Best Before? Expiring Central Bank Digital Currency and Loss Recovery*

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Abstract

Many central banks are considering issuing digital cash substitutes. An important property of physical cash payments is *resilience*—for example, imperviousness to power outages and independence of electronic/network coverage. These properties also make cash payments important in remote communities. Policy makers are considering building similar offline payment functionality into digital cash substitutes, while their digital nature allows for novel features that could make them more desirable than physical cash. This paper analyzes the possibility of introducing an expiry date for offline digital currency balances to automate personal loss recovery. Our results show this functionality could have a substantial positive impact on consumer demand for offline digital currency balances. We also examine the welfare effects of adjustments to the expiry date: small increases from the optimum cause little damage, but small decreases from the optimal expiry date can have a large negative impact. More information sharing between consumers and the central bank can improve loss recovery but has an ambiguous impact on social welfare.

Keywords: Central Bank Digital Currency, Design, Offline Payments, Outage, Financial Inclusion

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1 Introduction

Many central banks are considering issuing digital cash substitutes as the transactional demand for physical cash wanes (Boar et al., 2020). An important property of a digital cash substitute is its resilience: Physical cash allows for economic exchange even in conditions without electrical power or network access. Policy makers are seeking to build similar offline payments functionality into digital cash substitutes, which could be used in remote communities and serve as a backstop system when any disruption occurs (Bank of Canada, 2020; Bank of England, 2020; Group of Seven Central Banks, 2020).[1] At the same time, a digital cash substitute with offline functionality could have features that would make it more desirable to use than physical cash.

This paper considers one possible feature: the introduction of an expiry date on offline digital currency balances. Although such functionality might seem inconvenient at first sight, it has one major advantage, previously unconsidered: It would facilitate personal recovery of funds accidentally lost.[2] An inconvenience of a bearer instrument such as cash is that it is easily lost with little possibility of recovery by the owner.[3] One reason for the lack of opportunity for recovery is that it is usually difficult for the owner to prove that cash is truly lost and will never be used for payments in the future.[4] This is different for a digital currency that is allowed to expire over time. Since money balances that remain unspent after their expiry date cannot be spent in the future, it would be safe for the central bank to reimburse the (most likely) owner in terms of online balances. The reimbursement of expired funds could be implemented in a fully automatic fashion without the

[1] Offline payments functionality may be even more important in developing countries where substantial shares of the population have unreliable access to electricity or no access at all (data collected by the World Bank suggest that about 10 per cent of the world population had no access to electricity in 2019).

[2] Protecting individuals against accidental loss of cash is a new rationale for an expiry date. Others have investigated alternative rationales, such as stimulating spending at the macro level; see the literature review for details.

[3] In bitcoin, the problem is even more dramatic. According to the *New York Times*, "Of the existing 18.5 million Bitcoin, around 20 percent — currently worth around $140 billion — appear to be in lost or otherwise stranded wallets, according to the cryptocurrency data firm Chainalysis." (Popper, Jan. 12, 2021).

[4] This statement does not consider mutilated and contaminated bank notes (in which case, often, the loss can be proven). Some central banks spend considerable effort to reimburse owners of damaged bank notes. An infamous anecdote is the story about a Dutch cow who ate a wallet without realizing that it would be her most expensive, and alas, also her last meal. The cow was slaughtered and her tripe was delivered to the Dutch Central Bank where currency recovery experts retrieved the remains of seven bank notes of a thousand guilders each (Dutch News Report, 1974).

need for the owner to file a loss claim. Similarly, the expiry date could automatically be refreshed whenever users' devices connect to the network before the funds expire.

The features that are necessary to rule out double-spending in a payment system for offline payments also cause digital currency balances to be subject to loss events. To completely rule out double-spending in offline environments, it is necessary to uniquely store offline digital currency balances in a single local device.[5] To see this, consider the situation where copies of the same offline digital currency balances could be stored in multiple local devices. In this situation, the payer could simply double-spend the same funds by using different devices to pay different offline payees.[6] By definition, offline payments do not allow verification of a payment based on real-time information in a central ledger, so it would be impossible for offline payees to be informed in a timely manner as to whether the payer is attempting to double-spend.[7] Similarly, it is necessary to separate (or "earmark"/"lock") digital currency balances available for offline spending with a local device from balances that can be spend without that device.[8] Otherwise, after storing funds in a local device for offline payments, a payer could continue to spend the same funds to pay an online payee while using the offline device to pay a different offline payee. There would be no way for either payee to be aware of the attempt to double-spend at the time of the payment. The separation of offline digital currency balances while storing them uniquely in a single local device makes those funds subject to loss due to, e.g., malfunctioning, physical theft, or loss of the device.

[5]In practice, the design of a payment instrument for a generally accepted money is subject to a trilemma. As we will see, it can only have two out of three properties: "offline payment functionality", "no double-spending" and "loss of funds not implied by device loss".

[6]Note that uniquely storing funds in a single local device also rules out the possibility of the owner to create a back-up. The absence of the possibility to restore a back-up is necessary to prevent double-spending. Otherwise, an agent could pretend to have lost a local device and restore the funds on a second local device, which would allow the agent to use the two devices to double-spend at two different offline locations.

[7]A separate but related point is that the device to store funds needs to be tamper-resistant so that it is prohibitively expensive to restore a previous level of balances after making a payment, or to copy its contents to another device. Physical cash achieves this through its physical nature and security features that make it hard to copy (i.e., counterfeiting is difficult). An offline CBDC system may require additional security measures to mitigate the impact of a breach of tamper-resistance (Minwalla, 2020).

[8]E.g., when withdrawing physical cash, this occurs when the bank debits the withdrawer's account with the amount withdrawn.

The present paper starts from the position that the central bank would like to rule out double-spending, so that offline money balances will be separated and uniquely stored in a single local device, and, hence, will be subject to loss. We do so for two reasons. First, financial inclusiveness and universal access are regularly raised as core public policy goals for issuing digital cash substitutes (Miedema et al., 2020). Limiting fraud in a system that does not rule out double-spending may be prohibitively expensive if there are no possibilities to exclude bad actors who abuse possibilities to double-spend.[9] Second, this is the combination of features that is most like the existing arrangements for cash. Currently, "offline money balances" consist of physical bank notes. They are "moved offline" by withdrawal from a bank account, and banks typically debit the account immediately when cash is withdrawn.

Given this design choice, the present paper studies the economics of introducing an expiry date to facilitate loss recovery for offline money balances. Consumers in our model need to choose the optimal distribution of their money between offline and online balances. Both types of balances can be used to pay in environments where consumers have network connectivity ("centralized meetings"). During an outage, consumers can exclusively trade in an environment without network connectivity ("decentralized meetings"), in which only offline money balances can be used. A difference with typical monetary models (for example the centralized/decentralized markets models such as Lagos and Wright, 2005) is that both the occurrence and the length of decentralized periods are stochastic. In our model, an outage occurs occasionally and last for a number of periods. A disadvantage of offline money balances is that they can be lost. Users may be reimbursed automatically for lost offline balances after they expire.[10] Without an expiry date, balances are irrecoverable, as is usually

[9]Some traditional systems for offline payments that do not completely rule out double-spending include cheques and store-and-forward payments with credit cards. In these settings, individuals can be excluded after abuse by denying them as client for a chequing account or credit card. Fraud may also be mitigated by introducing penalties for bad actors through law enforcement. Even with exclusion and law enforcement, the costs of fraud to payees can be considerable. Fraud with paper cheques continues to increase even as they are used less and less for payments (American Bankers Association, 2020). Moreover, Adyen (2020) indicates that authorization for offline store-and-forward payments with credit cards may be as low as 95 per cent.

[10]For this mechanism to be effective, it requires that spending electronic cash stored in a local device will require some form of user authorization (e.g., unlocking a phone when electronic cash is stored in a smart phone or entering a pin code when electronic cash is stored in a card), so that it is unlikely that someone finding or stealing a local device will result in the stored electronic cash being spent.

the case with cash. With expiry date, users can be reimbursed for losses, but it may not always be optimal for merchants to accept offline money.

Our calibration results suggest that the introduction of an expiry date to facilitate loss recovery can have a substantial positive impact on consumer demand for offline digital currency balances. The reason is straightforward: The ability to reimburse individuals for personal losses once the lost digital currency expires reduces the cost of losses from the full amount to the inconvenience of temporarily not having access to the funds (until after they expire). This addresses a potentially significant cost to the users of offline money balances: A small survey we conduct suggests that, without an expiry date, the device losses might add an annual cost to offline money users that is roughly estimated to be in a range between 8 per cent (for funds stored locally in a smart phone) to 16 per cent (for funds stored locally in a payment card) of the offline money balances.[11]

Starting from the optimal expiry date, we find that the cost of small deviations is strongly asymmetric. There is a high cost associated with setting an expiry date that is shorter than optimal, while the cost of setting an expiry date somewhat longer than optimal is limited. The high cost associated with setting an expiry date that is somewhat too short, is that it may prevent the ability to conduct any transactions when payees expect to remain offline for a period that exceeds the expiry date. Payees will refuse to accept offline transactions, and in the limit an extremely short expiry date is equivalent to a situation with no offline cash. The only inconvenience to the users from setting an expiry date that is longer than optimal is the additional delay in recovering lost offline digital currency balances. An infinitely distant expiry date is equivalent to physical cash (which does not facilitate loss recovery).

