Disclosing to informed traders*

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Abstract

We develop a model of costly voluntary disclosure in the presence of diversely-informed investors. The manager's disclosure strategy influences trading by investors, which in turn affects the manager's incentives to disclose. When the manager is known to be informed, there exists a unique threshold equilibrium in which only sufficiently good news is disclosed. This equilibrium exhibits two novel features. First, more public information can increase the likelihood of voluntary disclosure. Second, the firm is either over- or under-valued relative to fundamentals, depending on how investors use the information in prices. When investors are uncertain about whether the manager is informed and investors' information is sufficiently precise, this threshold equilibrium may break down.

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1 Introduction

Market prices reflect the interaction of two distinct sources of information: strategic disclosures by firms and private information dispersed across investors. The existing theoretical literature often focuses on one of these dimensions while abstracting from the other. Traditional models of voluntary disclosure restrict investors to (common) public information, while models of trading by informed investors either ignore disclosures or assume they are exogenous and non-strategic.[1] Yet, understanding how these two sources interact is important, especially for empirical and policy analysis. How does a firm's strategic disclosure decision depend on the public and private information available to investors, and how they interpret this information? And how well do prices reflect fundamentals in this case?

To answer these questions, we develop a model of costly voluntary disclosure in which a firm's price is determined through trade among privately-informed risk-averse investors and noise traders. The firm's incentives to disclose are lower in the presence of informed trading since the price reveals cash flow information even when there is no disclosure by the firm. We show that this implies that more ex-ante public information can *increase* voluntary disclosure, in contrast to the common intuition that mandatory disclosure "crowds out" voluntary disclosure. Moreover, the firm's strategy of withholding bad news affects how the price aggregates investor information. We show that the firm is "misvalued" relative to the expected value of its cash flows, and the degree of over- or under-valuation depends on how investors use the information in prices to update their beliefs.[2] Importantly, this misvaluation might be larger when investors rationally use the information in prices than when they ignore it.

[1] See the disclosure literature following Verrecchia (1983) and Dye (1985) for examples of the former, and literature on noisy rational expectations and difference of opinions for examples of the latter (e.g., Miller (1977), Hellwig (1980), Diamond (1985)). As we elaborate in Section 2, there are a small number of papers that explicitly model the interaction of endogenous firm disclosures and informed trading by investors (e.g., Goldstein and Yang (2019), Schneemeier (2019), Yang (2020), Cianciaruso, Marinovic, and Smith (2020)), when firms can commit (ex-ante) to a disclosure policy. We study verifiable disclosure without commitment.

[2] As we further in Section 3.1, the misvaluation does not arise from a traditional risk-premium because we assume the firm's stock is in zero net supply and so cash flows are effectively idiosyncratic.

Section 3 presents the model. The firm's manager is privately informed about its future cash flows, which are normally distributed. A claim to these cash flows (stock, or equity, of the firm) is traded by noise traders and informed investors with CARA utility. Each investor observes a conditionally independent, normally-distributed signal about the cash flows, which is noisier than the manager's information. Instead of imposing common priors on the joint distribution of signals and fundamentals, we allow each investor to "agree to disagree" about the informativeness of others' signals. As such, our model nests both the rational expectations benchmark (e.g., Hellwig (1980)), where investors correctly condition on the information in prices, and the difference of opinions benchmark (e.g., Miller (1977)), where investors dismiss the information in prices.

In Section 4, we show that there exists a unique equilibrium in which the manager discloses only when the firm's value exceeds a threshold. To do so, we first characterize the firm's price in such an equilibrium. When the manager discloses, the firm's price is fully determined by the disclosure, which is a sufficient statistic for investors' private information. When the manager does not disclose, the firm's price is determined by the trading behavior of the investors. Since investors infer that the manager observed news that is below the threshold for disclosure, their conditional beliefs about the firm's cash flows are no longer normal. While this rules out a "linear" equilibrium as in traditional models (e.g., Hellwig (1980)), we show that there exists a unique non-linear equilibrium price that reflects the firm's value with noise.[3] Even though the non-disclosure price partially reflects the manager's information, the manager strictly prefers to disclose when the cash flows are sufficiently high. This leads to the existence of a threshold disclosure equilibrium, as in traditional models of voluntary disclosure (e.g., Verrecchia (1983), Dye (1985)).

[3]Specifically, we build on the techniques developed by Breon-Drish (2015) to show that the price given non-disclosure is instead a concave function of the firm's cash flows and noise trade. The Online Appendix of Breon-Drish (2015) shows that, subject to some non-singularity conditions, there exists a unique equilibrium in which the price is a continuous, monotonic, generally non-linear function of a noisy, linear signal about fundamentals. This allows us to characterize how each investor uses the information in their private signal and the price to update their beliefs about the stock, and trade on this information. We abstract from equilibria with discontinuous prices as those considered by Pálvölgyi and Venter (2015).

The key feature that distinguishes our analysis from existing models is that the non-disclosure price aggregates dispersed private information across investors. Section 5 highlights two novel implications that arise as a result. First, we find that ex-ante public information can encourage voluntary disclosure. In standard voluntary disclosure models, ex-ante public information reduces the net benefit of disclosure because it decreases uncertainty over the firm's value. While this effect is also present in our model, there is a second, offsetting effect. When ex-ante public information is more precise, investors trade less intensely on their private information, which makes the non-disclosure price less sensitive to fundamentals and increases the net benefit from disclosure. We show that the second effect dominates when disclosure costs are sufficiently high and private information is sufficiently precise. In this case, public information provision crowds in additional voluntary disclosure.

Second, we show that the non-disclosure price generically exhibits over- or under-valuation relative to the conditional expectation of cash flows, given no disclosure, even when the manager's information is idiosyncratic. This finding results from the interaction between the strategic disclosure and trade on private information: existing models that focus on only one of these aspects while abstracting from the other imply that idiosyncratic cash flows are priced at their expected value. The wedge between the price and the conditional expectation arises because, conditional on non-disclosure, the value of the firm is truncated above.[4] On the one hand, this implies that investors who receive more optimistic signals perceive less uncertainty and trade on their information more aggressively, pushing prices up on average. On the other hand, the upper bound on the firm's value also implies that its price is bounded from above so that noise-trader purchases have a reduced impact on the price relative to noise-trader sales, which pushes prices down on average.

Whether the firm is over-valued or under-valued depends on the relative magnitude of these forces, which, in turn, depends on the extent to which investors condition on the

[4] As we discuss in the next section, our results are related to the wedge that Albagli, Hellwig, and Tsyvinski (2021) derive in a rational expectations setting. Our results imply that the sign of the wedge (i.e., whether there is over- or under-valuation) depends not only on the asymmetry in payoffs, but also on how investors process the information in prices.

information in prices. When investors condition on prices, noise trading not only affects the quantity of shares that informed investors need to absorb, but also their beliefs about the payoffs. As a result the impact of noise trading on the price is amplified, which leads to under-valuation on average. At the other extreme, when investors dismiss the information in prices and noise-trading volatility is not too large, the stock is over-valued.

In our benchmark analysis, the manager incurs a cost of disclosure but is commonly known to be informed about cash flows (e.g., Verrecchia (1983)). In Section 6, we explore how our analysis changes when investors face uncertainty about whether the manager is informed (e.g., Dye (1985)). We show that the analogous threshold equilibrium exists when the information available to investors (via private signals and price information) is not overly precise. In this case, the impact of public information on voluntary disclosure and our findings concerning over- and under-valuation are qualitatively similar to the benchmark case when disclosure costs are significant. However, when disclosure costs are low, the firm may become over-valued even when investors exhibit rational expectations.

In contrast, when investor information is sufficiently precise, the threshold equilibrium may break down. Intuitively, the existence of a threshold equilibrium requires that the non-disclosure price is not overly sensitive to the underlying fundamentals. While this is always true when the manager is known to be informed (as in our benchmark), we find this may not be true when investors are uncertain about the manager's information endowment and their information about cash flows is sufficiently precise. In this case, small changes in fundamentals can lead to large changes in investor beliefs.[5] For a conjectured disclosure threshold, this can imply that the net benefit from disclosure is hump-shaped in firm value

[5]Specifically, consider a candidate equilibrium in which an informed manager discloses information beyond a threshold T and investor signals are very informative. While signals are informative about whether the manager is informed, the sensitivity depends on how far they are from the threshold. For signals far below T, changes in signals are not very informative about whether or not the manager is informed. For signals sufficiently above T, investors are very confident that the manager is uninformed, so again beliefs are not very sensitive to changes in signals. However, the posterior likelihood of whether the manager is informed is extremely sensitive to changes in signals around the threshold T. (As an extreme, consider the limiting case as signals become infinitely precise: in this case the uncertainty about whether the manager is informed is revealed perfectly as signal changes from just below T to just above T.).

near the threshold, which then leads to non-existence of a threshold equilibrium.

Empirical and policy implications

Our results have important implications for empirical analysis and regulatory policy. Regulators often motivate disclosure requirements as means to mitigate adverse selection across investors and "level the playing field." A standard critique of such policies is that they "crowd out" voluntary disclosure by firms (e.g., Verrecchia (1990)). However, existing empirical evidence on the relation between public information and voluntary disclosures is mixed.[6] Our analysis helps reconcile this evidence and clarifies how the impact of regulatory changes varies across firms. We show that increases in mandatory disclosures are most likely to improve overall transparency for firms that face higher disclosure costs and greater adverse selection (i.e., investors' private information is very precise) by encouraging more voluntary disclosure. As such, focusing on the *average* effect of such regulations across all firms may be misleading for policy analysis.

Empirical studies often interpret pricing errors as evidence of informational frictions, but treat firm disclosures as exogenous.[7] However, given that a large fraction of the information released by firms has a voluntary component (e.g., see Beyer, Cohen, Lys, and Walther (2010)), our analysis recommends caution. In our model, stocks may appear to be mispriced even when investors process all available information efficiently. And we show that average pricing errors may be lower when investors (incorrectly) dismiss the information in prices rather than accounting for it correctly.

Our analysis also sheds light on the negative, cross-sectional relation between idiosyn-

[6] While some papers suggest that firms increase voluntary disclosures to mitigate reductions in external information quality (e.g., see Balakrishnan, Billings, Kelly, and Ljungqvist (2014), Guay, Samuels, and Taylor (2016), and Barth, Landsman, and Taylor (2017)), others argue that public information and voluntary disclosures are positively correlated (e.g., Francis, Nanda, and Olsson (2008), Ball, Jayaraman, and Shivakumar (2012), and Bischof and Daske (2013)).

[7] Given a terminal cash flow $\tilde{v}$ and equilibrium price P, we refer to $\mathbb{E}[(\tilde{v}-P)^2]$ as the average pricing error. In our analysis, we assume the stock is in zero net supply and information is firm specific (i.e., idiosyncratic). These assumptions facilitate the interpretation of $\tilde{v}-P$ as a pricing error, or (risk-adjusted) abnormal return, as is commonly estimated in the empirical literature.

cratic return skewness and expected returns (e.g., see Jiang, Xu, and Yao (2009), Conrad, Dittmar, and Ghysels (2013), Boyer and Vorkink (2014)). This relation is difficult to reconcile in traditional models, and existing theories often assume that investors have a preference for such skewness.[8] Our analysis uncovers a novel explanation. As discussed above, the firm is under-valued and thus earns higher average returns following non-disclosure when investors exhibit rational expectations. Moreover, since good news is disclosed but bad news is not, returns are more negatively skewed following non-disclosure. To the extent that voluntary disclosures drive significant variation in skewness across firms and over time, this implies a negative relation between average returns and skewness.

2 Related Literature

Our paper contributes to two strands of literature: models of voluntary disclosure and models of heterogeneously-informed investors. The literature on voluntary disclosure, starting with Jovanovic (1982), Verrecchia (1983), and Dye (1985), typically models financial markets in a stylized manner, assuming that investors are uninformed, risk neutral, or both.[9] Notable exceptions are Bertomeu, Beyer, and Dye (2011), Petrov (2016) and Einhorn (2018) who also study the interaction between informed trade and disclosure. Bertomeu et al. (2011) and Petrov (2016) analyze settings in which there is a single risk-neutral informed trader, while Einhorn (2018) considers trade based on private information only when non-disclosure is completely uninformative.

The literature on heterogeneously-informed investors has evolved along two directions. The standard paradigm has been the noisy rational expectations approach (e.g., Hellwig (1980), Admati (1985)), which assumes that investors share common priors on the joint

[8] In standard settings, idiosyncratic skewness should be diversifiable, and so irrelevant for expected returns. As we discuss in Section 5.2, existing explanations for the negative relation (e.g., Mitton and Vorkink (2007), Barberis and Huang (2008)) assume that (some) investors have a preference for positive skewness and so over-value "lottery-like" stocks.

[9] Examples of voluntary disclosure models with risk-averse, but uninformed traders include Verrecchia (1983), Cheynel (2013), Jorgensen and Kirschenheiter (2015), and Dye and Hughes (2018).

distribution of signals and fundamentals, and so correctly incorporate the information in prices. However, a growing literature has explored the implications of allowing for differences of opinion (e.g., Miller (1977), Morris (1994), Kandel and Pearson (1995), Scheinkman and Xiong (2003), Banerjee and Kremer (2010)), where investors "agree to disagree" about the informativeness of others' signals. In the limit, this can lead investors to dismiss all the information in prices. Building on the approach in Banerjee (2011), our model nests these two possibilities as special cases by allowing for sufficiently flexible subjective beliefs. Our results highlight that how investors condition on the information in prices has qualitatively important effects on disclosure choices and equilibrium outcomes.[10]

Analysis of disclosure in this literature has largely focused on either non-strategic disclosure or settings in which the manager can commit, ex-ante, to a public signal with chosen precision (see Goldstein and Yang (2017) for a recent survey). For example, Diamond (1985), Kurlat and Veldkamp (2015), and Banerjee, Davis, and Gondhi (2018) study how public signals influence the acquisition of private information, and Goldstein and Yang (2019) analyze how disclosure on different components of firm value influences real efficiency. Yang (2020) considers a setting where firms choose whether to disclose information before learning information from asset prices that inform their production decisions. Schneemeier (2019) studies how firms can use ex-ante disclosure policies to direct investor attention towards their firm. Cianciaruso et al. (2020), following the Bayesian persuasion tradition, study a setting where the firm maximizes the expected price by designing ex-ante a signal that investors will publicly observe prior to trading the stock.

To the best of our knowledge, our paper is the first to study voluntary disclosure to a market of heterogeneously-informed, risk-averse investors when the manager cannot commit to a disclosure strategy ex-ante. A key step is to allow investors to learn from prices in an environment where the price does not have a standard "linear-normal" form. We build on

[10] As such, our analysis also has implications for settings where investors dismiss the information in prices due to other reasons, including "cursedness" (e.g., Eyster, Rabin, and Vayanos (2018)), costly price information (e.g., Vives and Yang (2020)) and "wishful thinking" (e.g., Banerjee, Davis, and Gondhi (2019)).

the insights of Breon-Drish (2015) to overcome this challenge: as in his paper, we show that there exists a unique equilibrium in which the price is a generalized linear function of a noisy signal about fundamentals.[11]

Our analysis relates to the theoretical literature on the relationship between prior public information (e.g., mandatory disclosure or analyst reports) and voluntary disclosure. The common intuition in the existing literature (e.g., Verrecchia (1990), Bertomeu, Vaysman, and Xue (2019)) is that these two types of information are substitutes, especially when they are concerned with the same underlying fundamental shocks.[12] Our analysis suggests that these two types of information may instead be complementary when investors are privately informed. The existing literature has documented economically distinct channels through which prior public information and voluntary disclosure may be complements. Friedman, Hughes, and Michaeli (2020a,b) show that these information sources may be complements when firms experience a discrete gain should investors' expectations exceed a cutoff. Einhorn (2005) find that certain correlation structures between public information and voluntary disclosure lead them to be complements, and Frenkel, Guttman, and Kremer (2020) find that disclosure by an external party may crowd in firm disclosure when the external party and the firm possess information with correlated probabilities.

Our finding that the firm may be over- or under-valued appears similar to existing results in the literature, but the underlying economic mechanism is distinct. For instance, Miller (1977) shows that prices exhibit over-valuation when investors exhibit differences of opinion and are subject to short-sales constraints. Banerjee (2011) establishes that in a dynamic model, expected returns are positively related to disagreement in a rational expectations equilibrium, but negatively related to disagreement in a difference of opinions model. In our model, since disagreement is driven by private information across investors, a similar

[11] Other papers that have considered rational expectations equilibria with non-linear prices include Banerjee and Green (2015), Albagli et al. (2021), Chabakauri, Yuan, and Zachariadis (2017), Glebkin (2015), Smith (2019), and Glebkin, Malamud, and Teguia (2020).

[12] As Goldstein and Yang (2017) and Goldstein and Yang (2019) point out, this may not be the case if the two sources of information are about different components of payoffs.

relation between expected returns and disagreement obtains, but in the absence of short-sales constraints and in a static model.[13]

Our results are more closely related to those in Albagli et al. (2021) and Chabakauri et al. (2017), who show that a wedge between expected price and expected dividends can arise due to asymmetry in dividend payoffs. However, their analysis takes the distribution of payoffs as (exogenously) given, and assumes that all investors exhibit rational expectations. Our analysis, which we view as complementary to theirs, highlights a natural and economically important source of *endogenous* asymmetry in payoffs that results from voluntary disclosure. Moreover, we extend the analysis to allow investors to exhibit differences of opinions about the information of others, and show that holding fixed the asymmetry in payoffs, whether the firm is over- or under-valued depends on how investors process the information in prices.

