Token-Based Platforms and Speculators*

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October 4, 2021

Abstract

In a model of token-based platforms and cryptocurrencies (tokens), platform users hold tokens for transactions and speculators hold tokens for returns. The marginal token investor can be a user or speculator and, to maximize its dollar value, the platform caters to the marginal investor, creating conflicts of interest between the platform, its users, and speculators. Speculators affect users via two opposing effects, crowding-out and risk-sharing, and their token investments are substitutes in certain circumstances but complements in others. We also evaluate alternative platform structures, such as a fiat-based platform or a dual token structure featuring a stablecoin and governance token.

Many cryptocurrencies and tokens — such as Ether, Ada, or Filecoin — are intended to serve as a means of payment or transaction medium on a token-based platform.[1] At the same time, cryptocurrencies and tokens have become notorious as speculative assets. The fact that cryptocurrencies and tokens are held by both speculators and users raises several questions. How do speculators affect the adoption and pricing of cryptocurrencies and tokens? What are the potential conflicts of interest between speculators and users, and how to resolve them? Does the design of cryptocurrencies cater to speculators at the expense of its users and, if so, when and why?

To address these questions, we analyze a dynamic model of a token-based platform that settles transactions among platform users with its native cryptocurrency (referred to as "tokens"). Users hold tokens to transact on the platform. To capture that price stability is important for any transaction medium (Doepke and Schneider (2017)), we assume that users have a preference for token price stability and so are risk-averse. Speculators representing financial investors trade tokens for returns (subject to a short selling constraint) but do not transact on the platform. Speculators do not have a transaction-based preference for token price stability and so are less risk-averse than users. The benefits of transacting on the platform increase with platform productivity capturing the general usefulness or technology of the platform. Fluctuations in platform productivity cause token price volatility which hampers platform transactions and adoption by users. As token price volatility changes over time, there are "normal times" and (more) "volatile times".[2]

The token price depends on both the transactional demand from users and the investments from speculators. In normal times, token price volatility is low, platform transaction volume is high, and the marginal token investor is a user. Low expected token returns then discourage speculative investment, further stabilizing token price. In volatile times, token price volatility is high, and therefore platform transaction volume and token price are low, which implies high expected token returns going forward that encourage speculative token investment. Consequently, the marginal token investor in volatile times is a speculator. As speculators buy tokens and drive up token price, expected token returns decrease and users' (opportunity) costs of holding tokens and transacting on the platform increase, which further curbs platform usage in volatile times. In other words, speculators crowd out platform users in volatile times. Due to this crowding-out effect, speculative

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[1]Ether (Ada) is the native cryptocurrency and transaction medium of the blockchain platform Ethereum (Cardano), and its market capitalization (as of 09/02/2021) lies around 440 (95) billion USD. Filecoin is the transaction medium of the equally named cloud storage platform. Filecoin raised 257 million USD in its initial coin offering (ICO) in 2017, and its market capitalization (as of 09/02/2021) lies around 8 billion USD. We refer to such platforms that feature a cryptocurrency/token as native transaction medium broadly as token-based platforms.

[2]In practice, volatile times could correspond to periods of high uncertainty in the broader financial market (e.g., due to a pandemic or financial crisis) or in cryptocurrency markets only (e.g., due to impending regulation of cryptocurrencies or the hack of a crypto exchange). Token price volatility could also reflect platform-specific (risk) factors, such as technological or user demand uncertainty. Consistent with our model, time-varying token price/return volatility is empirically documented for cryptocurrency markets in (Hafner, 2020).

and transactional token investments can be viewed as (static) substitutes.

Providing liquidity in volatile times, speculators absorb token price risk and effectively share risk with users. This risk-sharing effect stimulates platform transactions and adoption in normal times. When token price volatility rises, transactional demand for tokens falls and users would like to sell tokens. Users can sell tokens in volatile times at more favorable terms precisely because speculators buy. The enhanced resale value of tokens in volatile times increases users' incentives to hold tokens for transactions and to adopt the platform in normal times. As, in addition, speculators only buy tokens due to the prospect of high future adoption, speculative and transactional token investments can be viewed as dynamic complements. Our model predicts that due to the static substitutes and dynamic complements features, the amount of speculative token investment correlates negatively with contemporaneous platform usage but positively with future platform usage.

The presence of speculators increases platform usage in normal times via the risk-sharing effect but curbs it in volatile times via the crowding-out effect, thereby amplifying fluctuations in platform usage. Owing to these two opposing effects, it is generally not optimal for user welfare and total surplus to ban speculative trading; notably, it can be even optimal to stimulate speculative trading. To reduce the barriers to token trading and so to encourage speculative trading, the platform could facilitate derivatives-based token trading or pursue the token listing on crypto-exchanges (e.g., via an Initial Exchange Offering (IEO)). Another way to stimulate speculative token trading is to facilitate limited short selling. Expecting the token price to drop, speculators then short sell tokens in normal times, thereby decreasing token price and the magnitude of a potential price drop in normal times. Consequently, short selling by speculators dampens price fluctuations and therefore fosters adoption as long as it is subject to a constraint preventing a token price crash due to excessive short selling pressure.

To study how platform development and strategies interact with speculative token trading and investment, we introduce (physical) investment in platform productivity. Investment improves platform productivity (on average) but is risky and increases token price volatility. To maximize platform dollar value, the platform chooses its investment to cater to the needs of the *marginal* token investor so as to maximize token price (given token supply). Hence, the platform's incentives are aligned with those of the marginal investor, which inevitably leads to conflicts of interest in the triangular relationship between the platform (owners), its users, and speculators. When the marginal token investor is a user, the token price internalizes the negative impact of price volatility on platform transactions and reflects a high degree of risk-aversion, so that investment is low. When the marginal token investor is a speculator, the token price does not internalize the impact of price volatility on transactions and reflects a low degree of risk-aversion, so that investment is high but

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platform transaction volume is low.

While the marginal token investor's (risk) preferences determine the platform's incentives to invest, the choice of investment also determines whether the marginal investor is a user or a speculator. In particular, high platform investment increases token price volatility, reduces platform transactions, and fuels speculative token trading. The rise in speculative token trading incentivizes the platform to further increase its investment. This two way feedback between platform investment and speculative token trading boosts platform development, productivity growth, and future platform usage, but amplifies price volatility and so depresses current platform usage.

In normal times, speculators do not hold tokens and users' preference for price stability limits investment, leading to under-investment. Speculators as the marginal token investors in volatile times, in turn, stimulate investment (in both normal and volatile times) and therefore mitigate this under-investment problem. Notably, occasional periods of high token price volatility (i.e., volatility spikes) can be seen as a feature rather than a bug of token-based platforms, as they invite speculative trading which stimulates platform development, investment, and long-run adoption. However, the investment-enhancing effect of speculators has a counterpart. When the platform's investment and growth opportunities are subject to high uncertainty, speculators induce excessively high investment (that is, over-investment), leading to extended periods of excessively low platform usage, high token price volatility, and pronounced speculative token trading.

As tokens serve as the platform transaction medium, the platform earns payoff from issuing these tokens which is called seigniorage. Seigniorage allows the platform to be profitable *without* i) charging users a transaction fee or ii) adopting other monetization models that may harm users (such as exploiting user/transaction data or putting advertisements). On the other hand, a token-based platform suffers from the volatility of its native transaction medium. The natural alternative to a token-based platform is a fiat-based platform featuring fiat money as a stable transaction medium. Under a fiat-based (platform) structure, users are not exposed to any platform-specific risk which is entirely absorbed by speculators as the platform's equity holders. As such, a fiat-based structure improves risk-sharing between users and speculators, but does so at the expense of seigniorage. Since fiat money is issued by a central bank which collects the seigniorage, a fiat-based platform does not earn seigniorage and must charge transaction fees (or adopt aforementioned monetization models that are outside of our model) to be profitable. Transaction fees increase the cost of transacting on the platform and hence reduce platform transactions and adoption.

The trade-off between a fiat-based or token-based structure is to obtain either stability of the platform transaction medium (i.e., improved risk-sharing) or seigniorage. In light of this trade-off, a token-based structure is preferred if and only if the platform's (technological or user demand)

uncertainty is low. Therefore, only developed and mature platforms which tend to face less uncertainty than early-stage platforms benefit from launching their own tokens. Thus, our model can explain why large technology and digital platforms such as Amazon, Facebook, or Uber consider to launch their own tokens, whereas the ICO market dominated by early-stage platform firms performed poorly in the past. We also find that from a total welfare point of view, platform owners do not have sufficient private incentives to implement a token-based structure. Optimal regulation should mitigate this under-provision of viable token-based platforms and stimulate token issuance by mature and developed platforms, while restricting token issuance by early-stage platforms.

We then propose a dual token structure that features a stable platform transaction medium and allows the platform to harness seigniorage. The dual token structure has two native tokens: i) a price-stable transaction token (i.e., stablecoin) held by users for transactions and ii) a governance (equity) token held by speculators (financial investors). Users are charged a transaction fee and the governance token pays out revenues from transaction fees and transaction token issuance as dividends. The proceeds from governance token issuance are used to buy back (and burn) transaction tokens to stabilize their price. This dual token structure resembles the one of the Decentralized Finance (DeFi) platform MakerDAO, featuring the stablecoin DAI and governance token MKR. De-coupling the investment and transaction function of tokens, the dual token structure enhances risk-sharing between users and speculators, thereby stimulating platform usage.

Major cryptocurrencies with high trading volume and market capitalization — such as Bitcoin, Ether, or Ada — i) have become popular among retail investors and ii) can be shorted easily/cheaply (e.g., on crypto-exchanges like Binance). To capture these features, we study a model extension with noise traders and without short selling constraints. Noise traders represent retail investors who are less skilled or informed than speculators representing professional investors. Noise traders cause unpredictable price fluctuations which i) hamper platform transactions and ii) induce a "short squeeze risk" for short sellers. Notably, this short squeeze risk prevents excessive short selling by speculators that could otherwise trigger a token price crash.[3] As a result, token price risk from noise trading helps the token and the platform to sustain value and so can be beneficial despite its negative impact on platform transactions. Our findings imply that in general, regulation should not restrict cryptocurrency and token investments to accredited and professional investors only.

Our work adds to the literature on blockchain economics and cryptocurrencies.[4] Abadi and

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[3]The recent examples of GameStop and AMC have emphasized that a short squeeze can lead to tremendous losses for short sellers. Thus, short squeeze risk is likely a relevant concern for short sellers across asset classes, which is broadly consistent with our finding that short squeeze risk limits speculative short selling.

[4]For an extensive literature review on blockchain economics, see Chen, Cong, and Xiao (2021), and for an empirical overview of the public blockchain ecosystem, see Irresberger, John, and Saleh (2020). Related, Allen, Gu, and Jagtiani (2020) provide a survey about FinTech research.

Brunnermeier (2018), Chiu and Koeppl (2019) , Cong and He (2019), Huberman, Leshno, and Moallemi (2019), Ebrahimi, Routledge, and Zetlin-Jones (2020), and John, Rivera, and Saleh (2020) study the economic implications of blockchains and decentralized ledger technology. Hu, Parlour, and Rajan (2019), Liu and Tsyvinski (2021), and Liu, Tsyvinski, and Wu (2021) empirically analyze the cryptocurrency return structure. Biais, Bisiere, Bouvard, Casamatta, and Menkveld (2018), Schilling and Uhlig (2019), Pagnotta (2021), and Sockin and Xiong (2021) provide a theoretical analysis of the pricing of cryptocurrencies. Notably, Sockin and Xiong (2021) micro-found the network effect ineherent to token adoption and show that tokenization allows platform owners to commit not to abuse platform users, thereby providing a conceptual framework for tokenization. Easley, O'Hara, and Basu (2019), Biais, Bisiere, Bouvard, and Casamatta (2019), Garratt and van Oordt (2020), Lehar and Parlour (2020), Hinzen, John, and Saleh (2021), Prat and Walter (2021), and Benetton, Compiani, and Morse (2021) analyze the economic implications of the proof-of-work consensus algorithm and mining, and Rosu and Saleh (2020) and Saleh (2021) focus on proof-of-stake as alternative protocol. A large subset of the literature on cryptocurrencies focuses on initial coin offerings (ICOs) (see Li and Mann (2019) for a literature review). Empirical studies on ICOs include Lyandres, Palazzo, and Rabetti (2019). Howell, Niessner, and Yermack (2020), and Davydiuk, Gupta, and Rosen (2020). Recent theoretical contributions on ICOs include Malinova and Park (2018), Catalini and Gans (2018), Bakos and Halaburda (2019a,b), Li and Mann (2020), Lee and Parlour (2020), Canidio (2021), Goldstein, Gupta, and Sverchkov (2021), Garratt and Van Oordt (2021), Gan, Tsoukalas, and Netessine (2021a,b), Gryglewicz, Mayer, and Morellec (2021), and Chod and Lyandres (2021). Our paper differs from these studies, as it focuses on the conflicts of interest between token users and speculators.

We model platform transactions following Cong, Li, and Wang (2021a,b). Cong et al. (2021b) analyze the token pricing implications of users' inter-temporal adoption decisions. Cong et al. (2021a) study a platform's optimal management of token supply and investment, and find that commitment through blockchain technology can mitigate under-investment. Danos, Marcassa, Oliva, and Prat (2021) provide a valuation framework for utility tokens with endogenous token velocity and show that early on during the token's adoption phase, the marginal investor holds tokens for purely speculative purposes. Different to Cong et al. (2021a,b) and Danos et al. (2021) that feature a single type of token investors (users), our paper introduces speculators as separate investor type so as to highlight the dynamic interactions between platform users and speculators. We find that speculators and users interact via the crowding-out and risk-sharing effect, and that the platform owners may cater to speculators at the expense of users, generating conflicts of interest between the platform, its users, and speculators. These effects are not identified in Cong et al.

([2021a,b](#)) and [Danos et al. (2021)](#).

Finally, our paper adds to the literature on speculation in financial markets. Notable contributions include Harrison and Kreps (1978), Scheinkman and Xiong (2003), and Hong, Scheinkman, and Xiong (2006) on speculation-induced asset price bubbles, Simsek (2013) on speculation-induced credit booms, Nathanson and Zwick (2018) and Gao, Sockin, and Xiong (2020) on speculation in the housing market, Sockin and Xiong (2015) on speculation in commodity markets, and Caballero and Simsek (2020) on the macro-economic implications of speculation. Our paper differs from these works mainly in the following aspects. First, it focuses on cryptocurrency markets and token-based platforms that feature unique trade-offs inherent to speculation and asset usage. Second, it considers rational financial investors as speculators and does not assume any belief disagreement.

1   The Model

Time $t \in [0, \infty)$ is continuous. We consider a platform that settles transactions among its users with its native cryptocurrency ("tokens"). That is, tokens serve as the (only) platform transaction medium. Tokens have supply $M_t > 0$ and equilibrium price P_t in terms of the numeraire ("dollars"). So, token market capitalization in dollars is $M_t P_t$. There are two types of agents: a unit mass of platform users (indexed by $i \in [0, 1]$), and a unit mass of speculators (indexed by $j \in (1, 2]$). All agents discount the future at (interest) rate $r > 0$ and have unlimited funds. On the platform, users transact with each other and so hold tokens for transactions. Speculators are financial investors who do not transact on the platform and hold tokens solely for investment/trading returns.

Platform productivity.   The platform is characterized by its productivity A_t which captures the general usefulness of the platform, the quality of the platform's technology, or user demand for platform services. Platform productivity evolves according to a Geometric Brownian Motion:

$$\frac{dA_t}{A_t} = \mu dt + \sigma_t dZ_t, \tag{1}$$

where dZ_t is a standard Brownian Motion, $\mu \geq 0$ is the constant drift with $\mu < r$, and $\sigma = \sigma_t \geq 0$ is the volatility.[5] To capture that volatility in cryptocurrency/token markets can be time-varying as observed in the data (Hafner, 2020), we introduce uncertainty shocks.[6] Volatility $\sigma = \sigma_t \in$

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[5]The Brownian shocks to platform productivity can capture various sources of uncertainty. For instance, dZ_t could capture technological risk, user demand uncertainty (Chod and Lyandres, 2021; Gan et al., 2021a), aggregate risk in the financial market, or uncertainty specific to cryptocurrency markets such as regulatory uncertainty or security risks (Budish, 2018; Pagnotta, 2021).

[6]Uncertainty shocks and/or stochastic and time-varying volatility are modelled in the macroeconomics literature (Bloom, 2009; Christiano, Motto, and Rostagno, 2014; Di Tella, 2017), the asset pricing literature (Stein and Stein,

$\{\sigma_B, \sigma_G\}$ follows a Markov switching process with states B ("volatile times") and G ("normal times"), and transition probabilities $\lambda_G, \lambda_B \geq 0$ so that $\mathbb{P}(\sigma_{t+dt} = \sigma_G | \sigma_t = \sigma_B) = \lambda_G dt$ and $\mathbb{P}(\sigma_{t+dt} = \sigma_B | \sigma_t = \sigma_G) = \lambda_B dt$.[7] State B describes *volatile times* and state G describes *normal times*, in that $\sigma_B > \sigma_G \geq 0$. Uncertainty shocks could capture the change in uncertainty in the broader financial market or economy, e.g., due to a financial crises or pandemic, or in cryptocurrency markets only, e.g., due to impending regulation of cryptocurrencies or due to a hack of a crypto-exchange. One can interpret the uncertainty shocks also as purely platform-specific reflecting changes in technological or user demand uncertainty.

Having characterized the sources of uncertainty in the model, we postulate that equilibrium token price in dollars P_t evolves according to

$$\frac{dP_t}{P_t} = \mu_t^P dt + \sigma_t^P dZ_t + \Delta_t^G dJ_t^G + \Delta_t^B dJ_t^B. \tag{2}$$

In (2), $dJ_t^G = 1$ ($dJ_t^B = 1$) if and only if σ jumps from σ_B to σ_G (from σ_G to σ_B) at time t; otherwise, $dJ_t^G = 0$ ($dJ_t^B = 0$). Price drift μ_t^P, price volatility σ_t^P, and the jump terms Δ_t^G and Δ_t^B are endogenous and determined in equilibrium.

Platform transactions and users. Because tokens serve as the platform transaction medium, platform users must acquire and hold tokens to transact. Following Cong et al. (2021b), we model platform transactions in reduced form and consider that any user i derives a convenience yield (transaction utility) from holding tokens. This convenience yield depends on the real value (i.e., dollar value) u_{it} of user i's token holdings.[8] That is, user i's instantaneous payoff (utility) from holding $u_{it} \geq 0$ dollars in tokens (i.e., u_{it}/P_t tokens) at time t is

$$dR_{it}^U \equiv \underbrace{\frac{u_{it}^\alpha}{\alpha} V_t^\beta A_t^{1-\alpha-\beta} dt}_{\text{Convenience yield}} + u_{it} \Big(\underbrace{\frac{dP_t}{P_t}}_{\substack{\text{Token} \\ \text{returns}}} - \underbrace{r dt}_{\substack{\text{Opportunity} \\ \text{cost}}} \Big) - \underbrace{\pi |u_{it}\sigma_t^P| dt}_{\substack{\text{Dis-utility} \\ \text{of bearing risk}}}. \tag{3}$$

In (3), $\alpha, \beta \in (0,1)$ are constants, with $\alpha + \beta < 1$. With our reduced form modelling of platform transactions, we interpret users' aggregate token holdings in dollars,

$$V_t = \int_0^1 u_{it} di,$$

1991; Bansal and Yaron, 2004; Campbell, Giglio, Polk, and Turley, 2018), and the dynamic corporate finance literature (Bolton, Chen, and Wang, 2013). In our model, uncertainty shocks lead to time-varying price and return volatility which is empirically documented in the stock market (Schwert (1989)) and cryptocurrency markets (Hafner (2020)).

[7]That is, in state B (G), when $\sigma = \sigma_B$ ($\sigma = \sigma_G$), σ changes to σ_G (σ_B) with instantaneous probability λ_G (λ_B).

[8]This modelling of transactions is akin to the "money-in-the-utility-function approach" employed in the monetary economics literature (see, e.g., Feenstra (1986)).

as the platform transaction volume. The specification in (3) captures network effects, in that any user's utility from transacting increases with transaction volume V_t. Intuitively, higher transaction volume V_t means that users are more active on the platform. As a result, transaction volume V_t quantifies the level of *platform usage* or *platform adoption* at time t.

Users' instantaneous payoff from holding tokens in (3) increases with platform productivity and token returns, but decreases with the opportunity costs of holding tokens. Moreover, because price stability is an important feature of any transaction medium (see, e.g., Rocheteau (2011) or Doepke and Schneider (2017) for a micro-foundation), users prefer that tokens as the platform transaction medium have stable price in terms of the numeraire ("dollars").[9] To model this transaction-based preference for token price stability, we assume that users are risk-averse and incur dis-utility of bearing token price risk $\pi|u_{it}\sigma_t^P|$ no matter whether the price moves with ($\sigma_t^P > 0$) or against ($\sigma_t^P < 0$) the shock dZ_t (i.e., risk aversion is defined on the absolute value of $u_{it}\sigma_t^P$). The parameter $\pi \geq 0$ quantifies the degree of users' risk-aversion. For simplicity, we do not consider risk-aversion with respect to the jump shocks dJ_t^B and dJ_t^G.[10]

Speculators. Speculators are (rational) financial investors who hold tokens for investment returns. Speculators do not transact on the platform and therefore do not derive convenience yield from holding tokens. Speculator j derives instantaneous payoff (utility)

$$dR_{jt}^S \equiv s_{jt}\Big(\underbrace{\frac{dP_t}{P_t}}_{\substack{\text{Token} \\ \text{returns}}} - \underbrace{rdt}_{\substack{\text{Opportunity} \\ \text{cost}}} \Big) - \underbrace{\theta\pi|s_{jt}\sigma_t^P|dt}_{\substack{\text{Dis-utility} \\ \text{of bearing risk}}} \tag{4}$$

from holding s_{jt} dollars in tokens (i.e., s_{jt}/P_t tokens). Speculators are (possibly) risk-averse, and $\theta\pi$ quantifies their degree risk-aversion. Unlike platform users, speculators do not have a transaction-based preference for token price stability, so we stipulate that $\theta \in [0,1]$ and speculators are less risk-averse than users.[11] Note that when $\theta = 0$, speculators are risk-neutral. We assume that users and speculators i) cannot short sell tokens (so that $u_{it} \geq 0$ and $s_{jt} \geq 0$) and ii) cannot write contracts with each other to share risk. We verify that in equilibrium, $\sigma_t^P \geq 0$. Thus, $|u_{it}\sigma_t^P| = u_{it}\sigma_t^P$ and $|s_{jt}\sigma_t^P| = s_{jt}\sigma_t^P$ (as $u_{it}, s_{jt} \geq 0$), and we drop the absolute value notation

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[9]Related, Moreira and Savov (2017) argue that safety and liquidity are important features of money-like securities.