Determining the likely owner of expired offline digital currency balances requires exchange of information between the central bank and the devices of consumers when the devices are online. We consider a low- and a high-information model for how the central bank infers whether an agent's expired digital currency balances remained unspent. The low-information model places the onus

[11] The details of the survey are reported in Appendix A.

fully on the payee to deposit offline digital currency balances before the expiry date. This model provides a higher level of privacy in that it does not require the payer's device to reveal whether and where offline digital currency balances have been spent. However, loss recovery in this model is less precise in that the payer may be reimbursed for expired funds that a payee failed to deposit in a timely manner.[12] The high-information model provides less privacy to the payer by requiring the payer's device to reveal whether and where funds have been spend when connecting to the central bank. Again, the payee is required to deposit received offline digital currency balances before the expiry date. However, whenever a payee fails to do so, then disclosure by the payer's device may allow the central bank to reimburse the payee rather than the payer. Our results suggest that more information sharing has an ambiguous impact on social welfare. Whether it improves social welfare depends on whether payers allow their devices to reconnect with offline digital currency balances after making offline payments. Whether they do so, depends on how the amount of foregone interest on unspent offline balances compares to the likelihood of a windfall gain if the payee fails to deposit the spent balances. If payers were to choose to reconnect their devices, then loss allocation and social welfare are higher in the high information model. However, if it were optimal for payers to not reconnect, then the high information model would lead to lower social welfare. Payers will forego interest over their offline balances for a longer period of time and are therefore inclined to carry lower offline balances resulting in lower spending during offline periods.

The remainder of the paper is organized as follows. Section 2 discusses related literature. Section 3 illustrates the major trade-offs involving cash related to outages and loss events in the context of simple finite-time model. Section 4 introduces a more complex infinite-time model with a stochastic length for offline periods to obtain a better understanding of the quantitative impact of introducing an expiry date to facilitate loss recovery. Section 5 discusses the results of calibrating the more complex model. Section 6 concludes.

[12]A comparable windfall profit for the payee occurs in a traditional payment setting when a payee fails to deposit a cheque written by the payer.

2 Related Literature

Our paper fits into the quickly expanding economic literature on CBDC.[13] Early economic research on this topic focused primarily on whether it would be beneficial if central banks were to issue CBDCs (Barrdear and Kumhof, forthcoming; Brunnermeier and Niepelt, 2019; Keister and Sanches, 2019; Andolfatto, 2021; Chiu et al., 2019; Fernández-Villaverde et al., 2021; Schilling et al., 2020). More recently, the focus has increasingly shifted towards design aspects of CBDC. These include the security features of a CBDC (Kahn et al., 2020), the privacy it provides to its users (Kahn et al., 2005; Garratt and Van Oordt, 2021; Lee and Garratt, 2021), whether CBDC should generally be more deposit-like or more cash-like (Agur et al., forthcoming), the programmability of payments (Kahn and Van Oordt, 2021), and whether CBDC balances should pay interest (Keister and Sanches, 2019; Jiang and Zhu, 2021; Garratt and Zhu, 2021). Li (2021), Bijlsma et al. (2021) and Huynh et al. (2020) estimate how some of these features could affect the demand for CBDC based on survey data. Auer et al. (2020) study the technological approaches and policy stances of central banks on the issuance of CBDC. Our paper contributes to this literature by studying whether a digital cash substitute should be designed with a potential expiry date where users are automatically reimbursed for expired balances in order to enable the recovery of lost balances.

The traditional Baumol-Tobin model to understand cash demand suggests that cash holdings should explode as the interest rate approaches zero (Baumol, 1952; Tobin, 1956). More recent literature has recognized that the cost of carrying cash consists not only of the foregone interest but also of the risk of losing cash balances. Alvarez and Lippi (2009) approximate the costs of carrying cash as the sum of the nominal interest rate and the probability of cash theft – as a source of cash losses – when estimating a dynamic cash inventory model.[14] Sanches and Williamson (2010) explain theoretically how credit can co-exist with cash in an environment with limited commitment when cash can be subject to theft. Moreover, the model of Williamson (2019) considers CBDC

[13] See Kiff et al. (2020) and Carapella and Flemming (2020) for early surveys.

[14] Kosse (2013) and Kahn and Liñares-Zegarra (2016) document some empirical evidence on the relationship between perceived safety and cash use.

while explicitly assuming that CBDC comes with the advantage over cash that it cannot be stolen. Arguably, in the current low-interest-rate environment, one could reasonably take the position that the risk of losing cash has become one of the major reasons why individuals don't carry around substantial amounts of cash (another being potential budget constraints). Our paper analyzes how what may have become one of the major cost of carrying funds for offline payments can be limited for a digital cash substitute.

The motivation for an expiry date on a digital cash substitute to enable recovery of lost balances is distinct from that of putting an expiry date on stimulus money in order to encourage consumer spending in recessions (Andolfatto, 2020).[15] Imposing an expiry date on a digital cash substitute without reimbursing the owner for expired balances effectively increases the cost of carrying those balances: It imposes the threat of a tax on its owner if the funds are not spend before the expiry date. This has quite the opposite effect of the introduction of an expiry date to enable recovery of lost balances, which reduces the cost of holding balances. Imposing an expiry date without reimbursing owners for expired balances leads to worse outcomes in our model, as our model includes no rationale for the government to stimulate spending.

3 A Model of Lost Cash and Outages

The major trade-offs involving cash related to outages and loss events can be illustrated in a simple discrete finite-time model. In this setup, we analyze the incentives of a consumer to hold cash based on the properties of the cash issued by the central bank in an environment where outages may occur. We consider a broad concept of "cash". The essence of cash in our model is that it is a bearer instrument that can be used for making offline payments during outages. It may be physical cash in the form of coins and bank notes, or it may be stored-value in a payment card or smartphone. Carrying cash provides the ability to purchase consumption goods during an outage,

[15] Some stimulus programs provided shopping vouchers with expiry dates (Kan et al., 2017).

but entails the cost of foregone interest as well as the risk of losing the cash. The properties of the cash issued by the central bank determine whether cash lost once is lost forever.

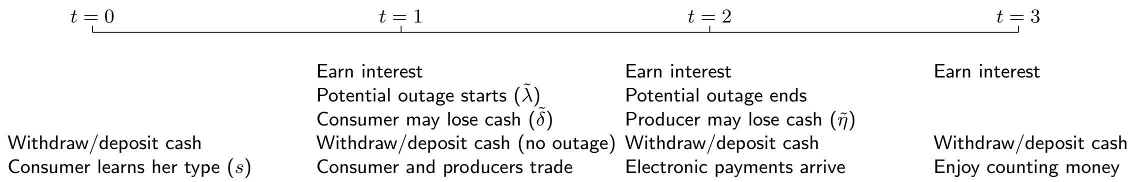
All agents are assumed to have quasi linear preferences; in the simple model this assumption will reduce to risk neutrality. Agents discount time with factor $\beta < 1$. There are two categories of agents–producers (she) and consumers (he)–and two types of consumers, denoted by $s \in \{1, 2\}$. A consumer of type s will have a demand for at most s units of the good manufactured by the producer. A consumer starts out with m units of money balances. He will divide them between cash and online money holdings. Cash pays no interest; online holdings pay interest at the rate $i = \beta^{-1} - 1$.

The timeline of the model is shown in Figure 1. At the initial date $t = 0$, the consumer decides what part of his money balances to hold as cash, denoted by z. (For simplicity, we treat cash holdings in excess of m as borrowings of online money balances at interest rate i.) At the end of $t = 0$, the consumer discovers his own type (we assume the two types are equally probable). Producers cannot observe the consumer's type.

At $t = 1$, there is a possibility for the consumer to purchase units of the good from producers and to consume them. Each producer can supply at most one unit of the good at a cost of β. The value of each unit to the consumer is v, up to the capacity determined by the consumer's type (1 or 2). Every consumer has the possibility to meet multiple producers at $t = 1$. Trading during a meeting works as follows: The consumer makes a take-it-or-leave-it offer that consists of a price per unit p_j and a method of payment $j \in \{c, d\}$, where c stands for cash and d stands for online money. Then the producer makes an acceptance decision $a \in \{0, 1\}$, where $a = 1$ stands for accepting the offer and producing the good. The transaction technology allows for an exchange of the good and money based on the agreed-upon price and method of payment.