3 Model Setup

Our model considers verifiable disclosure (e.g., Jovanovic (1982), Verrecchia (1983), Dye (1985)) in a market with privately informed investors (e.g., Hellwig (1980)).

Payoffs. Investors trade in both a risky and a risk-free security. The gross return on the risk-free security is normalized to 1. The risky security is the stock of a firm, which pays of a terminal dividend $\tilde{v}$ that is normally distributed with mean m and variance σ_v^2, i.e., $\tilde{v} \sim N(m, \sigma_v^2)$. We normalize the mean to zero (i.e., $m = 0$) without loss of generality. We assume that there are noise/liquidity traders who submit demands of $\tilde{z} \sim N(0, \sigma_z^2)$. We further assume that the firm is in zero net supply.

Preferences and Information. There is a continuum of investors indexed by $i \in [0, 1]$. Each investor i is endowed with initial wealth W_0 and zero shares of the stock, and exhibits

[13] In contrast to Banerjee (2011), the relation between disagreement and returns is not driven through the risk-premium in our model, since the aggregate supply of the asset is assumed to be zero.

CARA utility with risk-tolerance τ over terminal wealth W_i, where:

$$W_i = W_0 + D_i(\tilde{v} - P),$$

and D_i denotes his demand for the stock. Investor i observes a private signal $\tilde{s}_i$ of the form:

$$\tilde{s}_i = \tilde{v} + \tilde{\varepsilon}_i, \tag{1}$$

where the error terms $\tilde{\varepsilon}_i \sim N(0, \sigma_\varepsilon^2)$ are independent of all other random variables.

Subjective Beliefs. We allow for a flexible specification of subjective beliefs about the private information of others. Following Banerjee (2011), we assume that investor i's beliefs about his own signal is given by (1), but his beliefs about investor j's signal are given by:

$$\tilde{s}_j =_i \rho\, \tilde{v} + \sqrt{1-\rho^2}\, \tilde{\xi} + \tilde{\varepsilon}_j, \tag{2}$$

where the random variables $\tilde{\xi} \sim_i N(m, \sigma_v^2)$ and $\tilde{\varepsilon}_j \sim_i N(0, \sigma_\varepsilon^2)$ are independent of all other random variables and each other, and $\rho \in [0, 1]$ parametrizes the difference in opinions.

The above specification provides a tractable way to nest two natural benchmarks. When $\rho = 1$, investors exhibit rational expectations (as in Hellwig (1980)): this is equivalent to assuming that all investors share common priors about the joint distribution of fundamentals and signals. In this case, investors condition on the information in prices (in addition to their private information) when updating their beliefs about fundamentals. When $\rho = 0$, investors exhibit "pure" differences of opinion (as in Miller (1977)): each investor believes no other investor has payoff relevant information, and so prices are not incrementally informative about payoffs.[14] Moreover, when $\rho \in (0, 1)$, investors are partially dismissive of the information content of others' private signals. Given these subjective beliefs, investors take into account the information contained in the firm's price when determining their demands.

[14]This is analogous to the subjective beliefs of investors in other difference of opinions models (e.g., Scheinkman and Xiong (2003)) and in the "cursed equilibrium" of Eyster, Rabin, and Vayanos (2015)).

In what follows, we denote the subjective beliefs of investor i by $\mathbb{E}_i[\cdot]$ and $\text{var}_i[\cdot]$, and the objective expectation and variance by $\mathbb{E}[\cdot]$ and $\text{var}[\cdot]$, respectively.

Disclosure Decision. Prior to trade, the firm's manager privately observes $\tilde{v}$ and chooses whether or not to verifiably disclose this information to the market, subject to a private cost $c \geq 0$ on the manager. The manager has rational expectations about the joint distribution of $\{\tilde{s}_i, v\}$, and their disclosure choice maximizes their expectation of the equilibrium price net of disclosure costs.

3.1 Discussion of assumptions

Our benchmark analysis makes a number of simplifying assumptions for analytical tractability.

Disclosure frictions. Our benchmark model assumes the existence of disclosure costs (e.g., Jovanovic (1982) and Verrecchia (1983)) to prevent unravelling. While we assume the disclosure cost is privately incurred by the manager to simplify the exposition, our results are unchanged if the cost is instead incurred by the firm. In Section 6, we extend the analysis to allow for uncertainty over the discloser's information endowment (as in Dye (1985)). Specifically, in this case, we assume that the manager observes $\tilde{v}$ with probability $p \in [0, 1]$, and assume that at least one of $c > 0$ or $p < 1$ holds in order to prevent a trivial "unravelling" equilibrium. Moreover, we assume that if the manager does not learn $\tilde{v}$, they are unable to credibly convey their lack of information to the market.

Perfect verifiable disclosure. The assumption that disclosure is verifiable, as opposed to manipulable, is common in the literature. Einhorn and Ziv (2012) shows that the possibility of costly manipulation does not qualitatively affect the analysis; hence we rule it out for parsimony. Note we also assume the manager observes the value of the firm perfectly. The essential assumption that lends tractability to our analysis is that the manager has superior information to the market; qualitatively similar results hold when the firm's value also includes a component that is unknown to all agents.

Noise trade. The noise trade assumption is standard in the literature on informed trading as it is required for the price to not fully reveal the fundamentals. In our model, it also captures the notion that the manager faces uncertainty about the market reaction to the firm's disclosure decision (e.g., Suijs (2007)).

Net asset supply. We assume that on average, the stock is in zero net supply in order to rule out a direct effect of disclosure on the firm's risk-premium. This reflects the realistic case in which the the firm's disclosure is idiosyncratic information primarily used to update on its own value as opposed to market performance.[15,16] This also allows us to interpret our results about average over-valuation (under-valuation) as predictions about negative (positive, respectively) abnormal returns, and thus relate our results to the empirical literature. We expect our results to be qualitatively robust to an extension to a non-zero asset supply, after appropriate adjustments for the risk-premium given the non-linearity in prices. In related work, Dye and Hughes (2018) develop a multi-firm model voluntary disclosures and risk-averse investors who are not privately informed. They allow for a non-zero (but fixed) asset supply. Extending their analysis to allow for dispersed investor information is beyond the scope of the current paper, and left for future work.

4 Equilibrium

The timing is as follows. First, nature chooses the realization of cash flows $\tilde{v}$, private signals $\{\tilde{s}_i\}$ and noise trade $\tilde{z}$. Next, the manager learns the realization of cash flows and each investor learns their private signal. The manager then chooses whether or not to disclose their information. Conditional on disclosure, the price is completely determined by the disclosed information. Conditional on no disclosure, investors use their private signals and

[15] See Cianciaruso et al. (2020) for a proof in a setting with non-normal distributions which establishes that, under CARA utility, the price impact of disclosure in a single asset model with zero net supply is equivalent to the price impact of disclosure about idiosyncratic cash flows in a multi-asset model. Intuitively, in a large economy, idiosyncratic risk is in zero per-capita supply.

[16] Consistent with this assumption, Bonsall, Bozanic, and Fischer (2013) show that the firm's reaction to a common form of voluntary disclosure – earnings forecasts – is large, while the market reaction to these forecasts is very small.

the information in prices to submit demands and the price is determined by market clearing.

We focus on equilibria in which the manager discloses $\tilde{v}$ if and only if it exceeds a threshold T, which we refer to as a threshold equilibrium. In classical disclosure models, any equilibrium must take this form, as the manager's payoff to non-disclosure is constant and their payoff to disclosure increases in $\tilde{v}$. However, it is less clear that all equilibria must take this form in our model: not only does the manager's payoff given disclosure depend upon $\tilde{v}$, but so too does their payoff conditional on non-disclosure (through investors' trading behavior).

We can show that in any equilibrium, the manager discloses sufficiently large realizations and withholds sufficiently low realizations of $\tilde{v}$.[17] This rules out equilibria such as those in Clinch and Verrecchia (1997) and Kim and Verrecchia (2001), in which the manager discloses exclusively extreme or moderate values. However, we have not been able to rule out the existence of equilibria consisting of disjoint disclosure sets that are bounded from below.

To characterize a threshold equilibrium, our initial focus is on deriving the firm's price when they do not disclose; denote this event by ND. In contrast to standard models without private information, this price depends upon the firm's value through investors' private signals. Let P_{ND} denote equilibrium price given non-disclosure when investor i observes s_i and noise trade is $\tilde{z} = z$; we suppress the explicit dependence for expositional clarity.

4.1 Market pricing

Given the threshold nature of the firm's disclosure behavior, the absence of a disclosure leaves investors with a non-normal posterior. This implies that there does not exist an equilibrium

[17] Equilibria in which the manager discloses upon observing $\tilde{v}$ *below* some threshold T are easily ruled out: if the manager followed such a strategy, the firm's price when the manager does not disclose would be no less than T, for otherwise there would exist an arbitrage opportunity. Moreover, in any equilibrium, the firm's price conditional on disclosure is simply $\tilde{v}$. Thus, the manager would prefer to deviate, refraining from disclosure when they observe $\tilde{v} < T + c$. Likewise, in any equilibrium, managers observing sufficiently high $\tilde{v}$ always disclose. Intuitively, if managers observing $\tilde{v} > T$ did not disclose, then the firm's price conditional on non-disclosure would be bounded above by T. However, this implies that when the manager observes $\tilde{v} > T + c$, they would prefer to deviate to disclosing.

in which P_{ND} is a simple linear function of $\{s_i\}$ and z. We solve for the equilibrium by applying the techniques developed in Breon-Drish (2015). In particular, we conjecture and verify the existence of a "generalized" linear equilibrium in which, rather than a simple linear function, price is a monotonic transformation of a linear function of the private signals $\{s_i\}$ and z:[18]

$$P_{ND} = G\left(\bar{s} + \beta z\right), \tag{3}$$

where $\bar{s} = \int_i s_i di$ is the aggregation of the private signals, and $G(\cdot)$ is a strictly increasing, smooth function. Note that under the objective distribution $\bar{s} = \tilde{v}$, but under subjective beliefs, $\bar{s} = \rho\tilde{v} + \sqrt{1-\rho^2}\tilde{\xi}$.

The key feature of such an equilibrium is that, just as in a linear equilibrium, investor i can derive a "truth-plus-noise" signal from the price:

$$\tilde{s}_p \equiv \frac{1}{\rho} G^{-1}\left(P_{ND}\right) = \frac{1}{\rho}(\bar{s} + \beta\tilde{z}) \tag{4}$$

$$=_i \tilde{v} + \frac{\sqrt{1-\rho^2}}{\rho}\tilde{\xi} + \frac{\beta}{\rho}\tilde{z}. \tag{5}$$

Thus, conditional on $\tilde{v}$, investors perceive that $\tilde{s}_p \sim N(\tilde{v}, \sigma_p^2)$, where $\sigma_p^2 = \frac{\beta^2}{\rho^2}\sigma_z^2$. The inferred price signal $\tilde{s}_p$ has a variance σ_p^2 that decreases as ρ increases. In the rational expectations benchmark, $\rho = 1$, and so $\sigma_p^2 = \beta^2\sigma_z^2$. At the other extreme, when $\rho = 0$, the investors exhibit pure differences of opinions and so believe that the price is uninformative about fundamentals, i.e., $\sigma_p^2 \to \infty$ in this case.

The above characterization allows for the tractable calculation of investors' posterior beliefs given their private signals and the information in price. Specifically, investor i's posterior beliefs about cash-flows are given by a truncated normal distribution, where the truncation is determined by the disclosure threshold T, and the updated beliefs about the

[18] Note our framework fits into the exponential family of distributions that is necessary to apply the methodology in Breon-Drish (2015). Breon-Drish (2015) also demonstrates that the generalized linear equilibria we consider here are unique among the class of equilibria in which price is a continuous function.

underlying normal random variable $\tilde{v}$ are given by:

$$\tilde{\mu}_i \equiv \mathbb{E}_i[\tilde{v}|\tilde{s}_i, \tilde{s}_p] = \left(\frac{1}{\sigma_v^2} + \frac{1}{\sigma_\varepsilon^2} + \frac{1}{\sigma_p^2}\right)^{-1} \left(\frac{\tilde{s}_i}{\sigma_\varepsilon^2} + \frac{\tilde{s}_p}{\sigma_p^2}\right); \tag{6}$$

$$\sigma_s^2 \equiv \text{var}_i[\tilde{v}|\tilde{s}_i, \tilde{s}_p] = \left(\frac{1}{\sigma_v^2} + \frac{1}{\sigma_\varepsilon^2} + \frac{1}{\sigma_p^2}\right)^{-1}. \tag{7}$$

For simplicity, we abuse notation slightly and suppress the dependence of investors' posterior mean parameter $\tilde{\mu}_i$ on their signals $\tilde{s}_i$ and $\tilde{s}_p$. Let $\phi(x)$ and $\Phi(x)$ denote the density and distribution function of a standard normal distribution, respectively, so that $h(x) \equiv \frac{\phi(x)}{\Phi(x)}$ equals the inverse-Mills ratio. The following lemma characterizes the firm's price conditional on no disclosure.

Lemma 1. *Suppose there exists a $T \in \mathbb{R}$ such that the manager discloses if and only if $\tilde{v} > T$. Then, when the manager refrains from disclosure, there exists a unique equilibrium in the financial market. In this equilibrium, the firm's price equals:*

$$P_{ND} = H(\bar{s}, z) - \sigma_s h\left(\frac{T - H(\bar{s}, z)}{\sigma_s}\right), \tag{8}$$

where:

$$H(\bar{s}, z) \equiv \int_i \mu_i di + \frac{\sigma_s^2}{\tau} z = \frac{\sigma_v^2\left(\sigma_\varepsilon^2 + \rho\sigma_p^2\right)}{\rho\left(\sigma_\varepsilon^2\sigma_p^2 + \sigma_v^2\left(\sigma_\varepsilon^2 + \sigma_p^2\right)\right)}\left(\bar{s} + \frac{\sigma_\varepsilon^2}{\tau} z\right), \text{ and} \tag{9}$$

$$\sigma_p^2 = \frac{1 - \rho^2}{\rho^2}\sigma_v^2 + \frac{\sigma_\varepsilon^4 \sigma_z^2}{\tau^2 \rho^2}. \tag{10}$$

Moreover, P_{ND} is strictly increasing and concave in $\bar{s}$ and z.

The above result applies and extends Proposition 2.1 in the Online Appendix of Breon-Drish (2015) to our setting. We establish the result in a series of steps, which are outlined in detail in the appendix. First, taking as given the form of price in expression (3), we derive each investor's demand as a function of their information set, which includes the knowledge that the firm has not disclosed, their private signal $\tilde{s}_i$, and the signal contained in the price, $\tilde{s}_p$. Next, we apply the market-clearing condition to solve for the equilibrium price as a function of β. Finally, we verify that there exists a monotonic function $G(\cdot)$ and

a coefficient β such that this price aligns with the conjecture in (3).[19]

To develop intuition for the equilibrium non-disclosure price, it is helpful to consider two more familiar settings, which are limiting cases of our model: (1) the case in which the manager is informed and investors are uninformed (as captured by letting $\sigma_\varepsilon \to \infty$), and (2) the case in which the manager is uninformed but investors are informed (as captured by letting $T \to \infty$). In the first case, the price is simply equal to the firm's expected cash flows given non-disclosure, which reduces to (e.g., Verrecchia (1990)):

$$P_{ND} = \mathbb{E}\left[\tilde{v}|\tilde{v} < T\right] = \mathbb{E}\left[\tilde{v}\right] - \sigma_v h\left(\frac{T - \mathbb{E}\left[\tilde{v}\right]}{\sigma_v}\right). \tag{11}$$

Next, when the manager is uninformed, the absence of disclosure is entirely uninformative and the equilibrium price may be derived as in standard models of trade in which the firm's value is normally distributed and investors possess CARA utility (e.g., Hellwig (1980)). Specifically, the optimal demand for investor i is given by:

$$D_i(\mu_i) = \tau\,\frac{\mu_i - P}{\sigma_s^2}, \tag{12}$$

where P is the equilibrium price and μ_i is their realized posterior mean. Applying market clearing and substituting for μ_i, the price then equals the average investor's posterior mean plus a risk-adjustment term that is proportional to noise traders' demand z i.e.,

$$P = \int_i \mu_i di + \frac{\sigma_s^2}{\tau} z = H\left(\bar{s}, z\right), \tag{13}$$

where $H(\bar{s}, z)$ is a linear function of $\bar{s} + \frac{\sigma_\varepsilon^2}{\tau} z$.

Lemma 1 illustrates that the non-disclosure price when both investors and the manager have information combines features of expressions (11) and (13). Specifically, this price

[19] In particular, as illustrated in the lemma, $G\left(x\right) = kx - \sigma_s h\left(\frac{T-kx}{\sigma_s}\right)$ and $\beta = \frac{\sigma_s^2}{\tau}$, where $k = \frac{\sigma_v^2\left(\sigma_\varepsilon^2 + \rho\sigma_p^2\right)}{\rho\left(\sigma_\varepsilon^2\sigma_p^2 + \sigma_v^2\left(\sigma_\varepsilon^2 + \sigma_p^2\right)\right)}$.

equals the price that would arise were investors uninformed, where the mean parameter is adjusted to equal the price that would arise were the manager uninformed and variance parameter is equal to investors' common posterior variance. As a result, the equilibrium price satisfies a number of intuitive features; for instance, it increases in investors' average signal and decreases in the disclosure threshold. Moreover, it increases in noise-trader demand proportionally to investor uncertainty and investor risk tolerance.