[10]The Internet Appendix J generalizes our baseline setting by introducing risk-aversion with respect to the jump shocks (i.e., jump risk-aversion). The Internet Appendix shows that the model can easily accomodate this type of risk-aversion (without loosing tractability) and argues that the models with and without jump risk-aversion are likely to yield qualitatively similar results. The Internet Appendix also demonstrates that under certain parameter conditions, the model with jump risk-aversion is isomorphic to the model without jump risk-aversion and (risk) adjusted transition probabilities (λ_B, λ_G).

[11]Speculators' lower degree of risk-aversion could also reflect that they are less financially constrained or more optimistic than users (Caballero and Simsek (2020)).

whenever no confusion is likely to arise.[12]

2 Model Solution and Equilibrium

2.1 Users' problem and transaction volume

Any user i on the platform chooses her optimal token holdings in dollars, u_{it}, to solve

$$\max_{u_{it} \geq 0} \ \mathbb{E}[dR_{it}^U] = \max_{u_{it} \geq 0} \left[\underbrace{\frac{u_{it}^\alpha}{\alpha} V_t^\beta A_t^{1-\alpha-\beta} dt}_{\text{Convenience yield}} + u_{it} \left(\underbrace{\frac{\mathbb{E}[dP_t]}{P_t}}_{\substack{\text{Expected token} \\ \text{returns}}} - \underbrace{r dt}_{\substack{\text{Opportunity} \\ \text{cost}}} - \underbrace{\pi \sigma_t^P dt}_{\substack{\text{Dis-utility} \\ \text{of bearing risk}}} \right) \right]. \quad (5)$$

Holding tokens, users derive convenience yield (transaction utility) and earn expected token returns $\frac{\mathbb{E}[dP_t]}{P_t}$, but incur an opportunity cost of foregone interest at rate r and the dis-utility of bearing risk $\pi u_{it} \sigma_t^P$. All users $i \in [0,1]$ are symmetric and act the same, so that $V_t = u_{it}$. To solve for V_t, we calculate the first-order condition $\frac{\partial \mathbb{E}[dR_{it}^U]}{\partial u_{it}} = 0$, yielding for $u_{it} = V_t$:

$$V_t^{1-\alpha} = \frac{V_t^\beta A_t^{1-\alpha-\beta}}{r + \pi \sigma_t^P - \mathbb{E}[dP_t]/(P_t dt)}. \quad (6)$$

Note that equation (6) has two solutions. First, $V_t = 0$ or, second,

$$V_t = A_t \left(\frac{1}{r + \pi \sigma_t^P - \mathbb{E}[dP_t]/(P_t dt)} \right)^{\frac{1}{1-\gamma}}, \quad (7)$$

where we define $\gamma := \alpha + \beta < 1$ as the transformed network effect parameter. Due to the network effects, users' decisions to transact on the platform exhibit strategic complementarities, which implies the existence of a trivial equilibrium with $V_t = P_t = 0$.[13] As in Cong et al. (2021b), we focus on an equilibrium with positive transaction volume, so V_t is given in (7) (instead of $V_t = 0$).

Platform transaction volume increases with expected token returns $\mathbb{E}[dP_t]/(P_t dt)$. Expected token returns mitigate the (opportunity) costs of holding tokens and transacting on the platform. Intuitively, when tokens serve as the transaction medium, users can capitalize on platform success by earning token returns when transacting on the platform. While token price dynamics (i.e., $dP_t \neq 0$) are necessary for tokens to offer positive expected returns, price dynamics typically come

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[12]The fact that risk-aversion of both users and speculators is defined on the absolute value of token holdings becomes relevant when we allow for short selling in Sections 3.3 and 6 (so that $s_{jt} < 0$ becomes possible). For consistency, we therefore define risk aversion on the absolute value of token holdings already in the baseline model.

[13]The fact that strategic complementarities can induce multiple equilibria is also illustrated in Diamond and Dybvig (1983). As in (Morris and Shin, 1998; Goldstein and Pauzner, 2005), one can employ a global games approach to make the equilibrium unique in settings with strategic complementarities.

along with price volatility (i.e., $\sigma_t^P \neq 0$) which curbs platform usage.

2.2 Speculators' problem

Any speculator $j \in (1,2]$ maximizes the expected payoff from trading tokens subject to a short selling constraint $s_{jt} \geq 0$, that is, j solves

$$\max_{s_{jt} \geq 0} \mathbb{E}[dR_{jt}^S]. \tag{8}$$

For the token market to clear, optimal token holdings s_{jt} of speculator j must be finite. Thus, $\frac{\partial \mathbb{E}[dR_{jt}^S]}{\partial s_{jt}} \leq 0$ must hold, which is

$$\frac{\mathbb{E}[dP_t]}{P_t dt} \leq r + \theta \pi \sigma_t^P. \tag{9}$$

Inequality (9) is an equality if speculators hold tokens and $S_t > 0$, where

$$S_t = \int_1^2 s_{jt} dj$$

denotes speculators' aggregate token holdings in dollars. S_t quantifies speculative (token) invest-ment or, interpreted broadly, speculative (token) trading. We use the terms speculative (token) investment and speculative (token) trading interchangeably.[14] If $S_t > 0$, expected token returns are equal to speculators' required rate of return $r + \theta \pi \sigma_t^P$. The intuition behind (9) is that when $\frac{\mathbb{E}[dP_t]}{P_t dt} > r + \theta \pi \sigma_t^P$, speculators buy tokens and so drive up the demand for tokens, increasing the token price and decreasing expected token returns to the point that $\frac{\mathbb{E}[dP_t]}{P_t dt} = r + \theta \pi \sigma_t^P$. Limiting expected token returns, speculative investment reduces platform transaction volume V_t.

As we show later, a decrease in θ leads to an increase in speculative investment S_t, confirming that θ quantifies speculative investment.[15] Note that when $\theta = 1$, users and speculators have the same risk preferences. As users derive convenience yield from holding tokens but speculators do not, users value tokens strictly more than speculators and there is no speculative investment. Thus, considering sufficiently large values of θ is equivalent to removing speculators from the model.

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[14]Admittedly, speculative investment and trading need not be related, but they are in our framework. Section 2.4 shows that in equilibrium, there is a direct link between S_t and speculative token trading. Alternatively, one could micro-found a direct relation between speculative token investment and token trading by assuming that individual speculators are "short-lived" and arrive in overlapping generations. Thus, each generation of speculators trades tokens over an instant $[t, t + dt)$ only. Speculators who live from time t until time $t + dt$ buy tokens at time t and sell them to users or the following generation of speculators at time $t + dt$. Speculative token trading volume in dollars among speculators (at $t + dt$) is $S_t + dS_t$. Absent an uncertainty shock, dS_t is infinitesimal, so speculative token trading $S_t + dS_t \simeq S_t$ approximately equals aggregate speculative token holdings/investments.

[15]The parameter θ might also be related to the ease/costs of speculative token trading affected by platform/token design and regulation.

2.3 Token pricing and equilibrium concept

Recall that token market capitalization (in dollars) is $M_t P_t$, users' dollar token holdings (i.e., platform transaction volume) are V_t, and speculators' dollar token holdings are S_t. As a result, the token market clearing condition becomes

$$M_t P_t = V_t + S_t. \tag{10}$$

Expected token returns reflect whether the marginal token investor is i) a speculator or ii) a user. First, when the marginal token investor is a speculator (i.e., $S_t > 0$), then (9) holds in equality so[16]

$$P_t = \frac{\mathbb{E}[dP_t]}{(r + \theta \pi \sigma_t^P)dt}. \tag{11}$$

Second, when there is no speculative investment (i.e., $S_t = 0$), then the marginal token investor is a user. As a result, token market clearing implies $V_t = M_t P_t$, and — using (7) — we obtain the following expression for the token price:

$$P_t = \frac{A_t}{M_t} \left(\frac{1}{r + \pi \sigma_t^P - \mathbb{E}[dP_t]/(P_t dt)} \right)^{\frac{1}{1-\gamma}}. \tag{12}$$

We can combine expressions (11) and (12) to obtain

$$P_t = \max \left\{ \frac{A_t}{M_t} \left(\frac{1}{r + \pi \sigma_t^P - \mathbb{E}[dP_t]/(P_t dt)} \right)^{\frac{1}{1-\gamma}}, \frac{\mathbb{E}[dP_t]}{(r + \theta \pi \sigma_t^P)dt} \right\}. \tag{13}$$

One can rearrange (13) and solve

$$r + \theta \pi \sigma_t^P = \max \left\{ \left(\frac{A_t}{M_t P_t} \right)^{1-\gamma} - (1-\theta)\pi \sigma_t^P, 0 \right\} + \frac{\mathbb{E}[dP_t]}{P_t dt} \tag{14}$$

Token pricing equation (14) resembles a traditional valuation equation for financial assets. The left-hand side depicts financial investors' (i.e., speculators') risk-adjusted required returns. The right-hand side depicts expected token returns (second term) and the risk-adjusted convenience yield the marginal investor derives from holding tokens (first term). Note that the risk-adjusted convenience yield is positive if and only if the marginal token investor is a user (and $S_t = 0$),

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[16]In general, when $S_t = 0$, users hold tokens too and $V_t > 0$, because users' utility from holding tokens and the convenience yield satisfy the Inada conditions (implying an infinite marginal convenience yield/utility as u_{it} approaches zero). Strictly speaking, both users and speculators can then be considered marginal token investors. In what follows, we say that the marginal token investor is a speculator when $S_t > 0$, and the marginal token investor is a user when $S_t = 0$.

reflecting that speculators do not derive convenience yield.

In the baseline, token supply M_t is constant (i.e., $dM_t = 0$) with $M_0 = 1$. To ease the transition to the setup with endogenous platform investment and token supply (in Section 4), we nonetheless treat token supply M_t as state variable in the following characterization of a Markov equilibrium. We study a Markov equilibrium with state variables A_t, $\sigma \in \{\sigma_B, \sigma_G\}$, and M_t which is characterized by the following conditions. First, all agents act optimally. That is, users solve (5) so that platform transaction volume is characterized in (7), and speculators solve (8) so that (9) holds. Second, the token market clears, in that (10) is satisfied. As shown above, the relations (7), (9), and (10) imply that the token price P_t satisfies the equilibrium pricing relationship (13) (or equivalently (14)).

2.4 Solving for the equilibrium

In this Section, we solve for the Markov equilibrium described above. We demonstrate that token price P_t and platform transaction volume V_t can be expressed as functions of A_t, σ, and M_t as follows. The token price P_t scales with

$$X_t = \frac{A_t}{M_t}. \tag{15}$$

That is, token price can be written as $P_t = X_t p(\sigma)$, where $p(\sigma)$ only depends on σ and $p_G = p(\sigma_G)$ and $p_B = p(\sigma_B)$. Thus, token market capitalization $M_t P_t$ scales with A_t and is given by $A_t p(\sigma)$, so $p(\sigma)$ can be interpreted as scaled token market capitalization too. Platform transaction volume V_t scales with A_t, so that $V_t = A_t v(\sigma)$ where $v_G = v(\sigma_G)$ and $v_B = v(\sigma_B)$.

The argument to solve for the equilibrium follows a conjecture-and-verify approach. That is, we conjecture that token price satisfies $P_t = X_t p(\sigma)$ and transaction volume satisfies $V_t = A_t v(\sigma)$. Using $dM_t = 0$ and (1), it follows that

$$\frac{dX_t}{X_t} = \mu_X(\sigma)dt + \sigma_X(\sigma)dZ_t,$$

with $\mu_X(\sigma) = \mu$ and $\sigma_X(\sigma) = \sigma$. An application of Itô's Lemma yields

$$\mu_t^P = \left(\frac{\partial P_t}{\partial X_t}\right)\frac{X_t \mu_X(\sigma)}{P_t} + \frac{1}{2}\left(\frac{\partial^2 P_t}{\partial X_t^2}\right)\frac{X_t^2 \sigma_X(\sigma)^2}{P_t} = \mu_X(\sigma) \tag{16}$$

and

$$\sigma_t^P = \left(\frac{\partial P_t}{\partial X_t}\right)\frac{X_t \sigma_X(\sigma)}{P_t} = \sigma_X(\sigma). \tag{17}$$

Thus, $\mu_t^P = \mu$ and $\sigma_t^P = \sigma$. In addition, Itô's Lemma (in its version for jump processes) implies

that token price changes in response to uncertainty shocks are characterized by

$$\Delta_t^G = \frac{p_G - p_B}{p_B} = \frac{p_G}{p_B} - 1 \quad \text{and} \quad \Delta_t^B = \frac{p_B - p_G}{p_G} = \frac{p_B}{p_G} - 1 \tag{18}$$

respectively. We observe that μ_t^P, σ_t^P as well as Δ_t^G and Δ_t^B depend only on σ (and not on A_t, M_t, or X_t). Next, note that the law of motion (2) implies for expected token returns

$$\frac{\mathbb{E}[dP_t]}{P_t dt} = \begin{cases} \mu_t^P + \lambda_G \Delta_t^G, & \text{if} \quad \sigma = \sigma_B \\ \mu_t^P + \lambda_B \Delta_t^B, & \text{if} \quad \sigma = \sigma_G. \end{cases} \tag{19}$$

As μ_t^P and Δ_t^G, Δ_t^B only depend on σ, expected token returns $\frac{\mathbb{E}[dP_t]}{P_t dt}$ only depend on σ too.

We can now characterize the scaled token prices p_B and p_G as well as scaled transaction volume v_B and v_G. Using (7), (18), and (19), we obtain that in state G, scaled transaction volume reads

$$v_G = \left(\frac{1}{r - \mu + \pi\sigma_G - \lambda_B(p_B/p_G - 1)} \right)^{\frac{1}{1-\gamma}}. \tag{20}$$

The pricing equation (13) (or equivalently (14)) then implies the scaled token price[17]

$$p_G = \max \left\{ v_G, \frac{\lambda_B p_B}{r - \mu + \theta\pi\sigma_G + \lambda_B} \right\}. \tag{21}$$

Likewise, in state B, scaled transaction volume reads

$$v_B = \left(\frac{1}{r - \mu + \pi\sigma_B - \lambda_G(p_G/p_B - 1)} \right)^{\frac{1}{1-\gamma}} \tag{22}$$

and scaled token price is

$$p_B = \max \left\{ v_B, \frac{\lambda_G p_G}{r - \mu + \theta\pi\sigma_B + \lambda_G} \right\}. \tag{23}$$

Expressions (20), (21), (22), and (23) only depend on σ (and not on X_t, A_t, and M_t), as we have conjectured.[18] As a result, we have verified that in the Markov equilibrium, token price satisfies $P_t = X_t p(\sigma)$, and transaction volume satisfies $V_t = A_t v(\sigma)$.

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[17]For a derivation, recall (13) and note that the first term in curly brackets is V_t/M_t (compare (7)). First, when there is no speculative investment ($S_t = 0$), (12) holds, $P_t = V_t/M_t$ and $V_t = P_t M_t = A_t p(\sigma)$. As $V_t = A_t v(\sigma)$, we obtain $p(\sigma) = v(\sigma)$. Second, when there is speculative investment in state G ($S_t > 0$ and $P_t > V_t$), then (11) holds. As such, $\frac{\mathbb{E}[dP_t]}{P_t dt} = r + \theta\pi\sigma_t^P$. Using (18), (19), $\mu_t^P = \mu$ and $\sigma_t^P = \sigma$ yields $\mu + \lambda_B(p_B/p_G - 1) = r + \theta\pi\sigma_G$. We can solve this equation for $p_G = \frac{\lambda_B p_B}{r - \mu + \theta\pi\sigma_G + \lambda_B}$. Combining yields the expression (21) for scaled token price p_G The derivation for scaled token price p_B in state B is analogous.

[18]To obtain the equilibrium quantities (v_B, v_G, p_B, p_G), one must then solve (20), (21), (22), and (23), which — in general — must be done numerically.

Due to users' transaction-based preference for token price stability, the benefits of transacting on the platform decrease with token price volatility σ. Hence, all else equal, the platform is more valuable in normal times (state G) than in volatile times (state B), which implies $p_G > p_B$. Due to $p_G > p_B$ and $\mu < r$, expected token returns in state G are less than speculators' discount rate, r. As a result, speculators do not hold tokens in state G, leading to $p_G = v_G$. The intuition is that in normal times, tokens have low expected returns and low price volatility which supports their function as a transaction medium but discourages speculative token investment. Speculators hold tokens only in state B (if at all). In state B, token price volatility is high, which curbs transaction volume and token price. The prospect of high future adoption (upon reaching state G) implies high expected token returns going forward that encourage speculative investment.

Also note that in aggregate, speculators trade tokens only following uncertainty shocks, in that speculators' nominal token holdings S_t/P_t and scaled dollar token holdings S_t/A_t depend on σ only. When σ decreases (increases), speculators sell (buy) tokens worth S_t dollars — that is, S_t/P_t tokens — to (from) users. This confirms that indeed, S_t quantifies speculative token trading. We summarize our results in the following Proposition.

Proposition 1 (Token pricing). *There exists a unique Markov equilibrium with state variables A_t, σ, and M_t in which transaction volume V_t is characterized in (7). In this Markov equilibrium, the following holds:*

1. *Platform transaction volume satisfies $V_t = A_t v(\sigma)$, where $v_B = v(\sigma_B)$ is characterized in (22) and $v_G = v(\sigma_G)$ is characterized in (20).*

2. *Token price P_t satisfies the pricing equation (14) and scales with $X_t = \frac{A_t}{M_t}$. That is, $P_t = X_t p(\sigma)$, where the scaled token price $p_B = p(\sigma_B)$ is characterized in (23) and the scaled token price $p_G = p(\sigma_G)$ is characterized in (21). Expected token returns are characterized in (19). Token price drift is $\mu_t^P = \mu$ and token price volatility is $\sigma_t^P = \sigma$.*

3. *Token price satisfies $p_G > p_B$, transaction volume satisfies $v_G > v_B$, and speculators do not hold tokens in state G.*

4. *Speculative token investment in dollars S_t scales with A_t. That is, $S_t = s(\sigma)A_t$ where $s(\sigma_B) = p_B - v_B$ and $s(\sigma_G) = 0$. Speculators' nominal token holdings S_t/P_t depend on σ only. In state G, $S_t/P_t = 0$ and in state B, $S_t/P_t = M_t(1 - v_B/p_G)$ where $dM_t = 0$.*
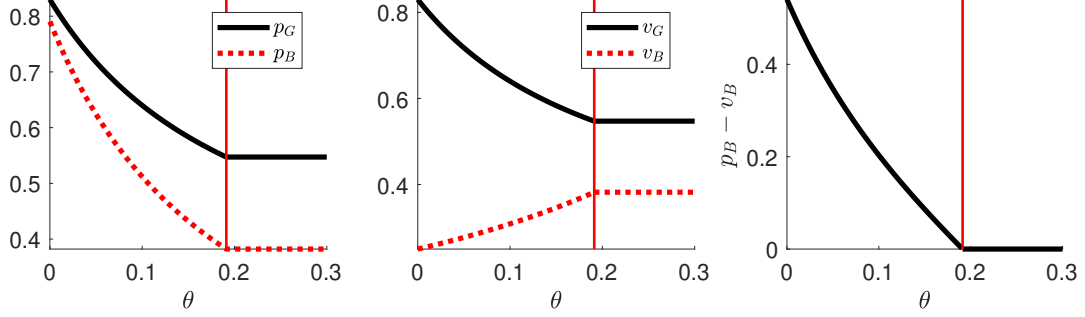
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Figure 1: The impact of speculators on token pricing and platform adoption. A lower value of θ implies more speculative trading/investment. The solid red line depicts $\bar{\theta}$: speculators hold tokens in state B if and only if $\theta < \bar{\theta}$. The parameters are $r = 0.06$, $\mu = 0.01$, $\gamma = 0.5$, $\sigma_G = 1 < \sigma_B = 2$, $\pi = 1$, and $\lambda_G = \lambda_B = 1$.

Figure 2: Expected token returns, token price fluctuations, and speculators. A lower value of θ implies more speculative trading/investment. The solid red line depicts $\bar{\theta}$: speculators hold tokens in state B if and only if $\theta < \bar{\theta}$. The parameters are $r = 0.06$, $\mu = 0.01$, $\gamma = 0.5$, $\sigma_G = 1 < \sigma_B = 2$, $\pi = 1$, and $\lambda_G = \lambda_B = 1$.
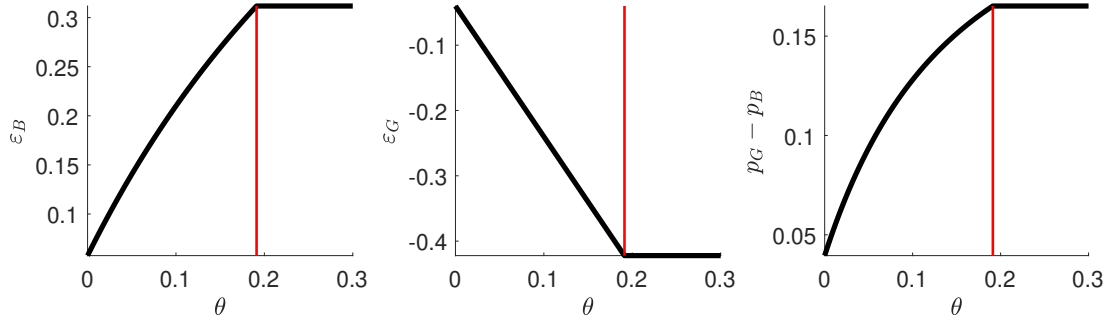
3 Analysis

3.1 Speculators and platform users: crowding-out vs. risk-sharing

We study the interactions between speculators and platform users as well as the effects on platform adoption and token pricing. To clearly illustrate and identify the impact of speculators, we conduct in Figures 1 and 2 comparative statics with respect to speculators' risk-aversion θ which quantifies speculative investment. Figure 1 plots scaled token prices p_G and p_B (left panel), scaled platform transaction volume v_G and v_B (middle panel), and scaled speculative dollar investment $p_B - v_B = S_t/A_t$ in state B (right panel) against θ.[19] Indeed, speculative investment in state B decreases with θ and reaches zero at some value $\bar{\theta}$ depicted by the solid red line. Thus, θ quantifies speculative investment, in that low (high) θ corresponds to high (low) speculative investment. For $\theta \geq \bar{\theta}$, there is no speculative investment. For $\theta < \bar{\theta}$, speculators hold tokens in state B, and both

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[19]Recall there is no speculative investment in state G.

p_G and p_B decrease in θ, meaning that speculative investment (in state B) boosts the token price (in both states). In addition, speculative investment reduces transaction volume in state B (i.e., v_B increases with θ) but raises transaction volume in state G (i.e., v_G decreases with θ).