Two types of adverse events may occur before the consumer gets to make an offer at $t = 1$. First, with probability δ the consumer may lose his cash. The occurrence of the loss is denoted

Figure 1: Model of Lost Cash and Outages: Timeline



by $\tilde{\delta} \in \{0, 1\}$. Second, with probability λ an outage may occur. The occurrence of the outage is denoted by $\tilde{\lambda} \in \{0, 1\}$. If there is no outage, then the consumer can pay with both online money balances and cash. Payments with online money take one period to settle, so that an online payment made by the payee at $t = 1$ starts earning interest for the payee from $t = 2$ onward. If there is an outage, then the consumer can pay with cash only. So, no consumption is possible in the unhappy state where there is an outage and the consumer has lost his cash. If the consumer pays with cash, then there is a probability η that the producer will lose the cash.[16] The occurrence of the loss is denoted as $\tilde{\eta} \in \{0, 1\}$. All chance events are drawn independently. We assume the following parameter restriction:

$$v > 1/(1 - \eta). \tag{1}$$

Otherwise, the probability of the producer losing cash would be so great that she would have no incentive to accept it.

Formally, the payoff of the consumer as a function of consumption at $t = 1$ and the consumer's terminal money holdings w_c at $t = 3$ is given as

$$u(q, w_c; s) = \beta \max\{q, s\} v + \beta^3 w_c, \tag{2}$$

where q denotes the number of accepted offers, which is the sum of the number of accepted cash offers, q_c, and the number of accepted offers involving online payments, q_d.

[16] Part of η may also be thought of as the cost of handling cash; this leaves the results in conditions (6) and (7) unaffected.

The consumer chooses a combination of (z, p_j, j) to maximize the payoff function in (2) subject to a cash-in-advance constraint for offers involving cash payments

$$q_c p_c \leq z(1 - \tilde{\delta}) \tag{3}$$

and a no-outage constraint for offers involving online payments

$$\tilde{\lambda} q_d p_d \leq 0. \tag{4}$$

The producers simply need to decide whether to accept the offer they receive; the payoff function of a producer is given by

$$u_p(a, w_p) = -\beta^2 a + \beta^3 w_p. \tag{5}$$

The income of the central bank consists of the seigniorage from cash losses, foregone interest on cash holdings and foregone interest on online money in transit. All the cost from money and payments for the consumer and producer are income for the central bank.

Welfare is defined as the sum of the payoffs of the consumers, the producers and the central bank. Welfare analysis in the model is simple and in essence boils down to analyzing the expected number of units that will be sold to each consumer.

3.1 Cash lost is lost forever

We first consider the case where, as with physical cash, there is no possibility for the consumer to recover lost balances. In this situation, the ultimate money holdings of the consumer are given by

$$w_c = \left[(m - z)(1 + i) + z(1 - \tilde{\delta}) - qp_j\right] \times (1 + i)^2,$$

where the elements within brackets are the sum of the online money holdings and the cash that has not been lost minus the money used to pay the producer. Similarly, the ultimate money holdings of the producer are given by the sales revenue—unless sales were paid with cash and lost—i.e.,

$$w_p = ap_j(1 - \tilde{\eta}\mathbf{1}_{j=c})(1 + i).$$

Online money will be the default payment instrument of choice for the consumer, since the producer needs to be reimbursed for the risk of losing cash: If the consumer were to offer an online payment, then the producer accepts any offer with a price greater or equal than $p_d = 1$. If the consumer were to offer a cash payment, then the producer rejects the offer unless the price is greater or equal than

$$p_c = \frac{1}{1 - \eta}.$$

It is optimal for the consumer to make a take-it-or-leave-it-offer of an infinitesimal amount above these prices. So, it is cheaper from the consumer's point of view to pay with online money whenever possible, that is, whenever there is no outage. Moreover, restriction (1) implies that, if an outage were to occur so that the consumer cannot pay with online money, he would always be willing to buy the quantity of goods he wants to consume at the higher price when carrying enough cash.

The only remaining question is whether the consumer is willing to carry enough cash to be able to pay during an outage. Carrying cash is costly because of foregone interest and the risk of losing it. So, the consumer will either choose not to carry cash at all, or to carry exactly enough to pay p_c for one unit during an outage, or the amount to pay $2p_c$ for two units during an outage. Which amount the consumer chooses depends on whether the cost of carrying cash is less than the benefit of the insurance for consumption during outages. The consumer is willing to carry enough cash to purchase at least one unit of the good during an outage if and only if

$$i + \delta < \lambda(1 - \delta) \times [\nu(1 - \eta) - 1]. \tag{6}$$

The condition means that, for the consumer to hold one unit of cash, the cost of doing so – the sum of foregone interest and expected costs of losing cash – on the left-hand-side needs to be less than the probability that it can be used in an outage multiplied with the marginal benefit of spending with cash during an outage on the right-hand-side.

For the consumer to be willing to carry more cash, a stronger condition needs to hold true. In particular, the consumer would be willing to carry enough cash to purchase two units of the goods during an outage if and only if

$$i + \delta < \frac{\lambda}{2}(1-\delta) \times \left[\nu(1-\eta) - 1 - \frac{i}{1+i}\right]. \tag{7}$$

This condition is stronger for two reasons. First, the probability that the consumer wants to consume the second unit during an outage is half the probability that he will want to purchase the first unit. So carrying additional cash insures against a smaller probability event. Second, and more subtly, the consumer will forego interest over two periods on the additional cash if he turns out not to need the larger amount during the outage.

Social welfare depends on the cost of carrying cash, i.e., the level of $i + \delta$. Social welfare is lowest if the cost of carrying cash is sufficiently high that condition (6) is violated. In this situation, consumers carry no cash and transactions between producers and consumers occur only when there is no outage. The expected number of units sold per consumer equals $\frac{3}{2}(1-\lambda)$. If the cost of carrying cash is at an intermediate level such that condition (6) is satisfied but (7) is not, then consumers carry sufficient cash to purchase only one unit when there is an outage. If there is no outage, then consumers buy all the goods they would like to consume, and when there is an outage and they haven't lost their cash, then they purchase one unit. The expected number of units sold per consumer equals $\frac{3}{2}(1-\frac{2}{3}\lambda\delta-\frac{1}{3}\lambda)$. Social welfare is highest if the cost of carrying cash is sufficiently low that condition (7) is satisfied. In this situation, consumers always purchase all the goods they would like to consume, except in the joint event where there is both an outage and the consumer loses his cash. The expected number of units sold per consumer equals $\frac{3}{2}(1 - \lambda\delta)$ in this situation.

3.2 Cash with loss recovery and no information exchange

The conditions in (6) and (7) show that the probability of cash losses may induce consumers to carry too little cash to insure themselves against outages. Next, we consider schemes involving an expiry date on cash that could help to weaken those constraints by alleviating the consequences of cash losses. The idea of introducing an expiry date on digital cash is that all cash that expires will be treated as lost and can be automatically reimbursed by the central bank in terms of online money balances to the agent who (most likely) lost it. Deriving which agent most likely lost the cash depends on the information exchange that occurs between the devices of consumers and the central bank when the devices are online.

We first consider the environment without any information sharing between the consumer's device and the central bank, so that connecting the consumer's device does not reveal to the central bank whether and where the consumer spent offline balances. As a consequence, the onus will be on the producer to deposit the cash received from the consumer before it expires (note that the producer may fail to do so if she lost the cash). If no one deposits the cash before the expiry date, then the central bank will infer that the consumer still owns the cash when it expires.

A shelf life of $t + 2$ allows the consumer to pay the merchant during an outage at $t = 1$ with money withdrawn at $t = 0$ while leaving enough time for the merchant to deposit the money in her online account at $t = 2$ before it expires. Since no information is provided to the central bank, the consumer's decision to deposit unspent balances after an outage at $t+2$ will not be affected by the expiry date. If the money is not deposited at $t = 2$ by the merchant or the consumer, then the central bank reimburses the consumer at $t = 3$ without any risk of losses to the central bank.

With an expiry date and no information sharing, the consumer would be willing to hold enough cash to purchase a single unit during outages if and only if

$$i + \delta \left(1 - \beta^2\right) < \lambda(1 - \delta) \times \left[\nu(1 - \eta) - \left(1 - \eta\beta^2\right)\right]. \tag{8}$$

This condition based on cash with an expiry date is weaker than condition (6) for two reasons. The main reason is the lower cost of cash losses to the consumer, which reduces the cost of holding cash on the left-hand-side of the condition. Before, cash lost would be lost forever. With loss recovery, there is only a cost of a delay during which the consumer cannot access the cash as he has to wait until after the expiry date before the central bank can reimburse her. The second reason is the more subtle impact of putting the onus on the producer to deposit the cash before it expires. The producer will fail to do so if the cash is lost by the producer, in which case the consumer will have the luck of being reimbursed for cash that was lost by the producer. Note from before that the potential of cash losses lead the producer to require higher prices for cash payments. With the expiry date, the consumer has a small chance of being reimbursed without losing cash, which reduces the wedge between the costs of cash and electronic payments to the consumer. The reduction in the wedge increases the marginal benefit of spending with cash during an outage, as shown on the right-hand-side.