4.2 Disclosure decision

Given our characterization of the firm's non-disclosure price, we can analyze the manager's disclosure choice. The manager who observes $\tilde{v} = v$ discloses if and only if their payoff given disclosure exceeds the expected non-disclosure price conditional on $\bar{s} = \tilde{v} = v$, i.e.,

$$v - c - \mathbb{E}\left[P_{ND}|\tilde{v} = v\right] \geq 0. \tag{14}$$

A disclosure threshold is incentive compatible if the manager is more inclined towards disclosure when their observed signal $\tilde{v} = v$ is greater. This would clearly be the case if the non-disclosure price were independent of the firm's value, as in voluntary disclosure models without informed trade. However, in our setting the non-disclosure price reflects the firm's value through investors' trading behavior, which may cause this condition to be violated.

An intuitive sufficient condition for there to exist a threshold equilibrium is that the non-disclosure price reacts to a marginal change in the firm's value only partially, i.e., $\frac{\partial P_{ND}}{\partial v} < 1$,[20] which implies the left-hand side of (14) increases in v. We show that this condition holds, and leads to the existence and uniqueness of a monotone threshold equilibrium.

Proposition 1. *There exists a unique equilibrium in which the manager discloses if and only if $\tilde{v} \geq T$. The equilibrium threshold satisfies:*

$$\mathbb{E}\left[P_{ND}|\tilde{v} = T\right] = T - c. \tag{15}$$

[20]To be clear, given our assumption that the manager correctly perceives the information contained in investors' signals, we refer here to the derivative of price when $\bar{s} = v$.

This proposition demonstrates that the disclosure equilibrium takes the same general form as in the canonical disclosure models in which investors are uninformed. However, we next illustrate that the presence of private information significantly alters the nature of prices and the level of disclosure in this equilibrium.

5 Model implications

In this section, we characterize properties of the equilibrium, focusing on the features that distinguish it from models of voluntary disclosure that do not permit private information. Section 5.1 describes how the quality of external public information influences the amount of disclosure that occurs in equilibrium. Section 5.2 characterizes how investors' use of price information affects firm valuation, average pricing errors and the relation between idiosyncratic skewness and expected returns.

5.1 Public information and the probability of disclosure

An extensive empirical literature has studied whether public information crowds in or crowds out voluntary disclosure, documenting mixed evidence. Using management forecasts as the proxy for voluntary disclosure and earnings volatility as the proxy for information quality, Imhoff Jr (1978), Cox (1985), and Waymire (1985) find that firms' forecast frequency is negatively related with their earnings volatility, consistent with a complementary relation. More recently, Francis et al. (2008) find that firms with more informative earnings have more expansive voluntary disclosures.

Other work suggests that public information and voluntary disclosure are substitutes. Balakrishnan et al. (2014) find that firms respond to a loss of public information by providing more earnings guidance. Barth et al. (2017) find that proprietary cost concerns that eliminate some of the previously mandatory disclosures, lead firms to provide additional disclosures. Finally, Billings, Jennings, and Lev (2015) find evidence consistent with the notion that

managers react to rising market volatility by providing guidance.

The ambiguous nature of this evidence is at odds with traditional models of disclosure, which suggest that public information crowds out disclosure (see Verrecchia (1990)). Furthermore, the direction of this relationship is critical to assessing the efficacy of disclosure regulations, as it determines their impact on the overall level of information available to market participants.

Our model suggests that whether public information substitutes or complements voluntary disclosure depends upon the amount of private information in the market as well as the extent of disclosure costs. To show this, we next characterize how the the amount of public information available to investors prior to the disclosure affects the probability of disclosure, $\Pr(\tilde{v} > T)$. For simplicity, we capture the amount of public information using the prior variance σ_v^2, where a lower variance captures greater public information.[21]

Proposition 2. *An increase in the amount of (ex-ante) public information may either increase or decrease the probability of disclosure. Moreover, there exist* $0 \leq \underline{\sigma}_\varepsilon^2 \leq \bar{\sigma}_\varepsilon^2$*, such that:*

i. ***(crowding-out)*** *When investors' private information is sufficiently noisy (i.e.,* $\sigma_\varepsilon^2 \geq \bar{\sigma}_\varepsilon^2$*), an increase in the amount of (ex-ante) public information decreases the probability of disclosure:* $\frac{\partial \Pr(\tilde{v}>T)}{\partial \sigma_v^2} > 0$.

ii. ***(crowding-in)*** *When investors' private information is sufficiently precise (i.e.,* $\sigma_\varepsilon^2 \leq \underline{\sigma}_\varepsilon^2$*) and disclosure is sufficiently expensive, an increase in the amount of (ex-ante) public information raises the probability of disclosure:* $\frac{\partial \Pr(\tilde{v}>T)}{\partial \sigma_v^2} < 0$.

The potential for both crowding in and crowding out arises because, in the presence of private information, better public information (i.e., lower σ_v^2) has two countervailing effects on the firm's incentives to disclose. On the one hand, as in disclosure models without private information (Verrecchia (1990)), investors interpret non-disclosure less negatively when ex-

[21] Equivalent results hold upon explicitly incorporating a normally-distributed public information signal. The realization of such a signal has no impact on the probability of disclosure, as it represents a location shift in the distribution of cash flows, and thus its only effect on the probability of disclosure is through prior uncertainty (Einhorn (2005)).

ante public information quality improves. Formally,

$$\frac{\partial}{\partial \sigma_v^2} \mathbb{E}\left[\tilde{v} | \tilde{v} < T\right] < 0. \tag{16}$$

As a result, better public information increases the non-disclosure price, which decreases the marginal firm's incentive to disclose ("crowding out" effect). On the other hand, as public information improves, investors place less weight on their private signals and rely more on their priors. This implies that the non-disclosure price P_{ND} more weakly reflects the firm's value, which amplifies the marginal firm's incentive to disclose (i.e., increases $(v - c) - \mathbb{E}\left[P_{ND} | \tilde{v} = v\right]$). This leads to crowding in of voluntary disclosure.

Whether public information crowds in or crowds out voluntary disclosure depends on which effect dominates. When both investors' private information quality (i.e., $1/\sigma_\varepsilon^2$) and disclosure costs (i.e., c) are sufficiently high, public information crowds in disclosure. The role of private information quality is transparent: investors' information must be sufficiently precise to ensure that their signals play a significant role in the determining the equilibrium price. The role of the disclosure cost c is to determine the location of the equilibrium threshold T. When c is higher, so is the disclosure threshold T. Intuitively, for T large, investors' beliefs given non-disclosure, $\tilde{v} | \tilde{v} < T$, are approximately normal. Thus, the firm's non-disclosure price approaches the standard "linear" price that arises when $\tilde{v}$ is normal:

$$P_{ND} \approx H\left(\bar{s}, z\right) = \int_0^1 \mu_i di + \frac{\sigma_s^2}{\tau} z,$$

which implies that:

$$\mathbb{E}\left[P_{ND} | \tilde{v} = T\right] = \int_0^1 \mathbb{E}\left[\tilde{\mu}_i | \tilde{v} = T\right] di = \beta\left(\sigma_v^2\right) T,$$

where $\beta\left(\sigma_v^2\right)$ denotes the weight on $\tilde{v}$ in investors' equilibrium posterior expectations as a function of σ_v^2. From equation (6), it can be verified that $\beta'\left(\cdot\right) > 0$, which simply reflects

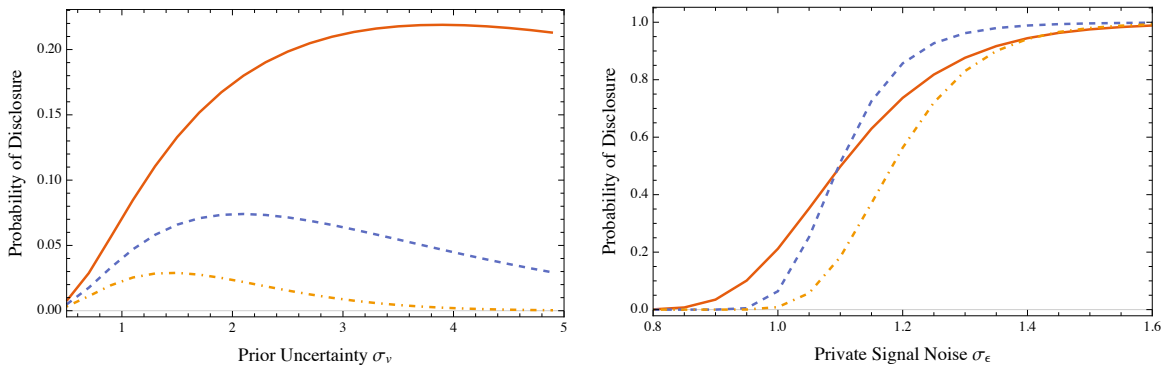
the fact that investors place more weight on their signals as their prior uncertainty σ_v^2 rises. Thus, the impact of σ_v^2 on the marginal firm's (i.e., the firm observing $\tilde{v} = T$) net benefit from disclosure equals:

$$\frac{\partial}{\partial \sigma_v^2} \left\{ T - c - \mathbb{E}\left[P_{ND} | \tilde{v} = T \right] \right\} = -\beta' \left(\sigma_v^2 \right) T,$$

which, for $T > 0$, is clearly negative.

Figure 1: Probability of Disclosure vs. Information Quality

The figure plots the probability of disclosure as a function of prior uncertainty σ_v (left) and private signal noise σ_ε (right). The solid, dashed and dot-dashed lines correspond to pure difference of opinions ($\rho = 0$), partial difference of opinions ($\rho = 0.5$), and rational expectations ($\rho = 1$), respectively. Unless otherwise mentioned, parameters are set to $\sigma_v = 3$, $\sigma_\varepsilon = 1$, $\sigma_z = 1.25$, and $c = 1$.



(a) Probability of Disclosure vs. σ_v (b) Probability of Disclosure vs. σ_ε

Figure 1 provides a numerical illustration of our results. Panel (a) illustrates that the probability of disclosure is increasing and then decreasing in prior uncertainty (i.e., σ_v).[22] For low prior uncertainty (when σ_v is low), the precision of private signals is relatively low (so that $\sigma_\varepsilon^2 > \bar{\sigma}_\varepsilon^2$), equilibrium voluntary disclosure features crowding out: the probability

[22]Note while Proposition 2 illustrates that crowding in arises for low σ_ε, ultimately, it is the level of σ_ε relative to σ_v that determines whether crowding in arises. Intuitively, if investors' priors are very noisy, even a small amount of private information has a significant impact on prices.

of disclosure increases with prior uncertainty. However, for sufficiently high levels of prior uncertainty, the relative precision of private information is sufficiently high, and equilibrium disclosure features crowding in. Panel (b) illustrates that, in contrast to public information, additional private information, as captured by the amount of noise in investors' private signals, always crowds out voluntary disclosure. This result is intuitive: additional private information causes cash flow information to be more strongly impounded into price even in the absence of disclosure, thereby decreasing the manager's incentive to disclose.

The result that public information can crowd in disclosure depends on both (i) the investors being informed about fundamentals and (ii) the strategic nature of voluntary disclosure. To reiterate, when investors do not have access to private signals, an increase in ex-ante public information leads to less voluntary disclosure (e.g., Verrecchia (1990)).[23] Moreover, in standard models with informed investors, disclosure is usually modeled as a non-discretionary commitment to release a public signal to the market. In these settings, better external information usually crowds out disclosure when both types of information are about the same dimension of fundamentals.[24] The above analysis implies that accounting for the discretionary nature of voluntary disclosure can have important implications for the impact of regulatory changes to the information environment.

A common measure of price informativeness in the literature is given by the posterior variance in payoffs, conditional on the information in prices, i.e., $\mathbb{E}[\text{var}[\tilde{v}|P]]$.[25] This measure is particularly useful from a policy perspective, because it reflects the average amount of information that an uninformed, rational investor can infer from prices. As such, empirical estimates are often used to evaluate the impact of changes in regulation, transaction costs, and disclosure requirements on how well market prices reflect fundamental information.[26]

[23] This is analogous to the limit of $\sigma_\varepsilon \to \infty$ in our model.

[24] See, e.g., Diamond (1985). See also Goldstein and Yang (2017), which discusses when this finding might not hold in such models.

[25] Weller (2017) distinguishes between the notion of *price informativeness*, which is an absolute measure of the total information content of prices, and *information efficiency*, which is a relative measure of how much of the private information dispersed across market participants is reflected in prices. We focus on the former measure.

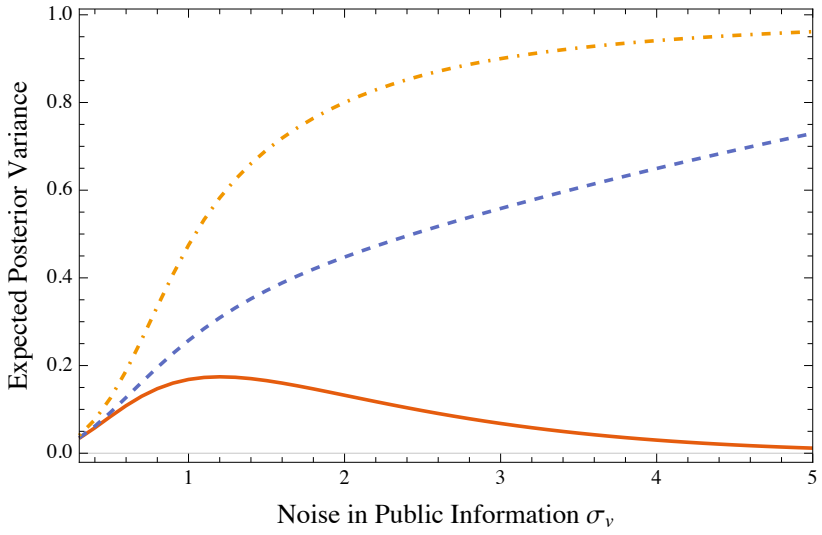
[26] Note that in our benchmark model, the price perfectly reveals whether or not the manager disclosed,

Figure 2 illustrates how this measure of price informativeness changes with prior uncertainty about fundamentals. Consistent with the results of Proposition 2, the plot shows a positive relation between the expected posterior variance $\mathbb{E}[\text{var}[\tilde{v}|P]]$ and prior uncertainty σ_v, except when disclosure costs are relatively low (solid line) and prior uncertainty is sufficiently high. In other words, price informativeness increases with (ex-ante) public information, except when disclosure costs are relatively low and the public information is not very precise (relative to private information). The figure illustrates how changes in the information environment can have different effects on price informativeness across firms, depending on the interaction between private information of investors and the firm's incentives to disclose. Thus, our analysis implies that accounting for this interaction is potentially important when interpreting empirical evidence and evaluating policy.

since conditional on disclosure (non-disclosure), the price must be greater than (less than, respectively) the equilibrium threshold. In particular, this implies one can express $\mathbb{E}[\text{var}[\tilde{v}|P]] = \Pr(ND)\mathbb{E}[\text{var}[\tilde{v}|P, ND]]$, since the posterior variance conditional on disclosure is zero i.e., $\text{var}[\tilde{v}|P, D] = 0$. As such, our measure of price informativeness is also equal to the expected posterior uncertainty about fundamentals for an econometrician who observes all public information (i.e., prices and disclosures).

Figure 2: Price Informativeness vs. Public Information

The figure plots the expected posterior variance $\mathbb{E}\left[\text{var}\left[\tilde{v}|P\right]\right]$ as a function of the amount of public information σ_v. The solid, dashed and dot-dashed lines correspond to low cost ($c = 0.65$), medium cost ($c = 0.75$), and high cost ($c = 1.5$), respectively. Parameters are set to $\sigma_\varepsilon = 1$, $\sigma_z = 1$, and $\rho = 1$.



5.2 Firm valuation

Next, we consider how the interaction between disclosure and dispersed private information influences the firm's average valuation. We show that the extent to which investors use the information in prices affects whether the firm is over-valued or under-valued relative to expected cash flows, and then explore the implications of this result for average mispricing, incentives for information acquisition and the relation between skewness and expected returns.

When investors are identically informed (and there is no noise trading), the firm's non-disclosure price is simply equal its expected value given investors' common information set (e.g., Verrecchia (1990)). In the following result, we show that the non-disclosure price may be higher or lower than its expected value when investors are privately informed. This result is particularly striking because information is firm-specific (idiosyncratic) and investors do

not bear aggregate risk when holding the stock.

