (24) for the value function is modified to

$$\lim_{C \rightarrow 0^+} V(C) = \max \left\{ \lim_{C \rightarrow 0^+} \frac{V'(C)}{\rho} \left\{ \underline{N}^\xi A^{1-\xi} - \eta \underline{N} \sigma \right\}, V(\hat{C}) - \hat{C} - \chi_0 - \chi_1 \hat{C} \right\}, \quad (34)$$

The first term in the max operator is the value obtained from off-loading risk to users, given by (24). The second term is the post-issuance value for existing shareholders. The results in Proposition 1 to 2, 3 and Corollary 1 still hold except that the boundary condition (24) is replaced by (34) with  $\hat{C}$  determined by  $V'(\hat{C}) = 1 + \chi_1$  as previously discussed.

**Proposition 5 (Optimal Equity Issuance).** *The platform raises external funds through equity issuances only if  $C_t = 0$  and when  $V(\hat{C}) - \hat{C} - \chi_0 - \chi_1 \hat{C} > \lim_{C \rightarrow 0^+} \frac{V'(C)}{\rho} \left\{ \underline{N}^\xi A^{1-\xi} - \eta \underline{N} \sigma \right\}$  (see (34)), where the optimal issuance amount  $\hat{C}$  is given by  $V'(\hat{C}) = 1 + \chi_1$ .*

Figure 8 illustrates the solution through a numerical example of costly equity issuance under our baseline parameters and  $\chi_0 = 1$  and  $\chi_1 = 0.1$ . The platform optimally raises new equity when  $C$  falls to zero. Notably, there are three lines of defense to avoid liquidation: i) platform (excess) reserves  $C$ , ii) debasement  $\sigma^P > 0$ , and iii) equity issuance. Under these parameter values, our model suggests a pecking order. First, the platform covers negative shocks by drawing on its

<sup>51</sup>To prove the concavity of value function stated in Proposition 1, we only need the HJB equation (22) and the upper boundary conditions (18) and (19), so recapitalization does not affect value function concavity.

reserves. Second, the platform debases token price to share risk with users. Third, the platform raises equity and incurs the issuance costs. If the equity issuance costs are sufficiently large, the platform does not raise equity and the solution is the one from the baseline without equity issuance. If, on the other hand, the equity issuance costs are sufficiently small, the platform may not debase the token prior to issuing equity and thus token price remains stable at all times.

Suppose that the equity issuance costs are such that the platform i) recapitalizes once  $C$  reaches zero and ii) debases token price prior to equity issuance. Note that when recapitalization happens,  $C_t$  jumps from zero to  $\widehat{C}$ , which then implies an upward jump in the token demand from  $N(0)$  to  $N(\widehat{C})$  (for  $N(0) < N(\widehat{C})$ , see Proposition 3). If the platform does not adjust the token supply,  $S_t$ , there will be an upward *predictable* jump in  $P_t$ , which implies an arbitrage opportunity. To preclude arbitrage, the platform must expand token supply simultaneously as it issues equity so that the token price stays at the pre-issuance level.<sup>52</sup> Let us revisit the results on the token price level in Corollary 2 and Proposition 4. Let  $P_j(C)$  denote the token price function after the  $j$ -th recapitalization. We have

$$P_j(\widehat{C}) = P_{j-1}(0), \quad (35)$$

which replaces (33) as the boundary condition for the price-level ODE (32). Token price level before the first recapitalization,  $P_0(C)$  is still solved under the boundary condition (33), i.e.,  $P_0(\overline{C}) = 1$ .

**Corollary 3 (Recapitalization and Token Price Level).** *Token price after the  $j$ -th recapitalization is solved by the ODE (32) subject to the boundary condition (35).*

In the baseline model without equity issuance, the debasement of token is temporary: token price level falls below 1 when  $C$  falls below  $\widetilde{C}$  due to negative shocks and it recovers back to 1 when the platform accumulates sufficient amount of dollar revenues so that  $C$  crosses above  $\widetilde{C}$  (Proposition 4). When recapitalization happens, the debasement is permanent. After the  $j$ -th recapitalization, token price level starts anew at a lower peg,  $P_j(\widehat{C}) = P_{j-1}(0)$ , and if negative shocks deplete the platform's reserves and triggers another recapitalization, token price level declines along the process and, right after recapitalization, stabilizes at an even lower peg,  $P_{j+1}(\widehat{C}) = P_j(0)$ .

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<sup>52</sup>Note that the expansion of token supply brings dollars of equal value into the reserve portfolio. This simultaneous expansion of assets and liabilities does not imply any variation in the excess reserves  $C_t$  at  $\overline{C}$ .

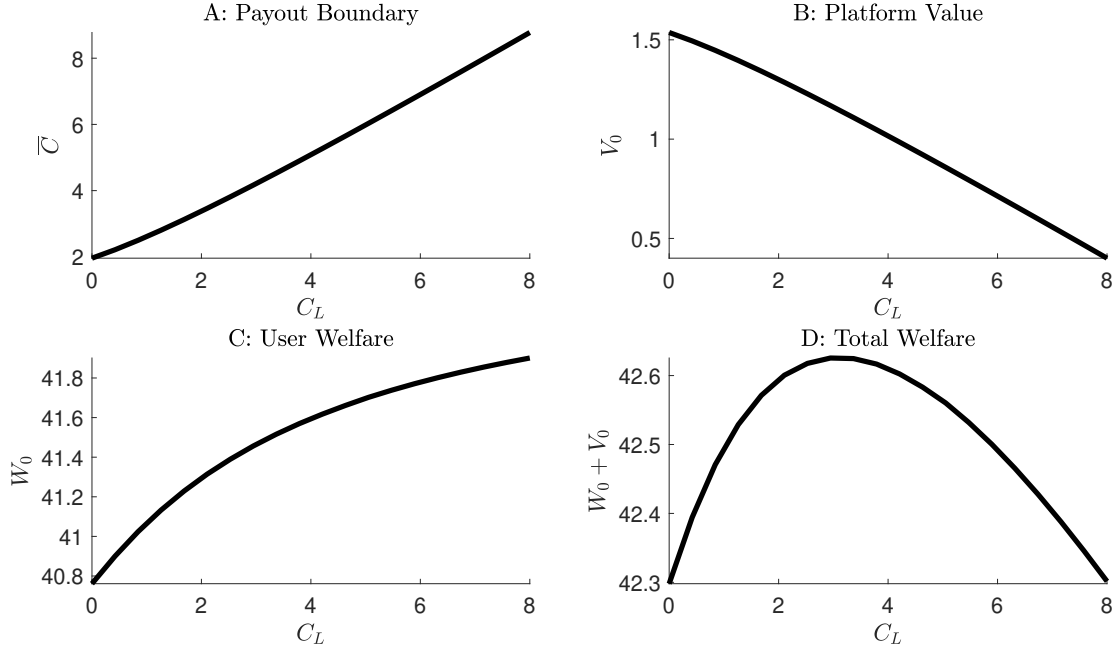


Figure 9: **Capital Requirement and Welfare.** We plot the numerical solutions of payout boundary  $\bar{C}$  (Panel A), the platform shareholders’ value at  $t = 0$ ,  $V_0$  (Panel B), users’ welfare at  $t = 0$ ,  $W_0$  (Panel C), and total welfare  $W_0 + V_0$  (Panel D) over the regulatory minimum of excess reserves,  $C_L$ . The parameterization follows Figure 2.

## 5 Regulating Stablecoins

We analyze three types of regulations. The first type, which is of our focus, stipulates a minimum level of excess reserves (“capital requirement”). The rationale behind is to generate a sufficient risk buffer so that the issuer is unlikely to debase the token. The second type (“stability regulation”) is more direct. It forces the platform to keep the token price fixed. The third type stipulates the riskiness of reserve assets. Our conclusion is that capital requirement, if carefully designed, improves welfare. Stability regulation, in contrast, destroys the economic surplus from risk-sharing, and regulating the riskiness of reserve assets may backfire and destabilizes token price due to the platform endogenous response in its reserve management decisions.

### 5.1 Capital Requirement

The regulator requires  $C \geq C_L$  and forces the platform to liquidate if the requirement is violated. Therefore,  $C_L$  replaces zero as the lower (liquidation) bound of excess reserves.<sup>53</sup> In Figure 9, we plot the payout boundary  $\bar{C}$  (Panel A), which is a measure of voluntary over-collateralization, and

<sup>53</sup>Because the stablecoins are over-collateralized so that coordination failure (or run) does not happen, unlike Carletti, Goldstein, and Leonello (2019), our model does not feature a need to introduce a separate liquidity requirement.

the welfare measures for different values of  $C_L$ . Not so surprisingly, when the capital requirement tightens, the whole region of excess reserves is pushed to the right, resulting in a higher payout boundary  $\bar{C}$  in Panel A. Because reserves earn an interest rate  $r$  that is below the shareholders' discount rate  $\rho$ , the platform shareholders' value,  $V_0$ , declines in  $C_L$ , as shown in Panel B. Panel C shows that users' welfare is improved by the capital requirement but there exists a significant degree of decreasing return as the regulator pushes up  $C_L$ . Appendix B.4 shows how we calculate user welfare,  $W_0 = \mathbb{E} [\int_0^\infty e^{-rt} dR_{i,t}]$ .

What is interesting is that, in Panel D of Figure 9, the total welfare is non-monotonic in  $C_L$ . When the regulator increases  $C_L$  from zero, the increase of users' welfare overwhelms the decrease of platform value, but as the capital requirement is tightened, the loss of platform value eventually dominates. This suggests the existence of an optimal level of  $C_L$  that maximizes the total welfare.

As long as the users' welfare increases faster than the platform value decreases, the regulator can administer a transfer from users to the platform, making the regulation Pareto-improving. For example, the regulator can allow the platform to charge users a membership fees, i.e., a fixed cost of access, and imposes a cap on such fees. This type of access fees is commonly seen in the literature on regulation of utility networks (Laffont and Tirole, 1994; Armstrong, Doyle, and Vickers, 1996).

In Figure 10, we further demonstrate the stabilization effects of capital requirement. In Panel A, we plot the ratio of  $\bar{C} - \tilde{C}$  to  $\bar{C} - C_L$  that measures the size of the stable subset of  $C$  where the platform maintains  $P(C) = 1$ . As  $C_L$  increases, the stable region enlarges. In Panel B, we plot the probability of  $C > \tilde{C}$  (i.e.,  $\sigma^P(C) = 0$ ) based on the stationary distribution of  $C$ , which shows that over the long run the platform spends more time in the stable region when  $C_L$  increases. In Panel C, we plot the long-run average value of  $\sigma_t^P$  using the stationary probability distribution. A declining pattern emerges, indicating that capital requirement is indeed effective in reducing the token volatility. In Panel D, we plot the expected number of years it takes to reach  $\tilde{C}$  from  $C_L$  (denoted by  $\tau(C_L)$ ). In Appendix B.1, we show how to calculate  $\tau(C_L)$ . This recovery time decreases when the capital requirement is tightened. As  $C_L$  increases, the platform at  $C_L$  still has abundant cash that can self-accumulate by via the interests earned on reserve holdings.

## 5.2 Stability Regulation

A key difference between stablecoins and bank deposits is that the issuers of stablecoins do not have any obligations to maintain  $P_t = 1$  while a large portion of bank deposits offer redemption at par through the deposit insurance mechanism and various regulatory backstops. Should stablecoins be



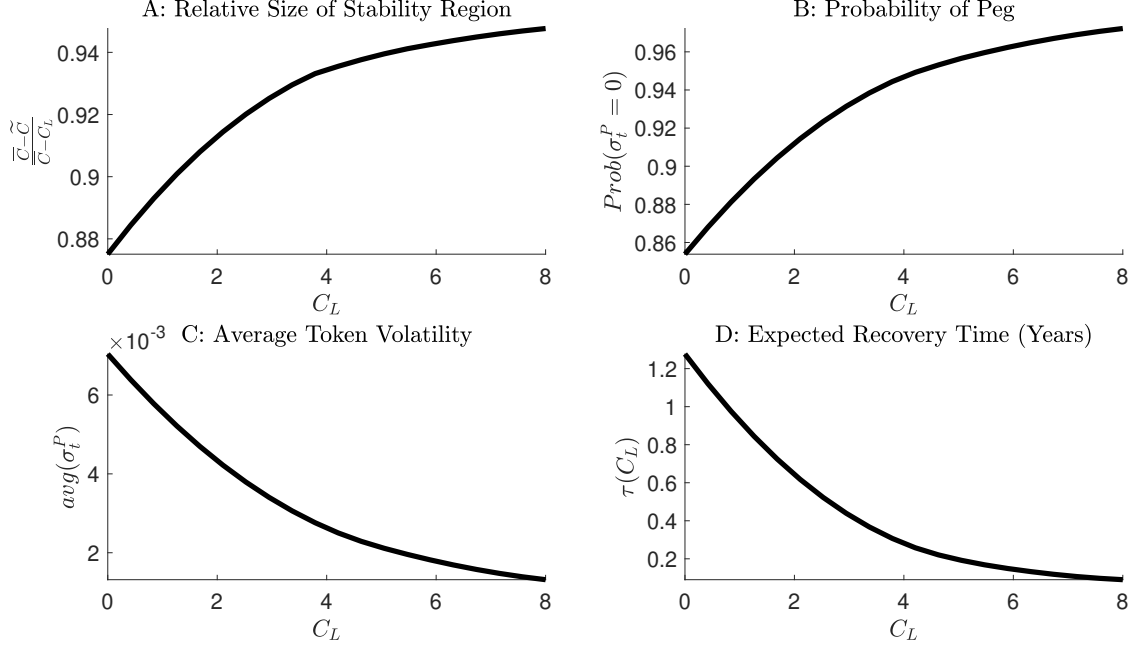


Figure 10: **Capital Requirement and Token Stability.** Using numeric solutions under different values of  $C_L$ , we plot the fraction of state space with  $P_t = 1$ , i.e.,  $\frac{\tilde{C}-C}{\tilde{C}-C_L}$  (Panel A), stationary probability of zero token volatility (Panel B), the long-run average (based on stationary probability density) of token volatility (Panel C), and the expected time to reach  $\tilde{C}$  from  $C_L$  (Panel D). The parameterization follows Figure 2.

more like regulated deposits and be legally required to maintain a perfectly stable value?

In Panel A of Figure 12, we show that under the zero-volatility requirement (i.e.,  $\sigma_t^P = 0$ ), the platform maintains a higher level of excess reserves to reduce the likelihood of liquidation because the option of off-loading risk to users is no longer available. Holding more reserves with an interest rate below the shareholders' discount rate reduces the platform value (see Panel B of Figure 12).

An interesting finding is that imposing the stability regulation even decreases users' welfare (Panel C of Figure 12) across all values of  $\eta$ . This seems to contradict the intuition that, by forcing the platform to maintain a perfectly stable token value, users will benefit, especially when they are more risk-averse. However, the argument ignores that, unable to off-load risk to users, the platform can compensate its risk exposure with higher fees and, in addition, there is liquidation ("run") when  $C = 0$ , causing the stablecoin to fail. Intuitively, while stability regulation precludes price volatility when excess reserves are positive, it also raises the likelihood that a run occurs, which causes eventual failure of the stablecoin and reduces welfare.