With an expiry date and no information sharing, the consumer would be willing to hold enough cash to purchase two units during outages if and only if

$$i + \delta \left(1 - \beta^2\right) < \frac{\lambda}{2}(1 - \delta) \times \left[\nu(1 - \eta) - \left(1 - \eta\beta^2\right) - \frac{i}{1+i}\right]. \tag{9}$$

This condition compares to condition (7) in a very similar manner as the conditions for holding enough cash to purchase a single unit.

In summary, the introduction of an expiry date with the objective of loss recovery has the potential to improve social welfare in an environment without information exchange between the consumer's devices and the central bank. A potential disadvantage of the model with an expiry date but no information exchange between the consumer's device and the central bank is the imprecise nature of loss recovery in that consumers may receive windfall reimbursements after spending cash when producers lose the cash.

3.3 Cash with loss recovery and information exchange

As a potential solution to the imprecise nature of loss recovery without information exchange, we explore an alternative scheme where the consumer's device reveals whether and where cash balances have been spent. The information released by the consumer's device can then be used by the central bank to reimburse the producer for cash she received and lost, rather than causing a windfall profit for the consumer. Whether this improves social welfare depends on the incentives for consumer's to connect their devices to the central bank since they cannot be required to do so.[17]

Suppose the environment is such that the consumer holds only enough cash to purchase a single unit. In this situation, the consumer has no incentive to reconnect after an outage. (The consumer would have spend all his money, so reconnecting only eliminates the probability to get the windfall profit of having the spent money returned when lost by merchants). So, information exchange or no information exchange between the consumer's device and the central bank does not change condition (8) which determines whether consumers carry any cash at all.

Things are different if the environment is such that consumers hold enough cash for two units. In this situation, consumers who spent all their cash still have no incentives to reconnect. However, a consumer who does not spend all his cash may have incentives to reconnect, since depositing unspent cash allows him to earn interest. This also comes at a cost to him as he foregoes the windfall from being reimbursed by the central bank for cash previously spent during an outage

[17] Requiring consumers to reconnect their device before receiving a reimbursement for cash losses defeats the purpose of loss recovery as they cannot connect lost devices.

that was lost by the producer. So, the consumer decides based on trading off foregone interest on unspent cash balances against the luck of getting back spent cash balances.

We first derive the condition under which consumers who did not spent all cash balances would be willing to reconnect to the central bank and, as a consequence, inform the central bank when and where they spend cash. Suppose those consumers who purchased a single unit were to reconnect at $t = 2$, then the lowest cash prices that risk-neutral producers would accept for any unit sold equals

$$p_c^I = \frac{1 - \frac{1}{3}\eta}{1 - \eta},$$

where the one-third in the numerator comes from the fact that the producer will be reimbursed by the central bank for lost cash when the consumers who purchased a single unit connect to deposit their unspent cash. The possibility of reimbursement for lost cash reduces the cost of accepting cash. Given the cash price p_c^I for products, it is optimal for a consumer who bought a single unit and held enough cash to purchase two units to reconnect if

$$i \geq \eta. \tag{10}$$

With information exchange and reconnect condition (10) holding true, then consumers would be willing to hold enough cash to purchase two units during an outage if:

$$i + \delta\left(1 - \beta^2\right) < \frac{\lambda}{2}(1 - \delta) \times \left[(\nu - 1)\frac{1 - \eta}{1 - \frac{2}{3}\eta} - \frac{i}{1 + i}\frac{1 - \frac{1}{3}\eta}{1 - \frac{2}{3}\eta} - \frac{2\beta^2\eta^2}{1 - \frac{2}{3}\eta}\right]. \tag{11}$$

If the reconnect condition (10) holds true, then condition (11) is weaker than condition (9).[18] In other words, individuals are more inclined to hold high cash balances with information exchange when it is optimal for them to re-connect than without information exchange. This comes from the fact that, thanks to the better targeted loss recovery, producers can accept more favorable

[18]This is not immediately clear from the expressions. One can easily derive that the right-hand-side of condition (11) is larger than the right-hand-side of condition (9) by ignoring the fact that the right-hand-side of condition (11) is divided by $(1 - \frac{2}{3}\eta)$ and using $i \geq \eta$ from condition (10).

prices when customers pay with cash. Hence, if customers reconnect condition (10) holds true, then enabling more targeted then enabling better targeted loss recovery through information exchange could improve social welfare.

If the reconnect condition (10) is violated, then consumers would not reconnect because they don't want to forego the possibility of being reimbursed for spent cash balances. In this situation, the cash prices will be unchanged from before, i.e., $1/(1-\eta)$. The average cost of carrying cash balances to consumers increases, because they have an incentive to delay depositing unspent cash balances after an outage. By not reconnecting and depositing unspent cash balances directly after the outage, they have a chance of obtaining cash lost by retailers. Under information exchange, if condition (10) violated, consumers will only hold enough cash to purchase two units of consumption if:

$$i + \delta\left(1-\beta^2\right) < \frac{\lambda}{2}(1-\delta) \times \left[\nu(1-\eta) - \left(1-\eta\beta^2\right) - \frac{i}{1+i}\frac{2+i}{1+i}\right]. \tag{12}$$

The higher cost of carrying cash compared to the case without information exchange is captured in the last element of the inequality which reflects the delay in depositing unspent cash balances, which makes this a stronger condition than the one without information exchange in (9). Hence, information exchange could reduce social welfare if the reconnect condition in (10) is violated.

In summary, the conclusion is that information exchange can make matters both better and worse. This comes from the fact that consumers strategically may chose to avoid reconnecting devices for offline payments in case of information exchange. Doing so increases the effective cost of carrying cash, which may induce consumers to carry less cash compared to the case where there is no information exchange. Finally, regardless of whether information exchange occurs, loss recovery is never fully precise as some consumers have no incentives to reconnect (in the model those who spent all their cash).

4 An Infinite Horizon Model

To better study the impact of an expiry date on the demand for cash and the optimal length of the period before cash expires, it is convenient to consider a model with no limit on the number of periods. Time is discrete as before and periods are numbered $0, 1, 2, ...$ An outage begins at a random date $t \geq 1$. Once the outage is over, no further outages occur. (This assumption allows us to avoid some technical complications arising when successive surprise outages occur with no gaps. Given that outages occur infrequently, this assumption is a reasonable approximation.) Conditional on the outage not having occurred by period $t-1$ the probability that the outage begins in period t is a constant λ. The outage is of stochastic length; g_τ is the probability it lasts τ periods. The length of the outage is revealed at the first period of the outage and known to all agents.

Following Lucas (1982) and King et al. (1992), we assume that every producer, who is in charge of selling, forms a household with a consumer, who is in charge of shopping. Every consumer can use only the offline cash that his partner accumulated in the last normal period. Consumers cannot consume the production by members of their household. In a normal period, agents derive utility $u(x)$ from consuming x units of the numeraire good, which can be produced a at constant marginal cost which is normalized to one. We assume that $u(x)$ is concave and strictly increasing in x and $u(0) = 0$. Agents decide the real value of offline money brought into the next period. To focus on the essence, we assume that post-outage, the real value of offline money is constant over time, i.e., inflation is zero. As in the normal periods, buyers want to consume the good in the outage, but the utility of the consumption of x units depends on the length of the outage. At the beginning of the outage, buyers buy all goods needed from sellers and then consume them during the outage. We assume each period in the outage, buyers consume the same amount.[19] Hence, their indirect utility from consuming x during the outage is $U(x, \tau)$, where

$$U(x, \tau) = u(x/\tau)\frac{1-\beta^\tau}{1-\beta}.$$

[19]This assumption is not necessary but makes computation simpler.

During the outage, any transaction must be facilitated by offline cash due to the lack of double coincidence. Offline cash expires after T periods and, unless it is deposited by another agent, it will be reimbursed to the account of the buyer in the first period after the outage as before. If a seller receives offline cash, then she can deposit it by connecting to the central bank before the cash expires. But if the seller has no opportunity to connect to the central bank before the money expires, then she would not be willing to accept cash. Therefore, no producer is willing to accept the offline money if $\tau \geq T$ in an outage (Figure 2, panel a). If, however, $\tau < T$, there can be trade (Figure 2, panel b). Each period, buyers may lose their devices with probability δ and sellers may lose their devices with probability η. The event of loss occurs at the beginning of each period before any other actions as in the simple model.