Figure 2 plots expected token returns in state B denoted $\varepsilon_B = \mu + \lambda_G(p_G/p_B - 1)$ (left panel), expected token returns in state G denoted $\varepsilon_G = \mu + \lambda_B(p_B/p_G - 1)$ (middle panel), and $p_G - p_B$ (right panel) against θ. For $\theta < \bar{\theta}$, expected token returns in state B (in state G) increase (decrease) in θ. This means that speculative investment reduces expected token returns in state B, but boosts expected token returns in state G. In addition, speculative token investment reduces token price downside risk in state G and upside potential in state B, in that $p_G - p_B$ increases in θ for $\theta < \bar{\theta}$. According to Figure 1, $v_G - v_B$ decreases with θ. Thus, the presence of speculators dampens fluctuations in token price but amplifies fluctuations in platform usage and adoption.

The intuition behind these outcomes is as follows. Recall that in normal times (state G), the marginal token investor is a user and there is no speculative investment, and consider $\theta < \bar{\theta}$ so that speculators are marginal in volatile times (state B). When token price volatility rises in response to an uncertainty shock, users' utility from transacting and holding tokens decreases. Consequently, transaction volume and token price fall, implying a larger potential for token price appreciation and hence high expected token returns going forward. High expected token returns encourage speculators to buy tokens, so that the marginal token investor in volatile times is a speculator. As speculators buy tokens, they drive up demand for tokens and token price p_B in volatile times.

Boosting token price p_B, speculators reduce the potential for token price appreciation and so expected token returns in volatile times. Lower expected token returns ε_B increase users' cost of holding tokens and transacting on the platform, which further reduces platform adoption and transactions in volatile times. In other words, speculators crowd out platform users (usage) in volatile times. Due to this crowding-out effect, speculative token investments S_t and transactional token investments by users V_t can be viewed as static substitutes.

However, pushing up token price p_B, speculators also decrease token price downside risk in normal times, thereby raising expected token returns ε_G, platform adoption v_G, and token price p_G. When token price volatility rises, transactional demand for tokens falls and users would like to sell tokens. Importantly, users can sell tokens in volatile times at more favorable terms precisely because speculators buy. That is, speculative investment (in volatile times) improves the resale option value of tokens and therefore increases users' incentives to hold tokens for transactions and to adopt the platform in normal times. We interpret this interaction between users and speculators as *risk-sharing*: providing liquidity in volatile times, speculators absorb token price volatility and therefore effectively share risk with users.

This risk-sharing effect stimulates platform adoption in normal times. As speculators buy tokens in volatile times only due to the prospect of high future platform adoption, speculative and transactional token investments can be viewed as dynamic complements.[20] Our model predicts that due to the static substitutes and dynamic complements feature, the amount of speculative token investments correlates negatively with contemporaneous platform usage but positively with future platform usage. We summarize our analytical findings regarding these effects discussed above in the following Corollary.

Corollary 1. *There exists $\overline{\theta} < 1$ such that there is no speculative investment ($S_t = 0$) when $\theta \geq \overline{\theta}$ and there is speculative investment ($S_t > 0$) in state B for $\theta < \overline{\theta}$. When $\theta \geq \overline{\theta}$, $\frac{\partial v_z}{\partial \theta} = \frac{\partial p_z}{\partial \theta} = \frac{\partial \varepsilon_z}{\partial \theta} = 0$ for $z = B, G$. When $\theta < \overline{\theta}$, the following holds:*

1. *Scaled speculative investment and token price decrease in θ, that is, $\frac{\partial(p_B - v_B)}{\partial \theta} < 0$ and $\frac{\partial p_z}{\partial \theta} < 0$.*

2. *Speculative investment boosts platform adoption in state G but reduces platform adoption in state B, that is, $\frac{\partial v_G}{\partial \theta} < 0$ and $\frac{\partial v_B}{\partial \theta} > 0$.*

3. *Speculative investment increases expected token returns in state G but reduces expected token returns in state B, that is, $\frac{\partial \varepsilon_G}{\partial \theta} < 0$ and $\frac{\partial \varepsilon_B}{\partial \theta} > 0$.*

3.2   Is it optimal to ban or restrict speculative trading?

The platform's dollar value/payoff at inception at $t = 0$ is the payoff from issuing tokens, that is, the initial token market capitalization $M_0 P_0 = A_0 p(\sigma)$.[21] As speculative investment boosts the token price in either state, the platform maximizing its dollar value benefits from the presence of speculators. However, the presence speculators stimulates platform adoption in normal times via the risk-sharing effect but curbs it in volatile times via the crowding-out effect. In light of these two opposing effects, it is ambiguous whether speculators improve user welfare and total surplus (which is the sum of user welfare and platform dollar value).[22]

Figure 3 plots scaled user welfare (in both states), scaled total surplus (in both states), and

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[20]In detail, due to the crowding out effect, speculative and transactional investment are static substitutes, in that an increase speculative investment at time t reduces transactional investment at time t. Due to the risk-sharing effect, speculative investment and transactional investment are dynamic complements, in that an increase speculative investment at time t positively correlates with transactional investment at future times $s > t$ and vice versa.

[21]As there are no other payoffs besides token issuance at $t = 0$, the platform equity value after time $t = 0$ is zero.

[22]Speculators earn zero utility in equilibrium, as they hold tokens only if (9) holds in equality. But, when (9) holds in equality, speculators are indifferent between holding tokens and not holding tokens (i.e., just break even).
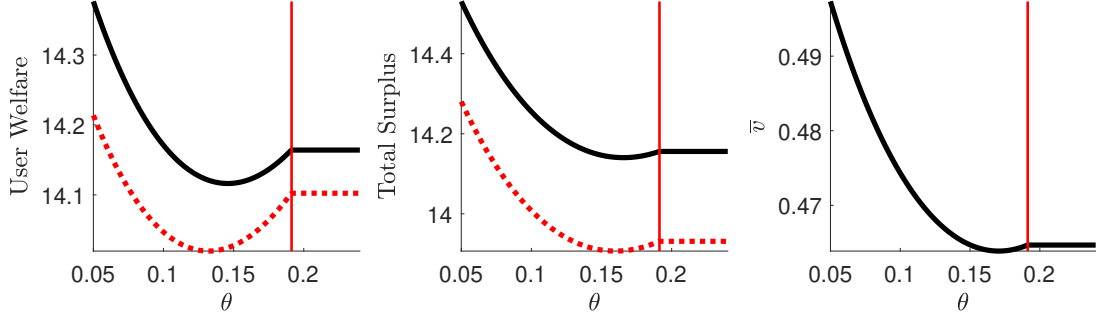
Figure 3: Do speculators improve user welfare, total surplus, and long-run adoption? A lower value of θ implies more speculative trading/investment. The solid red line depicts $\bar{\theta}$: speculators hold tokens in state B if and only if $\theta < \bar{\theta}$. In the left and middle panel, the solid black line depicts scaled user welfare (total surplus) in state G and the dotted red line depicts scaled user welfare (total surplus) in state B. The parameters are $r = 0.06$, $\mu = 0.01$, $\alpha = 0.25, \beta = 0.25$, $\gamma = \alpha + \beta = 0.5$, $\sigma_G = 1 < \sigma_B = 2$, $\pi = 1$, and $\lambda_G = \lambda_B = 1$.

scaled long-run (average) adoption[23]

$$\bar{v} \equiv= \lim_{t \to \infty} \mathbb{E}\left[\frac{V_t}{A_t}\right] = \frac{\lambda_G v_G + \lambda_B v_B}{\lambda_G + \lambda_B} \tag{24}$$

against θ quantifying speculative investment. To the right of the solid red line (depicting $\bar{\theta}$), there is no speculative investment and, to the left of the solid red line, speculators hold tokens in state B. Note that (scaled) user welfare, total surplus, and long-run adoption are non-monotonic in θ. For intermediate values of θ, the presence of speculators reduces user welfare, total surplus, and platform adoption (compared to a situation without speculators that pertains for larger values $\theta \geq \bar{\theta}$). Thus, Figure 3 highlights that under certain but not all circumstances, the presence of speculators improves user welfare, total surplus, and platform adoption. From the perspective of a social planner or regulator maximizing total surplus, it is generally not optimal to ban speculative trading for cryptocurrencies and tokens. In fact, it can even be optimal to adjust platform and token design to stimulate speculative trading so as to reach an optimal level that balances the crowding-out and risk-sharing effect.

But, how to encourage speculative token investment and trading in practice? We conjecture the following possibilities. First, if the platform could discriminate between token trading by users and speculators, the platform could effectively subsidize speculative trading or at least reduce the

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[23]User welfare is defined as

$$W_0 = \mathbb{E}\left[\int_0^\infty e^{-rt} dR_{it}^U\right],$$

under users' optimal choice of token holdings. Appendix C shows how to calculate user welfare and that user welfare scales with A_0, i.e., $W_0 = A_0 w(\sigma)$. Total surplus is $A_0(w(\sigma) + p(\sigma))$. Figure 3 plots scaled user welfare $w(\sigma)$ and scaled total surplus $w(\sigma) + p(\sigma)$ respectively.

barriers to speculative trading. Second, the platform could adjust technical parameters such as the block size — which determines the speed of settlement of token trades — according to speculators' needs. Third, the platform could facilitate or allow derivatives-based trading of its tokens, which may reduce the costs/barriers of trading tokens. The platform could offer the opportunity for derivatives trading on the platform itself. Because token derivatives trading primarily happens on crypto-exchanges (like Binance or Coinbase), the platform could pursue the listing of its tokens on such exchanges, e.g., through an IEO (Initial Exchange Offering).[24] Fourth, the next Section discusses how the platform could increase speculator participation by facilitating short selling of tokens. Fifth, in Section 4, we endogenize platform development and introduce physical investment in platform productivity. We show then that the platform can fuel speculative token trading by structuring its investment policies according to speculators' preferences.

3.3 Short selling and short selling constraints

In practice, short selling is possible for many cryptocurrencies and tokens but can be costly or limited. To capture these features, we introduce limited short selling possibilities. Suppose that each speculator j faces a short selling constraint of the following form: token holdings (in dollars) s_{jt} must exceed $\underline{S}_t \leq 0$. To preserve tractability, we assume that the short selling constraint scales with token market capitalization and takes the form

$$\underline{S}_t = -\eta M_t P_t, \tag{25}$$

for some constant $\eta \geq 0$. Thus, in nominal terms, any speculator cannot short sell more than ηM_t tokens. The short selling constraint could be also related to the costs of short selling tokens (i.e., short selling more than ηM_t becomes prohibitively costly). Setting $\eta = 0$ yields our baseline. Appendix H presents the detailed solution of the model with short selling.

Figure 4 illustrates the effects of short selling and presents comparative statics with respect to $\eta \geq 0$, observing that a higher value of η corresponds to less stringent short selling restrictions. The left panel plots scaled token prices p_G and p_B, and the middle panel plots scaled transaction volume v_G and v_B and long-run (average) transaction volume $\bar{v}$. The right panel displays speculators' (scaled) dollar token long position in state B and token short position in state G. Note that under all parameters considered, speculators short sell tokens in state G as much as the short selling

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[24]Likewise, the platform could stimulate speculative trading by making its token attractive for trading on decentralized exchanges like Uniswap.For instance, the platform could achieve this by adopting the ERC20 token specification, so that the token can be traded on-chain (i..e, on the Ethereum blockchain) via the decentralized exchange Uniswap. In addition, the platform could initially subsidize or found liquidity pools in the automated market maker protocol to "kickstart" token trading.
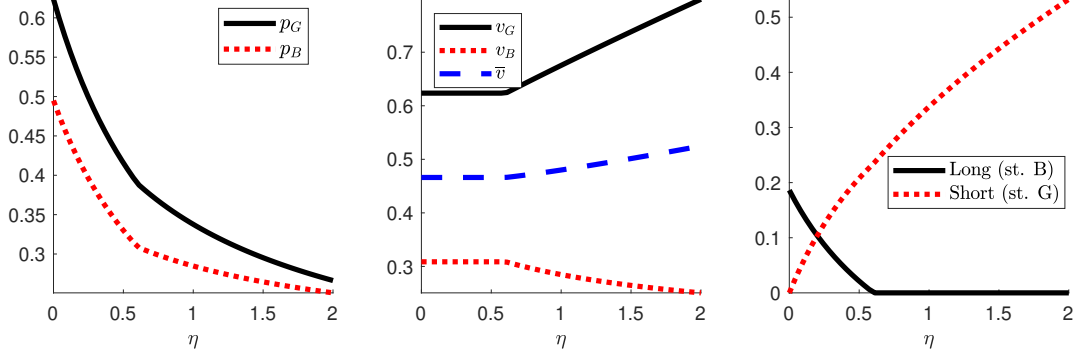
Figure 4: The effects of short selling constraints. The parameters are $r = 0.06$, $\mu = 0.01$, $\gamma = 0.5$, $\sigma_G = 1 < \sigma_B = 2$, $\pi = 1$, $\lambda_G = \lambda_B = 1$, $\theta = 0.1$.

constraint allows, so $S_t = -\eta M_t P_t$ and the dollar value of speculators' short position increases with η. In contrast, speculators do not short sell tokens in state B, and users never short sell tokens.[25]

Expecting the token price drop in response to an uncertainty shock, speculators short sell tokens in normal times, thereby decreasing token price in state G (i.e., $\frac{\partial p_G}{\partial \eta} < 0$). When η is low, speculators' short position is low, which implies a high token price p_G and so high expected token returns in state B. Then, speculators hold a (token) long position in state B, and platform transaction volume is insensitive to changes in η. As η increases, speculators' short position in state G expands, reducing token price p_G and expected token returns in state B to the point that there is no more speculative investment in state B. A lower token price p_G then limits the magnitude of a potential token price drop (i.e., $\frac{\partial(p_G - p_B)}{\partial \eta} \leq 0$) and so boosts expected token returns in normal times. Increasing expected token returns in state G and decreasing them in state B, short selling reduces platform usage in state B but increases platform usage in state G (i.e., $\frac{\partial v_G}{\partial \eta} \geq 0$ and $\frac{\partial v_B}{\partial \eta} \leq 0$). Overall, the positive impact of speculators' short selling on platform adoption/usage dominates, in that average (long-run) adoption $\bar{v}$ increases with η under our baseline parameters. The following Corollary presents analytical results regarding the effects discussed above.

Corollary 2. *Suppose that the short selling constraint is binding in state G (i.e., $S_t = \underline{S} = -\eta M_t P_t$), and that there is no short selling in state B (i.e., $S_t \geq 0$). Then, the scaled token price (in either state) decreases with η, i.e., $\frac{\partial p_z}{\partial \eta} < 0$ for $z = B, G$. If speculators hold tokens in state B (i.e., $S_t > 0$ and $p_B > v_B$), then $\frac{\partial v_z}{\partial \eta} = 0$. Otherwise, if speculators do not hold tokens in state B (i.e., $S_t = 0$ and $p_B = v_B$), then $\frac{\partial v_G}{\partial \eta} > 0$ and $\frac{\partial v_B}{\partial \eta} < 0$.*

However, as the next Corollary demonstrates, the positive effects of short selling pertain only if short selling is subject to a constraint that prevents excessive short selling pressure. Excessive

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[25]Users optimally do not short sell tokens, because the marginal convenience yield to holding tokens in (3) becomes infinity once u_{it} approaches zero (from above).

short selling can be detrimental and precipitate the collapse of the token price and platform (i.e., a token price crash), precluding platform transactions and rendering the platform useless.

Corollary 3. *If $\theta \geq 0$ is sufficiently small and $\eta \geq 0$ is sufficiently large, there exists no Markov equilibrium with $V_t = A_t v(\sigma)$ and $P_t = X_t p(\sigma)$ whereby token price $p(\sigma)$ and transaction volume $v(\sigma)$ are strictly positive in at least one state σ. The only Markov equilibrium with $V_t = A_t v(\sigma)$ and $P_t = X_t p(\sigma)$ features $v(\sigma) = p(\sigma) = 0$.*

Our findings imply that the facilitation of limited short selling dampens token price fluctuations, stimulates platform transactions and adoption, and therefore can be beneficial (for platform owners and users). Having characterized the effects of short selling, we abstract from short selling possibilities in the following analysis (except Section 6) so that (unless otherwise mentioned) $\eta = 0$.

4   Endogenous platform development and investment

4.1   Setup

To analyze how platform investment and strategies interact with speculative token trading, we endogenize platform development and introduce physical investment in platform productivity. Investment $a_t \in [0, \bar{a}]$ advances platform productivity but is risky and exacerbates token price volatility. Platform productivity evolves now according to

$$\frac{dA}{A_t} = (\mu + a_t \Delta\mu)dt + (\sigma + a_t \Delta\sigma)dZ_t, \tag{26}$$

with constants $\Delta\mu, \Delta\sigma \geq 0$. Increasing a_t is akin to following more ambitious and risky approaches and goals in platform development. The upper bound on investment $\bar{a}$ satisfies

$$\mu + \Delta\mu\bar{a} < r. \tag{27}$$

Condition (27) ensures that payoffs are well-defined and finite. Also note that $\bar{a} = 0$ yields our baseline setup without investment. Investment a_t is costly and entails quadratic dollar flow costs

$$I_t = A_t \frac{\kappa a_t^2}{2},$$

where $\kappa \geq 0$. As in Cong et al. (2021a), the costs of investment I_t are financed by issuing new tokens. Issuing dM_t tokens at time t yields $P_t dM_t$ dollars. The proceeds from token issuance

exactly cover investment costs, so $P_t dM_t = I_t dt$ and after dividing through M_t and P_t:[26]

$$\frac{dM_t}{M_t} = \left(\frac{A_t}{M_t P_t}\right) \frac{\kappa a_t^2}{2} dt. \tag{28}$$

The platform (or the platform owners/developers) dynamically chooses investment to maximize platform dollar value at $t = 0$, $M_0 P_0$. Notice that $M_0 P_0$ is the initial token market capitalization and hence represents the platform's revenue from issuing tokens at $t = 0$. This revenue can be also interpreted as the platform's seigniorage payoff from issuing tokens. Formally, the platform solves

$$\max_{(a_t)_{t \geq 0}} M_0 P_0, \tag{29}$$

subject to $a_t \in [0, \bar{a}]$. Analogous to the baseline, we study a Markov equilibrium with state variables A_t, $\sigma \in \{\sigma_B, \sigma_G\}$, and M_t in which i) all agents and the platform act optimally and ii) the token market clears.

4.2   Solution

In the Markov equilibrium, transaction volume scales with A_t (i.e., $V_t = A_t v(\sigma)$), token price scales with X_t (i.e., $P_t = X_t p(\sigma)$), and platform investment depends on σ only (i.e., $a_t = a(\sigma)$ with $a_B = a(\sigma_B)$ and $a_G = a(\sigma_G)$). To solve for this Markov equilibrium with platform investment, we recall (26) and (28) and invoke Itô's Lemma to calculate[27]

$$\frac{dX_t}{X_t} = \mu_X(\sigma) dt + \sigma_X(\sigma) dZ_t, \tag{30}$$

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[26]The law of motion (28) implies that token supply always increases but never decreases, which seems at odds with the fact that many cryptocurrencies have capped supply. However, in our setting, it is without loss of generality to consider uncapped token supply. To see this, note that the platform can always re-norm token supply M_t and token price P_t while leaving token market capitalization $M_t P_t$ unchanged via a "reverse coin split" that is analogous to a reverse stock split. For instance, at any point in time t, the platform could decide to halve token supply and double token price via a reverse coin split, which implies that each agent's nominal token holdings are halved but the real value (i.e., dollar value) of their token holdings remains unchanged. As agents only care about the real value (i.e., dollar value) of their token holdings, this reverse coin split is payoff irrelevant, so it can be used to cap token supply without affecting the model solution.

[27]For a derivation, first use $P_t = X_t p(\sigma)$ and $X_t = A_t / M_t$ to rewrite (28) as

$$\frac{dM_t}{M_t} = \frac{\kappa a_t^2}{2p(\sigma)} dt,$$

with $a_t = a(\sigma)$, and then invoke Itô's quotient rule to obtain,

$$\frac{dX_t}{X_t} = \frac{d(A_t/M_t)}{A_t/M_t} = \left[\mu + a(\sigma)\Delta\mu - \frac{\kappa a(\sigma)^2}{2p(\sigma)}\right] dt + (\sigma + a(\sigma)\Delta\sigma) dZ_t.$$

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with

$$\mu_X(\sigma) = \mu + a(\sigma)\Delta\mu - \frac{\kappa a(\sigma)^2}{2p(\sigma)} \quad \text{and} \quad \sigma_X(\sigma) = \sigma + a(\sigma)\Delta\sigma. \tag{31}$$

Invoking (16) and (17) (which are also valid in the model variant with investment), we obtain $\mu_t^P = \mu_X(\sigma)$ and $\sigma_t^P = \sigma_X(\sigma)$. Using (7), (18), and (19), we obtain scaled platform transaction volume in state G

$$v_G = \left(\frac{1}{r - \mu - a_G\Delta\mu + \pi(\sigma_G + a_G\Delta\sigma) + \kappa a_G^2/(2p_G) - \lambda_B(p_B/p_G - 1)} \right)^{\frac{1}{1-\gamma}}. \tag{32}$$

It then follows from the pricing equation (13) (or equivalently (14)) that scaled token price satisfies

$$p_G = \max\left\{ v_G, \frac{\lambda_B p_B}{r - \mu - a_G\Delta\mu + \theta\pi(\sigma_G + a_G\Delta\sigma) + \kappa a_G^2/(2p_G) + \lambda_B} \right\}. \tag{33}$$

Likewise, in state B, scaled transaction volume reads

$$v_B = \left(\frac{1}{r - \mu - a_B\Delta\mu + \pi(\sigma_B + a_B\Delta\sigma) + \kappa a_B^2/(2p_B) - \lambda_G(p_G/p_B - 1)} \right)^{\frac{1}{1-\gamma}} \tag{34}$$

and scaled token price is

$$p_B = \max\left\{ v_B, \frac{\lambda_G p_G}{r - \mu - a_B\Delta\mu + \theta\pi(\sigma_B + a_B\Delta\sigma) + \kappa a_B^2/(2p_B) + \lambda_G} \right\}. \tag{35}$$

Note that expressions (32), (33), (34), and (35) only depend on σ (and not on X_t, A_t, and M_t).