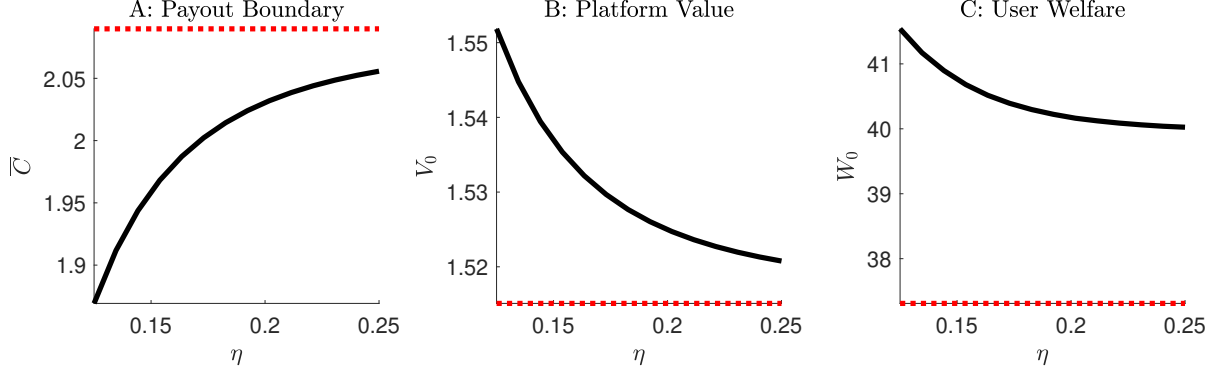


Figure 11: **Risk-Sharing, Stability Regulation, and Welfare.** Using the numerical solutions, we calculate the payout boundary  $\bar{C}$  (Panel A), the platform shareholders’ value at  $t = 0$ ,  $V_0$  (Panel B), users’ welfare at  $t = 0$ ,  $W_0$  (Panel C), and the long-run average fees based on stationary probability density (Panel D) over different values of users’ risk aversion  $\eta$  for both the baseline model (solid line) and the model under stability regulation (red dotted line). The parameterization follows Figure 2.

### 5.3 Regulating the Riskiness of Reserve Assets and Volatility Paradox

Another regulatory option is to restrict the riskiness of reserve assets. Many are concerned about the riskiness of reserve assets held by major stablecoin issuers as we discussed in Section 2. To analyze such regulation, we extend our model to incorporate a risk-return trade-off in reserve assets:

$$dM_t = rM_t dt + (P_t + dP_t)dS_t + N_t f_t dt + N_t \sigma dZ_t - dDiv_t + M_t(\hat{\mu} dt + \hat{\sigma} dZ_t), \quad (36)$$

where  $\hat{\mu}, \hat{\sigma} \geq 0$ . Relative to (8), the law of motion has an additional term that reflects the expected excess return on reserve holdings and the associated additional risk exposure. In Appendix B.2, we solve this extension. Regulating the riskiness of reserve assets corresponds to stipulating  $\hat{\sigma}$ . Therefore, to analyze its effects, we conduct comparative statics with respect to  $\hat{\sigma}$ . As we vary  $\hat{\sigma}$ , we change  $\hat{\mu}$  accordingly while fix the Sharpe ratio at a constant  $\omega \equiv \hat{\mu}/\hat{\sigma}$ .

In Figure 12, we report the results of two sets of comparative statics with  $\omega$  equal to 0 and 0.1 respectively.<sup>54</sup> For both cases, more reserve risk (i.e., a higher  $\hat{\sigma}$ ) is associated with a higher payout boundary  $\bar{C}$ . This suggests that forcing the platform to hold low-risk assets (i.e., lowering  $\hat{\sigma}$  through a regulatory mandate) reduces its incentive to hoard reserves. Moreover, as the platform endogenously responds to reduce reserves, token price becomes more volatile. In Panel B and C, we demonstrate such volatility paradox: Riskier reserves (i.e., a higher  $\hat{\sigma}$ ) is associated with more

<sup>54</sup>Throughout our analysis, the parameters must satisfy the condition  $r + \hat{\mu} < \rho$  to preclude a solution in which the platform indefinitely delays dividend payouts because returns on excess reserves exceed the shareholders’ discount rate, a standard condition in dynamic liquidity management models (e.g., Riddick and Whited, 2009).

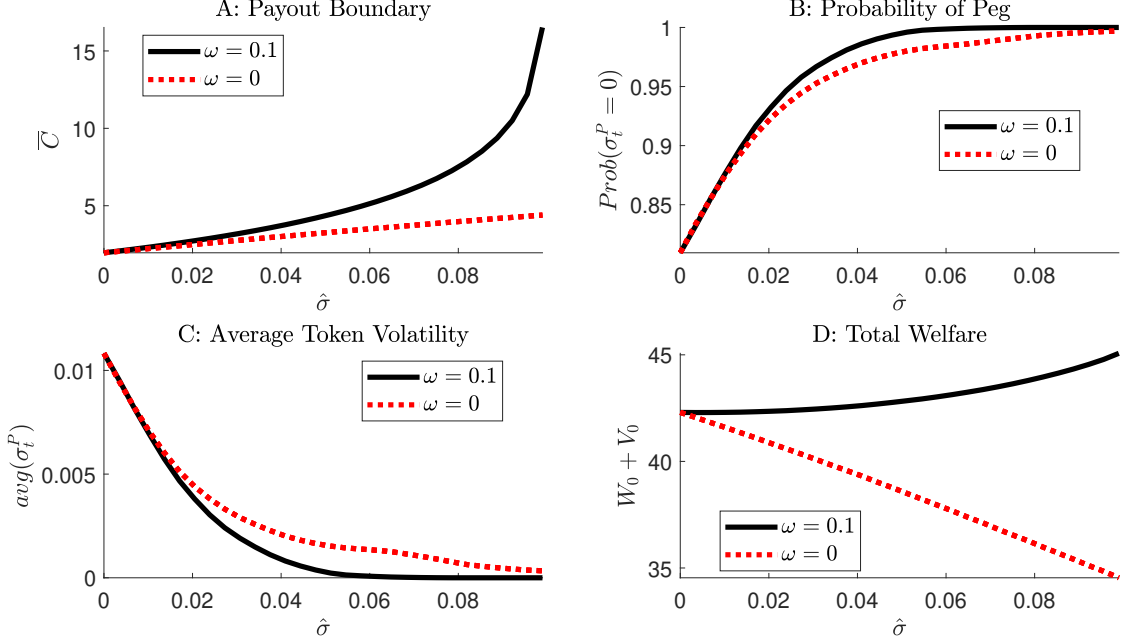


Figure 12: **Reserve Riskiness and Volatility Paradox.** Using the numerical solutions, we calculate the payout boundary  $\bar{C}$  (Panel A), the stationary probability of a fixed token price (Panel B), the average volatility  $\sigma_t^P$  (evaluated at the stationary distribution of  $C$ ), and the total welfare at  $t = 0$ ,  $V_0 + W_0$  (Panel D). The parameterization follows Figure 2. We set  $\hat{\mu} = \omega\mu$  and present solutions with  $\omega = 0.1$  (solid line) and  $\omega = 0$  (dotted line). We consider  $\hat{\sigma} \in [0.0999]$  so that the regularity condition  $r + \hat{\mu} < \rho$  is satisfied.

stable token price, reflected in both a higher probability of  $P_t = 1$  and a lower average  $\sigma_t^P$  (evaluated at the stationary distribution of  $C$ ). Therefore, mandating a decline in  $\hat{\sigma}$  destabilizes token price.<sup>55</sup>

The welfare implications of regulating the riskiness of reserve assets are ambiguous and depend on the Sharpe ratio  $\omega = \hat{\mu}/\hat{\sigma}$ . In Panel D, we show how the total welfare (the sum of platform value and users' value) varies with  $\hat{\sigma}$  under different Sharpe ratios. Under  $\omega = 0$ , total welfare declines in the riskiness of reserve assets, but the relation flips for  $\omega = 0.1$ . The intuition is simple. Under  $\omega = 0$ , an increase in  $\hat{\sigma}$  indicates more risk but no more return on reserve holdings. While token price becomes more stable thanks to the platform's build-up of precautionary savings, holding reserves is costly for the platform because the return is below shareholders' discount rate as in Riddick and Whited (2009).<sup>56</sup> The platform may charge higher fees on users to compensate the cost of holding more reserves, which offset the benefit of a more stable token price for users. As a result, both the platform's and users' welfare decline. Under  $\omega = 0.1$ , an increase in  $\hat{\sigma}$  brings in a

<sup>55</sup>The key to our mechanism of volatility paradox is the stablecoin issuer's response in liquidity hoarding decisions, which is different from the fire sale mechanism of volatility paradox in Brunnermeier and Sannikov (2014).

<sup>56</sup>As previously discussed, the wedge,  $\rho - r$ , can be microfounded with an exogenous Poisson-arriving liquidation with intensity  $\rho - r$ . The literature on agency cost of cash holdings also provides a rationale for why the return on liquidity holdings is below shareholders' discount rate (Nikolov and Whited, 2014; Nikolov, Schmid, and Steri, 2019).

higher expected return on reserve holdings, and with these additional revenues, the platform can reduce fees on stablecoin users and has more cash flows to buffer shocks, relying less on debasement to off-load risk to users. As a result, both parties' welfare can increase.

Our conclusion is two-fold. First, forcing the platform to hold low-risk assets triggers its response to maintain less reserves, which can destabilize the stablecoin. Second, the welfare implications depend on the risk-return trade-off.

## 6 Crypto Shadow Banking with User Collateral

The double-collateralization structure in Panel B of Figure 1 behind many stablecoins (e.g., DAI issued by MakerDAO) fits into our analytical framework. By requiring users to post collateral, the platform gains an additional degree of freedom (margin requirement). When setting the margin requirement, the platform faces the trade-off between reducing risk exposure and user participation.

For each dollar of stablecoins, the platform requires a user to post  $m_t$  dollars worth of collateral. In practice, many risky assets are eligible collateral, mainly cryptocurrencies such as Bitcoin and Ether, and thus are highly volatile. Let  $dZ_t$  denote a standard Brownian shock. Instead of interpreting it as a shock directly to reserves as in our baseline model, here we interpret the shock in the following way that is tied to the value of the user's collateral portfolio.

For simplicity, we do not model users' choice of collateral portfolio but rather assume that the collateral portfolio has a continuum of assets (indexed by  $a$ ) and, from  $t$  to  $t + dt$ , a fraction,  $2(\delta dt - \sigma dZ_t)$ , of these assets incur a percentage loss, denoted by  $\theta_a$ , which is drawn independently across assets  $a$  from a uniform distribution on  $[0, 1]$ .<sup>57</sup> At time  $t$ , the expected loss of the collateral portfolio is  $\mathbb{E} \left[ \frac{1}{2} \times 2(\delta dt - \sigma dZ_t) \right] = \delta dt$ , where the expected loss per asset,  $\frac{1}{2}$ , is multiplied by the fraction of assets in losses. The collateral portfolio also generates an expected value appreciation, denoted by  $\tilde{\mu}$ . Therefore, for each dollar of stablecoins, a user posts  $m_t$  dollars worth of collateral, with an expected net return equal to  $\tilde{\mu} - \delta - r$ , where the last term represents the cost of giving up the outside option of return  $r$  by locking wealth in the collateral portfolio.<sup>58</sup>

<sup>57</sup>Klimenko, Pfeil, Rochet, and Nicolo (2016) show that  $2(\delta dt - \sigma dZ_t)$  is the  $\Delta \rightarrow 0$  limit of a random variable whose value is  $2(\delta\Delta - \sigma\sqrt{\Delta})$  or  $2(\delta\Delta + \sigma\sqrt{\Delta})$  with equal probabilities. Before taking the limit, the parameters,  $\delta$  and  $\sigma$ , can be chosen so that the random fraction is well-defined within  $[0, 1]$ . The convergence is akin to that shown by Cox, Ross, and Rubinstein (1979) in their Binomial model of option pricing. In practice, the variation of a collateral asset (i.e.,  $\theta_a$  in our model) can be very large when the collateral is a cryptocurrency. What triggered the dramatic debasement of IRON was the almost 100% drop over two days of the collateral cryptocurrency, TITAN (Tiwari, 2021). The run on IRON in turn exacerbates the sell-off of TITAN.

<sup>58</sup>This expression is analogous to the user's cost of capital (Jorgenson, 1963) with the additional  $\tilde{\mu}$ .

Under the collateral requirement, a representative user  $i$ 's problem of choosing the optimal dollar value of stablecoin holdings,  $u_{i,t}$ , given by (5) in the baseline model, is now described below

$$\max_{u_{i,t}} \left\{ \frac{1}{\xi} u_{i,t}^\xi A^{1-\xi} dt + u_{i,t} (\mu_t^P - \eta |\sigma_t^P| - f_t) dt + u_{i,t} m_t (\tilde{\mu} - \delta - r) dt \right\}, \quad (37)$$

where the last term reflects the fact that the user's wealth is being locked in a risky collateral backing the stablecoins worth  $u_{i,t}$ . As in the baseline model, to solve the platform's optimal strategies, we first note that, given the token price dynamics (i.e.,  $\mu_t^P$  and  $\sigma_t^P$ ), the platform can directly set  $N_t$  through the fees  $f_t$ . Under the collateral requirement, users' optimal choice of  $u_{i,t}$  implies the following equation that connects  $N_t$  (i.e., the aggregated  $u_{i,t}$ ) and  $f_t$ :

$$f_t = \left( \frac{A}{N_t} \right)^{1-\xi} - m_t (r + \delta - \tilde{\mu}) + \mu_t^P - \eta |\sigma_t^P|. \quad (38)$$

Clearly, when  $m_t = 1$ ,  $\delta = 0$ , and  $\tilde{\mu} = 0$  (i.e., the platform does not impose a haircut and the collateral does not have expected losses or gains), equation (38) reduces to (12), the corresponding equation in the baseline model. Given  $f_t$ , a higher  $m_t$  leads to lower  $N_t$  according to (38), which reflects the fact that imposing a stricter collateral requirement leads to lower demand for stablecoins under the parameter restriction,  $r + \delta - \tilde{\mu} > 0$  (i.e., it is costly for users to post collateral).