We solve the model backwards. First, we consider the first period after the outage. Let a be the value in the buyer's online account, plus the expected value of reimbursements still due to be received for cash losses up to the present, discounted by the wait time until those reimbursements will be made, and let z be the current offline cash holding. Then the buyer's value function is

$$Q(a,z) = (1-\delta)\bar{Q}(a,z) + \delta\bar{Q}(\bar{a},0)$$

where $\bar{Q}(a,z)$ and $\bar{Q}(\bar{a},0)$ are respectively the value functions when the buyer keeps her device or loses her device at the beginning of this period. Suppose the buyer loses the remaining cash balances in the first period after the outage ends (that is, just before reconnection is established), then the buyer will be reimbursed for those losses by the central bank after $\max(T-\tau,0)$ periods, where τ is the realized length of the outage period. So, the discounted value of reimbursements increases by $z\beta^{\max(T-\tau,0)}$ when he loses the cash in the first period after an outage, hence, $\bar{a} = a + z\beta^{\max(T-\tau,0)}$. After the outage, there are no further incentives for the buyer to hold cash because no further outages occur and cash is costly to hold due to discounting and the possibility of a loss.

In a more general formulation it might be necessary to keep track of the entire history of cash losses by the buyer, since reimbursements of different losses could occur in different future periods.

Fortunately, given the quasi-linear utility, only the expected present value of the reimbursement is relevant: actual value functions are equal to those in a scenario where agents get offline money reimbursed immediately after the loss, as long as the reimbursed value equals the expected discounted value of the future reimbursement. This allows us to write

$$\bar{Q}(a,z) = \max_{x,\ell} u(x) - \ell + \beta\bar{Q}(0,0),$$
$$\text{st } x = \ell + z + a.$$

In this formulation, agents are reimbursed the expected discounted value a right away and have no future reimbursement; thus a enters the budget equation but not the continuation value function $\bar{Q}$. Also, notice that agents do not hold offline money after the outage. Therefore, the continuation value function is $\bar{Q}$. An immediate consequence is that $\bar{Q}_a(a,z) = \bar{Q}_z(a,z) = 1$. Therefore, $Q_a(a,z) = 1$ and $Q_z(a,z) = (1-\delta) + \delta\beta^{\max(T-\tau,0)}$.

Now we consider the problems before and during the outage. Let $W(a,z)$ be the value function at the beginning of a period. It equals the sum of the value function of a normal period and the value function of an outage weighted by the probability of an outage:

$$W(a,z) = (1-\lambda)W^n(a,z) + \lambda W^o(a,z),$$

where W^n and W^o are the value functions in a normal period and an outage period, respectively.

The value function of a buyer in a normal period can be written as

$$W^n(a,z) = (1-\delta)\bar{W}^n(a,z) + \delta\bar{W}^n(\bar{a},0),$$

where a buyer's value function is $\bar{W}^n(\bar{a},0)$ if he loses his device and $\bar{W}^n(a,z)$ otherwise. And $\bar{a} = a + \mathbb{E}\beta^{\phi(T)}z$ where $\phi(T)$ is the random time required to get reimbursement of lost cash. It depends on T and is random because it can be impacted by a potential outage, of which the arrival time and length are random. The calculation of the value of $\mathbb{E}\beta^{\phi(T)}$ is given in Appendix B. One can write

$$\bar{W}^n(a,z) = \max_{\hat{z},\ell,x} u(x) - \ell + \beta W(0,\hat{z}),$$
$$\text{st } x = \ell + a + z. \tag{13}$$

This implies that $W^n_a(a,z) = 1$ and $W^n_z(a,z) = (1-\delta) + \delta\mathbb{E}\beta^{\phi(T)}$.

Now move to the value function at the beginning of an outage. One implication of (13) is that we can assume that buyers carry no a into the next period if the outage does not occur, i.e., they use the discounted value of cash waiting for future reimbursement to consume now. Therefore, without loss of generality, we only need to consider the case where $a = 0$ at the beginning of the outage:

$$W^o(0,z) = \sum_\tau g_\tau J^\tau(z) \tag{14}$$

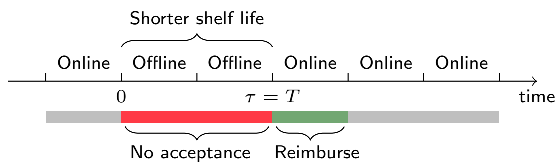
where J_τ is the value function at the beginning of an outage conditional on an outage length of τ periods. If $\tau \geq T$, offline cash expires in the middle of the outage and is reimbursed to the buyer after the outage (Figure 2, panel a). Therefore, no transaction occurs because a seller anticipates that she could not deposit the money before the expiration date. This implies that $J^\tau(z) = \beta^\tau Q(z,0)$. If $\tau < T$, a seller can deposit the offline balances she receives if she does not lose her device (Figure 2, panel b). Therefore, a buyer can buy goods if he does not lose his device. This implies that if $\tau < T$,

$$J^\tau(z) = (1-\delta)\max_{x,p}[U(x,\tau) + \beta^\tau(1-\delta_\tau)\bar{Q}(0,z-p) + \beta^T\delta_\tau\bar{Q}(z-p,0)] + \delta\beta^T z, \text{ st } p \leq z. \tag{15}$$

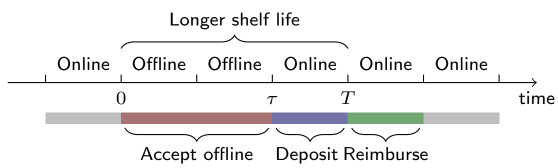
Here $\delta_\tau = 1-(1-\delta)^\tau$ is the probability that a buyer loses his device after trading in the first period of the outage but before he can deposit the cash. This formula follows because a buyer can lose his device with δ probability in each of the remaining $\tau - 1$ outage period and also in the first

Figure 2: Shelf life and outages

Panel (a): Shelf life does not exceed length of outage



Panel (b): Shelf life exceeds length of outage



period after the outage. The buyer decides his consumption x and payment p given that the terms of trade are determined by buyers making take-it-or-leave-it offer.

Sellers lose their devices with probability $\eta_\tau = 1 - (1-\eta)^\tau$ after the trade and before they can deposit the cash. If a seller loses her device, she cannot claim the money because it will be reimbursed to the buyer. The buyer makes the seller indifferent between a trade and no trade, which implies $\beta^\tau(1-\eta_\tau)p = x$. The β^τ captures the fact that sellers use the received funds to consume only after the outage is over. Then the envelope condition of J^τ is

$$\frac{\partial}{\partial z} J^\tau(z) = (1-\delta)(\Lambda(z,\tau) + \beta^\tau(1-\delta_\tau) + \delta_\tau\beta^T) + \delta\beta^T, \tag{16}$$

where

$$\Lambda(z,\tau) = \max\{\beta^\tau(1-\eta_\tau)U_x(\beta^\tau(1-\eta_\tau)z,\tau) - (\beta^\tau(1-\delta_\tau) + \delta_\tau\beta^T), 0\} \tag{17}$$

Then in a normal period before the outage, a buyer brings z^* into the next period, where z^* solves the following equation in z:

$$\begin{aligned} 1 = & \beta\lambda \sum_{\tau=1}^{T-1} g_\tau [(1-\delta)(\Lambda(z,\tau) + \beta^\tau(1-\delta_\tau) + \delta_\tau \beta^T) + \delta\beta^T] \\ & + \lambda \sum_{\tau=T}^{\infty} g_\tau \beta^{\tau+1} + (1-\lambda)\beta[(1-\delta) + \delta\mathbb{E}\beta^{\phi(T)}]. \end{aligned} \tag{18}$$

Because Γ is decreasing in z, the right-hand side of (18) is decreasing in z. Therefore, the solution to this equation is positive and unique if $T > 1$. The equation highlights the trade-offs involved in the choice of T. On the one hand a higher T, allows agents to trade in longer outages, which is reflected by the first summation on the right-hand side of (18). This increases the right-hand side of (18) and encourages the use of cash. On the other hand, it delays reimbursement of the lost offline cash, which is reflected by the terms β^T and $\beta^{\phi(T)}$. This decreases the right-hand side of (18) and discourages the use of cash. As a result, both the optimal cash holdings (z^*) and consumption in the outage may increase or decrease with T. Given z^*, the welfare of the buyer at period 0 is

$$W^n(0,0) = u(x^*) - x^* - z^* + \beta W(0, z^*), \tag{19}$$

where $u'(x^*) = 1$.