Proposition 3.

i. *(**rational expectations**) Suppose that $\rho = 1$. Then, conditional on non-disclosure, the firm's expected value exceeds its expected price i.e.,*

$$\mathbb{E}\left[P_{ND}|\tilde{v} < T\right] < \mathbb{E}\left[\tilde{v}|\tilde{v} < T\right].$$

ii. *(**differences of opinion**) Suppose that $\rho = 0$. Then, conditional on non-disclosure, the firm's expected price is greater than (less than) its expected value when $\frac{\tau^2}{\sigma_\varepsilon^2} > \sigma_z^2$ $\left(\frac{\tau^2}{\sigma_\varepsilon^2} < \sigma_z^2\right)$, i.e.,*

$$\mathbb{E}\left[P_{ND}|\tilde{v} < T\right] \lessgtr \mathbb{E}\left[\tilde{v}|\tilde{v} < T\right] \quad \Leftrightarrow \quad \frac{\tau^2}{\sigma_\varepsilon^2} \lessgtr \sigma_z^2.$$

To gain intuition for the above result, it is useful to compare the non-disclosure price to the conditional expectation of a typical investor. Denote the conditional expectation of a normal random variable $\tilde{x} \sim N(\mu, \sigma^2)$ truncated above at T by:

$$f(\mu; \sigma, T) \equiv \mathbb{E}[\tilde{x}|\tilde{x} < T] = \mu - \sigma h\left(\frac{T-\mu}{\sigma}\right), \tag{17}$$

where $f(\mu; \sigma, T)$ is an increasing, concave function of μ. Then, one can express the conditional expectation given non-disclosure of investor i as:

$$\mathbb{E}_i[\tilde{v}|\tilde{v} < T, \tilde{s}_i, \tilde{s}_p] = f(\tilde{\mu}_i; \sigma_s, T), \tag{18}$$

and the law of iterated expectations for investor i implies:

$$\mathbb{E}_i[f(\tilde{\mu}_i; \sigma_s, T)|\tilde{v} < T] = \mathbb{E}_i[\tilde{v}|\tilde{v} < T] = \mathbb{E}[\tilde{v}|\tilde{v} < T]. \tag{19}$$

Next, recall from Lemma 1 that we can express the non-disclosure price as the expected value given $\tilde{v} < T$ perceived by an investor whose mean and variance parameters are $H(\bar{s}, z)$

and σ_v^2, respectively:

$$P_{ND} = f\left(H(\bar{s}, z); \sigma_s, T\right), \quad \text{where } H(\bar{s}, z) = \int_i \tilde{\mu}_i di + \frac{\sigma_s^2}{\tau}\tilde{z}.$$

Given the concavity of $f(\cdot)$, the above implies that when $\rho \in \{0, 1\}$, whether the firm is over- or under-valued reduces to whether $H(\bar{s}, z)$ – which captures how the price aggregates investors' beliefs and noise trade – is more or less variable than the beliefs of a typical investor:[27]

$$\text{var}\left[\tilde{\mu}_i\right] \gtrless \text{var}\left[\int_i \tilde{\mu}_i di + \frac{\sigma_s^2}{\tau}\tilde{z}\right]. \tag{20}$$

The relative variability depends on two effects. On the one hand, $H(\bar{s}, z)$ is less variable than those of a typical investor because it reflects the aggregate (or average) valuation (i.e., $\text{var}[\int_i \tilde{\mu}_i di] < \text{var}[\tilde{\mu}_i]$). On the other hand, $H(\bar{s}, z)$ is more variable because it is more sensitive to noise-trading shocks via the "risk compensation" term $\frac{\sigma_s^2}{\tau}\tilde{z}$.

These effects reflect two economic forces. First, investors who observe more optimistic realizations of private information trade more aggressively on their signals. As we show in the proof of Lemma 4 in the Appendix, this is because an investor's optimal demand is a convex function of their conditional expectation of cash flows $\mathbb{E}_i[\tilde{v}|\tilde{v} < T, \tilde{s}_i, \tilde{s}_p]$. Intuitively, this is driven by the fact that investors who receive more optimistic signals perceive less uncertainty (i.e., $\frac{\partial \text{var}_i[\tilde{v}|\tilde{v}<T, \tilde{s}_i, \tilde{s}_p]}{\partial \tilde{s}_i} < 0$).[28] As a result, more optimistic signals are disproportionately reflected in the price, which pushes it higher on average. Note that this feature is absent in traditional settings where the payoff is conditionally normal and so investor

[27] This intuitive argument abstracts from several technicalities, which we address in the proof of Proposition 3. In particular, we show in this proof that the variance condition (20) yields a second-order stochastic dominance ordering between the conditional distributions of $\tilde{\mu}_i$ and $\int_i \tilde{\mu}_i di + \frac{\sigma_s^2}{\tau}\tilde{z}$ given ND.

[28] The fact that an investor's posterior variance declines in their signal follows from the observation that the variance of a truncated normal is given by

$$\text{var}_i\left[\tilde{v}|\tilde{v} < T, \tilde{\mu}_i\right] = \sigma_s^2\left[1 - \frac{T-\tilde{\mu}_i}{\sigma_s} h\left(\frac{T-\tilde{\mu}_i}{\sigma_s}\right) - \left(h\left(\frac{T-\tilde{\mu}_i}{\sigma_s}\right)\right)^2\right],$$

where $h(\cdot) = \frac{\phi(\cdot)}{\Phi(\cdot)}$. This expression declines in $\tilde{\mu}_i$. The proof of Lemma 4 clarifies that an investor's optimal demand is linear in their private signal, but since the conditional expectation of cash flows is concave in this signal, the optimal demand is concave in the conditional expectation.

demands (and prices) are linear in their conditional expectations.

Second, the impact of noise trading on prices is also asymmetric. When noise traders purchase the firm's shares, investors short the stock and demand a boost in price to do so. But because the firm's value is truncated from above, the downside from shorting is limited, and the price compensation is relatively small. In contrast, when noise traders sell, investors must bear the risk of being long. In this case, their downside is unlimited and so investors charge a larger compensation (discount) for bearing the risk. On average, this pushes prices down.[29]

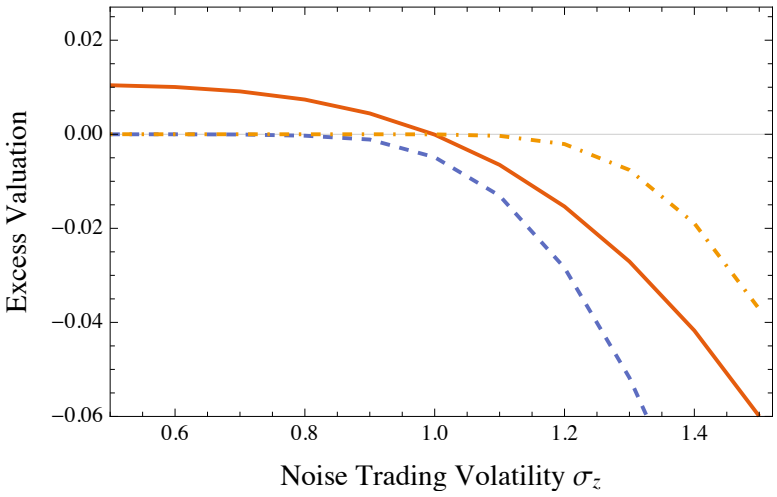
Interestingly, which of these two forces dominates depends on how investors learn about cash flows from the price. When investors exhibit rational expectations and condition on the information in prices, the detrimental effect of noise trade on the variance of $\int_i \tilde{\mu}_i di + \frac{\sigma_s^2}{\tau}\tilde{z}$ relative to $\tilde{\mu}_i$ is amplified, as noise trade has a correlated impact on the risk compensation and investors' expectations. As a result, the firm is always under-valued. In contrast, when investors do not update from price, whether the firm is under- or over-valued depends on the volatility of noise trade. As noise trade vanishes, the firm is over-valued, but when noise-trading volatility is sufficiently high, the firm is under-valued. Surprisingly, this implies that average mispricing may actually be higher under rational expectations, when investors condition correctly on all available information, than under differences of opinions when investors are dismissive of price information.

Figure 3 provides an illustration of the result in Proposition 3. The figure plots excess valuation $\mathbb{E}\left[P_{ND} - \tilde{v} | \tilde{v} < T\right]$, conditional on non-disclosure, as a function of noise-trading volatility σ_z for different levels of ρ. Note that the plots show that the firm can be over-valued when investors dismiss price information completely (i.e., $\rho = 0$), and is under-valued when they exhibit rational expectations (i.e., $\rho = 1$). The figure also depicts that the degree of under- vs. over-pricing may be non-monotonic in ρ: when $\rho = 0.5$, the firm's expected

[29]This asymmetric risk-compensation effect is absent in traditional models with linear prices because the value is symmetric and unbounded (usually normal). However, it is analogous to the "skewness effect" discussed in Albagli et al. (2021) and Cianciaruso et al. (2020).

Figure 3: Excess Valuation vs. Noise-Trading Volatility

The figure plots the firm's expected price less its expected cash flows conditional on non-disclosure, $\mathbb{E}\left[P_{ND} - \tilde{v} | \tilde{v} < T\right]$ as a function of noise-trading volatility σ_z. The solid, dashed and dot-dashed lines correspond to pure difference of opinions ($\rho = 0$), partial difference of opinions ($\rho = 0.5$), and rational expectations ($\rho = 1$), respectively. Unless otherwise mentioned, parameters are set to $\sigma_v = 3$, $\sigma_\varepsilon = 1$, $\sigma_z = 1.25$, and $c = 1$.



valuation falls below the case in which $\rho = 1$. This reflects the fact that the firm's value can become increasingly sensitive to noise trade for intermediate values of ρ.

Next, we highlight some implications of Proposition 3 for empirical and policy analysis, and for understanding a firm's incentives to acquire information.

Average pricing errors

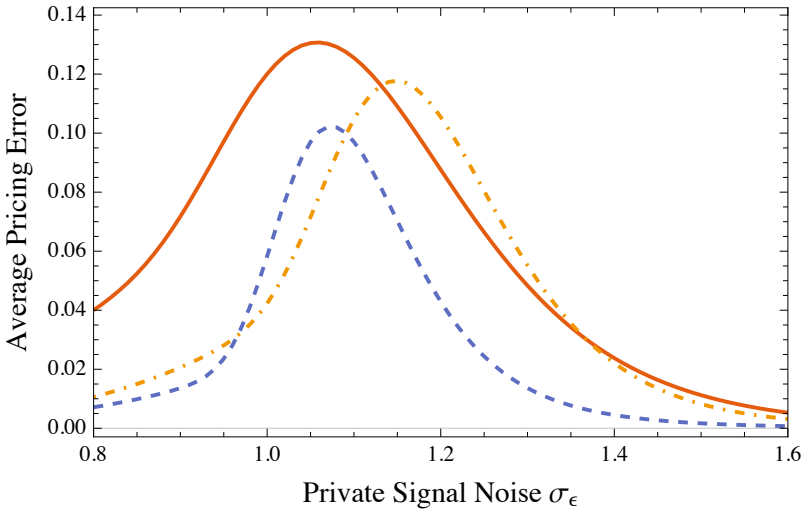
The existing literature often interprets average pricing errors as a measure of informational efficiency of prices.[30] Our analysis suggests that one must be careful in applying this interpretation to pricing errors in an economy with endogenous disclosure and informed trading.

[30]For instance, Frenkel et al. (2020) argue that $PEF = -\mathbb{E}[(\tilde{v} - P)^2]$ is a natural measure of the "social" benefit of having prices being close to fundamentals. As they discuss in greater detail, this interpretation can be motivated by the observation that a social planner maximizing allocative efficiency would set the equilibrium price equal to the conditional expected value, given all available information, in an economy with risk-neutral investors. This interpretation is not directly applicable in our setting, given that we have risk-averse investors with (potentially) heterogeneous priors. However, our analysis is important for the interpretation of empirical analysis based on this measure, as we discuss below. D'avila and Parlatore (2020) use a related measure, namely the excess variance in prices conditional on fundamentals (i.e., $\text{var}(P|\tilde{v})$), to quantify price informativeness in their (linear) equilibrium.

Figure 4 provides a numerical illustration. We plot the expected squared deviation between the firm's price and its value (i.e., $\mathbb{E}[(\tilde{v}-P)^2]$) as a function of private signal noise σ_ε for different values of ρ. Specifically, average pricing errors can be higher when investors correctly process the information in prices (i.e., $\rho = 1$) than when investors partially or completely dismiss this information (i.e., $\rho = 0.5$ or $\rho = 0$, respectively). As such, pricing errors (and related measures like abnormal returns or excess volatility) may not provide a reliable measure of how well investors process the information available to them.

Figure 4: Average Pricing Errors

The figure plots the average (squared) pricing error $\mathbb{E}\left[(\tilde{v}-P)^2\right]$ as a function of private signal noise σ_ε. The solid, dashed and dot-dashed lines correspond to pure difference of opinions ($\rho = 0$), partial difference of opinions ($\rho = 0.5$), and rational expectations ($\rho = 1$), respectively. Unless otherwise mentioned, parameters are set to $\sigma_v = 3$, $\sigma_\varepsilon = 1$, $\sigma_z = 1.25$, and $c = 1$.



Value of idiosyncratic information

In traditional models of voluntary disclosure with costs, the ability to disclose idiosyncratic information is strictly to a firm's detriment because disclosure is costly, but information has no real (or allocative) benefit. As a result, a firm would be better off if it could commit to not

acquiring information, since then it would be correctly valued on average (i.e., $\mathbb{E}[P] = \mathbb{E}[\tilde{v}]$).

However, in our setting, this is no longer the case. Specifically, Proposition 3 (ii) implies that when investors dismiss the information in prices, the firm may be better off by acquiring information than committing not to do so because it can exploit the over-valuation that results from non-disclosure. In this case, the firm may optimally engage in information acquisition and costly disclosure even though it is socially inefficient to do so.[31]

An additional implication of Proposition 3 (ii) is that a firm can have a *lower* cost of capital, on average, when it chooses not to disclose information. This is in contrast to the common intuition from existing models that suggests more disclosure leads to a lower cost of capital (e.g., Dye and Hughes (2018)). Our model predicts that the relation between voluntary disclosure and cost of capital depends on the extent to which investors learn from prices, and conditioning on this is important when trying to understand the impact of disclosure on cost of capital.

The relation between skewness and expected returns

The negative relation between idiosyncratic skewness and average returns (e.g, Jiang et al. (2009), Conrad et al. (2013), Boyer and Vorkink (2014)) is difficult to reconcile in a model with rational investors and standard utility functions. In traditional representative agent models, coskewness with the market may be priced (e.g., Kraus and Litzenberger (1976)), but idiosyncratic skewness is diversified away. As such, earlier explanations of the negative relation have relied on investors exhibiting a preference for firm-specific skewness (e.g., Mitton and Vorkink (2007), Barberis and Huang (2008)).[32]

Our model provides a complementary, rational explanation for the negative relation. Proposition 3 (i) suggests that average returns following non-disclosure are higher than

[31] We have verified that this result holds even if the disclosure cost is incurred by the firm rather than the manager. That is, expected over-valuation may dominate expected disclosure costs.

[32] Mitton and Vorkink (2007) show that a negative relation can arise in an economy where investors have heterogeneous preferences for skewness, while Barberis and Huang (2008) rely on investors having cumulative prospect theory preferences.

those following disclosure. This is because when investors exhibit rational expectations, the firm is under-valued when information is not disclosed, but correctly priced when it is. Moreover, consistent with empirical evidence (e.g., McNichols (1988), Kothari, Shu, and Wysocki (2009)), returns exhibit positive skewness when a firm discloses, but negative skewness when it does not (also, see Acharya, DeMarzo, and Kremer (2011)). This implies a negative cross-sectional relation between idiosyncratic skewness and average returns, to the extent that variation in skewness across firms and over time depends sufficiently on variation in voluntary disclosures.[33] Proposition 3 also suggests that the relation between skewness and returns depends on how investors condition on prices, and can be positive when investors dismiss price information. This provides a novel prediction of the model that distinguishes it from existing explanations.

6 Extension: Random information endowment

We now extend our model to consider the case in which, as in Dye (1985) and Jung and Kwon (1988), the manager possesses information with probability p, for $p \in [0, 1]$. That is, we now allow for either or both commonly-used frictions to prevent an "unravelling" equilibrium: a disclosure cost and a probabilistic information endowment. We assume that at least one of these two frictions is present, i.e., that at least one of $c > 0$ and $p < 1$ holds. We establish that the conclusions from our primary analysis continue to hold in this case when investors' information is not overly precise.

To demonstrate these results, we begin by extending the characterization of the non-disclosure price in Lemma 1 to accommodate the presence of information uncertainty. Let $P_{ND,p}$ now denote the firm's non-disclosure price, and let P_{ND} and $H(\bar{s}, z)$ be as defined in Lemma 1.

Lemma 2. *Suppose there exists a $T \in \mathbb{R}$, such that when the manager observes $\tilde{v}$, they*

[33] Since Beyer et al. (2010) find that 66% of return variation attributed to accounting information is driven by voluntary disclosures, this is plausible.

disclose if and only if $\tilde{v} > T$*. Then, when the manager refrains from disclosure, there exists a unique equilibrium in the financial market. In this equilibrium, the firm's price equals:*

$$P_{ND,p} = \frac{p\Phi\left(\frac{T-H(\bar{s},z)}{\sigma_s}\right) P_{ND} + (1-p)\, H\left(\bar{s}, z\right)}{p\Phi\left(\frac{T-H(\bar{s},z)}{\sigma_s}\right) + 1 - p}. \tag{21}$$

Lemma 2 shows that firm's price is now a weighted average of the price if the manager was known to be uninformed (i.e., $H\left(\bar{s}, z\right)$) and the price if the firm's value was known to fall below T (i.e., P_{ND}). The weights reflect the perceived likelihood that the manager is informed, presuming again that the prior mean over firm value is $H\left(\bar{s}, z\right)$. Thus, in contrast to the Dye (1985) - Jung and Kwon (1988) model, these weights depend upon the investors' private signals: a more optimistic signal indicates that the absence of a disclosure more likely resulted from an uninformed manager, as opposed to an informed manager who observed negative news.