To derive the law of motion of the state variable  $C_t$ , the excess reserves, we first derive the platform's flow cost per dollar of stablecoins created:

$$\begin{aligned} & 2(\delta dt - \sigma dZ_t) \times \mathbb{P}(\{m_t(1 - \theta_a) < 1\}) \mathbb{E}[1 - m_t(1 - \theta_a) | m_t(1 - \theta_a) < 1] \\ &= 2(\delta dt - \sigma dZ_t) \times \left( \int_{1-\frac{1}{m_t}}^1 (1 - m_t(1 - \theta_a)) d\theta_a \right) = \frac{1}{m_t} (\delta dt - \sigma dZ_t). \end{aligned} \quad (39)$$

In the first line, the fraction of users' collateral assets that incur losses is multiplied by the probability of a sufficiently large loss that leads to the violation of the margin requirement, and the last component is the platform's loss upon receiving and liquidating the collateral (with a remaining value of  $m_t(1 - \theta_t)$ ) and repurchasing the one dollar worth of stablecoins out of circulation. Therefore, given  $N_t$ , the dollar value of all stablecoins issued,  $-\frac{N_t}{m_t}(\delta dt - \sigma dZ_t)$  enters into the law of motion of reserves (8), replacing  $N_t \sigma dZ_t$  (which is essentially the case where  $m_t = 1$  and  $\delta = 0$ ). This flow cost is essentially the consequence of the stablecoin issuer extending an guarantee of the stablecoins' value, which is a contingent liability akin to the guarantee that a bank extends to its

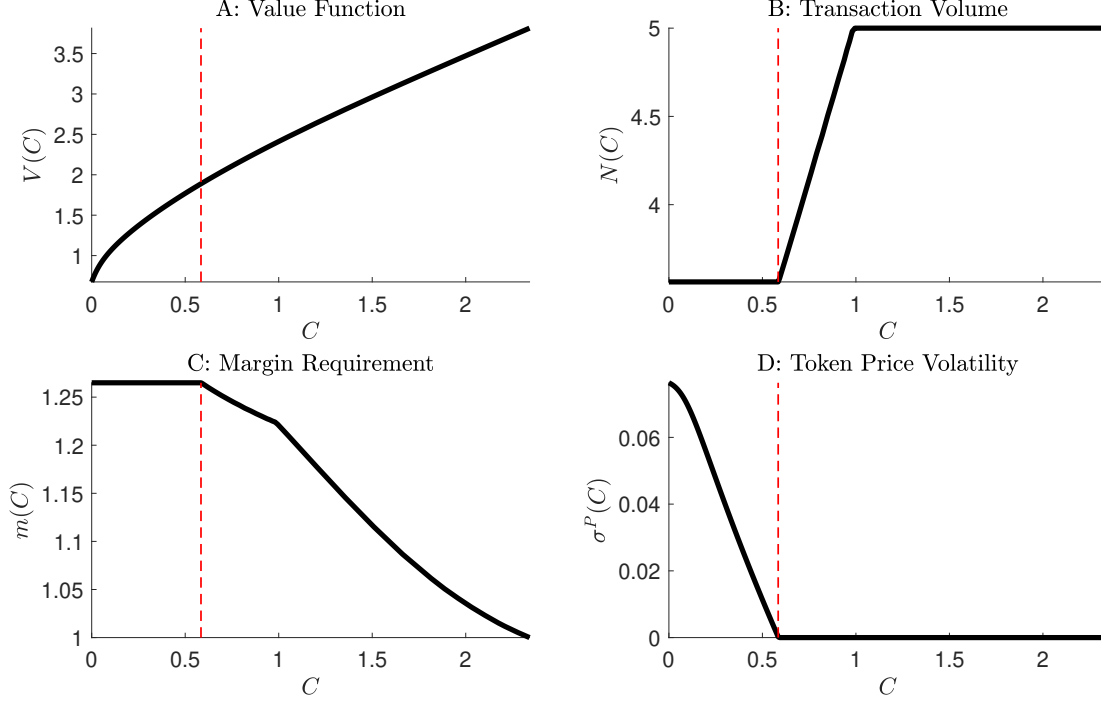


Figure 13: **Margin Requirement.** This figure plots the platform value function (Panel A), token demand or transaction volume (Panel B), optimal marginal requirement (Panel C), and token-price volatility (Panel D) as functions of excess reserves  $C$ . The parameterization follows Figure 2 with  $\tilde{\mu} = 0.05$  and  $\delta = 0.025$ .

off-balance-sheet conduits as discussed in Section 2.

Following the derivation in Section 4, we use (38) to substitute out  $f_t$  in the law of motion of reserves to obtain the law of motion of excess reserves,  $C_t$ :

$$dC_t = \left( rC_t - r(m_t - 1)N_t + m_t(\tilde{\mu} - \delta)N_t + N_t^\xi A^{1-\xi} - N_t\eta|\sigma_t^P| - \frac{N_t\delta}{m_t} \right) dt + N_t \left( \frac{\sigma}{m_t} - \sigma_t^P \right) dZ_t. \quad (40)$$

When  $m_t = 1$ ,  $\delta = 0$ , and  $\tilde{\mu} = 0$ , equation (40) reduces to (16) in the baseline model. In Appendix B.5, we provide all omitted solution details and derive the HJB equation of the value function,  $V(C_t)$  as well as the platform's optimal choices of fees (or equivalently,  $N(C_t)$ ), token price dynamics (or equivalently,  $\sigma^P(C_t)$ ), and the margin requirement  $m(C_t)$ . Figure 13 reports the numeric solutions.

In the model with user collateral, the shock to the platform's reserves,  $dZ_t$ , originates from the fluctuation of users' collateral value, and the platform's exposure is directly and inversely linked to the margin requirement,  $m_t$ , as shown in (40). Therefore, we expect the optimal margin requirement to be higher when the platform's excess reserves run down. This is shown in lower-left Panel of Figure 13. Introducing user collateral does not change the qualitative dynamics of the platform's

franchise value,  $V(C)$ , the transaction volume,  $N(C)$ , and the token price volatility,  $\sigma^P(C)$ .

**Discussion: immediate liquidation of collateral.** When users violate the margin requirement, the platform immediately liquidates users' collateral and repurchase stablecoins out of circulation. A question naturally arises: instead of liquidating the collateral and repurchasing stablecoins right away, why not incorporate the collateral assets into the platform's reserve portfolio? Doing so will create two types of stablecoins, one with the backing of both users' collateral and reserves (users of these stablecoins have not yet violated the margin requirements) and the other only backed by the platform's reserves (users of these stablecoins have violated the margin requirement). This is not done in practice, and analytically, it complicates the model by introducing a new stable variable, that is the fraction of stablecoins only backed by the platform's reserves.

## 7 Stablecoins, Digital Platforms, and Data Privacy

The interest in stablecoins among practitioners and regulators skyrocketed after Facebook announced its stablecoin project Libra (recently renamed to Diem and sold to Silvergate, a crypto-focused bank ([Yahoo Finance, 2022](#))). More recently, PayPal, a large payment provider, announced to venture into stablecoin issuance too (see, for instance, [Coindesk.com](#) and [Bloomberg](#)).

The stablecoin project of Facebook attracted enormous attention also because of the big data ambition of Facebook. Large platforms and payment providers (e.g., PayPal) profit from user-generated data and use targeted content to secure and grow user base. A widely adopted payment system enabled by a stablecoin allows the platform to gather transaction data. We extend the model to incorporate transaction data as a productive asset for the platform and explore how data acquisition affects a platform's stablecoin strategies.

Data is now a major asset of digital platforms. Social networks, such as Facebook and Twitter, utilize user-generated data to target users for content delivery. Payment platforms, such as PayPal and Square, have become data centers ([Bank for International Settlements, 2019](#)). They provide services beyond payment, for example, extending loans to consumers and businesses based on data-driven credit analysis. We follow [Veldkamp \(2005\)](#), [Ordoñez \(2013\)](#), [Fajgelbaum, Schaal, and Taschereau-Dumouchel \(2017\)](#), [Parlour, Rajan, and Zhu \(2020\)](#), and [Jones and Tonetti \(2020\)](#) to model data as a by-product of user activities.<sup>59</sup> We show that data acquisition incentive destabilizes

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<sup>59</sup>[Veldkamp and Chung \(2019\)](#) provide an excellent survey of the literature of data and aggregate economy.

the stablecoin. Our results demonstrate a connection between data privacy and stablecoins, two areas that have both attracted enormous attention but have been analyzed separately so far.

We now interpret  $A$  in (2) as a measure of effective data units that enhance platform productivity in locking in users' attention and stimulating user activities through user profiling. As previously discussed, we interpret  $N_t$  as the transaction volume and specify the following law of motion:

$$dA_t = \kappa A_t^{1-\xi} N_t^\xi dt. \quad (41)$$

Users' transactions generate a flow of raw data,  $\kappa N_t^\xi dt$ , where the parameter  $\kappa$  captures the technological efficiency of data processing and storage. To what extent the raw data contributes to the effective data units depends on the current amount of effective data via  $A_t^{1-\xi}$ . The complementarity between the old and new data captures the fact that the value of new data increases in the quality of statistical algorithms, which in turn depends on the amount of existing data that are needed to train the algorithms.<sup>60</sup> The Cobb-Douglas form is chosen for analytical convenience. To guarantee the convergence of the objective function, we impose the parametric restriction  $\rho > \kappa \bar{n}^\xi$ .

As platform productivity improves, we assume transaction capacity to increase accordingly, i.e.,  $\bar{N}_t = \bar{n}A_t$ , where  $\bar{n} > 0$  is constant. User optimization is static and follows the baseline model. As shown in (11), the transaction volume (or token demand)  $N_t \equiv n_t A_t$  where

$$n_t = \frac{1}{(r + f_t - \mu_t^P + \eta|\sigma_t^P|)^{\frac{1}{1-\xi}}} \wedge \bar{n}. \quad (42)$$

As in the baseline model, the platform sets  $n_t$  through the fees,  $f_t$ , and sets the dynamics of token price through its choice of  $\sigma_t^P$ . The model now has three natural state variables, reserves  $M_t$ , token supply  $S_t$ , and data stock  $A_t$ . Similar to the baseline model,  $C_t = M_t - S_t P_t$  and  $A_t$  summarize payoff-relevant information, driving the platform value,  $V_t = V(C_t, A_t)$ , and the dollar value of token,  $P_t = P(C_t, A_t)$ . To simplify the notations, we will suppress the time subscripts.

We conjecture that the system is homogeneous in  $A$ , and in particular, the platform's value function and dollar value of token are given by  $V(C, A) = v(c)A$  and  $P(C, A) = P(c)$ , respectively, where the excess reserves-to-data ratio is the key state variable for the platform's optimal strategies:

$$c \equiv \frac{C}{A}. \quad (43)$$

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<sup>60</sup>Related, in Farboodi, Mihet, Philippon, and Veldkamp (2019), data have increasing return to scale.



We will confirm the conjecture as we solve the platform's optimization problem in the following. First, to derive the law of motion of  $c_t$ , we follow the derivation of the baseline model to obtain

$$dC_t = \left( rC_t + A_t n_t^\xi - \eta A_t n_t |\sigma_t^P| \right) dt + A_t n_t (\sigma - \sigma_t^P) dZ_t - dDiv_t. \quad (44)$$

Given (41) and (44), the law of motion of  $c_t$  reads

$$dc_t = \left( rc_t + n_t^\xi - \eta n_t |\sigma_t^P| - \kappa n_t^\xi c_t \right) dt + n_t (\sigma - \sigma_t^P) dZ_t - \frac{dDiv_t}{A_t}. \quad (45)$$

Under the value function conjecture,  $V(C, A) = v(c)A$ , and the laws of motion of  $A$  (41) and  $c$  (45), the HJB equation for  $v(c)$  in the interior region (where  $dDiv_t = 0$ ) is given by

$$\rho v(c) = \max_{n \in [0, \bar{n}], \sigma^P} \left\{ [v(c) - v'(c)c] \kappa n^\xi + v'(c) \left( rc + n^\xi - \eta n |\sigma^P| \right) + \frac{1}{2} v''(c) n^2 (\sigma - \sigma^P)^2 \right\}. \quad (46)$$

The first term on the right side contains the marginal value of user-generated data (“data  $q$ ”)

$$q(c) = \frac{\partial V(C, A)}{\partial A} = v(c) - v'(c)c. \quad (47)$$

When the marginal value of reserves,  $V_A(C, A) = v'(c)$ , falls to one, the platform pays out dividends. We define the payout boundary as  $\bar{c}$  through  $v'(\bar{c}) = 1$ . The optimality of  $\bar{c}$  also implies  $v''(\bar{c}) = 0$ . Note that as in the baseline model, when  $C$  (or  $c$ ) approaches zero, the platform can avoid liquidation by setting  $\sigma^P(c) = \sigma$  to off-load risk to its users and gradually replenish reserves.<sup>61</sup> For simplicity, we do not consider recapitalization (equity issuance). In sum, the platform's excess reserves,  $C_t$ , move in  $[0, \bar{c}A]$ . As data grows, the platform accumulates more excess reserves.

**Proposition 6 (Platform Optimization under Data-Driven Productivity).** *The value function takes the form  $v(c_t)A_t$ , where  $v(c_t)$  solves the HJB equation (46) subject to the conditions  $v'(\bar{c}) = 1$ ,  $v''(\bar{c}) = 0$ , and  $\lim_{c \rightarrow 0} \sigma^P(c) = \sigma$ . The amount of excess reserves,  $C_t$ , stays below  $\bar{c}A_t$  where the upper bound increases with  $A_t$ , the effective data units. At  $C_t = \bar{c}A_t$ , the platform pays dividends when  $dC_t > 0$  so that dividend payments cause  $c_t$  to reflect at  $\bar{c}$ .<sup>62</sup>*

Next, we characterize the optimal transaction volume and volatility. Following our analysis of

<sup>61</sup>The boundary condition for  $v(c)$  is that as  $c$  approaches zero,  $-v''(c)$  approaches infinity (see footnote 43).

<sup>62</sup>When  $dC_t > 0$  at  $C_t = \bar{C}$ , the dividend amount is equal to  $dC_t$  (i.e., exactly the amount needed to avoid  $C_t > \bar{C}$ ).

the baseline model, we define the effective risk aversion based on  $v(c)$ :

$$\Gamma(c) = -\frac{v''(c)}{v'(c)}. \quad (48)$$

The following proposition summarizes the optimal choices of  $n(c)$  and  $\sigma^P(c)$ .

**Proposition 7 (Data  $q$ , Token Volatility, and Transaction Volume).** *At  $c$  where the platform maintains  $P(c) = 1$  (and  $\sigma^P(c) = 0$ ), the optimal transaction volume (token usage) is*

$$N = n(c)A = \left[ \frac{\xi}{\Gamma(c)\sigma^2} \left( 1 + \frac{\kappa q(c)}{v'(c)} \right) \right]^{\frac{1}{2-\xi}} A \wedge \bar{n}A; \quad (49)$$

otherwise, the optimal token volatility is

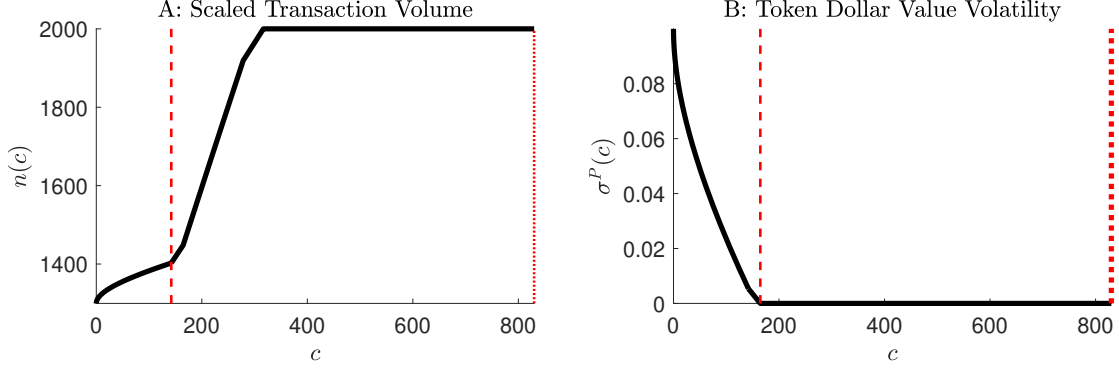
$$\sigma^P(c) = \sigma - \frac{\eta}{\Gamma(c)n(c)} \in (0, \sigma), \quad (50)$$

and the optimal transaction volume (token usage) is

$$N = n(c)A = \left[ \frac{\xi}{\eta\sigma} \left( 1 + \frac{\kappa q(c)}{v'(c)} \right) \right]^{\frac{1}{1-\xi}} A \wedge \bar{n}A. \quad (51)$$

The optimal transaction volume is proportional to  $A$ , the effective data units. Therefore, as the platform gathers more user-generated data following (41), it induces more transactions. With data as a productive asset, the platform faces a new trade-off. It can accumulate more reserves through higher fees or, by reducing fees, boost the transaction volume to accumulate more data. Therefore, the ratio of marginal value of data (the data  $q$ ) and marginal value of reserves,  $q(c)/v'(c)$ , emerges in both (49) and (51). When the data  $q$  is high relative to the marginal value of reserves, the platform implements a high transaction volume through low fees. As a reminder, given the token price dynamics, the monotonic relationship between transaction volume and fees is given by (42).