5 Quantitative Analysis

Theoretically, offline cash with a longer T enables consumers to consume in more outage states, but consumers also need to wait longer to retrieve it if they lost their device. It remains an empirical question as to how to optimally set T. In this section, we calibrate the infinite horizon model to data and try to provide some insights to this question.

5.1 Calibration

We first parameterize the model. For the functional form of the utility function, we choose the familiar form $u(x) = x^{1-\sigma}/(1-\sigma)$. The distribution of τ follows a Poisson distribution with a parameter γ. Then the model is calibrated by specifying values for the unknown parameters of the model $(\beta, \lambda, \sigma, \delta, \eta, \gamma)$.

We consider a daily calibration. We set β to match an annual discount factor of 0.96. The risk aversion parameter σ determines the demand for offline cash. It is analogous to the curvature of the utility function in the decentralized market in the literature on money search, e.g. Lagos and Wright (2005). Many papers in this literature try to calibrate the curvature of this utility function. And the resulting risk aversion parameter ranges from around 0.2 to more than 0.9. We set σ to be 0.7 in our benchmark calibration, but we also experimented with other values.

Next, we calibrate the loss probability for consumers, δ. To pin down this parameter, we conduct an online survey to estimate the probability of a consumer losing offline digital currency balances that would be locally stored in a payment card (see Appendix A for more details). The results suggest that the annual probability of a consumer losing digital currency balances stored locally in a payment card is around 16%.[20] We calibrate δ to match this probability. We assume that the probability that a seller loses cash stored in her device (η) is the same as that for the buyer. In a sensitivity analysis, we calibrate the model such that the loss probability corresponds to that when digital currency balances were locally stored in a phone and obtain qualitatively similar results (see Appendix C).

Lastly, we calibrate the parameters related to the likelihood and the length of an outage, λ and γ. For the likelihood, we choose λ such that someone is expected to enter into an extended offline

[20] This is higher than the number used by Alvarez and Lippi (2009) to approximate the probability of losing physical crash. They calibrate their model based on the annual probability that someone loses physical cash as a consequence of crime, which was around 2 per cent in Italy in 2002. The difference can be explained by the substantial probability of losing cash as a consequence of chance or carelessness.

Table 1: Calibration

Parameter	Daily value	Annualized level
Discount factor (β)	0.99990	0.96
Risk aversion (σ)	0.70	
Loss probability consumer (δ)	0.0004845	0.162
Loss probability producer (η)	0.0004845	0.162
Outage probability (λ)	0.00061	0.200
– Length: Poisson distribution (γ)	6.375	0.0175

period about once every five years. For the length of the outage, we choose γ such that the offline money balances are well over twice the level of daily spending as $T \to \infty$. The idea is that if $T = \infty$ — that is, if it takes infinitely long before lost cash is returned to the owners — that then the costs of carrying offline digital currency balances in the model should be equivalent to the cost of carrying physical cash. Survey evidence of Greene and Stavins (2020) suggests that, on average, the level of precautionary cash holdings is well over twice the amount of daily spending on purchases.[21] Table 1 summarizes the calibration.

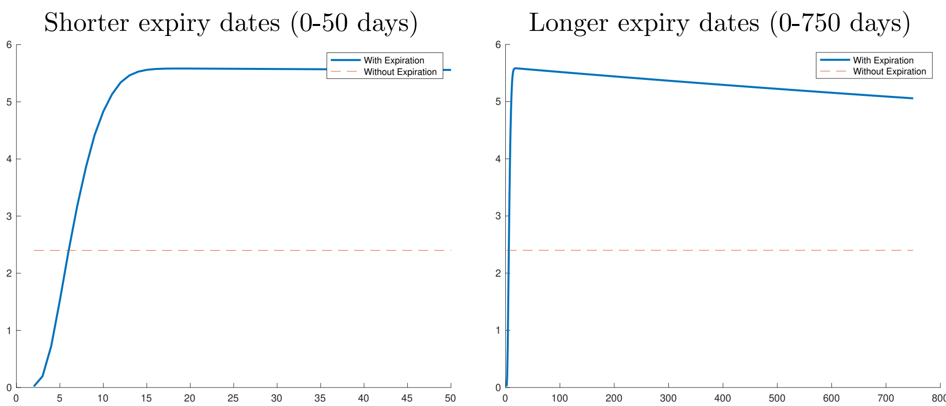
5.2 Effects of the expiry date

We now use the calibrated model to analysis how T affects the demand for offline money balances and the welfare of buyers. Figure 3, panel (a) shows the demand for offline balances as T increases. Recall that a higher T enables consumers to consume in outages that last longer but also makes consumers wait longer to be reimbursed if they lose their device. The former effect increases the demand for offline money while the latter effect reduces the demand. If T is small, the acceptance-during-outages effect dominates because it is very likely that the outage is longer than T. Therefore,

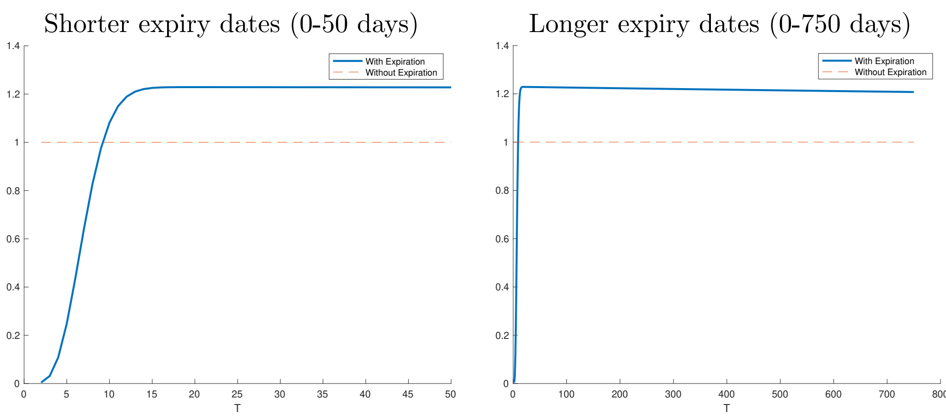
[21] Survey evidence suggests that consumers in the United States spend on average \$50.32 per day on purchases (\$1559.9 divided by 31) while the level of precautionary cash holdings measured as cash held elsewhere is estimated at on average \$120.20 (Greene and Stavins, 2020, Tables 3a and 7). This suggests a ratio of about $120.20/50.32 \approx 2.4$.

Figure 3: Cash holdings with expiry date and privacy

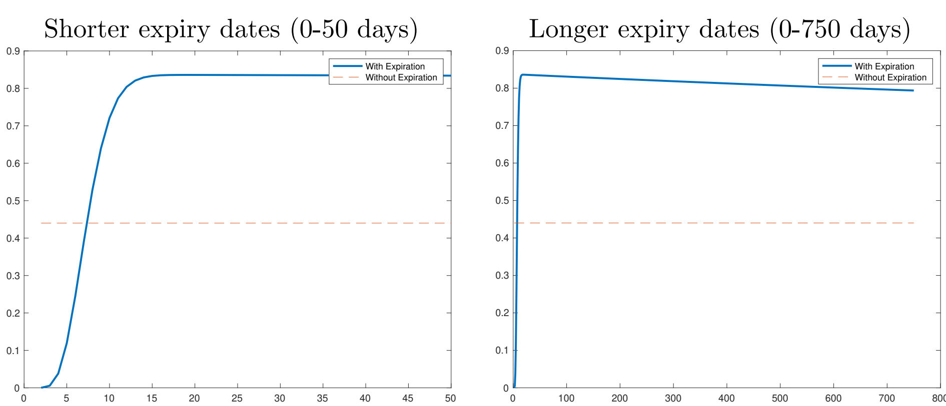
Panel (a): Optimal cash holdings as a function of the expiry date



Panel (b): Daily utility during outages as a function of the expiry date



Panel (c): Daily consumption during outages as a function of the expiry date



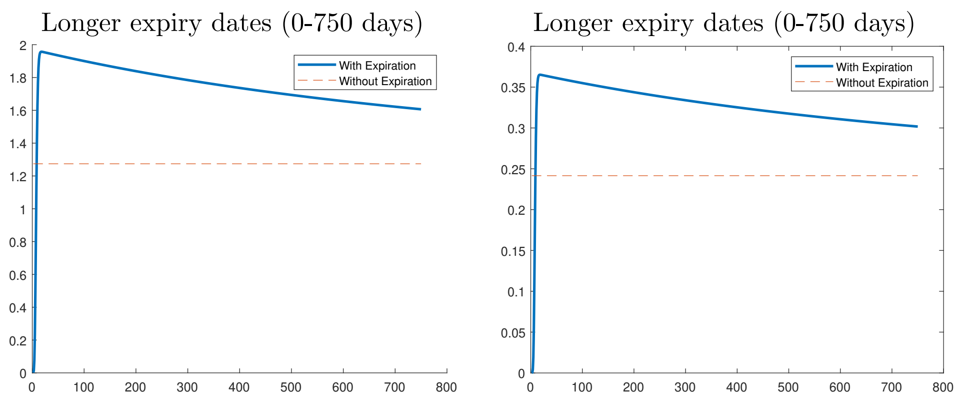
Note: Panel (a) shows the optimal cash holdings as a multiple of daily consumption for different lengths of the expiry dates. Panel (b) measures the expected utility in the outage minus the cost of bringing offline money. Panel (c) measures the expected daily consumption in an outage. The calibration of the model is reported in Table 1.