The platform solves its optimization in (29) using the dynamic programming principle. As a result, in state $z \in \{B, G\}$, the platform chooses a_z to maximize token price $X_t p_z$. Taking the level of X_t as given, the platform chooses investment a_z to maximize scaled token price p_z. The token price expressions (33) and (35) imply then that the platform's investment choice solves

$$a_z = \arg\max_{a \in [0,\bar{a}]} \left(a p_z (\Delta\mu - \theta_z(a)\pi\Delta\sigma) - \frac{\kappa a^2}{2} \right), \tag{36}$$

where $\theta_z(a_z) = \theta$ if and only if the marginal token investor in state z is a speculator and $\theta_z(a_z) = 1$ if and only if the marginal token investor in state z is a user. If interior (i.e., if $a_z \in (0, \bar{a})$), optimal investment satisfies

$$a_z = \frac{(\Delta\mu - \theta_z(a_z)\pi\Delta\sigma)p_z}{\kappa}. \tag{37}$$

Note that token price p_z, risk-aversion $\theta_z(a_z)$, and the platform's investment choice a_z reflect the preferences of the marginal token investor in state z.

Importantly, the objective in (36) only depends on σ, implying that the optimal choice of investment also depends on σ only. As a result, we have verified that in the Markov equilibrium, token price satisfies $P_t = X_t p(\sigma)$, transaction volume satisfies $V_t = A_t v(\sigma)$, and investment satisfies $a_t = a(\sigma)$. To conclude this Section, we summarize our findings in the following Proposition.

Proposition 2 (Markov equilibrium with investment). *There exists a unique Markov equilibrium with state variables A_t, σ, and M_t in which transaction volume V_t is characterized in (7). In this Markov equilibrium, the following holds:*

1. *Platform transaction volume satisfies $V_t = A_t v(\sigma)$, where $v_B = v(\sigma_B)$ is characterized in (34) and $v_G = v(\sigma_G)$ is characterized in (32).*

2. *Token price P_t satisfies the pricing equation (14) and scales with $X_t = \frac{A_t}{M_t}$. That is, $P_t = X_t p(\sigma)$, where the scaled token price $p_B = p(\sigma_B)$ is characterized in (35) and the scaled token price $p_G = p(\sigma_G)$ is characterized in (33). Expected token returns are characterized in (19), with token price drift $\mu_t^P = \mu_X(\sigma)$ and token price volatility $\sigma_t^P = \sigma_X(\sigma)$ from (31).*

3. *Platform investment only depends on σ, in that $a_t = a(\sigma)$ with $a_B = a(\sigma_B)$ and $a_G = a(\sigma_G)$. Optimal investment is characterized in (36).*

4.3 Optimal investment choice

To maximize token market capitalization (i.e., platform dollar value), the platform chooses its investment to cater to the marginal token investor — who can be a user or a speculator — so as to maximize token price (given token supply). Notably, the outcome that the platform's incentives are perfectly aligned with those of the marginal token investor inevitably leads to conflicts of interest in the triangular relationship between the platform (owners), platform users, and speculators.

When the marginal token investor is a user and $\theta_z(a_z) = 1$ in (36), the token price equals platform transaction volume (i.e., $p_z = v_z$) and so internalizes the negative impact of investment risk on platform transactions. As such, the token price reflects a high degree of risk-aversion, so that investment is low. When the marginal token investor is a speculator and $\theta_z(a_z) = \theta$, $p_z > v_z$, the token price does not internalize the impact of investment risk on transactions (up to a first order) and reflects a low degree of risk-aversion, so that investment is high but platform transaction volume is low.

While the marginal token investor's (risk) preferences determine the platform's incentives to invest, the choice of investment also determines whether the marginal investor is a user or a speculator. In particular, high platform investment increases token price volatility, reduces platform
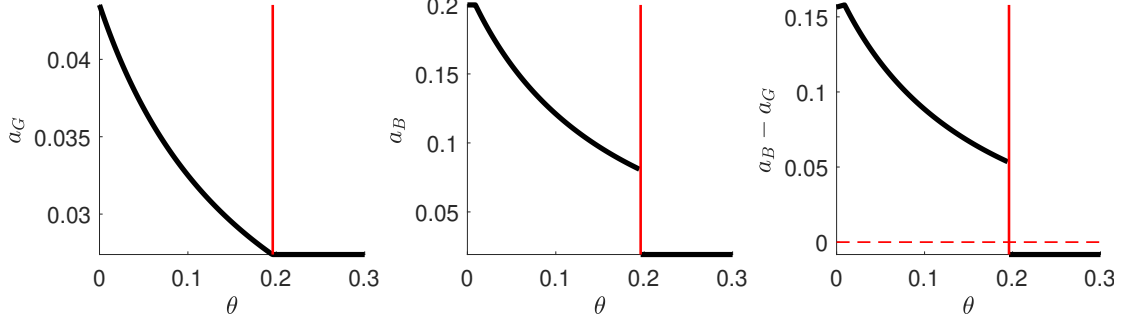
Figure 5: Platform investment and speculative token trading. A lower value of θ implies more speculative trading/investment. The solid red line depicts $\bar{\theta}$: speculators hold tokens in state B if and only if $\theta < \bar{\theta}$. The parameters are $r = 0.06$, $\mu = 0.01$, $\gamma = 0.5$, $\sigma_G = 1 < \sigma_B = 2$, $\pi = 1$, $\lambda_G = \lambda_B = 1$, $\kappa = 1$, $\Delta\sigma = 0.2$, $\Delta\mu = 0.25$, $\bar{a} = 0.2$.

transactions, and fuels speculative token trading. Formally, if the platform were to choose a sufficiently large investment level a_z, then the marginal token investor in state z is a speculator and $\theta_z(a_z) = \theta$.[28] And, the subsequent rise in speculative trading incentivizes the platform to further increase its investment, in that a_z decreases with $\theta_z(a_z)$ (see (36) and (37)). This two way feedback between platform investment and speculative trading advances platform development and productivity growth thereby boosting future (long-run) adoption, but amplifies token price volatility and therefore limits contemporaneous platform usage.

Figure 5 illustrates these effects and plots the optimal platform investment levels a_B and a_G against θ quantifying speculative investment/trading. Token prices satisfy $p_G > p_B$ (not displayed), and there is no speculative investment in state G. To the right of the solid red line (i.e., for $\theta \geq \bar{\theta}$), there is no speculative investment in state B, so the marginal token investor is a user. Then, investment levels satisfy $a_G > a_B$. In contrast, when the marginal token investor in state B is a speculator (i.e., when $\theta < \bar{\theta}$), platform investment and productivity growth is highest in volatile times (i.e., $a_B > a_G$). Thus, seemingly excessive token price volatility can be a positive sign suggesting high platform growth and future adoption.

Observe that speculative trading in volatile times boosts investment in both states, in that a_B and a_G decrease with θ when $\theta < \bar{\theta}$. In state B, speculators as the marginal token investors have

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[28]To see this, note that taking the limit $a_G \to \infty$ (ignoring the platform's optimization and the bound $\bar{a}$) and using the expressions (32) and (33)) yields

$$\lim_{a_G \to \infty} \frac{p_G}{v_G} = \lim_{a_G \to \infty} \frac{\kappa a_G^{\frac{2}{1-\gamma}}}{\kappa a_G^2} = \lim_{a_G \to \infty} \frac{a_G^{\frac{2}{1-\gamma}}}{a_G^2} = \lim_{a_G \to \infty} a_G^{\frac{2\gamma}{1-\gamma}} = \infty. \tag{38}$$

As a result, we can conclude that for a_G sufficiently large, $p_G > v_G$ and the marginal token investor is a speculator. Repeating the same argument in state B, we obtain that for a_B sufficiently large, $p_B > v_B$ and the marginal token investor is a speculator.
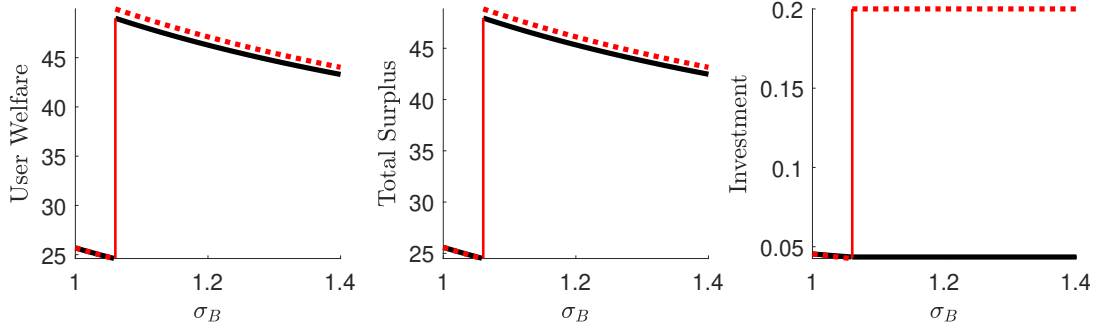
Figure 6: Volatility spikes and platform investment. This figure plots scaled user welfare and total surplus against σ_B (with $\sigma_B \geq \sigma_G$) in both states. In the left and middle panel, the solid black (dotted red) line depicts scaled user welfare and total surplus in state G (B). In the right panel, the solid black line depicts investment in state G, a_G, and the dotted red line depicts investment in state B, a_B. The vertical solid red line depicts the cutoff above which speculators buy tokens in state B: that is, the marginal token investor in state B is a speculator if and only if σ_B lies to the right of the solid red line. The remaining parameters are $r = 0.06$, $\mu = 0.01$, $\alpha = 0.25, \beta = 0.25$, $\gamma = \alpha + \beta = 0.5$, $\sigma_G = 1$, $\theta = 0$, $\pi = 1$, $\lambda_G = \lambda_B = 1$, $\kappa = 1$, $\Delta\sigma = 0.2$, $\Delta\mu = 0.25$, $\bar{a} = 0.2$.

direct (positive) impact on investment, increasing investment a_B and token price p_B in state B. The increase in token price p_B reduces the token price downside risk in state G, increasing adoption and token price p_G in normal times. And, a higher (scaled) token price p_G translates into a higher investment level a_G (see (36) and (37)).

In normal times, speculators do not hold tokens and users' transaction-based preference for token price stability curbs platform investment, effectively causing under-investment (compared to a setting with perfect risk-sharing between users and speculators).[29] Speculators as the marginal token investors in volatile times then boost investment (in both normal and volatile times), which mitigates this under-investment problem. Therefore, occasional periods of high volatility (i.e., volatility spikes) are desirable, as they invite speculative token trading that stimulates platform development, investment, and future adoption. Figure 6 illustrates these outcomes and plots scaled user welfare and total surplus as well as platform investment against volatility $\sigma_B \in [\sigma_G, 2\sigma_G]$ both in state G (solid black line) and B (dotted red line). When σ_B is small and close to σ_G, there are effectively no periods of high volatility. Then, speculators never buy tokens and low investment prevails in both states, which can be interpreted as under-investment. Once σ_B surpasses a critical level (denoted by the vertical solid red line), speculators become the marginal token investors in state B, and their presence boosts platform investment (in state B), improving user welfare and total surplus. As, at the cutoff (vertical solid red line), the increase in investment a_B induced by speculators comes along with an increase in user welfare and total surplus, one concludes that

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[29]Section 5.2 and Appendix G.1 discuss the model solution with perfect risk-sharing.
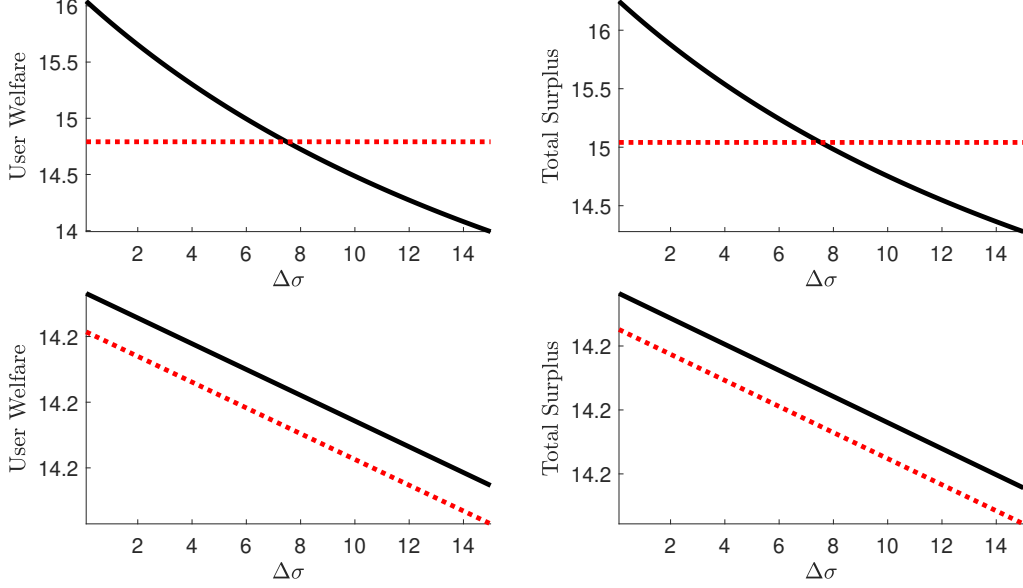
Figure 7: Investment uncertainty and over-investment. This figure plots scaled user welfare and total surplus against investment uncertainty $\Delta\sigma$ both when $\Delta\mu = 0.1$ (solid black line) and $\Delta\mu = 0$ (dotted red line). The upper two panels use $\theta = 0$. The lower two panels use $\theta = 1$ and so consider the model without speculators. Note that the axis labels are rounded to one decimal, so in the lower two panels any changes are quantitatively small. The remaining parameters are $r = 0.06$, $\mu = 0.01$, $\alpha = 0.25$, $\beta = 0.25$, $\gamma = \alpha + \beta = 0.5$, $\sigma_G = 1 < 2 = \sigma_B$, $\theta = 0$, $\pi = 1$, $\lambda_G = \lambda_B = 1$, $\kappa = 1$, $\bar{a} = 0.2$.

speculators mitigate the under-investment problem that prevails for lower levels of σ_B. Any further increase in σ_B, however, reduces user welfare, total surplus, and token price, so there is an interior optimal level of volatility $\sigma_B > \sigma_G$.

Crucially, the investment-stimulating effect of speculators has a counterpart: the presence of speculators can induce excessively high investment in state B, which can be seen as over-investment (compared to a setting without speculators). This over-investment leads to extended periods of excessively low platform usage and excessively high token volatility and speculative trading. The combination of speculative token investors and profitable growth (investment) opportunities leads to the curse of over-investment when the platform's growth opportunities are subject to high uncertainty and risk (i.e., $\Delta\sigma$ is sufficiently large).

To illustrate this result, Figure 7 plots user welfare and total surplus against $\Delta\sigma$ both in a setting with investment opportunities (i.e., $\Delta\mu = 0.1$; solid black line) and without investment opportunities (i.e., $\Delta\mu = 0$; dotted red line), using $\theta = 0$ in the upper two panels and $\theta = 1$ in the lower two panels. In the upper two panels, the marginal token investor in state B is a speculator under all parameters considered, whereas in the lower two panels there is no speculative investment. To improve overview and reduce the number of figure panels, we display quantities in state G only. First, consider the upper two panels. Observe that when $\theta = 0$ and $\Delta\sigma$ is

sufficiently large, the platform and its users would be better off without investment opportunities (i.e., $\Delta\mu = 0$), which implies that platform investment must be inefficiently high and there must be over-investment (when $\Delta\mu = 0.1$). Crucially, these effects cannot be seen in the lower two panels (without speculators), so it must be that speculators cause this over-investment.

5 Alternative platform structures and perfect risk-sharing

A token-based platform suffers from the volatility of its native transaction medium. As the previous sections have highlighted, speculators can alleviate this problem by engaging in (imperfect) risk-sharing with platform users. In this Section, we propose two alternative platform structures that feature a stable platform transaction medium and, notably, improve risk-sharing between platform users and speculators: a fiat-based (platform) structure and a dual token structure. We show that a fiat-based and dual token structure improve risk-sharing by transferring (platform) productivity risk from platform users to speculators (financial investors) who act as the platform's equity holders.

5.1 Fiat-based platform

Recall that under the token-based (platform) structure, tokens serve as the platform transaction medium and the platform earns initially $M_0 P_0$ dollars from issuing tokens. In other words, the platform earns *seigniorage* from issuing tokens. Notably, seigniorage monetizes platform transactions and allows the platform to earn payoffs and to be profitable without i) charging users a transaction fee or ii) adopting other monetization models that are likely to harm users (such as selling or exploiting user data or putting advertisements).

This changes under a fiat-based (platform) structure which features the numeraire ("dollars") as the platform transaction medium. When fiat money serves as the platform transaction medium, the platform does not earn seigniorage.[30] To earn payoffs and to monetize platform transactions, the platform charges users a fee for platform services and transactions, which curbs platform usage.[31] In general, an optimal transaction fee structure on platforms can be complex (Rochet and Tirole (2003)) and is beyond the scope of the paper. To maintain tractability, we restrict attention to variable transaction fees ϕ_t per dollar transaction, modelled in reduced form as a proportional fee

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[30]Strictly speaking, seigniorage accrues to the central bank issuing fiat money (which is outside of our model).

[31]Admittedly, other monetization models for platform transactions, that are outside of our model and beyond the scope of the paper, exist — such as putting/targeting advertisements or exploiting user/transaction data. Similar to transaction fees, these monetization models may limit the usefulness of the platform and therefore may harm platform users and adoption. For simplicity, we restrict attention to transaction fees as the platform's revenue source under a fiat-based structure. In addition, we abstract from transaction fees under a token-based structure. Token market capitalization $M_0 P_0$ is therefore a lower bound on platform dollar value under a token-based structure and potentially could be improved upon by levying transaction fees.

on users' holdings of fiat money.[32]

As in the baseline, we model platform transactions in reduced form following the money-in-the utility-function approach, in that any user i derives utility from holding $u_{it} \geq 0$ dollars:

$$dR_{it}^U := \underbrace{\frac{u_{it}^\alpha}{\alpha} V_t^\beta A_t^{1-\alpha-\beta} dt}_{\text{Convenience yield}} + u_{it} \Big(\underbrace{\phi_t dt}_{\substack{\text{Transaction} \\ \text{fee}}} - \underbrace{r dt}_{\substack{\text{Opportunity} \\ \text{cost}}} \Big). \tag{39}$$

Compared with users' utility on a token-based platform (see (3)), few differences are noteworthy. First, by definition, fiat money as a transaction medium is price-stable in terms of the numeraire ("dollars"). Price stability is a desirable feature of a transaction medium, but also precludes that users earn returns and capitalize on platform success when transacting. Second, users incur a proportional transaction fee ϕ_t (modelled as a fee on users' dollar holdings u_{it}). We assume that users do not earn interest when holding dollars, giving rise to the opportunity cost in (39).

The platform's equity value Q_t is the risk-adjusted expected discounted stream of fee revenues net the cost of investment, that is,

$$Q_t = \max_{(a_t)_{t \geq 0}, (\phi_t)_{t \geq 0}} \mathbb{E}_t \left[\int_t^\infty e^{-r(s-t)} \left(\phi_s V_s - A_s \frac{\kappa a_s^2}{2} - \theta \pi Q_s |\sigma_s^Q| \right) ds \right], \tag{40}$$

where σ_t^Q is the volatility of dQ_t/Q_t.[33] There are no financial frictions, and investment can be financed by issuing new equity units. Platform equity is held by speculators (i.e., financial investors) who exhibit a similar type of risk-aversion as in the baseline: speculators are risk-averse with respect to the volatility σ_t^Q of equity value Q_t and their risk-aversion is governed by $\theta \pi$. The platform dynamically chooses transaction fees ϕ_t and investment a_t to maximize its equity value. Note that at $t = 0$, the platform's dollar value/payoff under a fiat-based structure is the platform equity value Q_0, while the platform's dollar value/payoff under a token-based structure is the token market capitalization $M_0 P_0$.[34]

Appendix E presents the solution and equilibrium under a fiat-based platform structure. As

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[32]Note that a similar transaction fee structure is adopted by large fiat-based platforms, such as Uber or AirBnB, where users pay a service fee that increases in the dollar value of the transaction.

[33]Formally, platform equity value/price evolves according to:

$$\frac{dQ_t}{Q_t} = \mu_t^Q dt + \sigma_t^Q dZ_t + \Delta_t^{GQ} dJ_t^G + \Delta_t^{BQ} dJ_t^B,$$

where price drift μ_t^Q, price volatility σ_t^Q, and the jump terms Δ_t^{zQ} are endogenous. Platform equity value and price are the same under the assumption that there is one unit of equity. See Appendix E for the solution and the equilibrium under the fiat-based platform structure.