The optimal choice of token volatility resembles that of the baseline model. In the region where  $\sigma^P(c) > 0$ , it is the ratio of users' risk aversion to the platform's risk aversion that drives  $\sigma^P(c)$ . And in this region, the optimal transaction volume in (51), even scaled by  $A$ , is no longer the constant as in the baseline model but depends on  $q(c)/v'(c)$  instead, showing the trade-off between investing in data and accumulating reserves. Moreover, the optimal transaction volume depends on users' risk aversion  $\eta$  as  $\eta$  determines the cost of obtaining insurance from users (losing transaction volume after off-loading risk to users). When the platform absorbs all risk (i.e.,  $\sigma^P(c) = 0$ ), the



**Figure 14: Transaction Volume and Token Volatility.** This figure plots the  $A_t$ -scaled transaction volume  $n(c)$  in Panel A and token return volatility  $\sigma^P(c)$  in Panel B. In both panels, the red dotted lines mark the payout boundary  $\bar{c}$ , and the red dashed line marks  $\tilde{c}$ , the threshold that separates the regions of volatile and constant token prices. The parameterization follows Figure 2 with the additional parameters  $\bar{n} = 2000$  and  $\kappa = 0.00025$ . Note that  $\bar{n} = 2000$  implies that for  $A_0 = 0.0025$ ,  $\bar{N}_t = \bar{n}A_t = 5$  as under the parameterization in the baseline (see Figure 2).

optimal transaction volume varies with its own risk aversion  $\Gamma(c)$  (49) because  $\Gamma(c)$  drives the required risk compensation through higher fees that causes the transaction volume to decline.

Panel A of Figure 14 reports the optimal transaction volume. In contrast to Panel A of Figure 4 where the transaction volume is constant in the region where  $\sigma^P(c) > 0$ , the  $A$ -scaled volume now increases in  $c$ . The intuition is that as reserves become more abundant relative to data, the platform lowers fees to acquire more data through users' transactions at the expense of less dollar revenues for reserve accumulation. Panel B of Figure 14 shows a similar token volatility dynamics as Panel A of Figure 3 from the baseline model but in the space of  $c = C/A$  (instead of  $C$ ).

In our model, the technological advance in data acquisition and analysis can be captured by an increase of the parameter  $\kappa$ . In Figure 15, we examine the impact of data technology improvement on the operation of stablecoin platforms. In Panel A, we show that in response to an increase in  $\kappa$ , the platform optimally raises the ( $A$ -scaled) payout boundary,  $\bar{c}$ , which suggests a greater degree of over-collateralization. However, this does not translate into a more stable token price. As shown in Panel B, the long-run (stationary) probability of sustaining the peg decreases as the platform becomes more efficient in acquiring and utilizing user-generated data.

Therefore, our analysis reveals a paradox—if a digital platform introduces stablecoin to enhance its payment system and acquisition of transaction data, its stablecoin becomes more volatile precisely when data becomes more important. Conversely, stablecoins issued by platforms that respect user privacy and refrain from data usage are more stable in value according to our model.

To understand the mechanism, we plot the average fees and average transaction volume (both

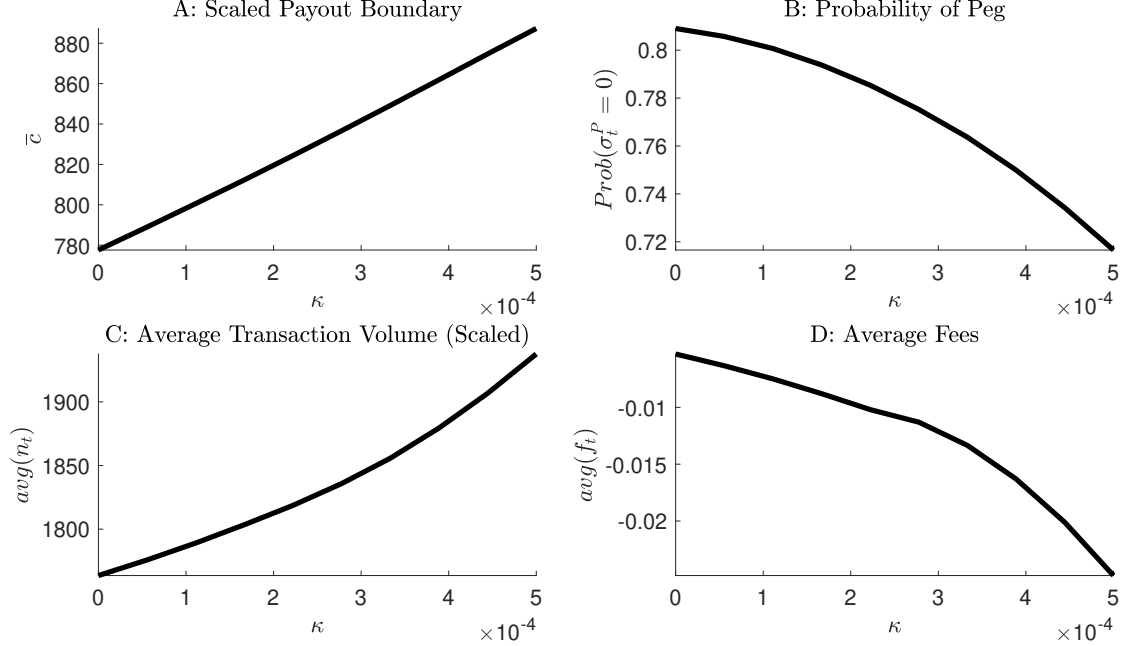


Figure 15: **Data Technology Progress and Platform Operation.** We plot the  $A$ -scaled payout boundary (Panel A), the probability of  $P(c) = 1$  (Panel B), the average transaction volume (Panel C), and the average fees per dollar of transactions (Panel D) over  $\kappa$  (the efficiency of data technology). The moments in Panel B, C, and D are based on the stationary distribution of  $c$ . The parameterization follows Figure 2 with  $\bar{n} = 2000$ .

calculated from the stationary distribution of  $c$ ) against  $\kappa$  in Panel C and D of Figure 15. To accumulate transaction data, the platform would like to increase the transaction volume. This is achieved through lower fees. In fact, the average fees per dollar of transaction even dips increasingly into the negative territory (i.e., becoming subsidies to users), a prediction in line with the practice that large digital platforms offer subsidies to grow user activities (Rochet and Tirole, 2006; Rysman, 2009). However, lowering fees reduce cash flows to the reserve buffer so that even though the platform hold more reserves at the payout boundary (see Panel A of Figure 15), it accumulates reserves at a slower pace on average over the state ( $c$ ) space, which destabilizes the token. In sum, data acquisition incentive makes the stablecoin issuer more aggressive in subsidizing users at the expense of its own precautionary savings that are key to the stabilization of token price.

Our model highlights an unintended benefit of privacy regulation. Regulations that restrict the stablecoin issuer’s ability to collect and utilize user-generated data can be interpreted as a decrease in  $\kappa$ .  $Prob(\sigma_t^P = 0)$  decreases in  $\kappa$  as shown in Panel B of Figure 15, so privacy regulations improve the stability of token value (i.e., reducing the probability of the stablecoin breaking the buck). Finally, recall that data collection improves the quality of the payment system and so benefits users to some extent. However, the destabilizing effect of the stablecoin issuer’s data collection and

so the stabilizing effect of privacy regulation would also arise (likely in stronger form) in a setting in which data collection only benefits the platform and possibly even harms users.

## 8 Conclusion

As decentralized finance develops rapidly, stablecoin initiatives arise to meet the demand for stable means of payment in the blockchain space. Stablecoins are issued by private entities or decentralized autonomous organizations (DAOs) that promise to maintain price stability by holding reserves for open market operations and users' redemption. However, as issuers maximize their own payoffs rather than total welfare, conflicts of interests naturally arise between the issuers and stablecoin users, making room for welfare-enhancing regulations. Digital networks (e.g., Facebook) plan to introduce their own stablecoins. Behind such initiatives, the incentives are complex especially when a payment system allows the platform to gather and profit from users' transaction data.

In spite of the enormous attention from both regulators and practitioners, to this date, there has not been a unified framework to address these issues. In this paper, we fill this gap and develop a dynamic model of stablecoin management. The equilibrium rationalizes a rich set of strategies and features two endogenous regimes. When the issuer's reserves are sufficiently high, the stablecoin price is fixed. When the reserves fall below a critical threshold, the stablecoin price comoves with the issuer's reserves, allowing risk sharing between the issuer and stablecoin users.

The system is bimodal and exhibits a unique instability mechanism. Above the reserve threshold, the issuer credibly sustains a fixed price, which induces a strong token demand that allows the issuer to profit from open market operations and further grow reserve holdings. This virtuous cycle turns into a vicious cycle when reserves fall below the threshold after negative shocks. As the stablecoin price becomes volatile, the users' token demand declines, so the issuer has to either drain its reserves further to stabilize price through open market operations or let debasement continue. The vicious cycle can be broken by issuing equity (governance tokens) to replenish reserves.

We evaluate several regulatory proposals and find that capital requirement improves welfare and stability. In contrast, a legally binding commitment to price stability destroys welfare. We also demonstrate a volatility paradox: Forcing a stablecoin issuer to hold low-risk assets may destabilize the stablecoin. Its welfare implications depend on how the expected return on reserve assets comove with riskiness. Finally, our model can be easily extended to incorporate a q-theory of data acquisition. Investing in data crowds out reserve hoarding and thus destabilizes the stablecoin

price. Therefore, data privacy regulation has an unintended benefit of improving the price stability of stablecoins issued by data-rich platforms.

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# A Proofs

## A.1 Proof of Propositions 1 and 2

The proof of Propositions 1 and 2 is split in three parts. Part I derives the HJB equation and its boundary conditions. Part II establishes the concavity of the value function. Part III shows that there is no liquidation.

### Part I — HJB equation and dividend payouts

Recall that the platform chooses dividends  $\{dDiv_t\}$ , transaction volume  $\{N_t\}$  (or equivalently transaction fees  $\{f_t\}$ ), and token price volatility  $\{\sigma_t^P\}$  (which implicitly pins down token price and the choice of token supply via the market clearing condition  $N_t = S_t P_t$ ) to maximize the future expected discounted value of dividends. By the dynamic programming principle, the platform solves (17) subject to  $dDiv_t \geq 0$  and the law of motion (16). As such, platform value  $V(C) = V(C_t)$  satisfies the following HJB equation (in differential form):

$$\rho V(C)dt = \max_{N \in [0, \bar{N}], \sigma^P, dDiv \geq 0} \{dDiv + \mathbb{E}[dV(C)]\}. \quad (\text{A.1})$$

In what follows, we assume that (A.1) admits a unique and twice differentiable solution  $V(C)$ .

Using Ito's Lemma and expanding the right-hand-side, we obtain

$$\rho V(C)dt = \max_{\{N \in [0, \bar{N}], \sigma^P, dDiv \geq 0\}} \left\{ (1 - V'(C))dDiv + V'(C) \left( rC + N^\xi A^{1-\xi} - \eta N |\sigma_t^P| \right) dt \right. \quad (\text{A.2})$$

$$\left. + \frac{1}{2} V''(C) N^2 (\sigma - \sigma^P)^2 dt \right\}. \quad (\text{A.3})$$

It follows that dividend payouts are optimal if and only if  $V'(C) \geq 1$ . As in Bolton et al. (2011), the optimal dividend policy therefore follows a barrier strategy, so that (in optimum) dividend payouts  $dDiv$  cause  $C_t$  to reflect at  $\bar{C}$ , i.e.,  $dDiv_t = \max\{C_t - \bar{C}, 0\}$ . And, the threshold  $\bar{C}$  satisfies smooth pasting and super contact conditions (for details, see, e.g., Dumas (1991)), i.e.,

$$V'(\bar{C}) - 1 = V''(\bar{C}) = 0.$$

Given this dividend policy, the HJB equation (A.2) simplifies to (22) whenever  $C_t \leq \bar{C}$ , as stated in Proposition 2. In addition, the optimal dividend policy also implies  $C_t \leq \bar{C}$  for all  $t \geq 0$ .

### Part II — Value function concavity

We prove the concavity of value function in Proposition 1. Recall the HJB equation (22), that is,

$$\rho V(C) = \max_{\{N \in [0, \bar{N}], \sigma^P\}} \left\{ V'(C) \left( rC + N^\xi A^{1-\xi} - \eta N |\sigma^P| \right) + \frac{1}{2} V''(C) N^2 (\sigma - \sigma^P)^2 \right\}.$$

Using the envelope theorem, we differentiate both sides of the HJB equation (evaluated under the optimal controls  $N$  and  $\sigma^P$ ) with respect to  $C$  to obtain

$$\rho V'(C) = rV'(C) + V''(C) \left( rC + N^\xi A^{1-\xi} - \eta N |\sigma_t^P| \right) + \frac{1}{2} V'''(C) N^2 (\sigma - \sigma^P)^2.$$

We can solve for

$$V'''(C) = \frac{2}{N^2 (\sigma - \sigma^P)^2} \left[ (\rho - r) V'(C) - V''(C) \left( rC + N^\xi A^{1-\xi} - \eta N |\sigma^P| \right) \right]$$

Using the smooth pasting condition,  $V'(\bar{C}) = 1$ , and the super-contact condition,  $V''(\bar{C}) = 0$ , we obtain  $V'''(\bar{C}) > 0$ . As  $V''(\bar{C}) = 0$ , it follows that  $V''(C) < 0$  in a left-neighbourhood of  $\bar{C}$ , in that there exists  $\varepsilon > 0$  so that  $V''(C) < 0$  for  $C \in (\bar{C} - \varepsilon, \bar{C})$ .