We next analyze when a threshold equilibrium similar to the one in our baseline model exists. Recall that an intuitive sufficient condition for there to exist such a equilibrium is that the non-disclosure price reacts to a marginal change in the firm's value only partially, i.e., $\frac{\partial P_{ND,p}}{\partial v} < 1$; this ensures that the manager is more inclined towards disclosure as their signal rises. While this condition may seem natural given that investors observe noisy signals, it is in fact possible that the price responds more than one-for-one with a change in the value of the firm. To determine when this is the case, we next characterize $\frac{\partial P_{ND,p}}{\partial v}$; to state this in an intuitive manner, let ND denote the event of non-disclosure.

Lemma 3. *Suppose there exists a threshold* $T \in \mathbb{R}$ *such that the manager discloses if and only if* $\tilde{v}$ *exceeds* T*. Then, when the manager does not disclose, the price response to a marginal change in the firm's value satisfies:*

$$\frac{\partial P_{ND,p}}{\partial v} = var_j\left[\tilde{v}\,|ND, \tilde{\mu}_j = H\left(\bar{s}, z\right)\right]\left(var_j^{-1}\left[\tilde{s}_j|\tilde{v}\right] + var_j^{-1}\left[\tilde{s}_p|\tilde{v}\right]\right). \tag{22}$$

The price response to a shift in $\tilde{v}$ is equal to the posterior variance perceived by an investor whose posterior mean parameter $\tilde{\mu}_j$ to equal $H\left(\bar{s}, z\right)$, multiplied by the combined precision

of their private signal and the signal they receive from price. To gain intuition, consider the case when the manager is known to be uninformed ($p = 0$), as in standard models of trade with normal distributions. In this case,

$$\frac{\partial P_{ND,0}}{\partial v} = \frac{\partial H(v,z)}{\partial v} = \frac{\partial}{\partial v}\left[\int_0^1 \mu_i di + \frac{\sigma_s^2}{\tau} z\right].$$

Upon substituting for μ_i and applying Bayes' rule for normal distributions, this reduces to:

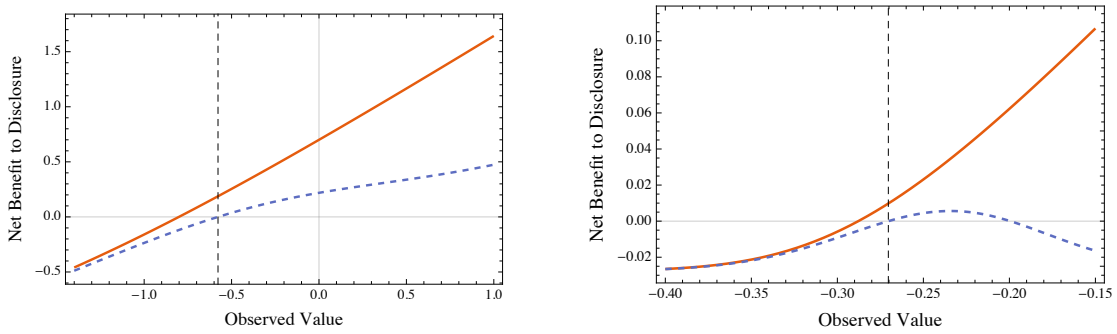
$$\text{var}_j\left[\tilde{v}|\tilde{s}_j, \tilde{s}_p\right]\left(\text{var}_j^{-1}\left[\tilde{s}_j|\tilde{v}\right] + \text{var}_j^{-1}\left[\tilde{s}_p|\tilde{v}\right]\right). \tag{23}$$

One can verify that this is always less than one, and so the price responds only partially to an increase in firm value. Intuitively, the price response is driven by the product of investors' posterior uncertainty, and the total precision of their information signals.

When the manager may be informed, the posterior variance that appears in expression (23), $\text{var}_j\left[\tilde{v}|\tilde{s}_j, \tilde{s}_p\right]$, is replaced by $\text{var}_j\left[\tilde{v}\,|ND, \tilde{\mu}_j = H(\bar{s}, z)\right]$, which conditions on the event of non-disclosure (under the belief $\tilde{\mu}_j = H(\bar{s}, z)$). Therefore, when the manager is informed, the event of non-disclosure changes the marginal reaction to the firm's information by adjusting investors' posterior variance. When the manager is *always* informed (i.e., $p = 1$) as in our benchmark, observing non-disclosure reveals that $\tilde{v} < T$. Because this strictly reduces the possible outcomes for the firm's value, investors' posterior variances fall short of the prior variance, and thus the marginal price response $\frac{\partial P_{ND}}{\partial v}$ falls short of the response when the manager is uninformed. Thus, a fortiori, this response is less than 1. In contrast, when $p < 1$, the posterior variance following the observation of non-disclosure may *increase* relative to the prior variance (see Dye and Hughes (2018) for further discussion). As discussed before, investors face uncertainty about whether non-disclosure resulted from an informed or uninformed manager, which leads to very different inferences regarding firm value. Nevertheless, we show that if the combined precision of investors' price and private signals are not excessively large – i.e., when σ_z^2 and σ_ε^2 are large and τ is small – the sensitivity of the

Figure 5: Existence and non-existence of a threshold equilibrium.

The plot shows the net benefit to disclosure $v - \mathbb{E}\left[P_{ND,p}|\tilde{v} = v\right] - c$ as a function of the observed value v for $p = 1$ (solid) and $p = 0.95$ (dashed) respectively. The left panel illustrates an example of low investor information precision ($\sigma_\varepsilon = 0.75$) while the right panel illustrates the case of high information precision ($\sigma_\varepsilon = 0.2$). The remaining parameters are $c = 0.025$, and $\sigma_z = 1$.



(a) Low Private Information ($\sigma_\varepsilon = 0.75$) (b) High Private Information ($\sigma_\varepsilon = 0.2$)

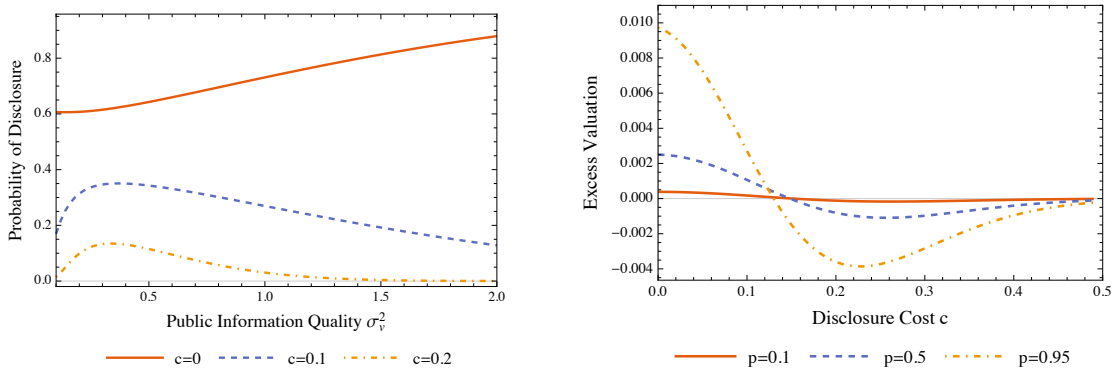
non-disclosure price to v is below 1.

Figure 5 demonstrates an example of how a threshold equilibrium may break down when investors' signals are highly precise. The left-hand figure illustrates the benefit to a manager of disclosing relative to not disclosing, $v - \mathbb{E}\left[P_{ND,p}|\tilde{v} = v\right] - c$, as a function of their observed signal, for low private information (high σ_ε), while the right hand plot depicts the analogous case for high private information precision (low σ_ε). To be precise, it plots this benefit when investors believe the manager discloses when $\tilde{v} > T^*$, where T^* is the minimum solution to $T^* - \mathbb{E}\left[P_{ND,p}|\tilde{v} = T^*\right] = c$, i.e., the threshold that would arise as an equilibrium if it were the case that $\frac{\partial P_{ND,p}}{\partial v} < 1$. When investors' signals are sufficiently noisy (left panel), the benefit to disclosure is always increasing, which implies the existence of a threshold equilibrium. However, when investors' signals are sufficiently precise (right panel), the benefit to disclosure can decline for $v > T^*$, returning to negative values, which implies that the manager prefers not to disclose for some values $v > T^*$. This, in turn, rules out the existence of a threshold equilibrium.

The next proposition states these results formally and establishes that when a threshold equilibrium exists, it is unique.

Proposition 4. *Suppose that either* $p = 1$ *and/or* $\frac{1}{\sigma_\varepsilon^2} + \frac{1}{\rho\sigma_p^2}$ *is sufficiently small. Then, there exists a unique equilibrium in which the manager discloses if and only if* $\tilde{v} \geq T$.

Figure 6: Model implications when $p < 1$ for the rational expectations equilibrium. The left panel plots probability of disclosure versus prior uncertainty (σ_v) for different values of cost c. The right panel plots excess valuation $\mathbb{E}\left[P_{ND,p} - \tilde{v} | \tilde{v} < T\right]$ as function of costs for different values of p. Unless otherwise specified, the parameters are given by $\sigma_v = 0.25$, $\sigma_\varepsilon = 0.5$, $\rho = 1$, and $\sigma_z = 1.5$.



(a) Crowding In Vs. Crowding Out

(b) Under- Vs. Over-valuation

In Figure 6, we numerically explore how our results on firm valuation and the relationship between public information and firm disclosure change when $p < 1$. We focus on the rational expectations benchmark (i.e., when $\rho = 1$). First, note that the plots suggest that when c is sufficiently large and σ_ε^2 is sufficiently small relative to σ_v^2, public disclosure can crowd in voluntary disclosure.[34] These results imply that the "crowding in" channel from our benchmark model is robust to allowing for uncertainty about the manager's information endowment.

Next, recall from Section 5.2, that when $p = 1$, prices always exhibit under-valuation in the rational expectations equilibrium because the non-disclosure price is concave in $\tilde{v}$. Figure

[34]Note that when $p < 1$, the existence of an equilibrium requires large σ_ε^2 and this complementary relationship requires small σ_ε^2. Under the parameters used to produce Figure 6, both the equilibrium exists and this complementary relationship arises.

6 illustrates that when $p < 1$, prices can exhibit over-valuation even if investors condition on prices efficiently. This is because, when $p < 1$, it is possible that the non-disclosure price is no longer a concave function, and thus the impact of non-disclosure on the firm's expected valuation is more subtle.

To understand this result, we may extend Lemma 3 to characterize the curvature in the non-disclosure price as follows:[35]

$$\frac{\partial^2 P_{ND,p}}{\partial^2 v} \propto \frac{\partial \text{var}_j \left[\tilde{v} \left| ND, \tilde{\mu}_j = H\left(\bar{s}, z\right)\right.\right]}{\partial v} \tag{24}$$

$$= \mathbb{E}_j \left[(\tilde{v} - H\left(\bar{s}, \tilde{z}\right))^3 | ND, \tilde{\mu}_j = H\left(\bar{s}, z\right)\right]. \tag{25}$$

This equation demonstrates that the marginal price response to v increases if the uncertainty faced by an investor increases in their perception of the mean, $\tilde{\mu}_j$ (and decreases otherwise). Moreover, this occurs if and only if they perceive that the firm's value is positively skewed. Intuitively, when the payoff is positively (negatively) skewed, an increase in an investor's signal pushes their beliefs towards the region of the distribution in which there is more (less) uncertainty.

This result offers us two lenses through which to view the potential convexity of price when $p < 1$. First, convexity may arise because investors become more, as opposed to less, uncertain as v rises. The reason is that for low levels of v, which lead investors to observe low signals, they perceive a higher probability that the manager was informed and chose to withhold their information. As v approaches the threshold T, their uncertainty over the firm's value may be compounded by an increase in uncertainty over whether the manager was informed. A second, mathematically equivalent explanation for price convexity is that when p is less than but close to 1, the firm's payoff distribution given non-disclosure can take a lottery-like form (and thus, may be positively skewed). In particular, with a high likelihood, the manager is informed and has a very low value, but with a small probability,

[35] This result may be formally derived by applying a similar argument to the proof of Lemma 3.

the manager is uninformed and has (in expectation) significant value.

Overall, while many implications from the benchmark analysis extend to this setting, allowing for uncertainty about whether the manager is informed leads to additional predictions. For instance, even if all investors process information correctly, firms may be over-valued relative to fundamentals when disclosure costs are sufficiently high. As before, this implies that abnormal returns are not indicative of information processing frictions or errors.

7 Conclusions

Standard voluntary disclosure models assume that investors do not have access to private information. We show that this assumption is an economically important restriction, and relaxing it has qualitatively novel implications. First, in contrast to traditional models, ex-ante public information can "crowd in" more voluntary disclosure. Second, prices generically exhibit over-valuation or under-valuation relative to expected cash-flows.

These implications are important for interpreting empirical evidence and evaluation regulatory policy. A standard criticism of policy changes that mandate more public disclosure is based on the common intuition that implies more public information crowds out voluntary disclosure. Our results show that this adverse effect is more likely to arise for firms with low disclosure costs. Importantly, however, mandatory ex-ante disclosure can lead to greater voluntary disclosure for firms with high disclosure costs and greater adverse selection (i.e., when investors have precise private information). This suggests that even though the impact of such policy changes is heterogeneous across firms, they may be most effective at "leveling the playing field" for firms that would rarely disclose in their absence.

Our analysis also suggests that one must exercise caution when using proxies of pricing errors as evidence of limited, or inefficient, information processing. In our model, abnormal returns are non-zero even when investors efficiently use all the information in prices. More importantly, the mispricing may be larger in this case than when investors dismiss the

information in prices. Similarly, studies that use changes in the cost of capital to evaluate the effectiveness of policy changes must be interpreted with caution. Finally, our results provide a novel interpretation of the negative empirical relation between idiosyncratic skewness and average returns, and our model provides novel predictions about how this relation varies with investor behavior.

Our model is stylized but suggests a number of natural extensions. First, investors and the manager are endowed with information in our model. It would be interesting to study how the interaction of their behavior affects the incentives to acquire information for either party. In traditional models of costly disclosure, the manager usually prefers to commit not to acquire information (ex-ante) because disclosure is costly but has no real effects. However, as we discuss in Section 5.2, our analysis implies that managers may find it valuable to acquire information when investors dismiss price information, since the possibility of voluntary disclosure can lead to over-valuation on average.

Second, the relative timing of public information and voluntary disclosure is potentially important. In our analysis, we focus on the effect of pre-disclosure public information, but preliminary analysis suggests that post-disclosure public information may have different implications (e.g., it appears to crowd out voluntary disclosure). This suggests that a dynamic model that allows for timing of voluntary disclosure (as in Guttman, Kremer, and Skrzypacz (2014)) could yield richer interactions between (exogenous) public information, private information and voluntary disclosure.

Finally, we consider a model without real and feedback effects. As an interesting extension, one could consider the possibility that managers use their disclosure policy to elicit information from the market and inform their investment choices. Alternatively, one might consider how voluntary disclosure influences the incentives of managers to invest, as in Ben-Porath, Dekel, and Lipman (2018), when investors possess private information.

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Appendix

Proof of Lemmas 1 and 2. We prove both of these lemmas by deriving the price given that the manager is informed with probability $p \in [0,1]$; Lemma 1 is then a special case when $p = 1$. We start by deriving the investors' demands. To state these demands in intuitive manner, note that a sufficient statistic for investor j's information signals $\{\tilde{s}_j, \tilde{s}_p\}$ is $\tilde{S}_j \equiv \frac{\tilde{s}_j}{\sigma_\varepsilon^2} + \frac{\tilde{s}_p}{\sigma_p^2}$, where $\sigma_p^2 \equiv var_j\left[\tilde{s}_p|\tilde{v}\right]$.