[34]As, under a token-based structure, there are no payoffs besides token issuance at $t = 0$, platform equity value after time $t = 0$ is zero under a token-based structure. For simplicity, we abstract from transaction fees which would imply positive equity value after $t = 0$.
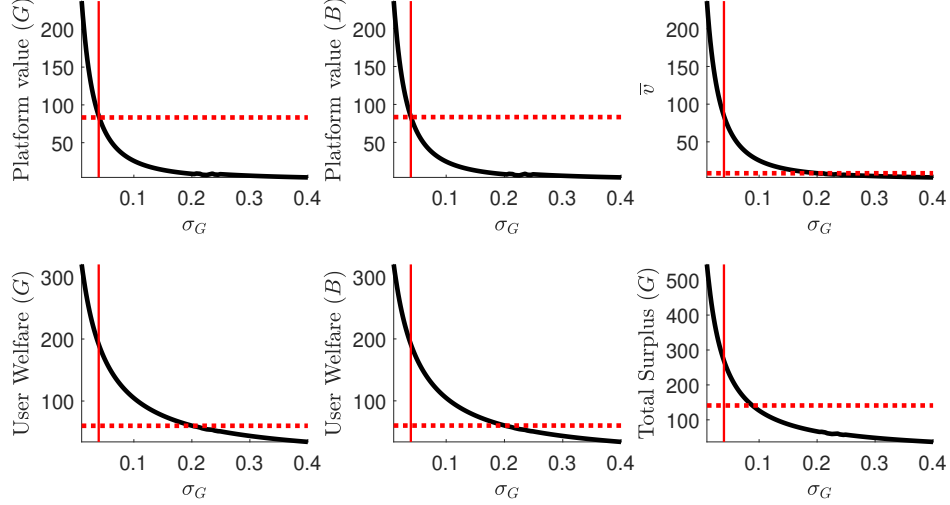
Figure 8: Token-based vs. fiat-based structure. This figure plots the scaled platform dollar value, long-run adoption $\bar{v}$, user welfare, and total surplus under a token-based structure (solid black line) and a fiat-based structure (dotted red line) against σ_G (with $\sigma_B = 2\sigma_G$. The vertical solid red line depicts the unique point at which the scaled platform dollar value in state G is the same under a token-based and fiat-based structure. At this point, scaled platform dollar value is approximately the same under both structures in state B too. The parameters satisfy $r = 0.06$, $\mu = 0.01$, $\alpha = 0.25$, $\beta = 0.25$, $\gamma = \alpha + \beta = 0.5$, $\sigma_B/\sigma_G = 2$, $\theta = 0$, $\pi = 1$, $\lambda_G = \lambda_B = 1$, $\bar{a} = 0$.

we show, a fiat-based structure improves risk-sharing between users and speculators (relative to a token-based structure), in a sense that under a fiat-based structure, users are not exposed to any platform-specific risk which is entirely absorbed by speculators as the platform's equity holders.[35] However, this improved risk-sharing comes at the expense of seigniorage.

5.1.1 Comparison: token-based platform vs. fiat-based platform

From the perspective of an entrepreneur or platform owner, the choice between implementing a fiat-based or token-based platform structure reflects the trade-off to either obtain price stability of the transaction medium (i.e., improved risk-sharing) or seigniorage. A token-based structure allows the platform to earn seigniorage and to dispense with transaction fees but features a volatile transaction medium. A fiat-based structure features a stable transaction medium but foregoes seigniorage and requires the platform to charge transaction fees.

We analyze under which circumstances it is optimal to adopt a token-based rather than a fiat-based platform structure. For this sake, Figure 8 plots scaled platform dollar value, scaled long-run (average) adoption $\bar{v}$, scaled user welfare, and scaled total surplus under a token-based structure (solid black line) and a fiat-based structure (dotted red line) against σ_G in both states B and G.

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[35]Note that this result not necessarily implied by the stability of the platform transaction medium, as potentially users could be exposed to platform risk via transaction fees and their dynamics.

We stipulate $\sigma_B = 2\sigma_G$, so the ratio σ_B/σ_G remains constant. According to Figure 8, a token-based structure implies higher platform dollar value, user welfare, total surplus, and adoption if and only if σ_G is sufficiently small. Thus, a token-based platform is by all means preferred over a fiat-based platform if and only if platform risk and uncertainty — as captured by σ_B and σ_G — is low. The intuition is that low platform risk implies low token price volatility, which limits the additional benefits of achieving full price stability through a fiat-based structure.

That is, according to our model, only developed and mature platforms that tend to face less (technological or user demand) uncertainty than early-stage platforms benefit from issuing their own tokens and implementing a token-based structure. Consequently, our findings rationalize why large technology and digital platform firms such as Amazon, Facebook, or Uber plan to launch their own tokens, while — at the same time — the ICO market dominated by young and early-stage platform firms performed poorly in the past. Likewise, our model implies that a token-based structure tends to be more beneficial for platform applications facing relatively low technological or user demand uncertainty (e.g., marketplace platforms), as opposed to platform applications facing relatively high uncertainty (e.g., "Internet of Things" or quantum computing platforms).

The platform (owner) finds it privately optimal to implement a token-based structure if and only if σ_G lies to the left of the solid red line depicting the unique point at which scaled platform dollar value in state G is the same under a token-based and fiat-based structure.[36] However, from the perspective of platform users or a social planner (regulator) maximizing total surplus, it is desirable to implement a token-based structure for even higher levels of σ_G that lie to the right of the solid red line. That is, from a social welfare point of view, platform owners do not have sufficient private incentives to implement a token-based structure and therefore decide "too often" for a fiat-based structure. Intuitively, platform owners do not fully internalize that tokenization spurs transaction activity, causing an under-provision of viable token-based platforms. According to our model, optimal regulation should address this under-provision problem and facilitate token issuance by mature/developed (and less risky) platforms, while restricting token issuance by early-stage (and risky) platforms. We summarize our analytical findings in the following Corollary.

Corollary 4. *If σ_G and σ_B are sufficiently large and $\theta \geq 0$ is sufficiently small, a fiat-based platform yields higher (scaled) platform dollar value, user welfare, total surplus, and long-run adoption than a token-based platform.*

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[36]At this point, scaled platform value is approximately the same under both structures in state B too.

5.1.2 Platform structure and agency conflicts

Under a token-based structure, the platform's incentives are not necessarily aligned with those of its users, giving rise to agency conflicts (i.e., conflicts of interest) between the platform and its users (see Section 4). We now study how the platform structure affects these agency conflicts. To do so in the most tractable and illustrative way, we present the following stylized model variant.

We introduce that at any time t, the platform chooses action $b_t \in \{0, 1\}$ (next to its investment a_t) that boosts the growth rate of platform productivity A_t at the expense of users. Specifically, setting $b_t = 1$ increases the growth rate of A_t by arbitrarily small amount $\varepsilon > 0$, but implies that the platform does not facilitate transactions at time t in that tokens offer zero convenience yield and platform transaction volume becomes $V_t = 0$. Throughout, we consider that the boost in growth rate is negligible, that is, the analysis is carried out in the limit $\varepsilon \to 0$. Hence, choosing $b_t = 1$ unambiguously harms platform users and therefore is akin to "abusing platform users" as in Sockin and Xiong (2021). Formally, under a token-based structure, any user's utility from holding u_{it} dollars in tokens becomes

$$dR_{it}^U \equiv \underbrace{(1 - b_t) \frac{u_{it}^\alpha}{\alpha} V_t^\beta A_t^{1-\alpha-\beta} dt}_{\text{Convenience yield}} + u_{it} \Big(\underbrace{\frac{dP_t}{P_t}}_{\substack{\text{Token} \\ \text{returns}}} - \underbrace{rdt}_{\substack{\text{Opportunity} \\ \text{cost}}} \Big) - \underbrace{\pi |u_{it} \sigma_t^P| dt}_{\substack{\text{Risk} \\ \text{premium}}}. \tag{41}$$

Platform productivity evolves according to

$$\frac{dA}{A_t} = (\mu + a_t \Delta \mu + b_t \varepsilon) dt + (\sigma + a_t \Delta \sigma) dZ_t, \tag{42}$$

where $b_t \in \{0, 1\}$. The platform chooses $(b_t)_{t \geq 0}$ to maximize platform dollar value at $t = 0$, that is, initial token market capitalization $M_0 P_0$ under a token-based structure. In equilibrium, the optimal choice of b_t depends on σ only, so that $b_t = b_z$ for $z = B, G$. And, the platform then chooses b_z to maximize scaled token price p_z. All other model elements remain unchanged compared with the baseline. Appendix F presents the model solution and equilibrium under this model specification, considering both a token-based and fiat-based structure.

Although choosing $b_z = 1$ has sizable negative impact on platform transaction volume and only negligible positive impact on productivity growth, the platform may nevertheless choose $b_z = 1$ under a token-based structure. Under a token-based structure, the platform's incentives are aligned with those of the marginal token investor who is either a user or a speculator, which inevitably leads to conflicts of interest in the triangular relationship between the platform, its users, and speculators. When the marginal investor is a speculator, the token price and hence the platform

do not internalize that choosing $b_z = 1$ reduces token convenience yield and transaction volume. Then, the platform sets $b_z = 1$ so as to boost token price and platform growth at the expense of users. Intuitively, the platform caters to speculators at the expense of users. We show that when σ_B is sufficiently large and θ is sufficiently low, the marginal token investor in volatile times is indeed a speculator, implying $b_B = 1$ (and $b_G = 0$) by the preceding arguments.

Crucially, the inefficient outcome $b_z = 1$ does not arise under a fiat-based platform structure. The reason is that under a fiat-based structure, the platform charges a transaction fee to users. If the platform ignored users' preferences, platform transaction volume and fee revenues would drop. Thus, the platform internalizes any negative effects on platform users. Transaction fee revenues depend on platform usage and directly reflect user preferences, while revenues from token issuance (i.e., seigniorage) may reflect speculators' investments and preferences too. In the presence of speculators, a fiat-based structure better aligns the incentives of platform owners and users than a token-based structure, thereby alleviating agency conflicts between platform owners and users.[37]

We can conclude that a fiat-based structure is preferred over a token-based structure, when agency conflicts between platform owners' and users' are severe and a first-order concern. This prediction is consistent with our previous finding that only developed and mature platforms benefit from implementing a token-based structure, as one would expect developed and mature platforms to suffer from less severe agency problems than early-stage platforms. To conclude this Section, we summarize these findings in the following Proposition.

Proposition 3. *Under a fiat-based structure, the platform chooses $b_z = 0$. Under a token-based structure, the platform chooses $b_z = 1$ if and only if the marginal token investor in state z is a speculator. When σ_B is sufficiently large and θ is sufficiently small, the platform chooses $b_B = 1$ in state B (and $b_G = 0$ in state G) under a token-based structure.*

5.2   Dual token structure

In this Section, we propose a dual token structure that allows platform owners to harness the advantages of both a fiat-based and token-based platform structure: i) price stability of the platform transaction medium (implementing improved risk-sharing between users and speculators), ii) seigniorage, and iii) the alignment of the platform's incentives with those of its users. The dual token structure features two native tokens: i) a price-stable transaction token (stablecoin) held by users for transactions and ii) a governance token (equity) held by speculators for returns. The price of the stablecoin is normalized to one and stablecoin supply is adjusted accordingly to keep the

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[37]Admittedly, our arguments suggest that transaction fees on a token-based platform could potentially solve this agency problem and align the platform's incentives with those of its users. We leave this possibility for future research.

price stable. Therefore, for the transaction token market to clear, transaction token supply must equal transaction volume V_t (which is users' aggregate transaction token holdings in dollars).

The governance token pays revenues from transaction fees and transaction token issuance as dividends $dDiv_t$. The platform can always issue new governance tokens to raise funds (without any frictions), in which case $dDiv_t < 0$. These funds are used i) to buy back (and burn) transaction tokens to adjust supply to maintain price stability and ii) to finance physical investment. Thus, governance token dividends read

$$dDiv_t = \underbrace{dV_t}_{\substack{\text{Stablecoin supply} \\ \text{adjustment}}} + \underbrace{\phi_t V_t dt}_{\substack{\text{Fee} \\ \text{revenues}}} - \underbrace{\frac{A_t \kappa a_t^2}{2} dt}_{\substack{\text{Investment} \\ \text{costs}}} . \tag{43}$$

The aggregate value of governance tokens — which is the value of platform equity — is the risk-adjusted expected discounted stream of future dividends, i.e.,

$$Q_t = \max_{(a_t)_{t\geq 0},(\phi_t)_{t\geq 0}} \mathbb{E}_t \left[\int_t^\infty e^{-r(s-t)} \left(dDiv_s - \theta\pi |Q_s \sigma_s^Q + \sigma_s^{Div}|ds \right) \right], \tag{44}$$

where σ_t^P is the volatility of dQ_t/Q_t and σ_t^{Div} the volatility of $dDiv_t$.[38] Governance tokens (equity) are held by speculators who have a similar type of risk-aversion as in the baseline: speculators are risk-averse with respect to the volatility of equity value Q_t and dividends Div_t, with risk-aversion coefficient $\theta\pi$. The platform chooses transaction fees and investment to dynamically maximize platform dollar value at time zero, given by $V_0 + Q_0$. Notably, under a dual token structure, platform (dollar) value stems from two sources: i) seigniorage (i.e., the proceeds from issuing transaction tokens) yielding V_0 dollars at $t = 0$ and ii) equity or governance token issuance yielding Q_0 dollars at $t = 0$. Appendix G presents the detailed solution under a dual token structure.

Under the baseline token-based platform structure, tokens have both a transaction and investment function; a dual token structure separates these two functions. As such, the dual token structure minimizes platform users' risk exposure by offloading risk to speculators, which enhances risk-sharing between users and speculators relative to the baseline token-based structure. Unlike a fiat-based structure, the dual token structure improves risk-sharing while collecting seigniorage from transaction token issuance. As argued in Appendix G, the transaction fees and commitment to transaction token price stability under a dual token structure align the platform's incentives

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[38]As under a fiat-based structure (see Section 5.1), platform equity value/price evolves according to:

$$\frac{dQ_t}{Q_t} = \mu_t^Q dt + \sigma_t^Q dZ_t + \Delta_t^{GQ} dJ_t^G + \Delta_t^{BQ} dJ_t^B,$$

where price drift μ_t^Q, price volatility σ_t^Q, and the jump terms Δ_t^{zQ} are endogenous. See Appendix G for the solution and the equilibrium under the dual token structure.

with those of its users, thereby alleviating agency conflicts between the platform and its users.

Appendix G.1 shows that under certain circumstances, the dual token structure implements perfect risk-sharing between users and speculators.[39] Then, the dual token structure leads to higher adoption and payoffs than the baseline token-based structure. In addition, risk-sharing between users and speculators eliminates the under-investment problem that is induced by users' transaction-based preference for price stability. However, the dual token structure is no free lunch. The reason is that in practice, implementing a dual token structure may be challenging, costly, or even infeasible due to various factors omitted in our analysis. Price stability of the stablecoin may be difficult to achieve, for instance, because the platform cannot credibly commit to token price stability, the issuance of governance tokens is subject to frictions and/or the stablecoin is under-collateralized (Routledge and Zetlin-Jones, 2021; Li and Mayer, 2021). Note that the proposed stablecoin system operates without reserves, and would be best described as "algorithmic stablecoin." In contemporaneous work, Bourany, d'Avernas, and Vandeweyer (2021) show that especially algorithmic stablecoins are exposed to demand fluctuations that can threaten price stability.

6 Noise traders and short squeeze risk

Major cryptocurrencies with high trading volume, market capitalization, and liquidity — such as Bitcoin, Cardano, or Ether — i) have become popular among retail investors and ii) can be shorted relatively easily/cheaply (e.g., via centralized exchanges like Binance or decentralized exchanges like Uniswap). To capture these features, we consider a model variant with i) no short selling constraints and ii) noise traders who represent retail traders. Note that speculators are modelled as fully rational and informed financial investors, a description that may not be suitable for a large share of retail investors. Therefore, speculators rather represent professional traders or professional and accredited investors, whereas noise traders rather represent less informed/skilled retail traders. We model noise traders following Kyle (1985) and Back, Collin-Dufresne, Fos, Li, and Ljungqvist (2018). Noise traders trade randomly according to a Brownian Motion $d\tilde{Z}_t$ that is independent of dZ_t. Noise traders' aggregate nominal token holdings $\tilde{N}_t$ evolve according

$$d\tilde{N}_t = \tilde{\sigma}_t d\tilde{Z}_t.$$

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[39]As for a fiat-based platform, the price stability of the platform transaction does not necessarily imply perfect risk-sharing, because users could be exposed to platform risk via transaction fees and their dynamics.

Noise traders' aggregate token holdings in dollars are $\tilde{N}_t P_t$. As a result, the token market clearing condition in the presence of noise traders becomes[40]

$$M_t P_t = V_t + S_t + \tilde{N}_t P_t \quad \Longleftrightarrow \quad N_t P_t = V_t + S_t, \tag{45}$$

where we define the token supply net of noise trader demand as

$$N_t \equiv M_t - \tilde{N}_t.$$

In principle, there are now four state variables: i) token supply M_t, ii) noise trader demand $\tilde{N}_t$, iii) productivity A_t, and iv) volatility σ. To reduce the dimensionality of the state space, we assume

$$\tilde{\sigma}_t = N_t \tilde{\sigma}$$

for some constant volatility parameter $\tilde{\sigma} \geq 0$. The assumption that $\tilde{\sigma}_t$ scales with N_t is economically sensible, as it prevents noise traders' demand for tokens from exceeding token supply M_t.[41] To clearly illustrate the economic forces that are specific to noise traders, we eliminate uncertainty shocks and investment, in that we consider that $\sigma = \sigma_G$ at all times (i.e., $\lambda_B = 0$) and $\bar{a} = 0$. The general solution with noise traders, uncertainty shocks, and investment is left for further research. We expect the same key implications of noise trading to arise in a more general setup too.

As there are now two types of Brownian shocks, equilibrium token price evolves according to

$$\frac{dP_t}{P_t} = \mu_t^P dt + \sigma_t^P dZ_t + \tilde{\sigma}_t^P d\tilde{Z}_t. \tag{46}$$

We stipulate that users and speculators are risk-averse with respect to price volatility from noise trading: user i incurs dis-utility $\pi_U |u_{it} \tilde{\sigma}_t^P|$ and speculator j incurs dis-utility $\pi_S |s_{jt} \tilde{\sigma}_t^P|$ for bearing token price risk, where $\pi_U, \pi_S \geq 0$ (with $\pi_U \geq \pi_S$) are constants. In what follows, we look for a Markov equilibrium with state variables A_t and N_t that is similar to the one considered in the baseline (except that σ is constant). In this Markov equilibrium, token price scales with $Y_t = A_t/N_t$ so that $P_t = Y_t p_N$, and transaction volume scales with A_t so that $V_t = A_t v_N$. Appendix I presents the detailed solution and equilibrium.

In equilibrium (provided it exists), $\frac{\mathbb{E}[dP_t]}{P_t dt} = \mu_t^P = \mu$, $\sigma_t^P = \sigma$, $\tilde{\sigma}_t^P = \tilde{\sigma}$, and platform transaction volume v_N decreases with $\tilde{\sigma}$. Generating additional price volatility, noise traders hamper platform

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[40]As there is no limit to short selling, $\tilde{N}_t < 0$ and $S_t < 0$ are possible. Users do not short sell tokens, because the marginal convenience yield (in (3)) becomes infinite once u_{it} approaches zero (from above).

[41]If it were $\tilde{N}_t > M_t$, then $N_t < 0$ and the joint aggregate token holdings of users and speculators is negative.

transactions and adoption. Note that due to $\frac{\mathbb{E}[dP_t]}{P_t dt} < r$, speculators do not take a token long position. Importantly, there exists a Markov equilibrium with positive token price $p_N > 0$ and platform transaction volume $v_N > 0$ if and only if

$$\tilde{\pi}_S \tilde{\sigma} \geq r - \mu - \theta \pi \sigma. \tag{47}$$

Condition (47) states that when $r - \mu - \theta \pi \sigma > 0$, a certain extent of noise trading is necessary for the token and the platform to possess value. The reason is that when (47) does *not* hold, expected token returns are so low (relative to the risk of short selling) that any speculator j would find it optimal to take an arbitrarily large short position (i.e., $s_{jt} \to -\infty$). This excessive short selling pressure would cause the token price to crash down to zero, triggering that there are no more platform transactions and the platform essentially collapses.

Noise trading generates unpredictable price fluctuations, thereby inducing a "short squeeze risk" that discourages such large short positions (when $\tilde{\pi}_S > 0$). As the recent examples of GameStop or AMC have emphasized, short squeeze risk is likely a concern for short sellers across asset classes. When (47) holds, the short squeeze risk from noise trading prevents excessive short selling by rational speculators that could cause the token price to crash. That is, noise traders help the token and the platform to sustain value and therefore can be beneficial for token-based platforms and its users. According to our model, it can be important that cryptocurrencies and tokens are tradeable for retail investors. In general, optimal regulation should not restrict cryptocurrency and token investments for retail investors and to accredited and professional investors only. We formalize our findings in the following Proposition.

Proposition 4 (Noise Trading). *There exists a Markov equilibrium with $V_t = A_t v_N$ and $P_t = Y_t p_N$ whereby $p_N > 0$ and $v_N > 0$ if and only if (47) holds; otherwise, if (47) does not hold, there exist only a "degenerate" Markov equilibrium with $p_N = v_N = 0$.*

7   Conclusions

Although often intended to serve as a means of payment, cryptocurrencies and tokens have become notorious for rampant speculation. In this paper, we develop a dynamic model of token-based platforms with both platform users and speculators as token investors. The token price reflects the demand from both users and speculators, and the marginal token investor can be a user or a speculator. The interaction between platform user and speculators can be described by two opposing effects, i) crowding-out and ii) risk-sharing. Via their investment, speculators crowd-out

platform users in volatile times, curbing platform adoption. However, providing liquidity in volatile times, speculators increase the resale option value of tokens and effectively share risks with users, which incentivizes users to acquire tokens and to adopt the platform in normal times. As speculators stimulate platform transactions and adoption via risk-sharing, it is in general not optimal for the regulator or platform to ban speculative trading in tokens and cryptocurrencies.

Next, we endogenize platform development and investment and show that the platform chooses its investment policies to cater to the marginal token investor who can be a user or a speculator, giving rise to conflicts of interest between the platform, its users, and speculators. Notably, speculators as the marginal token investors incentivize platform investment and mitigate the under-investment problem induced by users' preferences for token price volatility. Occasional periods can be seen as a feature rather than a bug of token-based platforms: volatility spikes invite speculative trading that boosts platform investment, growth, and long-run adoption.

We then discuss alternative platform structures, such as a fiat-based and dual token structure. Under a token-based structure, the platform earns seigniorage and can dispense with transaction fees but the platform transaction medium is volatile. Under a fiat-based structure, the platform features a stable transaction but does not earn seigniorage and therefore must charge a transaction fee to be profitable. The choice between a fiat-based and token-based platform structure is subject to the trade-off of obtaining stability of the platform transaction medium or seigniorage. A dual token structure with a stablecoin and governance token allows the platform to harness the benefits of both a token-based and fiat-based platform structure, namely, seigniorage and stability of the transaction medium.

Finally, we also demonstrate that noise traders help the platform and token to sustain value, because their trading generates unpredictable price fluctuations (leading to short-squeeze risk) that discourages excessive short positions by rational speculators. Overall, our results have implications for token and platform design as well as for the empirical relationship between speculative token trading and platform usage. Our model also provides guidance for the optimal regulation of i) cryptocurrency trading for speculators and retail investors and ii) token issuance (e.g., via ICOs and STOs).