We show now that  $V''(C) < 0$  for all  $C \in [0, \bar{C})$ . Suppose to the contrary that there exists  $\hat{C} < \bar{C}$  with  $V''(\hat{C}) \geq 0$  and set without loss of generality

$$\hat{C} = \sup\{C \in (0, \bar{C} - \varepsilon) : V''(C) \geq 0\}. \quad (\text{A.4})$$

As  $V''(C) < 0$  on the interval  $(\bar{C} - \varepsilon, \bar{C})$  and the value function is twice continuously differentiable, it follows that  $V''(\hat{C}) = 0$  and therefore the optimization in the HJB equation (22) implies  $\sigma^P(\hat{C}) < \sigma$ . In addition,  $V'(\hat{C}) \geq 1$ , so that  $V'''(\hat{C}) > 0$ . Thus, there exists  $C' > \hat{C}$  with  $V''(C') \geq 0$ , a contradiction. Therefore, the value function is strictly concave on  $[0, \bar{C})$ .

### Part III — There is no liquidation

Consider that  $C_t$  approaches zero, i.e.,  $C_t \rightarrow 0$ . If the volatility of  $dC_t$ ,  $\sigma_C(C_t) = N_t(\sigma - \sigma^P(C_t))$  does not tend to zero as  $C_t$  approaches zero,  $C_t$  drops below zero and the platform is liquidated with probability one in which case the platform owners' value becomes zero. To prevent liquidation as  $C_t$  approaches zero, it must be that i) the volatility of  $dC_t$ ,  $\sigma_C(C_t) = N_t(\sigma - \sigma_t^P)$ , tends to zero and ii) the drift of  $dC_t$ ,  $\mu_C(C_t) = rC_t + N_t^\xi A^{1-\xi} - N_t \eta |\sigma_t^P|$ , remains positive positive. Formally,

$$\lim_{C \rightarrow 0^+} \mu_C(C) > 0 = \lim_{C \rightarrow 0^+} \sigma_C(C), \quad (\text{A.5})$$

must hold.

Thus, if the platform prevents liquidation, then — by the law of motion (16) — it must be  $\lim_{C \rightarrow 0^+} \sigma^P(C) = \sigma$ . As  $V(C)$  is concave with  $V'(\bar{C}) = 1$ , it follows that  $V'(C) > 0$  for all  $C \in [0, \bar{C}]$ . As such, when  $\sigma^P(C) \rightarrow \sigma$ , then

$$V(C) \rightarrow \frac{1}{\rho} V'(C) \mu_C(C).$$

Therefore, when  $\lim_{C \rightarrow 0^+} \sigma^P(C) \rightarrow 0$ , the equivalence

$$\lim_{C \rightarrow 0^+} V(C) > 0 \iff \lim_{C \rightarrow 0^+} \mu_C(C) > 0$$

holds.

Next, using the HJB equation (22), we obtain

$$\begin{aligned} V(C) &\geq \frac{V'(C)}{\rho} \left( rC + \max_{\{N \in [0, \bar{N}]\}} \{N^\xi A^{1-\xi} - \eta N \sigma\} \right) \\ &\geq \frac{1}{\rho} \left( \max_{\{N \in [0, \bar{N}]\}} \{N^\xi A^{1-\xi} - \eta N \sigma\} \right) > 0. \end{aligned}$$

The first inequality uses that setting  $\sigma^P = \sigma$  is always possible (but not necessarily optimal) and the second inequality uses  $C \geq 0$  and  $V'(C) \geq 1$ . As such, the platform obtains strictly positive value from continuation, implying that liquidation is not optimal and the platform optimally prevents liquidation. Thus, liquidation never occurs, and (A.5) holds.

## A.2 Proof of Proposition 3

The proof of Proposition 3 is split in three parts. Part I characterizes the optimal controls  $N(C)$  and  $\sigma^P(C)$ . Part II shows that platform risk-aversion  $\gamma(C)$  decreases with  $C$ . Part III demonstrates that there exists  $\tilde{C}$  so that for  $C < \tilde{C}$  ( $C \geq \tilde{C}$ ),  $\sigma^P(C) > 0$  ( $\sigma^P(C) = 0$ ).

### Part I — Optimal control variables

We characterize the optimization in (22) and solve for the optimal control variables  $N = N(C)$  and  $\sigma^P = \sigma^P(C)$  in Proposition 3. To start with, we define

$$\underline{N} = \arg \max_{N \leq \bar{N}} \{N^\xi A^{1-\xi} - \eta N \sigma\}, \quad (\text{A.6})$$

which yields

$$\underline{N} = \min \left\{ \left( \frac{\xi A^{1-\xi}}{\eta \sigma} \right)^{\frac{1}{1-\xi}}, \bar{N} \right\}.$$

Now, we first optimize the HJB equation (22) over  $\sigma^P$  or equivalently over  $N\sigma^P$ .

If interior (i.e.,  $\sigma^P > 0$ ), the choice of  $\sigma^P$  satisfies the first order optimality condition

$$\frac{\partial V(C)}{\partial \sigma^P} = 0 \iff -\eta V'(C) - V''(C)(N\sigma - N\sigma^P) = 0.$$

We can rearrange the above first order condition to derive

$$N\sigma^P = \frac{-\eta V'(C) - N\sigma V''(C)}{-V''(C)}. \quad (\text{A.7})$$

It is clear from the maximization in the HJB equation (22) that setting  $\sigma^P < 0$  is never optimal. As such, to obtain the optimal choice of  $\sigma^P$  we truncate the expression in (A.7) from below by zero and obtain

$$N\sigma^P = \max \left\{ 0, \frac{-\eta V'(C) - N\sigma V''(C)}{-V''(C)} \right\} = \max \left\{ 0, -\frac{\eta V'(C)}{-V''(C)} + N\sigma \right\}. \quad (\text{A.8})$$

Note that by (11), users' aggregate token holdings are always positive (i.e.,  $N_t > 0$  at all times  $t \geq 0$ ) so that  $\sigma^P > 0 \iff N\sigma^P > 0$  and  $\sigma^P = 0 \iff N\sigma^P = 0$ . We distinguish between two different cases: 1)  $\sigma^P = 0$  and 2)  $\sigma^P > 0$ .

1. First, consider  $\sigma^P > 0$ . Then, we can insert the relation (A.7) (or (A.8) noting that  $N\sigma^P > 0$ ) into (22) to get

$$\rho V(C) = \max_{N \in [0, \bar{N}]} \left\{ V'(C) \left[ rC + N^\xi A^{1-\xi} - \eta N\sigma - \frac{\eta^2 V'(C)}{V''(C)} \right] + \frac{1}{V''(C)} \left[ \frac{(\eta V'(C))^2}{2} \right] \right\}.$$

Thus, by (A.6),  $N = \underline{N} > 0$  is the optimal choice of  $N$ , so that by means of (A.8):

$$\sigma^P = \max \left\{ 0, -\frac{\eta V'(C)}{-V''(C)\underline{N}} + \sigma \right\} = \max \left\{ 0, \sigma - \frac{\eta}{\gamma(C)\underline{N}} \right\}, \quad (\text{A.9})$$

where the last equality uses the definition  $\gamma(C) = -\frac{V''(C)}{V'(C)}$ .

2. Second, consider  $\sigma^P = 0$ . Inserting  $\sigma^P = 0$  into (22), the HJB equation becomes

$$\rho V(C) = \max_{N \in [0, \bar{N}]} \left\{ V'(C)[rC + N^\xi A^{1-\xi}] + V''(C) \left[ \frac{N^2 \sigma^2}{2} \right] \right\}. \quad (\text{A.10})$$

If interior (i.e.,  $N(C) < \bar{N}$ ), the optimal choice of  $N = N(C)$  must solve the first order condition

$$V'(C)\xi N^{\xi-1} A^{1-\xi} + V''(C)N\sigma^2 = 0 \iff V'(C)\xi N^{\xi-2} A^{1-\xi} + V''(C)\sigma^2 = 0.$$

Thus, optimal  $N = N(C)$  reads

$$N(C) = \min \left\{ \left( \frac{A^{1-\xi} \xi V'(C)}{-V''(C)\sigma^2} \right)^{\frac{1}{2-\xi}}, \bar{N} \right\}, \quad (\text{A.11})$$

where we truncate above by  $\bar{N}$ .

Overall, note that  $\sigma^P(C)$  (partially) decreases with  $\gamma(C)$ , the platform's risk-aversion, in that  $\frac{\partial \sigma^P(C)}{\partial \gamma(C)} \leq 0$ . When  $\sigma^P(C) > 0$ , this follows from (A.9), and, when  $\sigma^P = 0$ , this trivially holds.

## Part II — Effective risk-aversion

We prove  $\gamma'(C) < 0$ , i.e.,  $\frac{d(-V''(C)/V'(C))}{dC} < 0$ , in Proposition 3. To do so, we consider the following two cases, 1)  $\sigma^P = 0$  and 2)  $\sigma^P = 0$ :

1. Consider  $\sigma^P > 0$  so that  $N = \underline{N}$ . Then, the HJB equation (22) can be simplified to

$$\rho \frac{V(C)}{V'(C)} = rC + \underline{N}^\xi A^{1-\xi} - \eta \underline{N} \sigma - \frac{\eta^2}{2} \frac{V'(C)}{V''(C)}. \quad (\text{A.12})$$

Differentiating the equation above with respect to  $C$ , we obtain

$$\rho \left( 1 - \frac{V''(C)V(C)}{V'(C)^2} \right) = r - \frac{\eta^2}{2} \frac{d(V'(C)/V''(C))}{dC},$$

which can be rewritten as

$$\frac{d(V'(C)/V''(C))}{dC} = \frac{2}{\eta^2} \left[ (r - \rho) + \rho \left( \frac{V''(C)V(C)}{V'(C)^2} \right) \right].$$

Note that because  $\rho > r$  and  $V''(C) < 0$ , it follows that implies  $\frac{d(V'(C)/V''(C))}{dC} < 0$ , i.e.,  $\frac{d(-V''(C)/V'(C))}{dC} = \gamma'(C) < 0$ .

2. Consider  $\sigma^P = 0$ , so the HJB (22) simplifies to

$$\rho V(C) = \max_{N \in [0, \bar{N}]} \left\{ V'(C)[rC + N^\xi A^{1-\xi}] + V''(C) \left[ \frac{N^2 \sigma^2}{2} \right] \right\}, \quad (\text{A.13})$$

In this case, we further consider two cases, a)  $N = N(C) < \bar{N}$  and b)  $N = N(C) = \bar{N}$ :

- a)  $N(C) < \bar{N}$  and  $N = \left( \frac{A^{1-\xi} \xi V'(C)}{-V''(C) \sigma^2} \right)^{\frac{1}{2-\xi}}$ . In this case, the HJB can be simplified to

$$\rho \frac{V(C)}{V'(C)} = rC + \frac{1}{2} \left( \frac{\xi A^{1-\xi}}{\sigma^\xi} \right)^{\frac{2}{2-\xi}} \left( \frac{2-\xi}{\xi} \right) \left( \frac{V'(C)}{-V''(C)} \right)^{\frac{\xi}{2-\xi}}. \quad (\text{A.14})$$

Differentiating the equation above with respect to  $C$ , we obtain

$$\rho \left( 1 - \frac{V''(C)V(C)}{V'(C)^2} \right) = r - \frac{1}{2} \left( \frac{\xi A^{1-\xi}}{\sigma^\xi} \right)^{\frac{2}{2-\xi}} \left( \frac{V'(C)}{-V''(C)} \right)^{\frac{2\xi-2}{2-\xi}} \frac{d(-V'(C)/V''(C))}{dC}, \quad (\text{A.15})$$

implying  $\frac{d(V'(C)/V''(C))}{dC} < 0$  (because  $V''(C) < 0$  and  $\rho > r$ ), that is,  $\frac{d(-V''(C)/V'(C))}{dC} = \gamma'(C) < 0$ .

- b)  $N(C) = \bar{N}$ . In this case, the HJB can be simplified to

$$\rho \frac{V(C)}{V'(C)} = rC + \bar{N}^\xi A^{1-\xi} + \frac{\bar{N}^2 \sigma^2}{2} \frac{V''(C)}{V'(C)}. \quad (\text{A.16})$$

Differentiating the equation above with respect to  $C$ , we obtain

$$\rho \left( 1 - \frac{V''(C)V(C)}{V'(C)^2} \right) = r - \frac{\bar{N}^2 \sigma^2}{2} \frac{d(-V''(C)/V'(C))}{dC}, \quad (\text{A.17})$$

which implies  $\frac{d(-V''(C)/V'(C))}{dC} = \gamma'(C) < 0$  (because  $V''(C) < 0$  and  $\rho > r$ ).

### Part III — Existence of threshold $\tilde{C}$

In Part I, we have shown that  $\sigma^P(C)$  increases with  $\gamma(C)$  and, in Part II, we have shown that  $\gamma(C)$  decreases with  $C$  with  $\gamma(\bar{C}) = 0$ . Therefore,  $\sigma^P(C)$  decreases with  $C$ . As  $\gamma(\bar{C}) = 0$ , it must be that  $\sigma^P(C) = 0$  in a left-neighbourhood of  $\bar{C}$ . Because there is no liquidation, it holds that  $\lim_{C \rightarrow 0^+} \sigma^P(C) = \sigma$  and thus — by continuity —  $\sigma^P(C) > 0$  in a right-neighbourhood of  $C = 0$ . As  $\sigma^P(C)$  is continuous and decreases with  $C$  on  $[0, \bar{C}]$ , there exists unique  $\tilde{C} \in (0, \bar{C})$  so that  $\sigma^P(C) > 0$  for  $C < \tilde{C}$  and  $\sigma^P(C) = 0$  for  $C \geq \tilde{C}$  (while  $C \in [0, \bar{C}]$ ). The threshold  $\tilde{C}$  solves

$$\sigma - \frac{\eta}{\gamma(\tilde{C})\underline{N}} = 0,$$

which implicitly defines  $\tilde{C}$  (see (A.9)). This concludes the argument.

### A.3 Proof of Corollary 1

First, consider that  $C < \tilde{C}$ , so  $\sigma^P(C) > 0$  and  $N(C) = \underline{N}$ . Using (12), we obtain

$$f(C) = \left( \frac{A}{\underline{N}} \right)^{1-\xi} - r + \mu^P(C) - \eta |\sigma^P(C)|. \quad (\text{A.18})$$

Second, consider that  $C \geq \tilde{C}$  and  $N(C) < \bar{N}$ . Then,  $\sigma^P(C) = \mu^P(C) = 0$  and

$$N(C) = \left( \frac{\xi A^{1-\xi}}{\gamma(C)\sigma^2} \right)^{\frac{1}{2-\xi}}.$$

Using (12) and simplifying, we obtain

$$f(C) = \left( \frac{A\gamma(C)\sigma^2}{\xi} \right)^{\frac{1-\xi}{2-\xi}} - r.$$

Third, consider  $C \geq \tilde{C}$  and  $N(C) = \bar{N}$  so that  $\mu^P(C) = \sigma^P(C) = 0$ . Using (12), we obtain

$$f(C) = \left( \frac{A}{\bar{N}} \right)^{1-\xi} - r.$$

Finally, note that because  $\gamma(C)$  decreases with  $C$ ,  $N(C)$  increases with  $C$  for  $C \geq \tilde{C}$  with  $N(\bar{C}) = \bar{N}$ . Therefore, there exists  $\tilde{C}' \geq \tilde{C}$  so that  $N(C) = \bar{N}$  if  $C \in [\tilde{C}', \bar{C}]$ .