Lemma 4. *Let $g(x) \equiv \mathbb{E}_j\left[\tilde{v}|ND, \tilde{S}_j = x\right]$. Then, investor j's demand given $\tilde{S}_j$ equals:*

$$D_j = \tau\left[\tilde{S}_j - g^{-1}\left(P_{ND,p}\right)\right].$$

Proof. To begin, we derive some initial results regarding the investor j's subjective posterior distribution over $\tilde{v}$ given their signal $\tilde{s}_j = s_j$, the price signal $\tilde{s}_p = s_p$, and the event of non-disclosure ND, whose density we denote by $f_j\left(v|ND, s_j, s_p\right)$. Note that:

$$\begin{aligned} f_j\left(v|ND, s_j, s_p\right) &\propto f_j\left(v, s_j, s_p|ND\right) \\ &= f_j\left(s_j, s_p|v\right) f\left(v|ND\right) \\ &\propto \exp\left[-\frac{\left(s_j - v\right)^2}{2\sigma_\varepsilon^2} - \frac{\left(s_p - v\right)^2}{2\sigma_p^2}\right] f\left(v|ND\right) \\ &= \exp\left[\left(-\frac{1}{2\sigma_p^2} - \frac{1}{2\sigma_\varepsilon^2}\right)v^2 + \left(\frac{s_p}{\sigma_p^2} + \frac{s_j}{\sigma_\varepsilon^2}\right)v - \left(\frac{s_p^2}{2\sigma_p^2} + \frac{s_j^2}{2\sigma_\varepsilon^2}\right)\right] f\left(v|ND\right). \end{aligned}$$

Thus,

$$f_j\left(v|ND, s_j, s_p\right) = \frac{\exp\left[\left(-\frac{1}{2\sigma_p^2} - \frac{1}{2\sigma_\varepsilon^2}\right)v^2 + \left(\frac{s_p}{\sigma_p^2} + \frac{s_j}{\sigma_\varepsilon^2}\right)v\right] f\left(v|ND\right)}{\int_{-\infty}^{\infty} \exp\left[\left(-\frac{1}{2\sigma_p^2} - \frac{1}{2\sigma_\varepsilon^2}\right)v^2 + \left(\frac{s_p}{\sigma_p^2} + \frac{s_j}{\sigma_\varepsilon^2}\right)v\right] f\left(v|ND\right) dv}.$$

Now, let $M\left(t; x\right) = \mathbb{E}_j\left[\exp\left(t\tilde{v}\right)|ND, \tilde{S}_j = x\right]$ denote the moment-generating function of $\tilde{v}$ as perceived by investor j when $\tilde{S}_j = x$; see Breon-Drish (2015) for proofs that the integrals under consideration below in fact exist and that derivative-integral interchange is valid. Note that:

$$M\left(t; x\right) = \frac{\int_{-\infty}^{\infty} f\left(v|ND\right) \exp\left[\left(-\frac{1}{2\sigma_p^2} - \frac{1}{2\sigma_\varepsilon^2}\right)v^2 + \left(t + x\right)v\right] dv}{\int_{-\infty}^{\infty} f\left(v|ND\right) \exp\left[\left(-\frac{1}{2\sigma_p^2} - \frac{1}{2\sigma_\varepsilon^2}\right)v^2 + xv\right] dv}. \tag{26}$$

Now, define:

$$g(x) \equiv \frac{\int_{-\infty}^{\infty} v \exp\left[\left(-\frac{1}{2\sigma_p^2} - \frac{1}{2\sigma_\varepsilon^2}\right) v^2 + xv\right] f(v|ND)\, dv}{\int_{-\infty}^{\infty} \exp\left[\left(-\frac{1}{2\sigma_p^2} - \frac{1}{2\sigma_\varepsilon^2}\right) v^2 + xv\right] f(v|ND)\, dv}. \tag{27}$$

Then, we have:

$$\left[\frac{\partial M(t;x)}{\partial t}\right]_{t=0} = \mathbb{E}_j\left[\tilde{v}|ND, \tilde{S}_j = x\right] \equiv g(x). \tag{28}$$

We can now solve for investor j's optimal demand D_j, which solves:

$$D_j = \arg\max_y -\int_{-\infty}^{\infty} \exp\left\{-\frac{1}{\tau}\left(y\left(\tilde{v} - P_{ND,p}\right)\right)\right\} f_j(v|ND, s_j, s_p)\, dv. \tag{29}$$

It is easily verified that this function is concave and thus the first-order condition is sufficient for a solution. The first-order condition reduces to the following:

$$\begin{aligned}
P_{ND,p} &= \frac{\int_{-\infty}^{\infty} v \exp\left(-\tau^{-1} D_j \tilde{v}\right) f_j(v|ND, s_j, s_p)\, dv}{\int_{-\infty}^{\infty} \exp\left(-\tau^{-1} D_j \tilde{v}\right) f_j(v|ND, s_j, s_p)\, dv} \\
&= \frac{\int_{-\infty}^{\infty} v \exp\left[\left(-\frac{1}{2\sigma_p^2} - \frac{1}{2\sigma_\varepsilon^2}\right) v^2 + \left(-\frac{D_j}{\tau} + \frac{s_p}{\sigma_p^2} + \frac{s_j}{\sigma_\varepsilon^2}\right) v\right] f(v|ND)\, dv}{\int_{-\infty}^{\infty} \exp\left[\left(-\frac{1}{2\sigma_p^2} - \frac{1}{2\sigma_\varepsilon^2}\right) v^2 + \left(-\frac{D_j}{\tau} + \frac{s_p}{\sigma_p^2} + \frac{s_j}{\sigma_\varepsilon^2}\right) v\right] f(v|ND)\, dv} \\
&= g\left(\frac{s_p}{\sigma_p^2} + \frac{s_j}{\sigma_\varepsilon^2} - \frac{D_j}{\tau}\right).
\end{aligned}$$

In the proof of Lemma 3 below, we show that $g'(x) > 0$, and thus $g(\cdot)$ is invertible. Thus,

$$D_j = \tau\left[\frac{s_p}{\sigma_p^2} + \frac{s_j}{\sigma_\varepsilon^2} - g^{-1}\left(P_{ND,p}\right)\right].$$

□

Using this result, we may derive the firm's price by applying the market-clearing condition:

$$\begin{aligned}
0 &= z + \int_0^1 D_i di \\
&\iff -\frac{z}{\tau} = \frac{s_p}{\sigma_p^2} + \frac{v}{\sigma_\varepsilon^2} - g^{-1}\left(P_{ND,p}\right) \\
&\iff P_{ND,p} = g\left(\frac{v}{\sigma_\varepsilon^2} + \frac{s_p}{\sigma_p^2} + \frac{z}{\tau}\right).
\end{aligned} \tag{30}$$

To complete the proof, we verify that there exists a $\beta \in \Re$ such that the firm's price takes the conjectured generalized linear form with $G(x) = g(x)$. Note expression (30) satisfies the

conjectured form of equilibrium if and only if $\beta = \frac{\sigma_\varepsilon^2}{\tau}$. This, in turn, implies that:

$$\sigma_p^2 = \frac{1-\rho^2}{\rho^2}\sigma_v^2 + \frac{\sigma_\varepsilon^4 \sigma_z^2}{\tau^2 \rho^2},$$

and $s_p = \frac{1}{\rho}\left(v + \frac{\sigma_\varepsilon^2}{\tau} z\right)$. Substituting into expression (30), we may write:

$$P_{ND,p} = g\left(\pi\left(\bar{s}, z\right)\right) \text{ where } \pi\left(\bar{s}, z\right) \equiv \left(\frac{1}{\sigma_\varepsilon^2} + \frac{1}{\rho\sigma_p^2}\right)\left(\bar{s} + \frac{\sigma_\varepsilon^2}{\tau} z\right). \tag{31}$$

Now, note from expression (28) that:

$$P_{ND,p} = \mathbb{E}_j\left[\tilde{v} | ND, \tilde{S}_j = \pi\left(\bar{s}, z\right)\right].$$

Next, observe that:

$$\mathbb{E}_j\left[\tilde{v} | \tilde{S}_j = \pi\left(\bar{s}, z\right)\right] = \frac{\left(\frac{1}{\sigma_\varepsilon^2} + \frac{1}{\rho\sigma_p^2}\right)\left(\bar{s} + \frac{\sigma_\varepsilon^2}{\tau} z\right)}{\frac{1}{\sigma_v^2} + \frac{1}{\sigma_\varepsilon^2} + \frac{1}{\sigma_p^2}},$$

and so we may write:

$$P_{ND,p} = \mathbb{E}_j\left[\tilde{v} | ND, \tilde{\mu}_j = \frac{\left(\frac{1}{\sigma_\varepsilon^2} + \frac{1}{\rho\sigma_p^2}\right)\left(\bar{s} + \frac{\sigma_\varepsilon^2}{\tau} z\right)}{\frac{1}{\sigma_v^2} + \frac{1}{\sigma_\varepsilon^2} + \frac{1}{\sigma_p^2}}\right]. \tag{32}$$

Moreover, we have that:

$$\int_i \mu_i di - \frac{\sigma_s^2}{\tau} z = \frac{\frac{\bar{s}}{\sigma_\varepsilon^2} + \frac{s_p}{\rho\sigma_p^2}}{\frac{1}{\sigma_v^2} + \frac{1}{\sigma_\varepsilon^2} + \frac{1}{\sigma_p^2}} - \frac{1}{\tau}\frac{z}{\frac{1}{\sigma_v^2} + \frac{1}{\sigma_\varepsilon^2} + \frac{1}{\sigma_p^2}} = \frac{\left(\frac{1}{\sigma_\varepsilon^2} + \frac{1}{\rho\sigma_p^2}\right)\left(\bar{s} + \frac{\sigma_\varepsilon^2}{\tau} z\right)}{\frac{1}{\sigma_v^2} + \frac{1}{\sigma_\varepsilon^2} + \frac{1}{\sigma_p^2}}.$$

Thus, we may write expression (32) as:

$$P_{ND,p} = \mathbb{E}_j\left[\tilde{v} | ND, \tilde{\mu}_j = H\left(\bar{s}, z\right)\right],$$

where $H\left(\bar{s}, z\right) = \int_i \mu_i di - \frac{\sigma_s^2}{\tau} z$. To see that price can be expressed as in the lemmas, note that the event of non-disclosure ND results either from an informed manager who observed $\tilde{v} < T$ or an uninformed manager; denote the former event by $\tilde{\Lambda} = 1$ and the latter by $\tilde{\Lambda} = 0$.

Then, we have:

$$\mathbb{E}_j\left[\tilde{v}|ND,\tilde{\mu}_j=H\left(\bar{s},z\right)\right]=Pr_j\left(\tilde{\Lambda}=1|\tilde{\mu}_j=H\left(\bar{s},z\right)\right)\mathbb{E}_j\left[\tilde{v}|\tilde{v}<T,\tilde{\mu}_j=H\left(\bar{s},z\right)\right]+$$
$$Pr_j\left(\tilde{\Lambda}=0|\tilde{\mu}_j=H\left(\bar{s},z\right)\right)\mathbb{E}_j\left[\tilde{v}|\tilde{\mu}_j=H\left(\bar{s},z\right)\right]. \quad (33)$$

Note that $\Pr_j\left(\tilde{v}<T|\tilde{\mu}_j=H\left(\bar{s},z\right)\right)=\Phi\left(\frac{T-H(\bar{s},z)}{\sigma_s}\right)$. Therefore, we can apply Bayes' rule to arrive at:

$$Pr_j\left(\tilde{\Lambda}=1|\tilde{\mu}_j=H\left(\bar{s},z\right)\right)=1-Pr_j\left(\tilde{\Lambda}=0|\tilde{\mu}_j=H\left(\bar{s},z\right)\right)=\frac{p\Phi\left(\frac{T-H(\bar{s},z)}{\sigma_s}\right)}{p\Phi\left(\frac{T-H(\bar{s},z)}{\sigma_s}\right)+1-p}. \quad (34)$$

To explicitly derive $\mathbb{E}_j\left[\tilde{v}|\tilde{v}<T,\tilde{\mu}_j=H\left(\bar{s},z\right)\right]$, we may apply the formula for the mean of a truncated normal distribution and the fact that, by definition, $\sigma_s^2=\text{var}_j\left[\tilde{v}|\tilde{s}_j,\tilde{s}_p\right]=\text{var}_j\left[\tilde{v}|\tilde{\mu}_j\right]$. This yields:

$$\mathbb{E}_j\left[\tilde{v}|\tilde{v}<T,\tilde{\mu}_j=H\left(\bar{s},z\right)\right]=H(\bar{s},z)-\sigma_s h\left(\frac{T-H(\bar{s},z)}{\sigma_s}\right). \quad (35)$$

Substituting equations (34) and (35) into equation (33) yields:

$$P_{ND,p}=\frac{p\Phi\left(\frac{T-H(\bar{s},z)}{\sigma_s}\right)\left(H(\bar{s},z)-\sigma_s h\left(\frac{T-H(\bar{s},z)}{\sigma_s}\right)\right)+(1-p)H\left(\bar{s},z\right)}{p\Phi\left(\frac{T-H(\bar{s},z)}{\sigma_s}\right)+1-p}.$$

Finally, to see that the above expression equates to the price expressions stated in the lemmas, observe that when $p=1$, it reduces to $H(\bar{s},z)-\sigma_s h\left(\frac{T-H(\bar{s},z)}{\sigma_s}\right)$. □

Proof of Lemma 3. Observe from equation (31) that:

$$\begin{aligned}\frac{\partial P_{ND,p}}{\partial v} &= \left(\frac{1}{\sigma_\varepsilon^2}+\frac{1}{\rho\sigma_p^2}\right)g'\left(\pi\left(v,z\right)\right)\\ &= \left(var^{-1}\left[\tilde{s}_i|\tilde{v}\right]+\rho^{-1}var_j^{-1}\left[\tilde{s}_p|\tilde{v}\right]\right)g'\left(\pi\left(v,z\right)\right).\end{aligned}$$

Differentiating equation (27), we have that:

$$\begin{aligned} g'(x) &= \frac{\int_{-\infty}^{\infty} v^2 \exp\left[\left(-\frac{1}{2\sigma_p^2}+\frac{1}{2\sigma_\varepsilon^2}\right)v^2+xv\right] f(v|ND)\,dv}{\int_{-\infty}^{\infty} \exp\left[\left(-\frac{1}{2\sigma_p^2}+\frac{1}{2\sigma_\varepsilon^2}\right)v^2+xv\right] f(v|ND)\,dv} \\ &\quad - \left(\frac{\int_{-\infty}^{\infty} v \exp\left[\left(-\frac{1}{2\sigma_p^2}+\frac{1}{2\sigma_\varepsilon^2}\right)v^2+xv\right] f(v|ND)\,dv}{\int_{-\infty}^{\infty} \exp\left[\left(-\frac{1}{2\sigma_p^2}+\frac{1}{2\sigma_\varepsilon^2}\right)v^2+xv\right] f(v|ND)\,dv}\right)^2. \end{aligned}$$

Now, note from equation (26), this implies:

$$\begin{aligned} g'(x) &= \left[\frac{\partial^2 M(t;x)}{\partial t^2}\right]_{t=0} - \left\{\left[\frac{\partial M(t;x)}{\partial t}\right]_{t=0}\right\}^2 \\ &= \mathbb{E}_j\left[\tilde{v}^2|ND,\tilde{S}_j=x\right] - \mathbb{E}_j\left[\tilde{v}|ND,\tilde{S}_j=x\right]^2 \\ &= var_j\left[\tilde{v}|ND,\tilde{S}_j=x\right] > 0. \end{aligned}$$

Therefore, $g'(\pi(v,z)) = var_j\left[\tilde{v}|ND,\tilde{S}_j=\pi(v,z)\right]$. Applying the reasoning from the previous proof, from investor j's perspective, conditioning on $\tilde{S}_j = \pi(v,z)$ is equivalent to conditioning on $\tilde{\mu}_j = \int_i \mu_i di - \frac{\sigma_s^2}{\tau} z$, so that this may be written:

$$g'(\pi(v,z)) = var_j\left[\tilde{v}|ND,\tilde{\mu}_j = H(\bar{s},z)\right].$$

□

Proof of Propositions 1 and 4. We again prove these two results together given the considerable overlap in the arguments. We start by proving that there *exists* a threshold equilibrium when either $p=1$ and/or $\frac{1}{\sigma_\varepsilon^2}+\frac{1}{\rho\sigma_p^2}$ is sufficiently small. Note that the manager discloses if and only if:

$$v - \mathbb{E}\left[P_{ND,p}|\tilde{v}=v\right] \geq c.$$

Sufficient conditions for a threshold equilibrium to exist are that:

$$\begin{aligned} &\text{(i) } v - \mathbb{E}\left[P_{ND,p}|\tilde{v}=v\right] \text{ increases in } v; \\ \text{(ii) } \exists T \in \Re &\text{ such that } T - \mathbb{E}\left[P_{ND,p}|\tilde{v}=T\right] = c. \end{aligned} \tag{36}$$

We begin by showing that $p=1$ and/or $\frac{1}{\sigma_\varepsilon^2}+\frac{1}{\rho\sigma_p^2} < \left[\sigma_v^2\left(1+\frac{1}{2}p(1-p)\right)\right]^{-1}$ ensures that condition (i) holds. To do so, we show that $\frac{\partial P_{ND,p}}{\partial v} < 1\ \forall v,z$. To see why this is sufficient, note that, because $\frac{\partial P_{ND,p}}{\partial v} = \frac{\tau}{\sigma_\varepsilon^2}\frac{\partial P_{ND,p}}{\partial z}$, $\frac{\partial P_{ND,p}}{\partial v} \in (0,1)\ \forall v,z$ implies that $|P_{ND,p}|$ is sublinear in

z. Therefore, letting $\phi(\cdot)$ denote the PDF of a standard normal, $\left|\frac{1}{\sigma_z}P_{ND,p}\phi(z)\right| < \left|\frac{1}{\sigma_z}A\phi(z)\right|$ for some A that does not depend upon z, and, being the expectation of an absolute normal, $\int_{-\infty}^{\infty}\left|\frac{1}{\sigma_z}A\phi(z)\right|dz$ is finite. Thus, by the dominated convergence theorem,

$$\frac{\partial}{\partial v}\mathbb{E}\left[P_{ND,p}|\tilde{v}=v\right] = \frac{1}{\sigma_z}\int_{-\infty}^{\infty}\frac{\partial}{\partial v}P_{ND,p}\phi(z)\,dz < 1.$$

Now, to see that $\frac{\partial P_{ND,p}}{\partial v} < 1\ \forall v, z$, let $\Delta_v \equiv \sigma_s^{-2}\left(\frac{1}{\sigma_\varepsilon^2}+\frac{1}{\rho\sigma_p^2}\right)$ and $\Delta_z \equiv \frac{\sigma_\varepsilon^2}{\tau}\Delta_v$; this enables us to write $H(\bar{s},z)$ in the compact form $\Delta_v\bar{s}+\Delta_z z$. Let $\Phi(\cdot)$ denote the CDF of a standard normal and $h(x)=\frac{\phi(x)}{\Phi(x)}$ denote the inverse-mills ratio. Appealing to Lemma 1 and applying the notation in its proof, we have:

$$\begin{aligned} P_{ND,p} &= \mathbb{E}_j\left[\tilde{v}|ND,\tilde{\mu}_j = H(\bar{s},z)\right] \\ &= \frac{p\Phi\left(\sigma_s^{-1}\left(T-\Delta_v\bar{s}-\Delta_z z\right)\right)\mathbb{E}_j\left[\tilde{v}|\tilde{v}<T,\tilde{\mu}_j=H(\bar{s},z)\right]+(1-p)\left(\Delta_v\bar{s}+\Delta_z z\right)}{p\Phi\left(\sigma_s^{-1}\left(T-\Delta_v\bar{s}-\Delta_z z\right)\right)+1-p}. \end{aligned} \tag{37}$$