the demand for offline money increases. But if T is large, the slow-reimbursement effect dominates, and the demand decreases. Notice that when T is small, the demand rises sharply with T, while the demand decreases slowly if T is large. This is because consumption in outage is very valuable and delay in reimbursement is not very costly because agents are patient. The demand for offline money peaks at $T = 19$ days, but this is sensitive to the shape of the distribution function for the length of the offline periods. Importantly, the demand for offline balances is substantially higher with an expiry date and loss recovery. For our specific calibration, the maximum demand is about 136% higher than the demand for offline money when there is no expiry date, i.e., when $T = \infty$. Panel (b) shows consumer welfare measured by the expected daily utility during the outage. We also normalize the welfare without an expiry date to 1. The welfare maximizing expiry date T is 20 days. It increases welfare by 23% compared to the case of physical cash without an expiry date. Similar to the impact on demand, the cost of setting a longer than optimal expiry date is small, while setting an expiry date that is too short has a large negative impact. Lastly, panel (c) presents the expected per period consumption in an outage. Its pattern is similar to that of offline money balances and welfare. At the optimal T, the expected per period consumption is about 0.84 in an outage. It is less than 1, which is the consumption during a normal period. This is because of the cost associated with offline money balances. Compared to the case under $T = \infty$, the optimal T increases consumption during outages by about 90%.

The main insights are not very sensitive to the choice of the risk aversion parameter σ, but quantitative implications can be. Suppose, for example, $\sigma = 0.5$. Then, the γ needs to be set of 9.501 to match the diary data. The values of T that maximizes offline money demand and welfare are 23 and 24, respectively. Compared to the case without an expiry date, the welfare-maximizing T raises offline money holding by 194%, total welfare by around 65.3% and expected per period consumption in an outage by 168%. Again, setting T larger than the optimal value leads to small welfare loss, while setting it smaller than T is very costly.

Figure 4: Expiry date with a low discount factor

Panel (a): Optimal cash holdings *Panel (b): Consumption during outages*



Note: Panel (a) shows the optimal cash holdings as a multiple of daily consumption for different lengths of the expiry dates. Panel (b) measures the expected daily consumption in an outage. The discount factor is set at an annualized level of 0.76. The other parameter values are set as reported in Table 1.

The relative flatness of the optimal cash holdings for longer expiry dates is partially driven by the choice for the discount factor. Our baseline calibration with an annualized level of 0.96 implies that the waiting cost of the consumer for a refund after two years equals about 8 per cent of the lost balance. One may argue that this calibration does not reflect scenarios where consumers are financially-constrained or the situation in some developing countries with high inflation rates. As an alternative, we set the discount factor at the more extreme annualized level of 0.76 – which implies that the consumer's waiting cost for a refund after two years equals about 42 per cent of the lost balance – while keeping all other parameter values identical to those in Table 1. Figure 4 summarizes the results for this alternative calibration. The main difference with the baseline results is that the levels of the optimal cash holdings and consumption during outages with expiry date (blue lines) converge more quickly to their levels without expiry date (dashed lines). In other words, it becomes more costly to set an expiration date that is longer than optimal, so the discount factor is an aspect that the policy maker would need to take into account. That said, the asymmetry in deviations from the optimal expiry date remains because optimal cash holdings and consumption during outages converge to zero as the expiry date tends to zero on the left side of the chart.

6 Conclusion

The robustness of physical cash as a means of payment comes at a cost: it is essentially impossible for a user of cash to convincingly demonstrate to the issuer that it has been lost and should be replaced. In this paper we argue that central bank digital currency can be designed to improve on physical cash–combining offline robustness with loss recovery–by including an expiry date, automatically renewable whenever the holder is online.

We have provided a simple model of the process, and used it to examine the incentive issues entailed by such an arrangement. While a facility for recovering lost cash would be welfare improving, the details of its design matter. Increasing the information shared between consumers and the central bank in the loss recovery process could discourage consumers to carry cash.

We have also provided a more complicated dynamic model of outages and cash loss, one amenable to calibration. Our results show that including provision for loss recovery through expiry dates can have a significant welfare effect during outages, although it is clear that these calculations are only a first step in such an analysis. We have also examined the question of the optimal expiry date, and shown that the benefits are asymmetric: small upward deviations from the optimal duration have only minor welfare effects, while small deviations downward can entail major welfare losses. The preliminary conclusion then is that while a facility for limiting the life of offline CBDC is a desirable part of the design, given the inherent uncertainties it will be safest to make the offline CBDC relatively long-lasting.

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Appendix

A Survey

We obtain a rough estimate of the probability of individuals losing offline digital currency balances based on two single-question online surveys. We do so for two potential modes to store offline digital currency balances: when offline digital currency balances would be stored in a secure element in a payment card and when offline digital currency balances would be stored in a secure element in a smart phone. In either case, we will presume that the devices require some form of user authentication (e.g., a pin code or unlocking the device), so that the balances cannot be spend by others when the device is stolen or lost. The survey questions and the responses are reported in Table 2.

Our service provider is Google Surveys. The responses are provided by users on websites in the Google Surveys publisher network, who are asked to fill out a survey before they can continue reading the content they would like to view (a so-called "survey wall"). Methodological details are provided by Sostek and Slatkin (2018). Generally spoken, the service provider implements several mitigation strategies to deal with response biases and provides weights to weight responses by age, gender and region. Santoso et al. (2016) provide a relatively positive assessment of the service for academic research in social science, albeit with some cautions. One concern they identify is the potentially less substantive engagement of respondents when facing a survey wall. We include both a "Don't know" option and a "Don't want to answer" option as potential responses to our survey questions in order to mitigate the risk of this impacting our outcomes. We find that the percentage of respondents who choose one of these options in our surveys is comparable to the percentage of responses for the "Don't know" option observed in the assessment of Google Surveys by (Santoso et al., 2016, p. 364).

Table 2: Survey questions and response rates

Panel (a): Over the past 12 months, did you replace or cancel a payment card (for example, a debit or credit card) because it was damaged, physically stolen or lost?

Answer:	Canada: Weighted	Canada: Unweighted	United States: Weighted	United States: Unweighted
"No"	63.7%	60.6%	58.4%	59.2%
	(-2.6%, +2.5%)	(-2.2%, +2.1%)	(-2.9%, +2.8%)	(-2.2%, +2.1%)
"Once"	8.0%	7.1%	8.6%	8.2%
	(-1.3%, +1.6%)	(-1.1%, +1.2%)	(-1.5%, +1.8%)	(-1.1%, +1.3%)
"Twice, or more"	1.8%	1.6%	2.7%	2.2%
	(-0.6%, +0.8%)	(-0.5%, +0.7%)	(-0.8%, +1.1%)	(-0.6%, +0.8%)
"Don't know"	4.2%	4.3%	5.1%	5.1%
	(-0.9%, +1.2%)	(-0.8%, +1.0%)	(-1.1%, +1.4%)	(-0.9%, +1.1%)
"Don't want to answer"	22.3%	26.3%	25.2%	25.1%
	(-2.1%, +2.3%)	(-1.9%, +2.0%)	(-2.4%, +2.6%)	(-1.9%, +1.9%)
Respondents	1,376	2,001	1,146	2,001

Panel (b): Over the past 12 months, was your smart phone stolen, permanently lost, or broken so that you could no longer start it?

Answer:	Canada: Weighted	Canada: Unweighted	United States: Weighted	United States: Unweighted
"No"	70.2%	67.1%	71.6%	72.7%
	(-2.4%, +2.3%)	(-2.1%, +2.0%)	(-2.7%, +2.6%)	(-2.0%, +1.9%)
"Once"	4.4%	4.5%	5.7%	4.0%
	(-1.0%, +1.2%)	(-0.8%, +1.0%)	(-1.2%, +1.5%)	(-0.8%, +1.0%)
"Twice, or more"	2.4%	2.7%	0.9%	1.2%
	(-0.7%, +0.9%)	(-0.6%, +0.8%)	(-0.4%, +0.7%)	(-0.4%, +0.6%)
"Don't know"	2.9%	3.5%	2.4%	3.2%
	(-0.8%, +1.0%)	(-0.7%, +0.9%)	(-0.8%, +1.1%)	(-0.7%, +0.9%)
"Don't want to answer"	20.1%	22.2%	19.4%	18.8%
	(-2.0%, +2.2%)	(-1.8%, +1.9%)	(-2.2%, +2.4%)	(-1.7%, +1.8%)
Respondents	1,419	2,001	1,118	2,001

Note: The table reports the responses for two single-question surveys held in both Canada and the United States. Responses are provided by users of websites included in the Google Surveys publisher network over the period from 5 until 29 May 2021. The weighted responses weigh respondents by age, gender and region, and assign a zero weight to respondents for which this information is not fully available (hence, the higher count of respondents for the unweighted responses). The table reports the 95 per cent confidence intervals using the modified Wilson method (Brown et al., 2001) in parenthesis.