#### A.4 Proof of Corollary 2 and Proposition 4

The relevant arguments are already presented in the main text. In a Markov equilibrium with state variable  $C$ , token price  $P(C)$  and  $\sigma^P(C)$  are functions of  $C$  only. Ito's Lemma implies

$$\sigma^P(C) = \frac{P'(C)}{P(C)}N(C)(\sigma - \sigma^P(C)),$$

as desired. We normalize  $P(\bar{C}) = 1$ . For  $C \geq \tilde{C}$ , it holds that  $\sigma^P(C) = 0$  and thus  $P'(C) = P''(C) = 0$ , so  $\mu^P(C) = 0$ . For  $C < \tilde{C}$ , it holds that  $\sigma^P(C) > 0$  and so  $P'(C) > 0$ .

#### A.5 Proof of Proposition 5

Follows from the arguments presented in the main text.

#### A.6 Proof of Corollary 3

Follows from the arguments presented in the main text.

#### A.7 Proof of Proposition 6

To start with, recall the law of motion of the state variables  $C_t$  (see (44)),

$$dC_t = \left( rC_t + A_t n_t^\xi - \eta A_t n_t |\sigma_t^P| \right) dt + A_t n_t (\sigma - \sigma_t^P) dZ_t - dDiv_t, \quad (\text{A.19})$$

and  $A_t$ ,

$$\frac{dA_t}{A_t} = \kappa n_t^\xi dt.$$

Define  $n_t = N_t/A_t$ . Using Ito's Lemma, we can calculate

$$dc_t = \left( rc_t + n_t^\xi - \eta n_t |\sigma_t^P| - \kappa n_t^\xi c_t \right) dt + n_t (\sigma - \sigma_t^P) dZ_t - \frac{dDiv_t}{A_t}, \quad (\text{A.20})$$

with drift  $\mu_c(c_t) \equiv rc_t + n_t^\xi - \eta n_t |\sigma_t^P| - \kappa n_t^\xi c_t$  and volatility  $\sigma_c(c_t) = n_t (\sigma - \sigma_t^P)$ .

By the dynamic programming principle, the platform's value function  $V(C, A)$  solves the following HJB equation (in differential form):

$$\rho V(C, A) dt = \max_{\sigma^P, N \in [0, \bar{N}], dDiv \geq 0} \{ dDiv + \mathbb{E}[dV(C, A)] \}.$$

We can use Ito's Lemma to expand the right-hand-side of the HJB equation:

$$\begin{aligned} \rho V(C, A) dt = \max_{\sigma^P, N \in [0, \bar{N}], dDiv \geq 0} & \left\{ dDiv(1 - V_C(C, A)) + V_C(C, A) \left( rC + An^\xi - \eta An |\sigma^P| \right) dt \right. \\ & \left. + V_A(C, A) A \kappa n^\xi dt + \frac{V_{CC}(C, A) N^2 (\sigma - \sigma^P)^2}{2} dt \right\}, \quad (\text{A.21}) \end{aligned}$$



where a subscript denotes the partial derivative (e.g.,  $V_C(C, A) = \frac{\partial V(C, A)}{\partial C}$ ). As such, dividend payouts  $dDiv > 0$  are optimal if and only if  $V_C(C, A) \leq 1$ ; otherwise,  $dDiv = 0$ . Using the conjecture  $V(C, A) = Av(c)$ , we obtain

$$V_C(C, A) = v'(c), \quad V_A(C, A) = v(c) - v'(c)c, \quad \text{and} \quad V_{CC}(C, A) = \frac{v''(c)}{A}. \quad (\text{A.22})$$

As is standard (see, e.g., [Bolton et al. \(2011\)](#)), optimal dividend payouts cause  $c_t$  to reflect at  $\bar{c}$ , where the payout threshold  $\bar{c}$  satisfies  $v'(\bar{c}) - 1 = v''(\bar{c}) = 0$ . That is,  $dDiv = A \max\{c - \bar{c}, 0\}$ , and  $c_t \leq \bar{c}$  at all times  $t \geq 0$ .

When there are no dividend payouts, the HJB equation [\(A.21\)](#) therefore becomes (using [\(A.22\)](#), dividing both sides by  $dt$  and  $A$ , and simplifying):

$$\rho v(c) = \max_{n \in [0, \bar{n}], \sigma^P} \left\{ [v(c) - v'(c)c] \kappa n^\xi + v'(c) (rc + n^\xi - \eta n |\sigma^P|) + \frac{1}{2} v''(c) n^2 (\sigma - \sigma^P)^2 \right\}, \quad (\text{A.23})$$

which is [\(46\)](#).

As  $c$  approaches zero, the platform can either liquidate (yielding  $v(0) = 0$ ) or prevent liquidation by i) setting  $\sigma^P(c) \rightarrow \sigma$  and ii) ensuring that the drift of  $dc$ ,  $\mu_c(c)$ , remains positive. Formally, to prevent liquidation as  $c \rightarrow 0$ ,

$$\lim_{c \rightarrow 0^+} \mu_c(c) > 0 = \lim_{c \rightarrow 0^+} \sigma_c(c) \quad (\text{A.24})$$

must hold. Setting  $\sigma^P(c) \rightarrow \sigma$  as  $c \rightarrow 0$  yields

$$\lim_{c \rightarrow 0^+} \rho v(c) = \max_{n \in [0, \bar{n}]} \lim_{c \rightarrow 0^+} \left( v(c) \kappa n^\xi + v'(c) (n^\xi - \eta n \sigma) \right) > 0.$$

Note that because  $\kappa \bar{n}^\xi < \rho$  and  $v'(c) \geq 1$ ,  $\lim_{c \rightarrow 0^+} v(c) > 0$  implies  $\lim_{c \rightarrow 0^+} \mu_c(c) > 0$ , as (under the optimal controls)

$$\lim_{c \rightarrow 0^+} (\rho - \kappa n(c)^\xi) v(c) = \lim_{c \rightarrow 0^+} v'(c) \max_{n \in [0, \bar{n}]} \mu_c(c).$$

As  $v'(c) \geq 1$  for all  $c \leq \bar{c}$  and so

$$\lim_{c \rightarrow 0^+} \max_{n \in [0, \bar{n}]} v'(c) \mu_c(c) \geq \max_{n \in [0, \bar{n}]} (n^\xi - \eta n \sigma) > 0, \quad (\text{A.25})$$

it follows that  $\lim_{c \rightarrow 0^+} v(c) > 0$ , and the platform is better off averting liquidation. In optimum, liquidation never occurs and [\(A.24\)](#) holds, implying  $\lim_{c \rightarrow 0^+} \sigma^P(c) = 0$ .

## A.8 Proof of Proposition 7

The optimal control variables,  $n = n(c)$  and  $\sigma^P = \sigma^P(c)$ , are determined by the optimization in the HJB equation (46), that is,

$$\rho v(c) = \max_{n \in [0, \bar{n}], \sigma^P} \left\{ [v(c) - v'(c)c] \kappa n^\xi + v'(c) \left( rc + n^\xi - \eta n |\sigma^P| \right) + \frac{1}{2} v''(c) n^2 (\sigma - \sigma^P)^2 \right\}. \quad (\text{A.26})$$

We consider the following two cases, 1)  $\sigma^P > 0$  and 2)  $\sigma^P = 0$ .

1. If  $\sigma^P > 0$ , then the first order condition

$$\frac{\partial v(c)}{\partial \sigma^P} = 0 \iff -v'(c)\eta n(c) - v''(c)n(c)^2(\sigma - \sigma^P(c)) = 0$$

must hold. We can solve for

$$\sigma^P(c) = \sigma - \frac{\eta v'(c)}{v''(c)n(c)} = \sigma - \frac{\eta}{\Gamma(c)n(c)} \in (0, \sigma),$$

where

$$\Gamma(c) = -\frac{v''(c)}{v'(c)}.$$

Inserting the optimal choice of  $\sigma^P(c)$  back into (46), we obtain

$$\rho v(c) = \max_{n \in [0, \bar{n}], \sigma^P} \left\{ [v(c) - v'(c)c] \kappa n^\xi + v'(c) \left( rc + n^\xi - \eta \sigma n + \frac{\eta}{\Gamma(c)} \right) + \frac{1}{2} v''(c) \left( \frac{\eta}{\Gamma(c)} \right)^2 \right\}.$$

If interior (i.e.,  $n(c) < \bar{n}$ ), the optimal choice of  $n = n(c)$  solves the first order condition

$$\kappa \xi [v(c) - v'(c)c] n(c)^{\xi-1} + v'(c)(\xi n(c)^{\xi-1} - \eta \sigma) = 0.$$

We define

$$q(c) = v(c) - v'(c)c$$

and solve for

$$n(c)^{\xi-1} = \frac{v'(c)\eta\sigma}{\kappa\xi q(c) + \xi v'(c)} \iff n(c) = \left( \frac{\xi(v'(c) + \kappa q(c))}{\eta\sigma v'(c)} \right)^{\frac{1}{1-\xi}}.$$

Thus,

$$n(c) = \left[ \frac{\xi}{\eta\sigma} \left( 1 + \frac{\kappa q(c)}{v'(c)} \right) \right]^{\frac{1}{1-\xi}} \wedge \bar{n},$$

as desired.

2. Consider  $\sigma^P(c) = 0$ . If interior (i.e.,  $n(c) \in (0, \bar{n})$ ), optimal  $n = n(c)$  must solve the first

order condition

$$\xi[\kappa q(c) + v'(c)]n(c)^{\xi-1} + v''(c)n(c)\sigma^2 = 0.$$

Dividing both sides through  $n(c) > 0$ , we obtain

$$\xi[\kappa q(c) + v'(c)]n(c)^{\xi-2} + v''(c)\sigma^2 = 0 \iff n(c)^{\xi-2} = \frac{-v''(c)\sigma^2}{\xi(\kappa q(c) + v'(c))}$$

We can solve for

$$n(c) = \left( \frac{\xi(\kappa q(c) + v'(c))}{-v''(c)\sigma^2} \right)^{\frac{1}{2-\xi}}.$$

Thus,

$$n(c) = \left[ \frac{\xi}{\Gamma(c)\sigma^2} \left( 1 + \frac{\kappa q(c)}{v'(c)} \right) \right]^{\frac{1}{2-\xi}} \wedge \bar{n},$$

which was to show.

Finally, note that analogous to the baseline, there exist three regions and two thresholds  $\tilde{c}$  and  $\tilde{c}'$  such that i)  $\sigma^P(c) > 0$  if and only if  $c < \tilde{c}$  (otherwise,  $\sigma^P(c) = 0$ ) and ii)  $n(c) = \bar{n}$  if and only if  $c \geq \tilde{c}'$  (otherwise,  $n(c) < \bar{n}$ ).

## B Derivations

### B.1 Calculating the Expected Recovery Time

Note that there exists  $\tilde{C} \in (0, \bar{C})$  such that  $\sigma^P(C) = 0$ . Given  $C_t = C$  at time  $t$ , we define

$$\tau(C_t) = \mathbb{E}[\tau^* - t | C_t = C] \quad \text{with} \quad \tau^* = \inf\{s \geq t : C_s \geq \tilde{C}\},$$

which is the expected time until net liquidity reaches  $\tilde{C}$  and token price volatility vanishes.

We can rewrite  $\tau(C_t)$  as

$$\tau(C_t) = \mathbb{E}_t \left[ \int_t^{\tau^*} 1 dt \right]. \tag{B.27}$$

By definition, it holds that when  $C_t = C \geq \tilde{C}$ , then  $\tau^* = t$  and

$$\tau(C_t) = \tau(C) = 0.$$

By the integral expression (B.27) and the dynamic programming principle, it follows that For  $C \leq \tau(C)$ , the function  $\tau(C)$  solves the ODE

$$0 = 1 + \tau'(C)\mu_C(C) + \frac{\sigma_C(C)^2 \tau''(C)}{2}, \tag{B.28}$$

where

$$\begin{aligned}\mu_C(C) &= rC + N(C)^\xi A^{1-\xi} - \eta N(C)|\sigma^P(C)| \\ \sigma_C(C) &= N(C)(\sigma - \sigma^P(C))\end{aligned}$$

are drift and volatility of net liquidity  $C$  respectively. The ODE (B.28) is solved subject to the boundary condition

$$\tau(\tilde{C}) = 0 \tag{B.29}$$

at  $C = \tilde{C}$ . At  $C = C_L$  (possibly  $C_L = 0$ ), the lower boundary of the state space, the boundary condition

$$\lim_{C \rightarrow C_L} [1 + \tau'(C)\mu_C(C)] = 0$$

applies.

## B.2 Model with Additional Reserve Risk and Returns

**Setup.** We modify the dynamics of  $dM_t$  in (8) by considering that reserve shocks (partially) scale with the level of  $M_t$ , in that

$$dM_t = rM_t dt + (P_t + dP_t)dS_t + N_t f_t dt + N_t \sigma dZ_t - dDiv_t + M_t(\hat{\mu} dt + \hat{\sigma} dZ_t). \tag{B.30}$$

Using (12), (15), and  $M_t = C_t + N_t$  (i.e.,  $C_t = M_t - S_t P_t = M_t - N_t$ ), we obtain the dynamics of excess reserves:

$$dC_t = \left( rC_t + N_t^\xi A^{1-\xi} - N_t \eta |\sigma_t^P| \right) dt + N_t(\sigma - \sigma_t^P) dZ_t + (C_t + N_t)(\hat{\mu} dt + \hat{\sigma} dZ_t) - dDiv_t, \tag{B.31}$$

This model specification nests the baseline. The baseline is obtained for  $\hat{\mu} = \hat{\sigma} = 0$ , in which case (B.30) becomes (8) and (B.31) becomes (16). We solve for a Markov in which all quantities and the platform's value function  $V(C)$  are expressed in terms of excess reserves  $C$ . Unless necessary, we omit time subscripts.

**HJB Equation.** As in the baseline, dividend payouts occur once  $C$  reaches the payout boundary  $\bar{C}$ . The location of the payout boundary is determined by smooth pasting and super contact conditions, that is,  $V'(\bar{C}) - 1 = V''(\bar{C}) = 0$ . In the interior of the state space when  $C \in (0, \bar{C})$ , the

HJB equation for the value function becomes

$$\rho V(C) = \max_{\{N \in [0, \bar{N}], \sigma^P\}} \left\{ V'(C) \left( rC + \hat{\mu}(C + N) + N^\xi A^{1-\xi} - \eta N |\sigma^P| \right) + \frac{1}{2} V''(C) [N(\sigma - \sigma^P) + (C + N)\hat{\sigma}]^2 \right\}, \quad (\text{B.32})$$

subject to  $V'(\bar{C}) - 1 = V''(\bar{C}) = 0$ .

**Optimal Controls.** We solve now for the optimal controls,  $N(C)$  and  $\sigma^P(C)$ , determined via the optimization in (B.32). For this purpose, define  $\gamma(C) = -\frac{V''(C)}{V'(C)}$  as the platform's effective risk-aversion, just as in the main text and baseline. If  $\sigma^P(C) > 0$ , then  $\sigma^P(C)$  satisfies the first order condition,  $\frac{\partial V(C)}{\partial \sigma^P} = 0$ . The first order condition with respect to  $\sigma^P$  reads

$$-\eta N V'(C) - V''(C) [N(\sigma - \sigma^P) + \hat{\sigma}(C + N)] N = 0.$$

Thus,

$$N(\sigma - \sigma^P) + \hat{\sigma}(C + N) = \frac{\eta}{\gamma(C)} \iff \sigma^P(C) = \sigma + \frac{\hat{\sigma}(C + N)}{N} - \frac{\eta \gamma(C)}{N}, \quad (\text{B.33})$$

where  $\sigma_C(C) = N(\sigma - \sigma^P) + \hat{\sigma}(C + N)$  is the volatility of excess reserves. When above expression for  $\sigma^P(C)$  in (B.33) is negative, then  $\sigma^P(C) = 0$ .