When $p=1$, this reduces to $\mathbb{E}_j\left[\tilde{v}|\tilde{v}<T,\tilde{\mu}_j=H(\bar{s},z)\right]$, which we can explicitly calculate as the expected value of a truncated normal distribution:

$$\lim_{p\to 1}P_{ND,p} = \Delta_v\bar{s}+\Delta_z z-\sigma_s h\left(\sigma_s^{-1}\left(T-\Delta_v\bar{s}-\Delta_z z\right)\right).$$

Note from the manager's perspective, $\bar{s}=v$. So, differentiating the above expression with respect to v yields:

$$\frac{\partial P_{ND,p}}{\partial v} = \Delta_v\left[1+h'\left(\frac{T-\Delta_v v-\Delta_z z}{\sigma_s}\right)\right].$$

It may be verified that $h'(x)\in(-1,0)$ and thus this belongs to $(0,1)$. Moving to the case in which $p<1$, Lemma 3 implies that:

$$\frac{\partial P_{ND,p}}{\partial v} = \left(\frac{1}{\sigma_\varepsilon^2}+\frac{1}{\rho\sigma_p^2}\right)var_j\left[\tilde{v}|ND,\tilde{\mu}_j=H(\bar{s},z)\right] > 0.$$

Now, let $\tilde{\Lambda}$ denote a random variable that captures whether the manager is informed. Then, applying the law of total variance:

$$\begin{aligned} var_j\left[\tilde{v}|ND,\tilde{\mu}_j=H(\bar{s},z)\right] &= \mathbb{E}_j\left\{var_j\left[\tilde{v}|\tilde{\Lambda},ND,\tilde{\mu}_j=H(\bar{s},z)\right]\right\} \\ &\quad + var_j\left\{\mathbb{E}_j\left[\tilde{v}|\tilde{\Lambda},ND,\tilde{\mu}_j=H(\bar{s},z)\right]\right\}. \end{aligned} \tag{38}$$

Now,

$$\begin{aligned} & \mathbb{E}_j\left\{var_j\left[\tilde{v}|\tilde{\Lambda}, ND, \tilde{\mu}_j = H\left(\bar{s}, z\right)\right]\right\} \\ = & \frac{p\Phi\left(\sigma_s^{-1}\left(T - \Delta_v\bar{s} - \Delta_z z\right)\right)\sigma_s^2 + \left(1-p\right)var_j\left[\tilde{v}|\tilde{v} < T, \tilde{\mu}_j = H\left(\bar{s}, z\right)\right]}{p\Phi\left(\sigma_s^{-1}\left(T - \Delta_v\bar{s} - \Delta_z z\right)\right) + 1 - p} < \sigma_s^2. \end{aligned} \tag{39}$$

Next, we have:

$$\begin{aligned} & var_j\left\{\mathbb{E}_j\left[\tilde{v}|\tilde{\Lambda}, ND, \tilde{\mu}_j = H\left(\bar{s}, z\right)\right]\right\} \\ = & \frac{p\left(1-p\right)\Phi\left(\sigma_s^{-1}\left(T - \Delta_v\bar{s} - \Delta_z z\right)\right)\sigma_s^2 h\left(\sigma_s^{-1}\left(T - \Delta_v\bar{s} - \Delta_z z\right)\right)^2}{\left(p\Phi\left(\sigma_s^{-1}\left(T - \Delta_v\bar{s} - \Delta_z z\right)\right) + 1 - p\right)^2} \\ = & \frac{p\left(1-p\right)\sigma_s^2\phi\left(\sigma_s^{-1}\left(T - \Delta_v\bar{s} - \Delta_z z\right)\right)^2}{\Phi\left(\sigma_s^{-1}\left(T - \Delta_v\bar{s} - \Delta_z z\right)\right)\left(p\Phi\left(\sigma_s^{-1}\left(T - \Delta_v\bar{s} - \Delta_z z\right)\right) + 1 - p\right)^2} \\ < & p\left(1-p\right)\sigma_s^2\frac{\phi\left(\sigma_s^{-1}\left(T - \Delta_v\bar{s} - \Delta_z z\right)\right)^2}{\Phi\left(\sigma_s^{-1}\left(T - \Delta_v\bar{s} - \Delta_z z\right)\right)}. \end{aligned} \tag{40}$$

Note that $\frac{\phi(x)^2}{\Phi(x)}$ is bounded above by $\frac{1}{2}$ and thus the above expression is bounded over all realizations of $\bar{s}$ and z by $\frac{p(1-p)\sigma_s^2}{2}$. Combining (38), (39), and (40), we have:

$$\frac{\partial P_{ND,p}}{\partial v} < \sigma_s^2\left(\frac{1}{\sigma_\varepsilon^2} + \frac{1}{\rho\sigma_p^2}\right)\left(1 + \frac{p\left(1-p\right)}{2}\right) < \sigma_v^2\left(\frac{1}{\sigma_\varepsilon^2} + \frac{1}{\rho\sigma_p^2}\right)\left(1 + \frac{p\left(1-p\right)}{2}\right),$$

such that, for $\frac{1}{\sigma_\varepsilon^2} + \frac{1}{\rho\sigma_p^2} < \left[\sigma_v^2\left(1 + \frac{1}{2}p\left(1-p\right)\right)\right]^{-1}$, $\frac{\partial P_{ND,p}}{\partial v} \in (0,1)$.

We next show that condition (ii) in (36) always holds. It is easily seen that $T - \mathbb{E}\left[P_{ND,p}|\tilde{v} = T\right]$ is a continuous function of T. Thus, we can prove the existence of an equilibrium by showing $\lim_{T\to-\infty}\left\{T - \mathbb{E}\left[P_{ND,p}|\tilde{v} = T\right]\right\} < 0$ and $\lim_{T\to\infty}\left\{T - \mathbb{E}\left[P_{ND,p}|\tilde{v} = T\right]\right\} > \infty$. Letting $\kappa\left(v, T\right) \equiv v - \Delta_v T - \Delta_z z$ and $\omega\left(T\right) \equiv \kappa\left(T, T\right)$ (where dependence upon z is suppressed for convenience), we have:

$$\begin{aligned} & T - \mathbb{E}\left[P_{ND,p}|\tilde{v} = T\right] \\ = & \mathbb{E}\left\{\frac{\begin{array}{c} p\left[\Phi\left(\frac{\omega(T)}{\sigma_s}\right)\left(\left(1-\Delta_v\right)T - \Delta_z z\right) + \sigma_s\phi\left(\frac{\omega(T)}{\sigma_s}\right)\right] \\ + \left(1-p\right)\left(\left(1-\Delta_v\right)T - \Delta_z z\right) \end{array}}{p\Phi\left(\frac{\omega(T)}{\sigma_s}\right) + 1 - p}\right\} \equiv \mathbb{E}\left[\Psi\left(T, \tilde{z}\right)\right]. \end{aligned}$$

We now show this converges to $-\infty$ as $T \to -\infty$. To see this, note first that $\Psi\left(T, z\right)$ converges pointwise to infinity as $T \to -\infty$. To see this, note that $\Delta_v = \frac{1}{\rho}\frac{\sigma_v^2\left(\rho\sigma_p^2 + \sigma_\varepsilon^2\right)}{\sigma_v^2\left(\sigma_p^2 + \sigma_\varepsilon^2\right) + \sigma_p^2\sigma_\varepsilon^2} < 1$;

thus, $\lim_{T\to-\infty}\omega(T)=-\infty$, and we have:

$$\lim_{T\to-\infty}\Psi(T,z)=\lim_{T\to-\infty}\left\{\frac{1}{1-p}\left[p\Phi\left(\frac{\omega(T)}{\sigma_s}\right)+1-p\right]\left[(1-\Delta_v)T-\Delta_z z\right]\right\}=-\infty.$$

Moreover, note that $\Psi(T,z)$ is decreasing in z. This implies that, for any $K,Q\in\Re$, $\exists\bar{T}\in\Re$ such that $T>\bar{T}, z<Q \implies \Psi(T,z)<K$. Thus,

$$\begin{aligned} T &> \bar{T} \implies \mathbb{E}\left[\Psi(T,z)\right]=\int_{-\infty}^{Q}\frac{\Psi(T,z)}{\sigma_z}\phi\left(\frac{z}{\sigma_z}\right)dz+\int_{Q}^{\infty}\frac{\Psi(T,z)}{\sigma_z}\phi\left(\frac{z}{\sigma_z}\right)dz \\ &< K\Phi(Q)+\int_{Q}^{\infty}\frac{\Psi(T,z)}{\sigma_z}\phi\left(\frac{z}{\sigma_z}\right)dz. \end{aligned}$$

Now, it is easy to see that $\Psi(T,z)$ is sublinear in z and thus, choosing Q sufficiently large, the second term in this expression can be made arbitrarily small. Thus, $\mathbb{E}\left[\Psi(T,z)\right]$ can be made arbitrarily negative by choosing K sufficiently negative and Q sufficiently large. Next, note that:

$$\begin{aligned} &\lim_{T\to\infty}\left\{T-\mathbb{E}\left[P_{ND,p}\left(\{s_i\},\tilde{z}\right)|\tilde{v}=T\right]\right\} \\ &\qquad p\left[\Phi\left(\tfrac{\omega(T)}{\sigma_s}\right)\left((1-\Delta_v)T-\Delta_z z\right)+\sigma_s\phi\left(\tfrac{\omega(T)}{\sigma_s}\right)\right] \\ &\qquad +(1-p)\left((1-\Delta_v)T-\Delta_z z\right) \\ =\ &\lim_{T\to\infty}\frac{}{p\Phi\left(\tfrac{\omega(T)}{\sigma_s}\right)+1-p}=\infty. \end{aligned}$$

Thus, $\Psi(T,\tilde{z})$ converges pointwise to ∞. Applying similar reasoning to the case in which $T\to-\infty$, this implies that $\lim_{T\to\infty}\mathbb{E}\left[\Psi(T,\tilde{z})\right]=\infty$. To complete the proof, we show that when a threshold equilibrium exists, it is unique. It is sufficient to show that $\mathbb{E}\left[\Psi(T,\tilde{z})\right]$ strictly increases in T. Again applying the fact that $\Psi(T,z)$ is sublinear in z, we may again apply the dominated convergence theorem to arrive at $\frac{\partial}{\partial T}\mathbb{E}\left[\Psi(T,\tilde{z})\right]=\mathbb{E}\left[\frac{\partial}{\partial T}\Psi(T,\tilde{z})\right]$. Absorbing T into the numerator of $\mathbb{E}\left[P_{ND,p}|\tilde{v}=T\right]$ and expressing $\mathbb{E}_j\left[\tilde{v}|\tilde{v}<T,\tilde{\mu}_j=H(T,z)\right]$ in its integral form, we may write $\frac{\partial}{\partial T}\Psi(T,\tilde{z})$ as:

$$\frac{\partial}{\partial T}\left[\frac{p\left[\Phi\left(\frac{\omega(T)}{\sigma_s}\right)T-\int_{-\infty}^{T}\frac{v}{\sigma_s}\phi\left(\frac{\kappa(v,T)}{\sigma_s}\right)dv\right]+(1-p)\omega(T)}{p\Phi\left(\frac{\omega(T)}{\sigma_s}\right)+1-p}\right]. \tag{41}$$

Now, integration by parts yields:

$$\begin{aligned}\int_{-\infty}^{T}\frac{v}{\sigma_s}\phi\left(\frac{\kappa\left(v,T\right)}{\sigma_s}\right)dv &= \left[v\Phi\left(\frac{\kappa\left(v,T\right)}{\sigma_s}\right)\right]_{-\infty}^{T}-\int_{-\infty}^{T}\Phi\left(\frac{\kappa\left(v,T\right)}{\sigma_s}\right)dv\\ &= T\Phi\left(\frac{\omega\left(T\right)}{\sigma_s}\right)-\int_{-\infty}^{T}\Phi\left(\frac{\kappa\left(v,T\right)}{\sigma_s}\right)dv.\end{aligned}$$

Thus, expression (41) equals:

$$\begin{aligned}&\frac{\partial}{\partial T}\left[\frac{p\int_{-\infty}^{T}\Phi\left(\frac{\kappa(v,T)}{\sigma_s}\right)dv+\left(1-p\right)\omega\left(T\right)}{p\Phi\left(\frac{\omega(T)}{\sigma_s}\right)+1-p}\right]\\ &\left[p\Phi\left(\frac{\omega(T)}{\sigma_s}\right)+\left(1-p\right)\left(1-\Delta_v\right)\right]\left(p\Phi\left(\frac{\omega(T)}{\sigma_s}\right)+1-p\right)\\ =\quad &\frac{-\frac{1-\Delta_v}{\sigma_s}p\phi\left(\frac{\omega(T)}{\sigma_s}\right)\left[p\int_{-\infty}^{T}\Phi\left(\frac{\kappa(v,T)}{\sigma_s}\right)dv+\left(1-p\right)\omega\left(T\right)\right]}{\left[p\Phi\left(\omega\left(T\right)\right)+1-p\right]^2}.\end{aligned}$$

Manipulating the numerator of this expression yields:

$$\begin{aligned}&p^2\left[\Phi\left(\frac{\omega(T)}{\sigma_s}\right)^2-\frac{1}{\sigma_s}\phi\left(\frac{\omega(T)}{\sigma_s}\right)\int_{-\infty}^{T}\Phi\left(\frac{\kappa(v,T)}{\sigma_s}\right)dv\right]+p\left(1-p\right)\left(2-\Delta_v\right)\Phi\left(\frac{\omega(T)}{\sigma_s}\right)\\ &+\frac{p^2\Delta_v}{\sigma_s}\phi\left(\frac{\omega(T)}{\sigma_s}\right)\int_{-\infty}^{T}\Phi\left(\frac{\kappa(v,T)}{\sigma_s}\right)dv-\frac{1-\Delta_v}{\sigma_s}p\left(1-p\right)\omega\left(T\right)\phi\left(\frac{\omega(T)}{\sigma_s}\right).\end{aligned}$$

Now, note that the normal distribution is log concave, which implies that $\Phi\left(\frac{\omega(T)}{\sigma_s}\right)^2-\frac{1}{\sigma_s}\phi\left(\frac{\omega(T)}{\sigma_s}\right)\int_{-\infty}^{T}\Phi\left(\frac{\kappa(v,T)}{\sigma_s}\right)dv>0$ (Bagnoli and Bergstrom (2005)). So, the above expression exceeds:

$$\begin{aligned}&p\left(1-p\right)\left(2-\Delta_v\right)\Phi\left(\frac{\omega(T)}{\sigma_s}\right)\\ &+\frac{p^2\Delta_v}{\sigma_s}\phi\left(\frac{\omega(T)}{\sigma_s}\right)\int_{-\infty}^{T}\Phi\left(\frac{\kappa(v,T)}{\sigma_s}\right)dv-\frac{1-\Delta_v}{\sigma_s}p\left(1-p\right)\omega\left(T\right)\phi\left(\frac{\omega(T)}{\sigma_s}\right)\\ >\quad &p\left(1-p\right)\left[\left(2-\Delta_v\right)\Phi\left(\frac{\omega\left(T\right)}{\sigma_s}\right)-\left(1-\Delta_v\right)\frac{\omega\left(T\right)}{\sigma_s}\phi\left(\frac{\omega\left(T\right)}{\sigma_s}\right)\right]\\ \propto\quad &p\left(1-p\right)\left[\frac{2-\Delta_v}{1-\Delta_v}-\frac{\omega\left(T\right)}{\sigma_s}\frac{\phi\left(\frac{\omega(T)}{\sigma_s}\right)}{\Phi\left(\frac{\omega(T)}{\sigma_s}\right)}\right]\\ =\quad &p\left(1-p\right)\left[\frac{2-\Delta_v}{1-\Delta_v}-\chi h\left(\chi\right)\right],\end{aligned}$$

where $\chi\equiv\frac{\omega(T)}{\sigma_s}$. Now, it may be verified that, $\forall\chi\in\Re$, the inverse-mills ratio $h\left(\cdot\right)$ satisfies $\chi h\left(\chi\right)<\frac{1}{2}$. Together with the fact that $\frac{2-\Delta_v}{1-\Delta_v}>2$, this implies the above expression is

positive, which completes the proof. □

Proof of Proposition 2. Part i. Let $\Omega(T;\sigma_v)$ denote the equilibrium condition as a function of T and σ_v:

$$\Omega(T;\sigma_v) \equiv T - c - \mathbb{E}\left[P_{ND}\left(\{s_i\},\tilde{z}\right)|\tilde{v}=T\right].$$