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Appendix

A   Proof of Proposition 1

Claims 1) and 2) (i.e., the solution and characterization of the Markov equilibrium) follow from the arguments presented in the main text. It remains to prove claim 3), that is, $p_G > p_B$ and $S_t = 0$ in state G, and claim 4).

A.1   Claim 3

To start with, first note that expected token returns in state B read

$$\varepsilon_B = \mu + \lambda_G \left(\frac{p_G}{p_B} - 1 \right) \tag{A.1}$$

and expected token returns in state G read

$$\varepsilon_G = \mu + \lambda_B \left(\frac{p_B}{p_G} - 1 \right). \tag{A.2}$$

Suppose to the contrary that $p_B \geq p_G$. As a result, by (A.1),

$$\varepsilon_B \leq \mu < r,$$

so — by (8) and (9) — speculators do not hold tokens in state B and $v_B = p_B$. We consider now two different cases: 1) $p_G > v_G$ and 2) $p_G = v_G$.

First, if $p_G > v_G$, speculators hold tokens in state G and — due to (9) and $\varepsilon_B \leq \mu$ — expected token returns in state G satisfy $\varepsilon_G \geq r > \mu \geq \varepsilon_B$. As a result,

$$p_B = v_B = \left(\frac{1}{r + \pi\sigma_B - \varepsilon_B} \right)^{\frac{1}{1-\gamma}} < \left(\frac{1}{r + \pi\sigma_G - \varepsilon_G} \right)^{\frac{1}{1-\gamma}} = v_G < p_G, \tag{A.3}$$

where the first inequality uses $\sigma_G < \sigma_B$ and $\varepsilon_G > \varepsilon_B$. But, $p_B < p_G$ yields a contradiction.

Second, we consider $p_G = v_G$, so $v_B = p_B \geq v_G$. Due to $\sigma_B > \sigma_G$, $v_B \geq v_G$ implies $\varepsilon_B > \varepsilon_G$ (see expressions (22) and (20)). The expressions (A.1) and (A.2) imply that

$$\varepsilon_B > \varepsilon_G \quad \Longleftrightarrow \quad p_G > p_B,$$

a contradiction.

Thus, it follows that $p_G > p_B$. Therefore, $\varepsilon_G < \mu < r$, so that speculators do not hold tokens in state G. So, $v_G = p_G$ and $p_B \geq v_B$, which implies $v_G > v_B$.

A.2   Claim 4

Note that by market clearing, speculative token investment (in dollars) satisfies

$$S_t = M_t P_t - V_t = M_t X_t p(\sigma) - A_t v(\sigma) = A_t(p(\sigma) - v(\sigma)),$$

using $X_t = A_t/M_t$. So, in state B,

$$\frac{S_t}{A_t} = p_B - v_B$$

is the scaled speculative investment in dollars. In state G, $\frac{S_t}{A_t} = p_G - v_G = 0$. It follows that $\frac{S_t}{A_t}$ only changes following uncertainty shocks.

Next, note that by market clearing, speculators' nominal token holdings satisfy

$$\frac{S_t}{P_t} = M_t - \frac{V_t}{MP_t} = M_t \left(1 - \frac{v(\sigma)}{p(\sigma)}\right).$$

In state B,

$$\frac{S_t}{P_t} = M_t \left(1 - \frac{v_B}{p_B}\right)$$

In state G, $\frac{S_t}{P_t} = 0$. The fact that $dM_t = 0$ implies that $\frac{S_t}{P_t}$ only changes following uncertainty shocks.

B   Proof of Corollary 1

Recall that by Proposition 2, speculators hold tokens only in state B (if at all). To begin with, suppose there is speculative investment in state B and $v_B < p_B$. Note that under these circumstances, token price in state B satisfies (see (23))

$$p_B = \frac{\lambda_G p_G}{r - \mu + \theta\pi\sigma_B + \lambda_G}. \tag{B.4}$$

Thus, $\frac{p_G}{p_B} = \frac{r - \mu + \theta\pi\sigma_B + \lambda_G}{\lambda_G}$ and, using (22), we obtain:

$$v_B = \left(\frac{1}{r - \mu + \pi\sigma_B - \lambda_G(p_G/p_B - 1)}\right)^{\frac{1}{1-\gamma}} = \left(\frac{1}{\pi(1-\theta)\sigma_B}\right)^{\frac{1}{1-\gamma}}, \tag{B.5}$$

where the second equality follows after inserting (B.4) and simplifying. Using (20), we obtain

$$p_G = v_G = \left(\frac{1}{r - \mu + \pi\sigma_G - \lambda_B(p_B/p_G - 1)}\right)^{\frac{1}{1-\gamma}} = \left(\frac{1}{r + \lambda_B - \mu + \pi\sigma_G - \frac{\lambda_B\lambda_G}{r - \mu + \theta\pi\sigma_B + \lambda_G}}\right)^{\frac{1}{1-\gamma}}, \tag{B.6}$$

where the third equality inserts $\frac{p_B}{p_G} = \frac{\lambda_G}{r - \mu + \theta\pi\sigma_B + \lambda_G}$ (see (B.4)) and simplifies.

Define

$$f_{p_B}(\theta) = \frac{\lambda_G \hat{p}_G}{r - \mu + \theta\pi\sigma_B + \lambda_G} \quad \text{and} \quad f_{v_B}(\theta) = \left(\frac{1}{\pi(1-\theta)\sigma_B}\right)^{\frac{1}{1-\gamma}},$$

where

$$\hat{p}_G = \left(\frac{1}{r + \lambda_B - \mu + \pi\sigma_G - \frac{\lambda_B\lambda_G}{r - \mu + \theta\pi\sigma_B + \lambda_G}}\right)^{\frac{1}{1-\gamma}}.$$

Note that $\hat{p}_G$ decreases in θ, that is, $\frac{\partial \hat{p}_G}{\partial \theta} < 0$. As a result, $f_{p_B}(\theta)$ strictly decreases in θ and $f_{v_B}(\theta)$ strictly increases in θ, with $\lim_{\theta\to 1} f_{v_B}(\theta) = \infty$. Thus, on the interval $(0,1)$, there exists at most one point $\overline{\theta}^*$ such that $f_{p_B}(\overline{\theta}^*) = f_{v_B}(\overline{\theta}^*)$.

If there does not exist $\overline{\theta}^* \in (0,1)$ such that $f_{p_B}(\overline{\theta}^*) = f_{v_B}(\overline{\theta}^*)$, then $f_{p_B}(\theta) \le f_{v_B}(\theta)$ for all $\theta \in [0,1]$. Then, in equilibrium, $p_B = v_B$ and there is no speculative investment for $\theta \in [0,1]$. In this case, we stipulate $\overline{\theta} = 0$ and the set of values $\theta \in [0,1]$ with $\theta < \overline{\theta}$ is empty.

If, on the other hand, there exists $\overline{\theta}^* \in (0,1)$ such that $f_{p_B}(\overline{\theta}^*) = f_{v_B}(\overline{\theta}^*)$, then $f_{p_B}(\theta) > f_{v_B}(\theta)$ for all $\theta \in [0,\overline{\theta}^*)$ and $f_{p_B}(\theta) \leq f_{v_B}(\theta)$ for $\theta \in [\overline{\theta}^*, 1]$. Then, in equilibrium, $p_B > v_B$ and there is speculative investment in state B for $\theta \in [0,\overline{\theta}^*)$. Likewise, $p_B = v_B$ and there is no speculative investment for $\theta \geq \overline{\theta}^*$. In this case, we stipulate $\overline{\theta} = \overline{\theta}^*$. Overall, we have shown that there exists $\overline{\theta} \in [0,1)$ such that there there is speculative investment (in state B) and $p_B > v_B$ if and only if $\theta \in [0,\overline{\theta})$.

In what follows, we prove the remaining claims stated in the Corollary under 1), 2), and 3). The case $\theta \geq \overline{\theta}$ is trivial. We therefore consider $\overline{\theta} > 0$ and $\theta < \overline{\theta}$. Then, scaled token price p_G is given by (B.6) and scaled token price p_B by (B.4); scaled platform transaction volume v_G is given by (B.6) and scaled platform transaction volume v_B is given by (B.5).

1. Expression (B.6) implies that p_G decreases in θ, that is, $\frac{\partial p_G}{\partial \theta} < 0$. In turn, expression (B.4) implies that p_B decreases in θ, that is, $\frac{\partial p_B}{\partial \theta} < 0$. Expression (B.5) implies that v_B increases in θ, that is, $\frac{\partial v_B}{\partial \theta} > 0$. As such, $p_B - v_B$ decreases in θ, that is, $\frac{\partial (p_B - v_B)}{\partial \theta} < 0$.

2. Expression (B.6) implies that v_G decreases in θ, that is, $\frac{\partial v_G}{\partial \theta} < 0$. Likewise, expression (B.5) implies that v_B increases with θ, that is, $\frac{\partial v_B}{\partial \theta} > 0$.

3. Recall that expected token returns in state B are given by (A.1) and expected token returns in state G are given by (A.2). Due to $\frac{p_G}{p_B} = \frac{r - \mu + \theta \pi \sigma_B + \lambda_G}{\lambda_G}$, it follows that expected token returns in state B increase in θ, that is, $\frac{\partial \varepsilon_B}{\partial \theta} > 0$. Due to $\frac{p_B}{p_G} = \frac{\lambda_G}{r - \mu + \theta \pi \sigma_B + \lambda_G}$, it follows that expected token returns in state G decrease in θ, that is, $\frac{\partial \varepsilon_G}{\partial \theta} < 0$.

C User welfare

User welfare at time t reads

$$W_t = \mathbb{E}\left[\int_0^\infty e^{-r(s-t)} dR_{is}^U\right],$$

under users' optimal choice of token holdings, $(u_{it})_{t \geq 0}$. Next, note that

$$\mathbb{E}[dR_{it}^U] = \frac{u_{it}^\alpha}{\alpha} V_t^\beta A_t^{1-\alpha-\beta} dt + u_{it}\left(\frac{\mathbb{E}[dP_t]}{P_t} - rdt - \pi\sigma_t^P dt\right)$$

Inserting $u_{it} = V_t$ and using $\gamma = \alpha + \beta$ and (7) yields

$$
\begin{aligned}
\mathbb{E}[dR_{it}^U] &= \frac{V_t^\gamma A_t^{1-\gamma}}{\alpha} dt + V_t\left(\frac{\mathbb{E}[dP_t]}{P_t} - rdt - \pi\sigma_t^P dt\right) \\
&= \frac{V_t^\gamma A_t^{1-\gamma}}{\alpha} dt + A_t\left(\frac{1}{r + \pi\sigma_t^P - \mathbb{E}[dP_t]/(P_t dt)}\right)^{\frac{1}{1-\gamma}}\left(\frac{\mathbb{E}[dP_t]}{P_t} - rdt - \pi\sigma_t^P dt\right) \quad \text{(C.7)} \\
&= \frac{V_t^\gamma A_t^{1-\gamma}}{\alpha} dt - V_t^\gamma A_t^{1-\gamma} dt = \left(\frac{1-\alpha}{\alpha}\right) V_t^\gamma A_t^{1-\gamma} dt = A_t\left(\frac{1-\alpha}{\alpha}\right) v(\sigma)^\gamma dt,
\end{aligned}
$$

where the last equality uses $V_t = A_t v(\sigma)$. We conjecture and verify that W_t takes the form $W_t = A_t w(\sigma)$, with $w_z = w(\sigma_z)$ for $z = B, G$.

By the dynamic programming principle, user welfare solves at any time $t \geq 0$:

$$rW_t dt = \mathbb{E}[dR_{it}^U + dW_t],$$

where $\mathbb{E}[dR_{it}^U]$ is characterized in (C.7). Thus, using $W_t = A_t w(\sigma)$, expanding and dividing both sides through $A_t > 0$ and dt, we obtain

$$rw(\sigma) = \left(\frac{1-\alpha}{\alpha}\right) v(\sigma)^\gamma + (\mu + a(\sigma)\Delta\mu)w(\sigma) + \frac{\mathbb{E}[dw(\sigma)]}{dt}. \tag{C.8}$$

Define the "flow term"

$$\rho(\sigma) \equiv \left(\frac{1-\alpha}{\alpha}\right) v(\sigma)^\gamma,$$

where $\rho_z = \rho(\sigma_z)$. Expanding (C.8) with respect to jump risk and noting that

$$\frac{\mathbb{E}[dw(\sigma)]}{dt} = \begin{cases} \lambda_G(w_G - w_B) & \text{if} \quad \sigma = \sigma_B \\ \lambda_B(w_B - w_G) & \text{if} \quad \sigma = \sigma_G, \end{cases}$$

we obtain that scaled user welfare in states w_B and w_G solves the system of equations

$$w_B = \frac{\rho_B + \lambda_G w_G}{r + \lambda_G - \mu - a_B \Delta\mu} \tag{C.9}$$

$$w_G = \frac{\rho_G + \lambda_B w_B}{r + \lambda_B - \mu - a_G \Delta\mu}. \tag{C.10}$$

Here, a_z is platform investment in state z.

D   Proof of Proposition 2

Follows from the arguments presented in the main text.

E   Model solution under a fiat-based platform structure

We look for a Markov equilibrium with state variables A_t and σ in which all agents act optimally. In this Markov equilibrium, platform transaction volume V_t — determined by the user optimization discussed below — scales with A_t, in that $V_t = A_t v(\sigma)$, and investment depends on σ only, in that $a_t = a(\sigma)$. The platform chooses transaction fees and investment to dynamically maximize platform equity value Q_t, given in (40). In equilibrium, platform equity value Q_t scales with A_t, in that $Q_t = A_t q(\sigma)$.

To begin with, note that any user solves

$$\max_{u_{it} \geq 0} \mathbb{E}[dR_{it}^U], \tag{E.11}$$

with dR_{it}^U from (39). This leads to the optimal platform transaction volume $V_t = A_t v(\sigma)$ with

$$v(\sigma) = \left(\frac{1}{r + \phi_t}\right)^{\frac{1}{1-\gamma}}. \tag{E.12}$$

The optimal transaction fee ϕ_t is chosen to maximize equity value, given in (40). As can be seen from (40), optimal transaction fee ϕ_t maximizes flow revenues $\phi_t V_t$. Therefore, using (E.12), the

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optimal transaction fee reads

$$\phi_E = \arg\max_\phi \left[\phi \left(\frac{1}{r+\phi} \right)^{\frac{1}{1-\gamma}} \right].$$

Solving this maximization problem yields closed-form expressions for the optimal transaction fee and transaction volume:

$$\phi_E = \frac{(1-\gamma)r}{\gamma} \quad \text{and} \quad v_E = \left(\frac{\gamma}{r} \right)^{\frac{1}{1-\gamma}}. \tag{E.13}$$

Note that the optimal transaction fee does not depend on the state σ, so — by (E.12) — transaction volume $v(\sigma)$ does not depend on σ either. Intuitively, as the platform transaction medium is price-stable, volatility σ does not directly affect users' decisions to transact on the platform, so transaction fee ϕ_E and transaction volume v_E do not depend on σ. This also implies that under a fiat-based structure, users are not exposed to any platform-specific risk which is entirely absorbed by speculators as the platform's equity holders.

Inserting $\phi_t = \phi_E$ into (E.12), we obtain the (scaled) revenue from transaction fees:

$$\Phi \equiv \phi_E v_E = (1-\gamma) \left(\frac{\gamma}{r} \right)^{\frac{\gamma}{1-\gamma}}. \tag{E.14}$$

Note that scaled fee revenues decrease with the discount rate r.

Finally, we can calculate platform (equity) value Q_t, given by (40), that is,

$$Q_t = \max_{(a_t)_{t\geq 0}, (\phi_t)_{t\geq 0}} \mathbb{E}_t \left[\int_t^\infty e^{-r(s-t)} \left(\phi_s V_s - A_s \frac{\kappa a_s^2}{2} - \theta \pi Q_s |\sigma_s^Q| \right) ds \right],$$

where $\phi_s V_s = A_s \Phi$ under optimal fee setting. We conjecture (and verify) that $Q_t = Q(A_t, \sigma)$ scales with A_t and satisfies $Q(A, \sigma) = A q(\sigma)$, where we denote $q_z = q(\sigma_z)$. We also show that $\sigma_t^Q \geq 0$.

By the dynamic programming principle, platform equity value in state ϕ solves

$$rQ(A, \sigma)dt = \max_{a \in [0, \bar{a}]} \left\{ \left(A\Phi - \frac{A\kappa a^2}{2} - \theta \pi |\sigma^Q| Q(A, \sigma) \right) dt + \mathbb{E}[dQ(A, \sigma)] \right\}. \tag{E.15}$$

Using the conjecture $Q(A, \sigma) = A q(\sigma)$ and invoking Ito's Lemma, we can calculate

$$\frac{\mathbb{E}[dQ(A, \sigma)]}{dt} = \begin{cases} A(\mu + a_G \Delta\mu)q_G + \lambda_B A(q_B - q_G) & \text{if } \sigma = \sigma_G \\ A(\mu + a_B \Delta\mu)q_B + \lambda_G A(q_G - q_B) & \text{if } \sigma = \sigma_B. \end{cases}$$

Likewise, Ito's Lemma yields

$$\sigma_t^Q = \sigma^Q(A_t, \sigma) = \left(\frac{\partial Q_t}{\partial A_t} \right) \frac{A_t \sigma_A(\sigma)}{Q_t} = \sigma_A(\sigma) = \sigma + a(\sigma)\Delta\sigma > 0,$$

where $\sigma_A(\sigma)$ is the volatility of dA_t/A_t. Inserting these relations and $Q(A, \sigma) = A q(\sigma)$ into (E.15), dividing both sides through A, and simplifying (in both states B and G), we obtain that scaled

(state-dependent) equity value solves the system of equations:

$$q_B = \max_{a_B \in [0,\bar{a}]} \left(\frac{\Phi + \lambda_G q_G - \kappa a_B^2/2}{r - \mu - a_B \Delta\mu + \theta\pi(\sigma_B + a_B\Delta\sigma) + \lambda_G} \right)$$

$$q_G = \max_{a_G \in [0,\bar{a}]} \left(\frac{\Phi + \lambda_B q_B - \kappa a_G^2/2}{r - \mu - a_G \Delta\mu + \theta\pi(\sigma_G + a_G\Delta\sigma) + \lambda_B} \right). \tag{E.16}$$

The optimization in (E.16) implies that optimal investment a_B and a_G solve

$$\max_{a_z \in [0,\bar{a}]} \left(a_z q_z (\Delta\mu - \theta\pi\Delta\sigma) - \frac{\kappa a_z^2}{2} \right) \tag{E.17}$$

for $z = B, G$. As such, optimal investment a_t depends on σ only, i.e., $a_t = a(\sigma)$. We summarize our results in the following Proposition.

Proposition 5 (Fiat-based platform). *Suppose that fiat money serves as the platform transaction medium and that the platform charges a proportional transaction fee ϕ_t (modelled as a fee on token holdings). In the Markov equilibrium, platform transaction volume is $V_t = A_t v(\sigma)$ where $v(\sigma)$ is characterized in (E.12). The platform chooses investment $(a_t)_{t\geq 0}$ and transaction fees $(\phi_t)_{t\geq 0}$ to maximize its (equity) value in (40). Optimal transaction fee is given in (E.13). Platform (equity) value Q_t scales with A_t, in that $Q_t = A_t q(\sigma)$ where $q_B = q(\sigma_B)$ and $q_G = q(\sigma_G)$ are characterized in (E.16). Optimal investment levels a_B and a_G solve (E.17) for $z = B, G$.*

E.1   Proof of Corollary 4

Under a token-based structure, scaled platform transaction volume v_B and v_G are given by (34) and (32) respectively. Taking the "double" limit $\sigma_B \to \infty, \sigma_G \to \infty$ (with $\sigma_G > \sigma_B$) in (34) and (32) yields

$$\lim_{\sigma_B \to \infty, \sigma_G \to \infty} v_B = \lim_{\sigma_B \to \infty, \sigma_G \to \infty} v_G = 0,$$

which implies (see (35) and (33))

$$\lim_{\sigma_B \to \infty, \sigma_G \to \infty} p_B = \lim_{\sigma_B \to \infty, \sigma_G \to \infty} p_G = 0$$

and (see (C.9))

$$\lim_{\sigma_B \to \infty, \sigma_G \to \infty} w_B = \lim_{\sigma_B \to \infty, \sigma_G \to \infty} w_G = 0.$$

Under a fiat-based structure, when $\theta = 0$, then q_B, q_G, Φ, and $v(\sigma)$ do not depend on σ_G and σ_B. As such, in the limit $\sigma_B \to \infty, \sigma_G \to \infty$, a fiat-based platform yields higher (scaled) platform value (that is, $q_G > p_G$ and $q_B > p_B$), higher platform transaction volume, and higher user welfare and total surplus. By continuity, a fiat-based platform yields higher (scaled) platform value, user welfare, total surplus, and long-run adoption $\bar{v}$ when σ_B and σ_G are sufficiently large and $\theta \geq 0$ is sufficiently small.

F   Details for Section 5.1.2 and Proof of Proposition 3

In this Section, we provide the model solution under the specification of Section 5.1.2 for both a token-based and fiat-based platform structure. The equilibrium concept is analogous to the one of

the baseline and Section 4 (i.e., Appendix E) respectively. In equilibrium, the optimal choice of b_t depends on σ only, i.e., $b_t = b(\sigma)$ where we write $b_z = b(\sigma_z)$.

F.1   Token-based platform

We can derive the model solution and equilibrium performing steps analogous to those in Section 2.4. Note that users' utility from holding tokens is characterized (41) and platform productivity evolves according (42). Any user i solves $\max_{u_{it} \geq 0} \mathbb{E}[dR_{it}^U]$ with dR_{it}^U from (41). Under user homogeneity, all users act the same, so that $u_{it} = V_t$. Going through the same steps as in the derivation of (7), we solve the platform transaction volume

$$V_t = A_t \left(\frac{1 - b_t}{r + \pi \sigma_t^P - \mathbb{E}[dP_t]/(P_t dt)} \right)^{\frac{1}{1-\gamma}}. \tag{F.18}$$

Platform transaction volume scales with A_t, in that $V_t = A_t v(\sigma)$. And, token price scales with $X_t = A_t/M_t$, in that $P_t = X_t p(\sigma)$. Following the steps as in Section 4, one can use Ito's Lemma to calculate

$$\frac{dX_t}{X_t} = \mu_X(\sigma)dt + \sigma_X(\sigma)dZ_t, \tag{F.19}$$

with

$$\mu_X(\sigma) = \mu + b(\sigma)\varepsilon + a(\sigma)\Delta\mu - \frac{\kappa a(\sigma)^2}{p(\sigma)} \quad \text{and} \quad \sigma_X(\sigma) = \sigma + a(\sigma)\Delta\sigma.$$

Using (16) and (17) (which are also valid in this extension), we obtain $\mu_t^P = \mu_X(\sigma)$ and $\sigma_t^P = \sigma_X(\sigma)$.