We distinguish now between two cases:

1. First, consider  $\sigma^P > 0$ . Inserting the expression for  $\sigma^P$  from (B.33) into the HJB equation (B.32) and simplifying, we calculate

$$\rho V(C) = \max_{N \in [0, \bar{N}]} \left\{ V'(C) \left( rC + \hat{\mu}(C + N) + N^\xi A^{1-\xi} - \eta [(\sigma + \hat{\sigma})N + \hat{\sigma}C - \eta \gamma(C)] \right) + \frac{1}{V''(C)} \left[ \frac{(\eta V'(C))^2}{2} \right] \right\}. \quad (\text{B.34})$$

If  $N = N(C)$  is interior (i.e.,  $N(C) \in (0, \bar{N})$ ), the first order condition  $\frac{\partial V(C)}{\partial N} = 0$  holds. Using (B.34), the first order condition with respect to  $N$  reads then

$$\hat{\mu} + \xi N^{\xi-1} A^{1-\xi} - \eta(\sigma + \hat{\sigma}) = 0,$$

so that

$$N = \underline{N} \equiv A \left( \frac{\xi}{\eta(\sigma + \hat{\sigma}) - \hat{\mu}} \right)^{\frac{1}{1-\xi}} \wedge \bar{N}. \quad (\text{B.35})$$

Re-inserting the expression for  $N$  (i.e.,  $N = \underline{N}$ ) into the expression (B.33) for  $\sigma^P$  yields

$$\sigma^P(C) = \sigma + \hat{\sigma} - \frac{\eta\gamma(C) - \hat{\sigma}C}{\underline{N}}.$$

2. Second, consider that  $\sigma^P = 0$ . Inserting  $\sigma^P = 0$  into (B.32), we can calculate the first order condition with respect to  $N$ ,  $\frac{\partial V(C)}{\partial N} = 0$ :

$$V'(C)\left(\hat{\mu} + \xi N^{\xi-1} A^{1-\xi}\right) + V''(C)[N\sigma + \hat{\sigma}(C + N)](\sigma + \hat{\sigma}) = 0. \quad (\text{B.36})$$

In general, this equation (B.36) cannot be solved for  $N$  in closed-form. Let  $N^*(C)$  the solution to (B.36). Then,

$$N(C) = N^*(C) \wedge \bar{N},$$

where we account for the exogenous upper boundary  $\bar{N}$  on  $N$ .

**Final Solution Steps.** Next, we characterize the boundary behavior of  $V(C)$  as  $C$  approaches zero. Recall that  $\sigma_C(C) = N(\sigma - \sigma^P) + \hat{\sigma}(C + N)$  is the volatility of excess reserves. Similar to the baseline, as  $C$  approaches zero,  $\sigma_C(C) \rightarrow 0$  which implies  $\sigma^P(C) \rightarrow \sigma + \hat{\sigma}$ . Recall that we have shown that  $N(C) = \underline{N} \iff \sigma^P(C) > 0$ . Therefore, we can use the simplified HJB equation (B.34) to derive

$$\lim_{C \rightarrow 0} V(C) = \lim_{C \rightarrow 0} \frac{V'(C)}{\rho} \left( A^{1-\xi} \underline{N}^\xi + \hat{\mu} \underline{N} - \eta \underline{N} (\sigma + \hat{\sigma}) \right).$$

To solve the model it is useful to solve an expression for  $V'''(C)$ . To do so, differentiate both sides of the HJB under the envelope theorem with respect to  $C$  to obtain:

$$(\rho - r - \hat{\mu})V'(C) = V''(C)\mu_C(C) + V''(C)\sigma_C(C)\hat{\sigma} + \frac{V'''(C)\sigma_C(C)^2}{2}.$$

For the value function to be concave and for a non-degenerate solution to exist, it must be that  $V'''(\bar{C}) > 0$ , which requires

$$r + \hat{\mu} < \rho.$$

That is, shareholders' discount rate  $\rho$  must exceed the rate of return on excess reserves,  $r + \hat{\mu}$ . Otherwise, holding excess reserves would not be costly and shareholders would indefinitely delay dividend payouts. For the formal proof of value function concavity in the baseline (i.e.,  $\hat{\mu} = \hat{\sigma} = 0$ ), see Appendix A.1. The arguments of this proof can easily be adjusted to extend the proof of value function concavity when  $\hat{\mu} > 0$  or  $\hat{\sigma} > 0$ .

### B.3 Solution when reserve shocks scale with $M_t$

Recall that the model of Appendix B.2 nests the baseline model. Also note that upon setting  $\sigma = 0$ ,  $\hat{\mu} = 0$ , and  $\hat{\sigma} > 0$ , we obtain that Brownian shocks to reserves  $M_t$  are  $M_t \hat{\sigma} dZ_t$  in (B.30) and scale with the level of reserves  $M_t$ . We present the numerical solution of this alternative

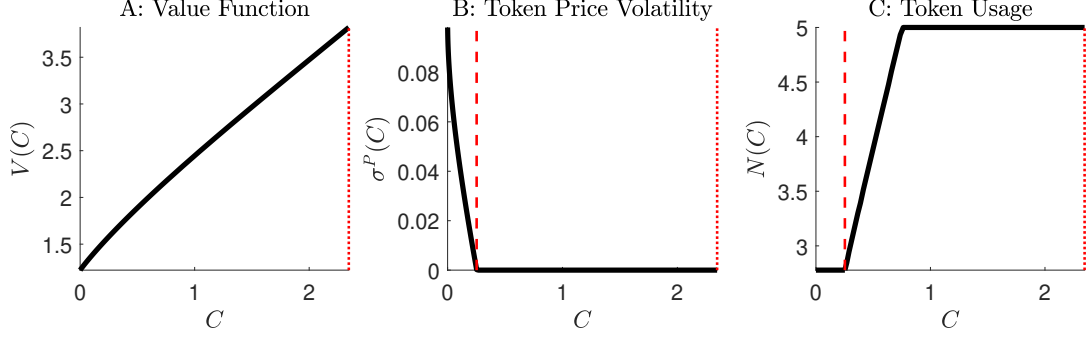


Figure B.1: **Model Solution when reserve shocks scale with  $M_t$ .** We use our baseline parameters from Figure 2 except that we set  $\sigma = 0$  and  $\hat{\sigma} = 0.1$ . We also set  $\hat{\mu} = 0$ .

model specification in Figure B.1. We use our baseline parameters, except that we set  $\hat{\sigma} = 0.1$ ,  $\sigma = 0$ , and  $\hat{\mu} = 0$ . The findings are qualitatively the same as in the baseline. The value function is increasing and concave in  $C$  (left panel A). The middle panel B shows that the token price is stable, if  $C$  is sufficiently large (i.e.,  $C \geq \tilde{C}$ ), and there is token price volatility for lower values of  $C$  and, in particular, as  $C$  approaches zero. Token price volatility decreases with excess reserves and becomes zero at some threshold  $C = \tilde{C}$  (vertical dashed red line). And, the right panel C illustrates that token usage increases with  $C$ , just as in the baseline. As such, we conclude that the specific specification of the Brownian shocks to reserves in (8) does not drive our results.

## B.4 Calculating User Welfare

### B.4.1 Baseline

To start with, recall that any users' utility flow is

$$dR_{it} \equiv N_t^\alpha A^{1-\xi} \frac{u_{it}^\beta}{\beta} dt + u_{it} \left( \frac{dP_t}{P_t} - r dt - f_t dt - \eta |\sigma_t^P| dt \right)$$

As such,

$$\mathbb{E}[dR_{it}] = N_t^\alpha A^{1-\xi} \frac{u_{it}^\beta}{\beta} dt + u_{it} \left( \mu_t^P dt - r dt - f_t dt - \eta |\sigma_t^P| dt \right).$$

Inserting  $u_{it} = N_t$  and (12) and using  $\xi = \alpha + \beta$  yields

$$\begin{aligned} \mathbb{E}[dR_{it}] &= \frac{N_t^\xi A^{1-\xi}}{\beta} dt + N_t \left( \mu_t^P dt - r dt - (N_t^{\xi-1} A^{1-\xi} + \mu_t^P - r - \eta |\sigma_t^P|) dt - \eta |\sigma_t^P| dt \right) \\ &= \frac{N_t^\xi A^{1-\xi}}{\beta} dt - N_t^\xi A^{1-\xi} dt = \frac{(1-\beta) A^{1-\xi}}{\beta} N_t^\xi dt. \end{aligned} \quad (\text{B.37})$$

As a next step, define the user welfare from time  $t$  onward, i.e.,

$$W_t := \mathbb{E} \left[ \int_t^\infty e^{-r(s-t)} dR_{is} \right]. \quad (\text{B.38})$$

As  $C$  is the payoff-relevant state variable, we can express user welfare as function of  $C$ , in that  $W_t = W(C_t)$ . The dynamic programming principle implies that user welfare solves on  $[0, \bar{C}]$  the ODE

$$rW(C_t)dt = \mathbb{E}[dR_{it}] + \mathbb{E}[dW(C_t)].$$

We can rewrite the ODE as

$$rW(C) = \frac{(1-\beta)A^{1-\xi}}{\beta}N(C)^\xi + W'(C)\mu_C(C) + \frac{W''(C)\sigma_C(C)^2}{2}, \quad (\text{B.39})$$

whereby

$$\begin{aligned} \mu_C(C) &= rC + N(C)^\xi A^{1-\xi} - \eta N(C)|\sigma^P(C) + \hat{\mu}(C + N(C))| \\ \sigma_C(C) &= N(C)(\sigma - \sigma^P(C)) + \hat{\sigma}(C + N(C)) \end{aligned}$$

are drift and volatility of net liquidity  $C$  respectively. For the baseline, we set  $\hat{\mu} = \hat{\sigma} = 0$ . For the model extension in Appendix B.2,  $\hat{\mu}$  and  $\hat{\sigma}$  are potentially positive.

The ODE (B.39) is solved subject to the boundary conditions

$$W'(\bar{C}) = 0$$

and

$$\lim_{C \rightarrow 0^+} W(C) = \frac{1}{r} \lim_{C \rightarrow 0^+} \left( \frac{(1-\beta)A^{1-\xi}}{\beta}N(C)^\xi + W'(C)\mu_C(C) \right).$$

#### B.4.2 Model extension with Big Data as a Productive Asset

In the model extension with big data as a productive asset, user welfare is a function  $W(C, A)$ , that is,  $W_t = W(C_t, A_t)$ . We conjecture and verify that  $W(C, A)$  scales with  $A$ , i.e.,  $W(C, A) = Aw(c)$  with  $c = C/A$ . First, we recall (B.37), that is,

$$\mathbb{E}[dR_{it}] = \frac{(1-\beta)A^{1-\xi}}{\beta}N_t^\xi dt = \frac{(1-\beta)A}{\beta}n_t^\xi dt,$$

and note that  $n_t = N_t/A_t$  is a function of  $c_t = C_t/A_t$  only, i.e.,  $n_t = n(c_t)$ . Second, the dynamic programming principle implies that user welfare solves the ODE

$$rW(C_t, A_t)dt = \mathbb{E}[dR_{it}] + \mathbb{E}[dW(C_t, A_t)]. \quad (\text{B.40})$$

Using the conjecture  $W(C, A) = Aw(c)$ , we obtain

$$W_C(C, A) = w'(c), \quad W_A(C, A) = w(c) - w'(c)c, \quad \text{and} \quad W_{CC}(C, A) = \frac{w''(c)}{A}. \quad (\text{B.41})$$



Expanding the right hand side of (B.40), using (B.41) and  $W(C, A) = Aw(c)$ , simplifying and dividing both sides of (B.40) by  $dt$ , one derives

$$(r - \kappa n(c)^\xi)w(c) = w'(c)\mu_c(c) + \frac{w''(c)\sigma_c(c)^2}{2}, \quad (\text{B.42})$$

with drift  $\mu_c(c) \equiv rc + n(c)^\xi - \eta n(c)|\sigma^P(c)| - \kappa n(c)^\xi c$  and volatility  $\sigma_c(c) = n(c)(\sigma - \sigma^P(c))$ . The ODE (B.42) is solved subject to the boundary conditions  $w'(\bar{c}) = 0$  and

$$\lim_{c \rightarrow 0^+} (r - \kappa n(c)^\xi)w(c) = \lim_{c \rightarrow 0^+} w'(c)\mu_c(c).$$

## B.5 Details on the Model with User Collateral

In this section, we provide the solution details under the model specification with user collateral requirements. To start with, take users' problem (37) (facing collateral requirements  $m_t$ ):

$$\max_{u_{i,t}} \left\{ \frac{1}{\xi} u_{i,t}^\xi A^{1-\xi} dt + u_{i,t} \left( \mu_t^P - \eta |\sigma_t^P| - f_t \right) dt + u_{i,t} m_t (\tilde{\mu} - \delta - r) dt \right\}.$$

All users act the same so that  $u_{i,t} = N_t$ . Analogous to the baseline, we then calculate optimal platform transaction volume (after solving users' optimization and invoking  $u_{i,t} = N_t$ ):

$$N_t = \frac{A}{(r + f_t - \mu_t^P + \eta |\sigma_t^P| + m_t(\tilde{\mu} - \delta - r))^{1-\xi}},$$

when  $N_t < \bar{N}$ . We can solve the above for  $f_t$ , yielding (38), i.e.,

$$f_t = \left( \frac{A}{N_t} \right)^{1-\xi} - m_t(r + \delta - \tilde{\mu}) + \mu_t^P - \eta |\sigma_t^P|.$$

As a next step in the solution, we calculate

$$\begin{aligned} \int_{1-\frac{1}{m_t}}^1 (1 - m_t(1 - \theta_a)) d\theta_a &= [\theta - m_t\theta + 0.5m_t\theta_a^2]_{1-\frac{1}{m_t}}^1 \\ &= 1 - 0.5m_t - \left( 1 - \frac{1}{m_t} - m_t + 1 + 0.5m_t \left[ 1 + \frac{1}{m_t^2} - \frac{2}{m_t} \right] \right) \\ &= \frac{1}{2m_t}, \end{aligned}$$

so

$$\begin{aligned} &2(\delta dt - \sigma dZ_t) \times \mathbb{P}(\{m_t(1 - \theta_a) < 1\}) \mathbb{E}[1 - m_t(1 - \theta_a) | m_t(1 - \theta_a) < 1] \\ &= 2(\delta dt - \sigma dZ_t) \times \left( \int_{1-\frac{1}{m_t}}^1 (1 - m_t(1 - \theta_a)) d\theta_a \right) = \frac{1}{m_t}(\delta dt - \sigma dZ_t), \end{aligned} \quad (\text{B.43})$$

which was to show.