The responses in Canada and the United States are generally quite comparable, as are the unweighted and weighted responses. Based on the weighted responses in the United States, the fraction of respondents who would not have lost stored-value in a payment card based on our survey question is estimated to be about $0.584/(0.584 + 0.086 + 0.027) \approx 0.837$ on an annual basis, which corresponds to an annual loss probability of 16.3%. For stored-value in a phone, the corresponding estimate is about $0.716/(0.716 + 0.057 + 0.009) \approx 0.916$ on an annual basis, which corresponds to an annual loss probability of about 8.4%. These are the loss probabilities that are used for the baseline calibration and the calibration of the robustness check. For Canadians, the estimated annual loss probabilities are respectively 13.3% for stored-value in a card and 8.8% for stored-value in a phone. The pattern in both jurisdictions is that correspondents are less likely to lose stored-value in phones, potentially due to features that allow them to locate their devices when lost.

B Value of recovery with stochastic outage length

This appendix derives the value of $\mathbb{E}\beta^{\phi(T)}$, which is the expected value of loss recovery for a single dollar to the consumer as a function of the expiry date in an environment where the occurrence and the length of the outage are stochastic. The value of loss recovery is not straightforward because outages with stochastic length introduce uncertainty around the moment when the consumer can access recovered funds. Three different scenarios may materialize that need to be accounted for: (i) no outage may occur until the moment of recovery, (ii) an outage may occur that ends before the moment of loss recovery, and (iii) an outage may start before the moment of recovery but may continue until after the moment of recovery. The probabilities in the equation related to these scenarios are indicated below the equation

$$\mathbb{E}\beta^{\phi(T)} = \beta^{T+1}\left(\underbrace{(1-\lambda)^{T+1}}_{\substack{\text{No outage}\\ \text{until } T+1\\ \text{(inclusive)}}} + \underbrace{\sum_{s=1}^{T}\sum_{t=1}^{T+1-s}\lambda(1-\lambda)^{s-1}g(t)}_{\substack{\text{Probability of outage}\\ \text{between now and } T+1\\ \text{that ends before } T+1}}\right) + \sum_{i=1}^{\infty}\beta^{T+1+i}\left(\underbrace{\sum_{s=1}^{T+1}\lambda(1-\lambda)^{s-1}g(T+1-s+i)}_{\substack{\text{Probability of outage}\\ \text{between now and } T+1 \text{ (inclusive)}\\ \text{that ends at } T+i}}\right).$$

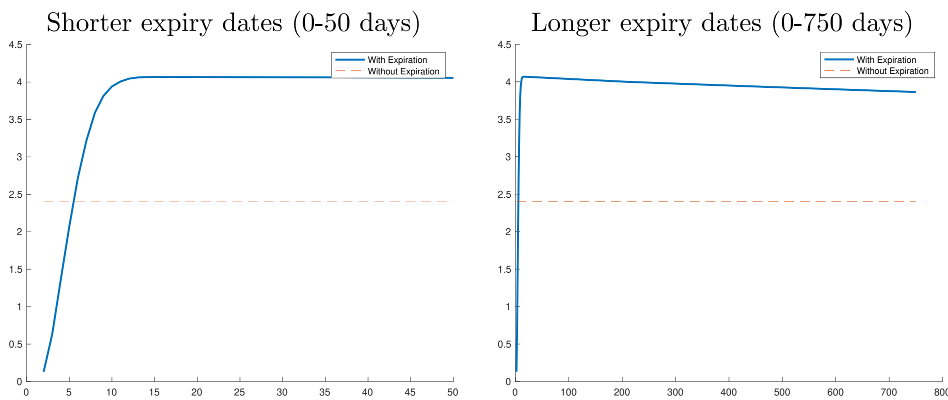
In scenarios (i) and (ii), the value of a recovered dollar simply equals β^{T+1}. In scenario (iii), the value of a recovered dollar depends on when it can accessed by the consumer (i.e., β^{T+1+i} for access at $t = T + 1 + i$). The function calculates the expected value of loss recovery by summing the product of β^{T+1+i} and the probability that an outage starts before $T + 1$ (inclusive) and ends at $t = T + 1 + i$ for each $i = 1, 2, ..., \infty$.

C Sensitivity Analysis

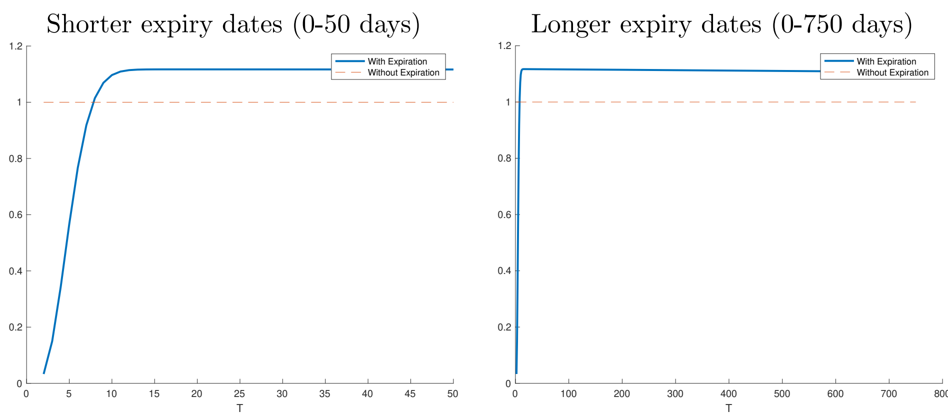
According to our online survey, the probability of losing value stored in a secure element in a smart phone is around 8.44%, which is lower than the probability of losing value stored in a payment card. In this appendix, we analyze the sensitivity of our results to a lower probability of losing digital cash. If the off-line money is store on a secure chip of the cellphone, a buyer loses offline balance daily with a probability of $\delta = 2.42 * 10^{-4}$. We again assume that $\delta = \eta$ and re-calibrate the model. We then obtain $\gamma = 4.47$ for $\sigma = 0.7$. The results are shown in Figure 5. The qualitative results are similar to those obtained using the baseline calibration. The value of T that maximizes money holding is 16 days, while the one that maximizes welfare is 17 days. Compared to the case without an expiry data, the optimal T increases money holding by about 70%, daily consumption in an outage by 40% and daily utility during outages by 12%.

Figure 5: Cash holdings with expiry date and privacy: Sensitivity to lower loss probability

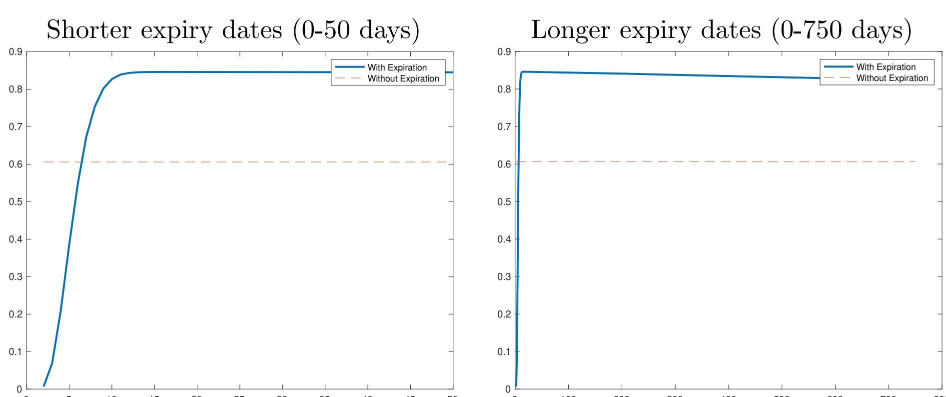
Panel (a): Optimal cash holdings as a function of the expiry date



Panel (b): Daily utility during outages as a function of the expiry date



Panel (c): Daily consumption during outages as a function of the expiry date



Note: Panel (a) shows the optimal cash holdings as a multiple of daily consumption for different lengths of the expiry dates. Panel (b) measures the expected utility in the outage minus the cost of bringing offline money. Panel (c) measures the expected daily consumption in an outage.