Using (F.18) and (19) (which is also valid in this extension), we obtain scaled transaction volume in state v_G,

$$v_G = \left(\frac{1 - b_G}{r - \mu - b_G\varepsilon - a_G\Delta\mu + \pi(\sigma_G + a_G\Delta\sigma) + \kappa a_G^2/(2p_G) - \lambda_B(p_B/p_G - 1)} \right)^{\frac{1}{1-\gamma}}, \tag{F.20}$$

and scaled token price in state G,

$$p_G = \max \left\{ v_G, \frac{\lambda_B p_B}{r - \mu - b_G\varepsilon - a_G\Delta\mu + \theta\pi(\sigma_G + a_G\Delta\sigma) + \kappa a_G^2/(2p_G) + \lambda_B} \right\}. \tag{F.21}$$

Note that (F.20) is analogous to (32) (where the factor $1 - b_G$ enters multiplicatively and the growth rate of platform productivity is augmented by $b_G\varepsilon$). Likewise, (F.21) is analogous to (33) (where the growth rate of platform productivity A_t is augmented by $b_G\varepsilon$). In state B, scaled transaction volume reads

$$v_B = \left(\frac{1 - b_B}{r - \mu - b_B\varepsilon - a_B\Delta\mu + \pi(\sigma_B + a_B\Delta\sigma) + \kappa a_B^2/(2p_B) - \lambda_G(p_G/p_B - 1)} \right)^{\frac{1}{1-\gamma}} \tag{F.22}$$

which is analogous to (34) (where the factor $1 - b_B$ enters multiplicatively and the growth rate of platform productivity is augmented by $b_B\varepsilon$). Scaled token price in state B is

$$p_B = \max \left\{ v_B, \frac{\lambda_G p_G}{r - \mu - b_B\varepsilon - a_B\Delta\mu + \theta\pi(\sigma_B + a_B\Delta\sigma) + \kappa a_B^2/(2p_B) + \lambda_G} \right\}, \tag{F.23}$$

which is analogous to (35) (where the growth rate of platform productivity is augmented by $b_B\varepsilon$).

Next, we discuss the platform's choice of $b_z \in \{0, 1\}$ in both states $z = B$ and $z = G$ in the limit $\varepsilon \downarrow 0$. This is akin to considering $\varepsilon = 0$ and assuming that the platform chooses $b_z = 1$ whenever it is indifferent between $b_z = 0$ and $b_z = 1$. It is clear that choosing $b_B = b_G = 1$ is not optimal, as it would imply $v_B = v_G = p_B = p_G = 0$. As such, when $b_B = 1$ ($b_G = 1$), then $b_G = 0$ ($b_B = 0$), leading to $p_B, p_G > 0$.

We show that the platform chooses $b_z = 1$ if and only if the marginal token investor is speculator. If the platform chooses $b_z = 1$, then platform transaction volume becomes zero (i.e., $V_t = v_z = 0$) and $p_z > v_z$ so that the marginal token investor is a speculator. If the marginal token investor is a speculator, then $p_z > v_z$ and — by expressions (F.21) and (F.23) — it follows that $\frac{\partial p_z}{\partial b_z} > 0$ where $\frac{\partial p_z}{\partial b_z} = o(\varepsilon)$ is "negligible." As such, the platform chooses $b_z = 1$.

Next, observe that when $\theta = 0$, expressions (F.22) and (F.23) imply

$$\lim_{\sigma_B \to \infty} v_B = 0 < \lim_{\sigma_B \to \infty} p_B.$$

By continuity, when σ_B is sufficiently large and $\theta \in [0, 1]$ is sufficiently small, the marginal token investor in state B is a speculator and $p_B > v_B$. By our previous results, $b_B = 1$ and $b_G = 0$.

F.2 Fiat-based platform

Under this alternative specification, users' payoff from holding tokens is

$$dR_{it}^U := \underbrace{\frac{u_{it}^\alpha}{\alpha}(1 - b_t)V_t^\beta A_t^{1-\alpha-\beta} dt}_{\text{Transaction utility}} + u_{it} \Big(\underbrace{\phi_t dt}_{\substack{\text{Transaction} \\ \text{fee}}} - \underbrace{r dt}_{\substack{\text{Opportunity} \\ \text{cost}}} \Big). \tag{F.24}$$

Any user solves $\max_{u_{it} \geq 0} \mathbb{E}[dR_{it}^U]$, with dR_{it}^U from (F.24). We go through the optimization and can perform the same steps as in Section 5.1 and Appendix E, which yields $V_t = A_t v(\sigma)$ where

$$v(\sigma) = \left(\frac{1 - b(\sigma)}{r + \phi(\sigma)} \right)^{\frac{1}{1-\gamma}}.$$

Optimal transaction fees ϕ_E do not depend on the state σ and are chosen to maximize fee revenues

$$\phi \left(\frac{1 - b_t}{r + \phi(\sigma)} \right)^{\frac{1}{1-\gamma}} = (1 - b(\sigma))\phi \left(\frac{1}{r + \phi(\sigma)} \right)^{\frac{1}{1-\gamma}}$$

It follows that optimal transaction fees are characterized in (E.13) and that the platform's fee revenues are characterized in (E.14).

Performing analogous steps as in Appendix E, we derive that scaled platform equity solves the system of equations:

$$q_B = \max_{a_B \in [0, \bar{a}], b_B \in \{0,1\}} \left(\frac{\Phi(1 - b_B) + \lambda_G q_G - \kappa a_B^2/2}{r - \mu - b_B \varepsilon - a_B \Delta\mu + \theta\pi(\sigma_B + a_B \Delta\sigma) + \lambda_G} \right) \tag{F.25}$$

$$q_G = \max_{a_G \in [0, \bar{a}], b_G \in \{0,1\}} \left(\frac{\Phi(1 - b_G) + \lambda_B q_B - \kappa a_G^2/2}{r - \mu - b_G \varepsilon - a_G \Delta\mu + \theta\pi(\sigma_G + a_G \Delta\sigma) + \lambda_B} \right). \tag{F.26}$$

As $\Phi > 0$, it follows from the optimization in (F.25) that for $\varepsilon \to 0$, setting $b_B = b_G = 0$ is optimal.

G Model solution under a dual token structure

We look for a Markov equilibrium with state variables A_t and σ in which all agents act optimally and the markets for tokens clear. We normalize the price of transaction tokens to one dollar. Transaction token market clearing requires that platform transaction volume in dollars, V_t, is equal to the market capitalization of transaction tokens. It follows that V_t is both the supply and market capitalization of transaction tokens.

In the Markov equilibrium, platform transaction volume V_t — determined by the user optimization discussed below — scales with A_t, in that $V_t = A_t v(\sigma)$ with $v_z = v(\sigma_z)$, and investment depends on σ only, i.e., $a_t = a(\sigma)$ with $a_z = a(\sigma_z)$. The platform chooses transaction fees and investment to dynamically maximize platform dollar value at time zero, which is $Q_0 + V_0$. We impose that platform equity Q_t must exceed a positive lower bound, in that

$$Q_t \geq A_t \underline{q} \tag{G.27}$$

with a constant $\underline{q} \geq 0$. The constraint (G.27) can be interpreted as limited liability or limited commitment constraint of equity holders (who are speculators/financial investors) and requires platform equity value to remain positive. For instance, a negative platform equity value would violate limited liability and preclude that platform equity (governance tokens) are tradeable under limited liabilities. In equilibrium, platform equity value Q_t scales with A_t, i.e., $Q_t = A_t q(\sigma)$.

To begin with, note that any user solves

$$\max_{u_{it} \geq 0} \mathbb{E}[dR_{it}^U], \tag{G.28}$$

with dR_{it}^U from (39). This leads to the platform transaction volume $V_t = A_t v(\sigma)$ with

$$v(\sigma) = \left(\frac{1}{r + \phi(\sigma)} \right)^{\frac{1}{1-\gamma}}, \tag{G.29}$$

which is identical to (E.12). Here, $\phi(\sigma)$ denotes the (possibly) state-dependent transaction fee.

Note that due to $V_t = A_t v(\sigma)$,

$$dV_t = \begin{cases} v_B dA_t + \lambda_G A_t (v_G - v_B) dt & \text{if} \quad \sigma = \sigma_B \\ v_G dA_t + \lambda_B A_t (v_B - v_G) dt & \text{if} \quad \sigma = \sigma_G, \end{cases}$$

where dA_t is characterized in (26). We can now rewrite (43) as

$$\frac{dDiv_t}{A_t} = \begin{cases} v_B \Big((\mu + a_B \Delta\mu + \phi_B) dt + (\sigma + a_B \Delta\sigma) dZ_t \Big) - \frac{\kappa a_B^2}{2} dt + \lambda_G (v_G - v_B) dt, & \text{if} \quad \sigma = \sigma_B \\ v_G \Big((\mu + a_G \Delta\mu + \phi_G) dt + (\sigma + a_G \Delta\sigma) dZ_t \Big) - \frac{\kappa a_G^2}{2} dt + \lambda_B (v_B - v_G) dt, & \text{if} \quad \sigma = \sigma_G, \end{cases} \tag{G.30}$$

with state-dependent investment and fees $(a_B, a_G, \phi_B, \phi_G)$. Thus, the volatility of $dDiv_t$ reads

$$\sigma_t^{Div} = A_t v(\sigma) \big(\sigma + a(\sigma)\Delta\sigma \big) > 0.$$

Recall that platform equity value reads (see (44))

$$Q_t = \mathbb{E}_t \left[\int_t^\infty e^{-r(s-t)} \left(dDiv_s - \theta\pi |Q_s \sigma_s^Q + \sigma_s^{Div}| ds \right) \right],$$

where dividends $dDiv$ are characterized in (43) and (G.30). We conjecture and verify that $Q(A_t, \sigma)$ scales with A_t, in that $Q(A_t, \sigma) = A_t q(\sigma)$ with $q_B = q(\sigma_B)$ and $q_G = q(\sigma_G)$. We also show that $\sigma_t^Q \geq 0$ and $\sigma_t^{Div} \geq 0$ at all times t. The platform chooses state-dependent transaction fees ϕ_B and ϕ_G as well as state-dependent investment a_B and a_G to dynamically maximize platform value $V_t + Q_t$. As both $V_t = A_t v_z$ and $Q_t = A_t q_z$ scale with A_t, it follows that the platform chooses a_z and ϕ_z to maximize scaled platform value $q_z + v_z$ in state $z = B, G$.

The dynamic programming principle implies that equity value solves — evaluated under the optimal controls — the ODE

$$rQ(A, \sigma)dt = \mathbb{E}\left[dDiv - \theta\pi(Q(A, \sigma)\sigma^Q + \sigma^{Div})dt + dQ(A, \sigma)\right]. \tag{G.31}$$

Using the conjecture $Q(A, \sigma) = Aq(\sigma)$ and invoking Ito's Lemma, we can calculate

$$\frac{\mathbb{E}[dQ(A, \sigma)]}{dt} = \begin{cases} A(\mu + a_G\Delta\mu)q_G + \lambda_B A(q_B - q_G) & \text{if } \sigma = \sigma_G \\ A(\mu + a_B\Delta\mu)q_B + \lambda_G A(q_G - q_B) & \text{if } \sigma = \sigma_B. \end{cases}$$

Likewise, Ito's Lemma yields

$$\sigma_t^Q = \sigma^Q(A_t, \sigma) = \left(\frac{\partial Q_t}{\partial A_t}\right)\frac{A_t\sigma_A(\sigma)}{Q_t} = \sigma_A(\sigma) = \sigma + a(\sigma)\Delta\sigma > 0,$$

where $\sigma_A(\sigma)$ is the volatility of dA_t/A_t. Inserting these relations, (G.30), and $Q(A, \sigma) = Aq(\sigma)$ into (E.15), dividing both sides through A, and simplifying (in both states B and G), we obtain that scaled (state-dependent) equity value solves the system of equations:

$$q_B = \frac{[\phi_B + \mu + a_B\Delta\mu - \theta\pi(\sigma_B + a_B\Delta\sigma)]v_B - \kappa a_B^2/2 + \lambda_G(q_G + v_G - v_B)}{r - \mu - a_B\Delta\mu + \theta\pi(\sigma_B + a_B\Delta\sigma) + \lambda_G} \tag{G.32}$$

$$q_G = \frac{[\phi_G + \mu + a_G\Delta\mu - \theta\pi(\sigma_G + a_G\Delta\sigma)]v_G - \kappa a_G^2/2 + \lambda_B(q_B + v_B - v_G)}{r - \mu - a_G\Delta\mu + \theta\pi(\sigma_G + a_G\Delta\sigma) + \lambda_B},$$

evaluated under the optimal controls $(a_B, a_G, \phi_B, \phi_G)$ which we determine below.

As a next step, we calculate

$$q_B + v_B = \frac{(\phi_B + r)v_B - \kappa a_B^2/2 + \lambda_G(q_G + v_G)}{r - \mu - a_B\Delta\mu + \theta\pi(\sigma_B + a_B\Delta\sigma) + \lambda_G}$$

$$q_G + v_G = \frac{(\phi_G + r)v_G - \kappa a_G^2/2 + \lambda_B(q_B + v_B)}{r - \mu - a_G\Delta\mu + \theta\pi(\sigma_G + a_G\Delta\sigma) + \lambda_B}. \tag{G.33}$$

To maximize $q_z + v_z$, optimal transaction fee ϕ_z in state $z = B, G$ maximizes

$$(r + \phi_z)v_z = (r + \phi_z)\left(\frac{1}{r + \phi_z}\right)^{\frac{1}{1-\gamma}} = \left(\frac{1}{r + \phi_z}\right)^{\frac{\gamma}{1-\gamma}},$$

which — absent any constraints — would imply the degenerate outcome $\phi_z \downarrow -r$. And, $\phi_z \downarrow -r$ implies $v_z \to \infty$ as well as $q_z \to -\infty$ which violates (G.27). As a result, the constraint (G.27) binds in equilibrium for both states $z = B, G$, so that $q_z = \underline{q}$ for $z = B, G$.

Now, we insert $q_z = \underline{q}$ back into (G.33) (or (G.32)) and solve for

$$\phi_B = (r - \mu - a_B \Delta\mu + \theta\pi(\sigma_B + a_B \Delta\sigma))\frac{\underline{q}}{v_B} + \frac{\kappa a_B^2}{v_B} \tag{G.34}$$

$$- \lambda_G \left(\frac{v_G}{v_B} - 1\right) - \mu - a_B \Delta\mu + \theta\pi(\sigma_B + a_B \Delta\sigma)$$

$$\phi_G = (r - \mu - a_G \Delta\mu + \theta\pi(\sigma_G + a_G \Delta\sigma))\frac{\underline{q}}{v_G} + \frac{\kappa a_G^2}{v_G}$$

$$- \lambda_B \left(\frac{v_B}{v_G} - 1\right) - \mu - a_G \Delta\mu + \theta\pi(\sigma_G + a_G \Delta\sigma).$$

Inserting (G.34) into (G.29), we obtain

$$v_B = \left(\frac{1}{[r - \mu - a_B \Delta\mu + \theta\pi(\sigma_B + a_B \Delta\sigma)](1 + \underline{q}/v_B) + \kappa a_B^2/(2v_B) - \lambda_G(v_G/v_B - 1)}\right)^{\frac{1}{1-\gamma}}$$

(G.35)

$$v_G = \left(\frac{1}{[r - \mu - a_G \Delta\mu + \theta\pi(\sigma_G + a_G \Delta\sigma)](1 + \underline{q}/v_G) + \kappa a_G^2/(2v_G) - \lambda_B(v_B/v_G - 1)}\right)^{\frac{1}{1-\gamma}}.$$

To complete the description of the Markov equilibrium under the dual token structure, we analyze platform investment. Platform investment in state z is chosen to maximize

$$\max_{a_z \geq 0} \left[q_z + v_z\right] \tag{G.36}$$

where $q_z = \underline{q}$. According to the objective in (G.36), optimal investment maximizes the joint (scaled) value of transaction tokens (v_z) and governance tokens (q_z).

Notice that maximizing $v_z + p_z$, the platform would never choose $b_z = 1$ under the specification of Section 5.1.2 with $\varepsilon \to 0$, as setting b_z causes v_z to drop to zero and has only negligible impact on platform productivity growth. Intuitively, the platform transaction fees and the commitment to price stability of transaction tokens align the platform's incentives with those of its users. If the platform chooses $b_z = 1$, fee revenues and demand for transaction tokens fall, which would require the platform to buy back transaction tokens to stabilize the transaction token price. Both the drop in fee revenues and the buybacks reduce platform revenues upon setting $b_z = 1$, so the platform directly internalizes its effects on platform users (and so the negative effects of setting $b_z = 1$). As such, we conclude that a dual token structure aligns the platform's incentives with those of its users, thereby alleviating conflicts of interest between the platform (owners) and its users.

Finally, we summarize the findings of this Section in the following Proposition.

Proposition 6 (Dual Token Structure)**.** *Under the dual token structure, there exists a Markov equilibrium with the following properties. The platform issues i) transaction tokens (stablecoins) with stable dollar price (normalized to one dollar) held by users for transactions and ii) governance (equity) tokens held by speculators for returns. Transaction volume V_t (in dollars) equals both transaction token supply and market capitalization. Platform transaction volume satisfies $V_t = A_t v(\sigma)$ where $v_G = v(\sigma_G)$ and $v_B = v(\sigma_B)$ are characterized in (G.35). The governance token price/platform equity value satisfies $Q_t = A_t q(\sigma)$, where $q_B = q(\sigma_B)$ and $q_G = q(\sigma_G)$ are characterized in (G.32). Optimal transaction fees only depend on σ (i.e., $\phi_t = \phi_z = \phi(\sigma_z)$) and are*

given in (G.34). *Scaled governance token price/platform equity value satisfies $q_z = \underline{q}$ under optimal fee setting. Optimal investment only depends on σ, and is characterized in (G.36).*

G.1 Dual token structure and perfect risk-sharing

In the following, we discuss the special case $\underline{q} = 0$. Under these circumstances, (G.35) simplifies to

$$v_B^* = \left(\frac{1}{r - \mu - a_B \Delta\mu + \theta\pi(\sigma_B + a_B\Delta\sigma) + \kappa a_B^2/(2v_B) - \lambda_G(v_G^*/v_B^* - 1)} \right)^{\frac{1}{1-\gamma}} \qquad \text{(G.37)}$$

$$v_G^* = \left(\frac{1}{r - \mu - a_G \Delta\mu + \theta\pi(\sigma_G + a_G\Delta\sigma) + \kappa a_G^2/(2v_G) - \lambda_B(v_B^*/v_G^* - 1)} \right)^{\frac{1}{1-\gamma}}.$$

Note that the expressions for (scaled) platform transaction volume attained under the dual token structure with $\underline{q} = 0$, v_B^* and v_G^*, resemble the respective expressions for (scaled) platform transaction volume attained under the baseline token-based platform structure (see expressions (22) and (20) respectively) with two important differences.

First, p_z is replaced by v_z. Second, users' risk-aversion coefficient (that is, $\pi\sigma_B$ in (22) and $\pi\sigma_G$ in (20)) is replaced by speculators' risk aversion coefficient, $\theta\pi\sigma_B$ and $\theta\pi\sigma_G$ respectively. In other words, the dual token structure achieves perfect risk-sharing between users and speculators. Perfect risk-sharing also removes the under-investment problem that is induced by users' transaction-based risk-aversion. In fact, when $\underline{q} = 0$, optimal investment satisfies

$$a_z = \arg\max_{a\in[0,\bar{a}]} \left(a p_z(\Delta\mu - \theta\pi\Delta\sigma) - \frac{\kappa a^2}{2} \right), \qquad \text{(G.38)}$$

which is (36) with $\theta_z(a_z)$ replaced by $\theta \leq \theta_z(a_z)$ (notice that investment decreases with $\theta_z(a_z)$).

H   Solution with short selling

In what follows, we characterize a Markov equilibrium with state variables A_t, M_t, and σ. In this Markov equilibrium, users solve the same problem as in the baseline, so that platform transaction volume is characterized in (7), and speculators' problem with short-selling possibilities is characterized below. Users optimally do not short sell tokens, as the marginal convenience yield in (3) becomes infinite as u_{it} approaches zero. In the Markov equilibrium, platform transaction volume scales with A_t so that $V_t = A_t v(\sigma)$, with $v_B = v(\sigma_B)$ and $v_G = v(\sigma_G)$. Token price P_t scales with $X_t = A_t/M_t$ so that $P_t = X_t p(\sigma)$, with $p_B = p(\sigma_B)$ and $p_G = p(\sigma_G)$. Under homogeneity, speculators' aggregate and individual token holdings are the same (i.e., $s_{jt} = S_t$) and scale with A_t so that $S_t = A_t s(\sigma)$, with $s_B = s(\sigma_B)$ and $s_G = s(\sigma_G)$. Analogous to the baseline, token price volatility satisfies $\sigma_t^P = \sigma \geq 0$ and token price drift satisfies $\mu_t^P = \mu$.

H.1 Speculators' optimization

In this Section, we characterize the speculators' optimization with short selling. Any speculator j faces the short selling constraint $s_{jt} \geq \underline{S}_t$ with $\underline{S}_t = -\eta M_t P_t$ and $\eta > 0$. Under this specification, any speculator j solves

$$\max_{s_{jt} \geq \underline{S}_t} \left[s_{jt} \left(\frac{\mathbb{E}[dP_t]}{P_t} - rdt \right) - \theta\pi |s_{jt}\sigma_t^P| dt \right]. \qquad \text{(H.39)}$$

The speculators' problem in (H.39) is a linear program, and recall $\sigma_t^P = \sigma \geq 0$.