Under this alternative specification, platform reserves follow

$$dM_t = rM_t dt + (P_t + dP_t)dS_t + N_t f_t dt - \frac{\delta N_t}{m_t} dt + \frac{N_t \sigma}{m_t} dZ_t - dDiv_t.$$

Inserting (38), we obtain

$$dM_t - (P_t + dP_t)dS_t = rM_t dt + N_t^\xi A^{1-\xi} dt - \frac{\delta N_t}{m_t} dt - m_t(r + \delta - \tilde{\mu})N_t dt \quad (\text{B.44})$$

$$+ N_t (\mu_t^P - \eta|\sigma_t^P|) dt - \frac{N_t \sigma}{m_t} dZ_t - dDiv_t. \quad (\text{B.45})$$

By Ito's Lemma,  $d(S_t P_t) = dS_t P_t + S_t dP_t + dS_t dP_t$ . As a result, we can calculate that  $C_t$  follows

$$dC_t = \mu_C(C_t)dt + \sigma_C(C_t)dZ_t - dDiv_t,$$

with

$$\begin{aligned} \mu_C(C) &= rC - r(m-1)N(C) + m(\tilde{\mu} - \delta)N(C) + N(C)^\xi A^{1-\xi} - N(C)\eta|\sigma^P(C)| - \frac{N(C)\delta}{m} \\ \sigma_C(C) &= N(C) \left( \frac{\sigma}{m} - \sigma^P(C) \right) \end{aligned}$$

and  $\sigma_t^P = \sigma^P(C_t)$  and  $N_t = N(C_t)$ .

As in the baseline, dividend payouts occur at the upper reflecting boundary  $\bar{C}$ , satisfying  $V'(\bar{C}) - 1 = V''(\bar{C}) = 0$ . Then, the dynamic programming principle, the HJB equation for  $C \in (0, \bar{C})$  can be written as

$$\rho V(C) = \max_{N \in [0, \bar{N}], m, \sigma^P} V'(C)\mu_C(C) + \frac{\sigma_C(C)^2 V''(C)}{2}.$$

To solve for the optimal controls, we distinguish between the cases 1)  $\sigma^P(C) > 0$  and 2)  $\sigma^P(C) = 0$ :

1. Suppose that  $\sigma^P(C) > 0$ . Then, the first order condition with respect to  $\sigma^P$  yields

$$\frac{\partial V(C)}{\partial \sigma^P} = 0 \iff -\eta V'(C) - V''(C) \left( \frac{N\sigma}{m} - N\sigma^P \right) = 0$$

so that

$$N\sigma^P = \frac{-\eta V'(C) - V''(C) \frac{N\sigma}{m}}{-V''(C)}.$$

Overall,

$$N\sigma^P = \max \left\{ 0, \frac{-\eta V'(C) - V''(C) \frac{N\sigma}{m}}{-V''(C)} \right\} = \max \left\{ 0, -\frac{\eta V'(C)}{-V''(C)} + \frac{N\sigma}{m} \right\}.$$

We can insert this expression for  $\sigma^P$  into (22) to get

$$\rho V(C) = \max_{N \in [0, \bar{N}], m} \left\{ V'(C) \left[ rC + N^\xi A^{1-\xi} - \frac{\eta N \sigma}{m} - r(m-1)N + m(\tilde{\mu} - \delta)N - \frac{N\delta}{m} - \frac{\eta^2 V'(C)}{V''(C)} \right] + \frac{1}{V''(C)} \left[ \frac{(\eta V'(C))^2}{2} \right] \right\}.$$

The choice of  $m$  is independent of  $N$ . One can calculate that optimal  $m$  solves the first-order condition

$$\tilde{\mu} - \delta - r + \frac{\delta}{m^2} + \frac{\eta\sigma}{m^2} = 0,$$

so that

$$\frac{1}{m^2} = \frac{r + \delta - \tilde{\mu}}{\delta + \eta\sigma} \iff m = \bar{m} \equiv \sqrt{\frac{\delta + \eta\sigma}{r + \delta - \tilde{\mu}}}.$$

Next, we can take the first-order condition with respect to  $N$  to obtain

$$\xi N^{\xi-1} A^{1-\xi} - \frac{\eta\sigma}{m} - r(m-1) + m(\tilde{\mu} - \delta) - \frac{\delta}{m} = 0.$$

Thus,

$$N(C) = \underline{N} = A \left( \frac{\xi}{\frac{\eta\sigma}{m} + r(\bar{m}-1) - \bar{m}(\tilde{\mu} - \delta) + \frac{\delta}{m}} \right)^{\frac{1}{1-\xi}} \wedge \bar{N} \quad (\text{B.46})$$

2. Suppose that  $\sigma^P = 0$ . Then, taking the derivative with respect to  $N$  yields

$$\frac{\partial V(C)}{\partial N} = \frac{1}{\rho} \left( V'(C) \left[ \xi N^{\xi-1} A^{1-\xi} - r(m-1) + m(\tilde{\mu} - \delta) - \frac{\delta}{m} \right] + N \left( \frac{\sigma}{m} \right)^2 V''(C) \right).$$

Taking the first-order condition with respect to  $m$  yields

$$\frac{\partial V(C)}{\partial m} = 0 \iff V'(C)N \left[ \tilde{\mu} - \delta - r + \frac{\delta}{m^2} \right] - N^2 V''(C) \frac{\sigma^2}{m^3} = 0. \quad (\text{B.47})$$

Thus,

$$N = N(C) = \frac{-V'(C)}{V''(C)} \left( \frac{r + \delta - \delta/m^2 - \tilde{\mu}}{\sigma^2/m^3} \right) = \frac{-V'(C)}{V''(C)} \left( \frac{(r + \delta - \tilde{\mu})m^3 - \delta m}{\sigma^2} \right)$$

Finally, we discuss the value function at the payout boundary  $\bar{C}$  where  $V'(\bar{C}) - 1 = V''(\bar{C}) = 0$ . At  $C = \bar{C}$ , we have — as in the baseline —  $\gamma(\bar{C}) = V''(\bar{C}) = 0$ . As such,

$$\sigma^P(\bar{C}) = 0$$

and

$$N(\bar{C}) = \bar{N}.$$

Using (B.47), we obtain

$$m(\bar{C}) = \sqrt{\frac{\delta}{r + \delta - \tilde{\mu}}}$$

as the margin requirement at  $C = \bar{C}$ . Analogous to the baseline, there exist three regions and two thresholds  $\tilde{C}$  and  $\tilde{C}'$  such that i)  $\sigma^P(C) > 0$  if and only if  $C < \tilde{C}$  (otherwise,  $\sigma^P(C) = 0$ ) and ii)  $N(C) = \bar{N}$  if and only if  $C \geq \tilde{C}'$  (otherwise,  $N(C) < \bar{N}$ ). The platform optimally prevents liquidation as  $C$  approaches zero, leading to the boundary condition

$$\lim_{C \rightarrow 0^+} \sigma^P(C) = 0 \iff \lim_{C \rightarrow 0^+} \sigma^P(C) = \frac{\sigma}{\bar{m}}.$$

This boundary condition can be manipulated to obtain a condition analogous to (??). Using that

$$\lim_{C \rightarrow 0^+} N(C) = \underline{N}, \quad \lim_{C \rightarrow 0^+} m(C) = \bar{m} \quad \text{and} \quad \lim_{C \rightarrow 0^+} \sigma_C(C) = 0,$$

we obtain

$$\lim_{C \rightarrow 0^+} \frac{V(C)}{V'(C)} = \frac{1}{\rho} \left\{ -r(\bar{m} - 1)\underline{N} + m(\tilde{\mu} - \delta)\underline{N} + \underline{N}^\xi A^{1-\xi} - \underline{N}\eta \frac{\sigma}{\bar{m}} - \frac{N\delta}{\bar{m}} \right\}.$$

## B.6 Negative Lower Bound in the State Space

In this Section, we provide the model solution without the assumption of over-collateralization,  $C_t \geq 0$ . Instead, the lower bound on  $C_t$ , denoted  $\underline{C}$ , can potentially be negative. To start with, note that regardless of the value of the lower bound  $\underline{C}$ , the law of motion (16) applies for  $C_t$ , and transaction volume  $N_t$  is characterized in (11). In a Markov equilibrium, all quantities can be expressed as functions of  $C$  only, so we omit time subscripts unless necessary. As is standard, dividend payouts are made at an endogenous payout boundary  $\bar{C}$ , with  $V'(\bar{C}) - 1 = V''(\bar{C}) = 0$ . On  $(\underline{C}, \bar{C})$ , the platform's value function  $V(C)$  solves the HJB equation (22). The optimal controls  $\sigma^P(C)$  and  $N(C)$  are determined according to the optimization in the HJB equation (22).

Once  $C < \underline{C}$ , there is liquidation and the platform owners' value falls permanently to zero. The platform operates under  $C \geq \underline{C}$  and  $M \geq 0$  (i.e., the value of reserves must remain positive), and, importantly, prevents liquidation.<sup>63</sup> We analyze the case of  $\underline{C} < 0$ . To begin with, note that

$$M = C + SP \geq 0 \iff C \geq -SP = -N,$$

where token market clearing implies  $SP = N$  (see (7)). Due to  $N \leq \bar{N}$  and  $M \geq 0$ , it follows that  $C \geq -\bar{N}$ . That is,  $\underline{C} \geq -\bar{N}$ . Another observation is that for  $C < 0$ , the optimization constraint  $N(C) \geq -C \iff M \geq 0$  applies to the optimization in the HJB equation (22) to ensure reserves

<sup>63</sup>Notably, if the platform is liquidated, then liquidation occurs at  $C \geq 0$ . Upon liquidation at time  $\tau$ , token price is zero,  $P_\tau = 0$ . As such,  $C_\tau = M_\tau - S_\tau P_\tau \geq 0$  and, because token price evolves continuously, liquidation cannot occur at  $C < 0$ .

$M$  remain positive. We numerically verify that this constraint never binds under our baseline parameters.

To ensure  $C_t \geq \underline{C}$  at all times  $t \geq 0$  and so to prevent  $C$  from dropping below  $\underline{C}$ , it is necessary that i) the drift of  $dC$  remains positive and ii) the volatility of  $dC$  vanishes, as  $C$  approaches  $\underline{C}$ . According to (16), the second requirement ii) is equivalent to

$$\lim_{C \rightarrow \underline{C}_L} \sigma^P(C) = \sigma.$$

And, recall that by maximization in the HJB equation (22), optimal transaction volume becomes  $N(C) = \underline{N}$  whenever  $\sigma^P(C) > 0$  and the constraint  $N(C) \geq -C$  does not bind, where  $\underline{N}$  is characterized in closed-form in (25) (for details, see Part I of the proof of Proposition 3). As a result,  $\lim_{C \rightarrow \underline{C}} N(C) = \max\{\underline{N}, -\underline{C}\}$ . Inserting  $C = \underline{C}$ ,  $\sigma^P(C) = \sigma$ , and  $N(C) = \max\{\underline{N}, -\underline{C}\}$  into (16), we obtain the following drift of  $dC$ , denoted  $\mu_C(C)$ , as  $C$  approaches  $\underline{C}$ :

$$\lim_{C \rightarrow \underline{C}} \mu_C(C) = \max\{\underline{N}, -\underline{C}\}^\xi A^{1-\xi} - \eta \max\{\underline{N}, -\underline{C}\} \sigma + r\underline{C}.$$

Equating above expression to zero, we obtain that the drift of  $dC$  remains zero as  $C$  approaches  $\underline{C}$  as long as

$$\underline{C} \geq \frac{1}{r} \left( \eta \max\{\underline{N}, -\underline{C}\} \sigma - (\max\{\underline{N}, -\underline{C}\})^\xi A^{1-\xi} \right) =: \hat{C}_L(\underline{C}). \quad (\text{B.48})$$

Combining this relation with  $\underline{C} \geq -\bar{N}$ , we obtain the lowest possible  $\underline{C}$ , which we call  $\underline{C}^{min}$ :

$$\underline{C}^{min} = \max \left\{ \hat{C}_L(\underline{C}^{min}), -\bar{N} \right\}.$$

We solve the model for the lowest possible  $\underline{C}$ , that is,  $\underline{C} = \underline{C}^{min}$ .<sup>64</sup> Because drift and volatility of excess reserves vanish as  $C$  approaches  $\underline{C} = \underline{C}^{min}$ , the boundary condition  $\lim_{C \rightarrow \underline{C}} V(C) = 0$  applies.

Figure B.2 presents the model solution with lower bound  $\underline{C} = \underline{C}^{min}$  under our baseline parameters. Under our baseline parameters,  $\underline{C}^{min} = -0.823$ ,  $\underline{N} = 2.778 > -\underline{C}$  and  $C_L > -\bar{N} = -5$ . We verify that the constraint  $N(C) \geq -C$  never binds under our baseline parameters. A novel finding is that depending on the circumstances, the same stablecoin may be under- or over-collateralized, in that  $C$  takes in equilibrium both positive and negative values. The other findings are qualitatively the same as in the baseline. The value function is increasing and concave in  $C$  (left panel A). The middle panel B shows that the token price is stable, if  $C$  is sufficiently large (i.e.,  $C \geq \tilde{C}$ ), and there is token price volatility for lower values of  $C$  and, in particular, as  $C$  approaches the lower bound  $\underline{C}$  and tokens become under-collateralized. Token price volatility decreases with excess reserves and becomes zero at some threshold  $C = \tilde{C}$  (vertical dashed red line). And, the right panel C illustrates that token usage increases with  $C$ , just as in the baseline. As such, we conclude that the focus on

<sup>64</sup>Note that a decrease of  $\underline{C}$  relaxes the platform's optimization constraint  $C \geq \underline{C}$  and therefore is beneficial for the platform.

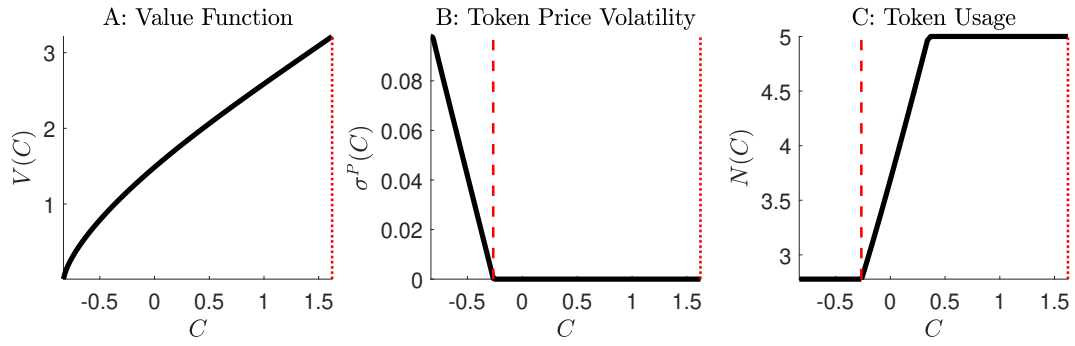


Figure B.2: **Solution with A Negative Lower Bound on  $C$ .** This figure plots the platform value function (Panel A), token price volatility (Panel B), and token demand and usage (Panel C) as functions of excess reserves  $C$ . The parameterization follows Figure 2, and the lower bound is  $\underline{C} = -0.823$ .

over-collateralization,  $C \geq 0$  (i.e.,  $\underline{C} = 0$ ), does not drive our results.