Note that speculator j chooses $s_{jt} > 0$ only if

$$\frac{\mathbb{E}[dP_t]}{P_t dt} \geq r + \theta\pi\sigma_t^P.$$

Likewise, speculator j chooses $s_{jt} < 0$ only if

$$\frac{\mathbb{E}[dP_t]}{P_t dt} \leq r - \theta\pi\sigma_t^P,$$

where $s_{jt} = \underline{S}$ if the above inequality is strict. And, when

$$r - \theta\pi\sigma_t^P < \frac{\mathbb{E}[dP_t]}{P_t dt} < r + \theta\pi\sigma_t^P,$$

then $s_{jt} = 0$.

H.2   Solving for the equilibrium

In what follows, we solve for the state-dependent scaled transaction volume v_B and v_G as well as the state-dependent scaled token prices p_B and p_G. We focus on an equilibrium in which speculators only short sell tokens in state G (if at all). As in the baseline, speculators do not take a token long position in state G. In state B, speculators either buy tokens (i.e., take a long position) or hold zero tokens, as in the baseline. Speculators indeed do not have incentives to short sell tokens in state B if expected token returns in state B satisfy $\frac{\mathbb{E}[dP_t]}{P_t dt} > r - \theta\pi\sigma_t^P$ which is equivalent to

$$\lambda_G\left(\frac{p_G}{p_B} - 1\right) + \mu > r - \theta\pi\sigma_B. \tag{H.40}$$

That is, we assume that parameters are such that (H.40) holds in equilibrium. Note that condition (H.40) can be checked and verified ex-post (i.e., after solving for the candidate equilibrium).

As such, scaled transaction volume and token price in state B are characterized by (22) and (23). That is, in state B, scaled transaction volume is

$$v_B = \left(\frac{1}{r - \mu + \pi\sigma_B - \lambda_G(p_G/p_B - 1)}\right)^{\frac{1}{1-\gamma}} \tag{H.41}$$

and scaled token price is

$$p_B = \max\left\{v_B, \frac{\lambda_G p_G}{r - \mu + \theta\pi\sigma_B + \lambda_G}\right\}. \tag{H.42}$$

If expected token returns in state G satisfy $\frac{\mathbb{E}[dP_t]}{P_t dt} > r - \theta\pi\sigma_t^P$ which is equivalent to

$$\lambda_B\left(\frac{p_B}{p_G} - 1\right) + \mu > r - \theta\pi\sigma_G, \tag{H.43}$$

then there is no short selling in state G so that $S_t = 0$. In this case, the model solution and equilibrium is the one from the baseline.

In what follows, we focus on the case with short selling in state G and conjecture (and verify) that speculators' aggregate token holdings scale with A_t. As such, speculators' aggregate token

holdings in state G can be written as $S_t = s_G A_t \leq 0$ with $s_G \leq 0$. Define

$$\eta_G = \frac{-S_t}{A_t p_G} = \frac{-s_G}{p_G},$$

so $s_G = -p_G \eta_G$. Recall $P_t = X_t p(\sigma) = \frac{A_t p(\sigma)}{M_t}$. Then, note that $S_t \geq \underline{S}_t = -\eta M_t P_t = -\eta A_t p_G$ (in state G) readily implies

$$\eta_G \leq \eta.$$

Token market clearing implies

$$M_t P_t = S_t + V_t$$

so that (in state G):

$$p_G = s_G + v_G = -p_G \eta_G + v_G.$$

Thus, $v_G = p_G(1 + \eta_G)$ which is equivalent to

$$p_G = \frac{v_G}{1 + \eta_G}. \tag{H.44}$$

Similar to the baseline, scaled platform transaction volume in state G satisfies

$$v_G = \left(\frac{1}{r - \mu + \pi \sigma_G - \lambda_B (p_B/p_G - 1)} \right)^{\frac{1}{1-\gamma}}. \tag{H.45}$$

When there is short selling in state G and $\eta_G > 0$, expected token returns in state G satisfy

$$\lambda_B \left(\frac{p_B}{p_G} - 1 \right) + \mu \leq r - \theta \pi \sigma_G. \tag{H.46}$$

Note that $\eta_G \in (0, \eta)$ only if (H.46) holds in equality in which case speculators are indifferent over the amount of tokens they short sell. Otherwise, when (H.46) is strict, then $\eta_G = \eta$.

In summary, the following three cases, 1) $\eta_G = 0$, 2) $\eta_G \in (0, \eta)$ and 3) $\eta_G = \eta$, can prevail in an equilibrium with short selling possibilities:

1. When $\eta_G = 0$, (H.43) generally holds (except in the knife-edge case that speculators are indifferent across levels η_G and $\eta_G = 0$). Then, $\eta_G = 0$ and the four equilibrium quantities v_B, v_G, p_B, and p_G solve the four equations (H.41), (H.42), (H.44), and (H.45). The equilibrium is identical to the one of the baseline.

2. When $\eta_G \in (0, \eta)$, speculators are indifferent between short selling tokens ($s_G < 0$) and not short selling tokens ($s_G = 0$), which is the case if and only if (H.46) holds in equality. Under these circumstances, the five equilibrium quantities v_B, v_G, p_B, p_G, and η_G solve the five equations (H.41), (H.42), (H.44), (H.45), and (H.46) which holds in equality.

3. When $\eta_G = \eta$, (H.46) generally holds as strict inequality (except in the knife-edge case that speculators are indifferent across levels of η_G and $\eta_G = \eta$). Then, $\eta_G = \eta$ and the four equilibrium quantities v_B, v_G, p_B, and p_G solve the four equations (H.41), (H.42), (H.44), and (H.45).

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H.3 Proof of Corollary 2

We consider two cases, 1) there is speculative investment in state B and $p_B > v_B$ and 2) there is no speculative investment in state B and $p_B = v_B$.

Case 1. We start by analyzing case 1), so that $p_B > v_B$ and speculators hold tokens in state B. Note that under these circumstances, token price in state B satisfies (see (H.41) and (H.42))

$$p_B = \frac{\lambda_G p_G}{r - \mu + \theta \pi \sigma_B + \lambda_G}. \tag{H.47}$$

Thus, $\frac{p_G}{p_B} = \frac{r - \mu + \theta \pi \sigma_B + \lambda_G}{\lambda_G}$ and, using (H.41), we obtain:

$$v_B = \left(\frac{1}{r - \mu + \pi \sigma_B - \lambda_G(p_G/p_B - 1)} \right)^{\frac{1}{1-\gamma}} = \left(\frac{1}{\pi(1-\theta)\sigma_B} \right)^{\frac{1}{1-\gamma}}. \tag{H.48}$$

As the above expression for v_B in (H.48) does not depend on η, it follows that $\frac{\partial v_B}{\partial \eta} = 0$.

Note that because speculators hold tokens in state B, the equilibrium pricing relationship (9) holds in equality in state B, which implies

$$\mu + \lambda_G \left(\frac{p_G}{p_B} - 1 \right) = r + \theta \pi \sigma_B. \tag{H.49}$$

The right-hand side of (H.49) does not depend on η and so does the left-hand side. This implies $\frac{\partial}{\partial \eta} \frac{p_G}{p_B} = 0$, which is equivalent to $\frac{\partial}{\partial \eta} \frac{p_B}{p_G} = 0$.

Therefore, by (H.45), v_G does not depend on η so that $\frac{\partial v_G}{\partial \eta} = 0$. Token market clearing in state G implies $p_G = v_G/(1 + \eta)$, so that p_G decreases with η (i.e., $\frac{\partial p_G}{\partial \eta} < 0$). By (H.47), it follows that p_B decreases with η too (i.e., $\frac{\partial p_B}{\partial \eta} < 0$).

Case 2. Next, we consider that speculators do not hold tokens in state B, that is, $p_B = v_B$. Suppose now to the contrary that v_B increases with η (i.e., $\frac{\partial v_B}{\partial \eta} \geq 0$ and $\frac{\partial p_B}{\partial \eta} \geq 0$). Then, expression (H.41) implies that p_G/p_B and, therefore, p_G must increase in η. And, due to $p_G = v_G/(1 + \eta)$, this requires v_G to strictly increase in η. But, by expression (H.45), v_G increases with η only if p_B/p_G strictly increases with η which cannot be because p_G/p_B increases with η, a contradiction. Thus, $v_B = p_B$ strictly decreases with η (i.e., $\frac{\partial v_B}{\partial \eta} < 0$). Note that by (H.41), $\frac{\partial v_B}{\partial \eta} < 0$ implies that p_G/p_B decreases with η. Equivalently, p_B/p_G increases with η, which — by (H.45) — implies that v_G increases with η. As p_B decreases with η and p_B/p_G increases with η, it follows that p_G decreases with η too.

H.4 Proof of Corollary 3

By continuity, it suffices to prove the Corollary assuming that there is no short selling constraint (i.e., $\eta = \infty$ with a slight abuse of notation) and $\theta = 0$. We show that under these circumstances, there exists no Markov equilibrium with $V_t = A_t v(\sigma)$ and $P_t = X_t p(\sigma)$ whereby token price $p(\sigma)$ and transaction volume $v(\sigma)$ are strictly positive in at least one state σ. The only remaining possibility is then the trivial equilibrium with $p(\sigma) = v(\sigma) = 0$ for all σ, implying $P_t = V_t = 0$ at all times $t \geq 0$. Suppose to the contrary there exists a Markov equilibrium with positive token price and

platform adoption, in that there exists $z \in \{B, G\}$ with $p_z > 0$ and $v_z > 0$. We distinguish between two cases: i) $p_G > p_B$ and ii) $p_B \geq p_G$.

We start with i), $p_G > p_B \geq 0$. Due to $p_B < p_G$, $\theta = 0$, and $\mu < r$

$$\frac{\mathbb{E}[dP_t]}{P_t dt} < r - \theta \pi \sigma_t^P \quad \Longleftrightarrow \quad \lambda_B \left(\frac{p_B}{p_G} - 1 \right) + \mu < r - \theta \pi \sigma_G \tag{H.50}$$

holds in state G. According to speculators' optimization in (H.39), any speculator j finds it optimal to set $s_{jt} \to -\infty$ in state G. This leads to $S_t \to -\infty$, and $p_G \to 0$, a contradiction.

Next, consider ii), $p_B \geq p_G$ so that $p_B > 0$. Then, due to $p_G \leq p_B$, $\theta = 0$, and $\mu < r$

$$\frac{\mathbb{E}[dP_t]}{P_t dt} < r - \theta \pi \sigma_t^P \quad \Longleftrightarrow \quad \lambda_G \left(\frac{p_G}{p_B} - 1 \right) + \mu < r - \theta \pi \sigma_B \tag{H.51}$$

holds in state B. According to speculators' optimization in (H.39), in state B, any speculator j finds it optimal to set $s_{jt} \to -\infty$. This leads to $S_t \to -\infty$ and $p_B \to 0$, yielding a contradiction.

Admittedly, there might be other Markov equilibria with $P_t > 0$ at some time t. However, these Markov equilibria do not fit into the class of equilibria considered, which requires platform transaction volume to satisfy $V_t = A_t v(\sigma)$ and token price to satisfy $P_t = Y_t p(\sigma)$ where $v(\sigma)$ and $p(\sigma)$ depend on σ only.

I   Solution with noise traders and proof of Proposition 6

We present the solution and equilibrium with noise traders under the simplifying assumption that there are no uncertainty shocks and no investment opportunities. That is, without loss of generality, $\lambda_B = 0$ and $\sigma = \sigma_G$ at all times, and we assume $\bar{a} = 0$. Under these circumstances, equilibrium token price follows

$$\frac{dP_t}{P_t} = \mu_t^P dt + \sigma_t^P dZ_t + \tilde{\sigma}_t^P d\tilde{Z}_t, \tag{I.52}$$

where the two Brownian Motions dZ_t and $d\tilde{Z}_t$ are independent, i.e., $dZ_t \cdot d\tilde{Z}_t = 0$. Price drift μ_t^P and the token price volatilities σ_t^P and $\tilde{\sigma}_t^P$ are determined in equilibrium, discussed below. We verify that in equilibrium, $\sigma_t^P \geq 0$ and $\tilde{\sigma}_t^P \geq 0$.

I.1   Users' optimization

We start by discussing user optimization. With noise traders, users' flow utility becomes

$$dR_{it}^U \equiv \underbrace{\frac{u_{it}^\alpha}{\alpha} V_t^\beta A_t^{1-\alpha-\beta} dt}_{\text{Convenience yield}} + u_{it} \left(\underbrace{\frac{dP_t}{P_t}}_{\text{Token returns}} - \underbrace{rdt}_{\substack{\text{Opportunity} \\ \text{cost}}} \right) - \underbrace{|u_{it} \pi \sigma_t^P| dt - |u_{it} \tilde{\pi}_U \tilde{\sigma}_t^P| dt}_{\substack{\text{Dis-utility} \\ \text{of bearing risk}}}. \tag{I.53}$$

Any user i solves

$$\max_{u_{it} \geq 0} \mathbb{E}[dR_{it}^U], \tag{I.54}$$

leading to

$$V_t = u_{it} = A_t \left(\frac{1}{r + \pi \sigma_t^P + \tilde{\pi}_U \tilde{\sigma}_t^P - \mathbb{E}[dP_t]/(P_t dt)} \right)^{\frac{1}{1-\gamma}}, \tag{I.55}$$

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where we omit the absolute value notation due to $\sigma_t^P \geq 0$ and $\tilde{\sigma}_t^P \geq 0$.

I.2 Speculators' optimization

Next, we discuss speculators' optimization problem. Recall that there are no short selling constraints for speculators, and $\sigma_t^P, \tilde{\sigma}_t^P \geq 0$. Any speculator j solves

$$\max_{s_{jt}} \left[s_{jt} \left(\frac{\mathbb{E}[dP_t]}{P_t} - rdt \right) - \theta\pi|s_{jt}\sigma_t^P|dt - \tilde{\pi}_S|s_{jt}\tilde{\sigma}_t^{\,P}|dt \right]. \tag{I.56}$$

The speculators' problem is a linear program. Note that speculator j optimally chooses $s_{jt} \to \infty$ if

$$\frac{\mathbb{E}[dP_t]}{P_t dt} > r + \theta\pi\sigma_t^P + \tilde{\pi}_S\tilde{\sigma}_t^P. \tag{I.57}$$

Likewise, speculator j optimally chooses $s_{jt} \to -\infty$ if

$$\frac{\mathbb{E}[dP_t]}{P_t dt} < r - \theta\pi\sigma_t^P - \tilde{\pi}_S\tilde{\sigma}_t^P. \tag{I.58}$$

In (a non-degenerate) equilibrium with positive token price, any speculator's token holdings s_{jt} and speculators' aggregate token holdings S_t must be finite, which requires

$$r - \theta\pi\sigma_t^P - \tilde{\pi}_S\tilde{\sigma}_t^P \leq \frac{\mathbb{E}[dP_t]}{P_t dt} \leq r + \theta\pi\sigma_t^P + \tilde{\pi}_S\tilde{\sigma}_t^P \tag{I.59}$$

to hold.

I.3 Solving for the Markov equilibrium

We look for a Markov equilibrium with state variables A_t and N_t that is similar to the one considered in the baseline (except that σ is constant). In this Markov equilibrium, users solve (I.54) and speculators solve (I.56) and the token market clears (i.e., (45) holds). In this Markov equilibrium, token price scales with $Y_t = A_t/N_t$ so that $P_t = Y_t p_N$, and transaction volume scales with A_t so that $V_t = A_t v_N$ for constants p_N and v_N.

Recall that $N_t = M_t - \tilde{N}_t$. As $dM_t = 0$, it follows that $dN_t = -d\tilde{N}_t = -\tilde{\sigma}_t d\tilde{Z}_t$. Under the assumption $\tilde{\sigma}_t = N_t\tilde{\sigma}$, we therefore obtain

$$\frac{dN_t}{N_t} = -\tilde{\sigma}d\tilde{Z}_t. \tag{I.60}$$

Using Ito's Lemma, we calculate for $Y_t = A_t/N_t$:

$$\frac{dY_t}{Y_t} = \mu dt + \sigma dZ_t + \tilde{\sigma}d\tilde{Z}_t. \tag{I.61}$$

We conjecture (and verify) that token price takes the form $P_t = Y_t p_N$, with the constant p_N to be determined. Under this conjecture, Itô's Lemma yields

$$\mu_t^P = \left(\frac{\partial P_t}{\partial Y_t} \right) \frac{Y_t\mu}{P_t} + \frac{1}{2} \left(\frac{\partial^2 P_t}{\partial Y_t^2} \right) \frac{Y_t^2(\sigma^2 + \tilde{\sigma}^2)}{P_t} = \mu, \tag{I.62}$$

and

$$\sigma_t^P = \left(\frac{\partial P_t}{\partial Y_t}\right)\frac{Y_t\sigma}{P_t} = \sigma \geq 0. \tag{I.63}$$

Likewise,

$$\tilde{\sigma}_t^P = \left(\frac{\partial P_t}{\partial Y_t}\right)\frac{Y_t\tilde{\sigma}}{P_t} = \tilde{\sigma} \geq 0. \tag{I.64}$$

The law of motion (I.52) implies that expected token returns satisfy $\frac{\mathbb{E}[dP_t]}{P_t dt} = \mu_t^P = \mu$.

Using above relations and (I.55), we obtain that platform transaction volume scales with A_t and takes the form $V_t = A_t v_N$, where

$$v_N = \left(\frac{1}{r - \mu + \pi\sigma + \tilde{\pi}_U\tilde{\sigma}}\right)^{\frac{1}{1-\gamma}}.$$

Note that because $\mu_t^P = \mu < r$, the second inequality of (I.59) holds and speculators never buy tokens (so that $S_t \leq 0$). As a result, the necessary equilibrium condition (I.59) is satisfied if and only if

$$r - \theta\pi\sigma - \tilde{\pi}_S\tilde{\sigma} \leq \mu,$$

which is equivalent to (47). As such, (47) is necessary for the prescribed Markov equilibrium with $p_N > 0$ and $v_N > 0$ to exist. In fact, it is also sufficient. If (47) does not hold, the only equilibrium with $V_t = A_t v_N$ and $P_t = Y_t p_N$ features $v_N = p_N = 0$

Admittedly, if (47) does not hold, there might be other Markov equilibria with $P_t > 0$ at some time t. However, these Markov equilibria do not fit into the class of equilibria considered, which requires platform transaction volume to satisfy $V_t = A_t v_N$ and token price to satisfy $P_t = Y_t p_N$ for constants v_N and p_N.

J Internet Appendix — Model variant with jump risk aversion

We consider a variant of our baseline model in Section 1 that features risk-aversion with respect to jump risk too. Specifically, consider that any user i incurs (flow) dis-utility of bearing token price risk $u_{it}C_t$ with:

$$C_t = \pi \sigma_t^P + \Pi_G |\Delta_t^G| \mathbb{I}\{\sigma = \sigma_B\} + \Pi_B |\Delta_t^B| \mathbb{I}\{\sigma = \sigma_G\},$$

with constants $\Pi_z \geq 0$ and Δ_t^z from (18) for $z = B, G$. Here, $\mathbb{I}\{\cdot\}$ denotes the indicator function which equals one if the argument $\{\cdot\}$ is true and zero otherwise. The absolute value $|\cdot|$ on Δ_t^z guarantees that user i incurs dis-utility for being exposed to token price fluctuations regardless of the sign of Δ_t^z. Any speculator j incurs flow dis-utility of bearing token price risk $\theta C_t |s_{jt}|$.

The model solution and equilibrium is analogous to the baseline. Platform transaction volume scales with A_t, that is, $V_t = A_t v(\sigma)$, and token price scales with X_t, that is, $P_t = X_t p(\sigma)$, where $v_z = v(\sigma_z)$ and $p_z = p(\sigma_z)$. Token price volatility is $\sigma_t^P = \sigma$ and token price drift is $\mu_t^P = \mu$. And, as in the baseline, $p_G > p_B$ (without proof) which implies $\Delta_t^G > 0$ and $\Delta_t^B < 0$. Depending on the state $z = B, G$, the (marginal) dis-utility of bearing token price risk reads:

$$C_B = \pi \sigma_B + \Pi_G \left(\frac{p_G}{p_B} - 1 \right) \quad \text{and} \quad C_G = \pi \sigma_G - \Pi_B \left(\frac{p_B}{p_G} - 1 \right).$$

To obtain the equilibrium expressions for scaled transaction volume (v_B, v_G) and scaled token price (p_B, p_G), we simply must replace in expressions (20), (21), (22), and (21) the baseline (marginal) dis-utility of bearing token price risk, $\pi \sigma_z$, with the (marginal) dis-utility of bearing token price risk from this Section, C_z.

This yields (noting that $p_G = v_G$):

$$
\begin{aligned}
v_B &= \left(\frac{1}{r - \mu + \pi \sigma_B - \lambda_G (1 - \Pi_G)(p_G/p_B - 1)} \right)^{\frac{1}{1-\gamma}} \\
p_B &= \max \left\{ v_B, \frac{\lambda_G (1 - \theta \Pi_G) p_G}{r - \mu + \theta \pi \sigma_B + \lambda_G (1 - \theta \Pi_G)} \right\} \\
v_G &= p_G = \left(\frac{1}{r - \mu + \pi \sigma_G - \lambda_B (1 + \Pi_B)(p_B/p_G - 1)} \right)^{\frac{1}{1-\gamma}}.
\end{aligned}
\tag{J.65}
$$

We impose the sensible parameter restriction $\Pi_G < 1$, which implies that the expected token returns from token price appreciation (upon reaching state G) outweigh users' dis-utility of bearing this upside risk. Under $\Pi_G < 1$, platform transaction volume increases with the magnitude of token price appreciation $\Delta^G = p_G/p_B - 1$ upon reaching state G.

We note that the equilibrium expressions in (J.65) are similar to those in (20), (21), (22), and (23). Comparing (J.65) to (20), (21), (22), and (23), we see that jump risk-aversion effectively alters the transition probabilities of a regime switch (whereby users and speculators apply different transition probabilities in valuing payoffs). Note that when $\theta = 0$, the models with and without jump risk-aversion are isomorphic to each other and their solutions are the same up to a change in the transition probabilities. Specifically, the model with jump risk aversion and transition probabilities (λ_B, λ_G) yields the same outcomes as the model without jump risk-aversion and transition probabilities $(\lambda_B (1 + \Pi_B), \lambda_G (1 - \Pi_G))$.[42] We expect the models with and without jump risk-aversion to generate qualitatively similar results for other values of θ too.

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[42]Assuming no jump risk aversion and accordingly adjusting the transition probabilities is akin to carrying out the analysis under a risk-neutral probability measure when there is a stochastic discount factor (Duffie (2010)).