Then, note that:

$$\begin{aligned}
\frac{\partial \Pr(\tilde{v}>T)}{\partial \sigma_v} &= \frac{\partial}{\partial \sigma_v}\left[1-\Phi\left(\frac{T}{\sigma_v}\right)\right] \\
&= -\phi\left(\frac{T}{\sigma_v}\right)\frac{\sigma_v\frac{\partial T}{\partial \sigma_v}-T}{\sigma_v^2} \\
&= -\phi\left(\frac{T}{\sigma_v}\right)\frac{-\sigma_v\left(\frac{\partial \Omega(T;\sigma_v)}{\partial T}\right)^{-1}\frac{\partial \Omega(T;\sigma_v)}{\partial \sigma_v}-T}{\sigma_v^2} \\
&\propto \sigma_v\frac{\partial \Omega(T;\sigma_v)}{\partial \sigma_v}+T\frac{\partial \Omega(T;\sigma_v)}{\partial T}.
\end{aligned}$$

Note moving forward, the arguments applied in the previous proof enable us to interchange the order of limits/derivatives and expectations. It can be verified that:

$$\lim_{\sigma_\varepsilon\to\infty}\Delta_v = 0;\ \lim_{\sigma_\varepsilon\to\infty}\Delta_z = \frac{\sigma_\varepsilon^2}{\tau};\ \lim_{\sigma_\varepsilon\to\infty}\sigma_s = \sigma_v.$$

Therefore, we have:

$$\begin{aligned}
\lim_{\sigma_\varepsilon\to\infty} T\frac{\partial \Omega(T;\sigma_v)}{\partial T} &= \lim_{\sigma_\varepsilon\to\infty}\left\{T * \frac{\partial}{\partial T}\mathbb{E}\left[T(1-\Delta_v)-\Delta_z z+\sigma_s h\left(\sigma_s^{-1}\left(T(1-\Delta_v)-\Delta_z\tilde{z}\right)\right)\right]\right\} \\
&= \lim_{\sigma_\varepsilon\to\infty}\mathbb{E}\left[T(1-\Delta_v)\left(1+h'\left(\sigma_s\left(T(1-\Delta_v)-\Delta_z\tilde{z}\right)\right)\right)\right] \\
&= \mathbb{E}\left[T\left(1+h'\left(\sigma_v^{-1}\left(T-\frac{\sigma_v^2}{\tau}\tilde{z}\right)\right)\right)\right].
\end{aligned}$$

Moreover,

$$\begin{aligned}
&\lim_{\sigma_\varepsilon\to\infty}\sigma_v\frac{\partial \Omega(T;\sigma_v)}{\partial \sigma_v} \\
= &\lim_{\sigma_\varepsilon\to\infty}\sigma_v\mathbb{E}\left[\begin{array}{c} T * \frac{\partial}{\partial \sigma_v}(1-\Delta_v)+\frac{\partial \sigma_s}{\partial \sigma_v}h\left(\sigma_s^{-1}\left((1-\Delta_v)T-\Delta_z\tilde{z}\right)\right) \\ +\sigma_s\frac{\partial}{\partial \sigma_v}\left(\sigma_s^{-1}\left((1-\Delta_v)T-\Delta_z\tilde{z}\right)\right)h'\left(\sigma_s^{-1}\left((1-\Delta_v)T-\Delta_z\tilde{z}\right)\right)\end{array}\right] \\
= &\lim_{\sigma_\varepsilon\to\infty}\sigma_v\mathbb{E}\left[-\sigma_v^{-1}\left(T+\frac{\sigma_v^2}{\tau}\tilde{z}\right)h'\left(\sigma_v^{-1}\left(T-\frac{\sigma_v^2}{\tau}\tilde{z}\right)\right)+h\left(\sigma_v^{-1}\left(T-\frac{\sigma_v^2}{\tau}\tilde{z}\right)\right)\right] \\
= &\mathbb{E}\left[\sigma_v h\left(\sigma_v^{-1}\left(T-\frac{\sigma_v^2}{\tau}\tilde{z}\right)\right)-\left(T+\frac{\sigma_v^2}{\tau}\tilde{z}\right)h'\left(\sigma_v^{-1}\left(T-\frac{\sigma_v^2}{\tau}\tilde{z}\right)\right)\right].
\end{aligned}$$

Next, note that $\lim_{\sigma_\varepsilon \to \infty} \Omega(T;\sigma_v) = T - c + \mathbb{E}\left[T + \sigma_v h\left(\sigma_v^{-1}\left(T - \frac{\sigma_v^2}{\tau}\tilde{z}\right)\right)\right]$. Thus, substituting $\Omega(T;\sigma_v) = 0$, we have:

$$\begin{aligned}
&\lim_{T\to-\infty}\left[\sigma_v \frac{\partial \Omega(T;\sigma_v)}{\partial \sigma_v} + T\frac{\partial \Omega(T;\sigma_v)}{\partial T}\right] \\
= \;& \mathbb{E}\left[T + \sigma_v h\left(\sigma_v^{-1}\left(T - \frac{\sigma_v^2}{\tau}\tilde{z}\right)\right) - \frac{\sigma_v^2}{\tau}\tilde{z}h'\left(\sigma_v^{-1}\left(T - \frac{\sigma_v^2}{\tau}\tilde{z}\right)\right)\right] \\
= \;& c - \frac{\sigma_v^2}{\tau}\mathbb{E}\left[\tilde{z}h'\left(\sigma_v^{-1}\left(T - \frac{\sigma_v^2}{\tau}\tilde{z}\right)\right)\right]
\end{aligned}$$

Now, since $h'' > 0$, $Cov\left[\tilde{z}, h'\left(\sigma_v^{-1}\left(T - \frac{\sigma_v^2}{\tau}\tilde{z}\right)\right)\right] > 0$, and this is positive.
Part ii. From the proof of Proposition 1, $\frac{\partial T}{\partial c} > 0$ and $\lim_{c\to\infty} T = \infty$. Therefore,

$$\lim_{c\to\infty}\left[\sigma_v \frac{\partial \Omega(T;\sigma_v)}{\partial \sigma_v} + T\frac{\partial \Omega(T;\sigma_v)}{\partial T}\right] = \lim_{T\to\infty}\left[\sigma_v \frac{\partial \Omega(T;\sigma_v)}{\partial \sigma_v} + T\frac{\partial \Omega(T;\sigma_v)}{\partial T}\right]. \tag{42}$$

Now, note that:

$$\lim_{T\to\infty}\mathbb{E}\left[h\left(\sigma_s^{-1}\left(\left(1-\Delta_v\right)T - \Delta_z\tilde{z}\right)\right)\right] = \mathbb{E}\left[h\left(\sigma_s^{-1}\left(\lim_{T\to\infty} T\left(1-\Delta_v\right) - \Delta_z\tilde{z}\right)\right)\right] = 0.$$

It can further be verified that $xh'(x) \to \infty$ as $x \to \infty$, and thus:

$$\lim_{T\to\infty}\mathbb{E}\left[Th'\left(\sigma_s^{-1}\left(\left(1-\Delta_v\right)T - \Delta_z\tilde{z}\right)\right)\right] = 0.$$

Now, calculating the derivatives in expression (42) and applying these results, we have that:

$$\begin{aligned}
&\lim_{T\to\infty}\sigma_v\frac{\partial \Omega(T;\sigma_v)}{\partial \sigma_v} \\
= \;& \lim_{T\to\infty}\sigma_v\mathbb{E}\left\{\frac{\partial}{\partial \sigma_v}\left[\left(1-\Delta_v\right)T - \Delta_z\tilde{z} + \sigma_s h\left(\sigma_s^{-1}\left(\left(1-\Delta_v\right)T - \Delta_z\tilde{z}\right)\right)\right]\right\} \\
= \;& \sigma_v \lim_{T\to\infty}\left[\begin{array}{c} T * \frac{\partial}{\partial \sigma_v}\left(1-\Delta_v\right) + \sigma_s\frac{\partial}{\partial \sigma_v}\left(\sigma_s^{-1}\left(\left(1-\Delta_v\right)T - \Delta_z\tilde{z}\right)\right)h'\left(\sigma_s^{-1}\left(\left(1-\Delta_v\right)T - \Delta_z\tilde{z}\right)\right) \\ +\frac{\partial \sigma_s}{\partial \sigma_v}h\left(\sigma_s^{-1}\left(\left(1-\Delta_v\right)T - \Delta_z\tilde{z}\right)\right) \end{array}\right].
\end{aligned}$$

Now, since $\frac{\partial}{\partial \sigma_v}\left(\sigma_s^{-1}\left(\left(1-\Delta_v\right)T - \Delta_z\tilde{z}\right)\right)$ is linear in T, this reduces to:

$$= \sigma_v \frac{\partial}{\partial \sigma_v}\left(1-\Delta_v\right)\lim_{T\to\infty} T.$$

Next, note:

$$\begin{aligned}
&\lim_{T\to\infty}\left[T\frac{\partial\Omega\left(T;\sigma_v\right)}{\partial T}\right]\\
=\;&\lim_{T\to\infty}T*\mathbb{E}\left\{\frac{\partial}{\partial T}\left[\left(1-\Delta_v\right)T+\sigma_s h\left(\sigma_s^{-1}\left(\left(1-\Delta_v\right)T-\Delta_z\tilde{z}\right)\right)\right]\right\}\\
=\;&\left(1-\Delta_v\right)\lim_{T\to\infty}T.
\end{aligned}$$

Therefore,

$$\begin{aligned}
&\lim_{T\to\infty}\left[\sigma_v\frac{\partial\Omega\left(T;\sigma_v\right)}{\partial\sigma_v}+T\frac{\partial\Omega\left(T;\sigma_v\right)}{\partial T}\right]\\
=\;&sign\left[1-\Delta_v+\sigma_v*\frac{\partial}{\partial\sigma_v}\left(1-\Delta_v\right)\right]*\infty.
\end{aligned}$$

Now, explicitly calculating $1-\Delta_v+\sigma_v*\frac{\partial}{\partial\sigma_v}\left(1-\Delta_v\right)$, we find it is proportional to:

$$\sigma_\varepsilon^{10}\sigma_z^4-\sigma_\varepsilon^8\sigma_z^4\sigma_v^2+\left(2-3\rho\right)\sigma_v^2\sigma_z^2\sigma_\varepsilon^6-\left(1-\rho\right)\left(2+3\rho\right)\tau^2\sigma_v^4\sigma_z^2\sigma_\varepsilon^4+\left(1-\rho\right)\tau^4\sigma_v^4\sigma_\varepsilon^2-\left(1-\rho\right)^2\tau^4\left(1+\rho\right)\sigma_v^6.$$

As $\sigma_\varepsilon\to 0$, this converges to $-\left(1-\rho\right)^2\tau^4\left(1+\rho\right)\sigma_v^6<0$. □

Proof of Proposition 3. Part i. Note first that: Since $\mathbb{E}\left[\tilde{z}\right]=0$, we can write the expected price given non-disclosure, $\mathbb{E}\left[P_{ND}\left(\left\{s_i\right\},z\right)|\tilde{v}<T\right]$, as:

$$\mathbb{E}\left[P_{ND}|\tilde{v}<T\right]=\int_i\mathbb{E}\left[\tilde{\mu}_i|\tilde{v}<T\right]di-\sigma_s\mathbb{E}\left[h\left(\sigma_s^{-1}\left(T-\int_i\tilde{\mu}_i di-\frac{\tilde{z}}{\tau}\sigma_s^2\right)\right)|\tilde{v}<T\right].$$

Now, note that, because $\tilde{\mu}_i$ aligns with the objective conditional expectation $\mathbb{E}\left[\tilde{v}|\tilde{v}<T,\tilde{s}_i,\tilde{s}_p\right]$ when $\rho=1$, we may write the firm's expected value conditional on non-disclosure as:

$$\begin{aligned}
\mathbb{E}\left[\tilde{v}|\tilde{v}<T\right]&=\mathbb{E}\left\{\mathbb{E}\left[\tilde{v}|\tilde{v}<T,\tilde{s}_i,\tilde{s}_p\right]|\tilde{v}<T\right\}\\
&=\mathbb{E}\left[\tilde{\mu}_i|\tilde{v}<T\right]-\sigma_s\mathbb{E}\left[h\left(\sigma_s^{-1}\left(T-\tilde{\mu}_i\right)\right)|\tilde{v}<T\right].
\end{aligned}$$

Given that investors' signals are homogeneously distributed, $\int_i\mathbb{E}\left[\tilde{\mu}_i|\tilde{v}<T\right]di=\mathbb{E}\left[\tilde{\mu}_i|\tilde{v}<T\right]$. This yields:

$$\begin{aligned}
&\mathbb{E}\left[P_{ND}|\tilde{v}<T\right]-\mathbb{E}\left[\tilde{v}|\tilde{v}<T\right]\\
\propto\;&\mathbb{E}\left[h\left(\sigma_s^{-1}\left(T-\tilde{\mu}_i\right)\right)|\tilde{v}<T\right]-\mathbb{E}\left[h\left(\sigma_s^{-1}\left(T-\int_i\tilde{\mu}_i di-\frac{\tilde{z}}{\tau}\sigma_s^2\right)\right)|\tilde{v}<T\right].
\end{aligned}$$

Next, note that the inverse-mills ratio $h\left(\cdot\right)$ is convex. Thus, to show that the above expression

is positive (negative), it is sufficient to show that, conditional on $\tilde{v} < T$, $\tilde{\mu}_i \succ_{SSD} \int_i \tilde{\mu}_i di + \frac{\tilde{z}}{\tau}\sigma_s^2$ ($\int_i \tilde{\mu}_i di + \frac{\tilde{z}}{\tau}\sigma_s^2 \succ_{SSD} \tilde{\mu}_i$), where $\succ_{SSD}$ denotes second-order stochastic dominance. Observe that the coefficients on $\tilde{v}$ in each of $\tilde{\mu}_i$ and $\int_i \tilde{\mu}_i di + \frac{\tilde{z}}{\tau}\sigma_s^2$ equal Δ_v. Therefore, the components of variation driven by $\tilde{v}$ in both $\tilde{\mu}_i$ and $\int_i \tilde{\mu}_i di + \frac{\tilde{z}}{\tau}\sigma_s^2$ are identical. Together with the normality of $\tilde{\varepsilon}_i$ and $\tilde{z}$ and their independence of $\tilde{v}$, this implies that second-order stochastic dominance reduces to the relative variance conditional on $\tilde{v}$, i.e.,

$$\tilde{\mu}_i \succcurlyeq_{SSD} \int_i \tilde{\mu}_i di + \frac{\tilde{z}}{\tau}\sigma_s^2 \Longleftrightarrow var\left[\tilde{\mu}_i|\tilde{v}\right] \lesseqgtr var\left[\int_i \tilde{\mu}_i di + \frac{\tilde{z}}{\tau}\sigma_s^2|\tilde{v}\right]. \tag{43}$$

Calculating these variances, we have:

$$\begin{aligned} var\left[\tilde{\mu}_i|\tilde{v}\right] &= var\left[\frac{\frac{1}{\sigma_\varepsilon^2}\tilde{\varepsilon}_i + \frac{\tau^2}{\sigma_\varepsilon^4\sigma_z^2}\frac{\sigma_\varepsilon^2}{\tau}\tilde{z}}{\frac{1}{\sigma_\varepsilon^2} + \frac{\tau^2}{\sigma_\varepsilon^4\sigma_z^2} + \frac{1}{\sigma_v^2}}\right] = \frac{\sigma_v^4\sigma_z^2\sigma_\varepsilon^4\left(\tau^2 + \sigma_z^2\sigma_\varepsilon^2\right)}{\left(\tau^2\sigma_v^2 + \sigma_v^2\sigma_z^2\sigma_\varepsilon^2 + \sigma_z^2\sigma_\varepsilon^4\right)^2}; \\ var\left[\int_i \tilde{\mu}_i di + \frac{\tilde{z}}{\tau}\sigma_s^2|\tilde{v}\right] &= var\left[\frac{\left(\frac{\tau^2}{\sigma_\varepsilon^4\sigma_z^2}\frac{\sigma_\varepsilon^2}{\tau} + \frac{1}{\tau}\right)\tilde{z}}{\frac{1}{\sigma_\varepsilon^2} + \frac{\tau^2}{\sigma_\varepsilon^4\sigma_z^2} + \frac{1}{\sigma_v^2}}\right] = \frac{1}{\tau^2}\frac{\sigma_v^4\sigma_z^2\sigma_\varepsilon^4\left(\tau^2 + \sigma_z^2\sigma_\varepsilon^2\right)^2}{\left(\tau^2\sigma_v^2 + \sigma_z^2\sigma_\varepsilon^2\left(\sigma_v^2 + \sigma_\varepsilon^2\right)\right)^2}. \end{aligned}$$

Taking the difference yields $-\frac{1}{\tau^2}\frac{\sigma_v^4\sigma_z^4\sigma_\varepsilon^6\left(\tau^2+\sigma_z^2\sigma_\varepsilon^2\right)}{\left(\tau^2\sigma_v^2+\sigma_v^2\sigma_z^2\sigma_\varepsilon^2+\sigma_z^2\sigma_\varepsilon^4\right)^2} < 0$.

Part ii. Applying the same reasoning as in the proof of part i., since, when $\rho = 0$, investors entirely ignore the information in price, $\tilde{\mu}_i$ aligns with the objective expectation of $\tilde{v}$ given the private signal $\tilde{s}_i$ only, $\mathbb{E}\left[\tilde{v}|\tilde{s}_i\right]$. Thus, we may write:

$$\begin{aligned} \mathbb{E}\left[\tilde{v}|\tilde{v} < T\right] &= \mathbb{E}\left\{\mathbb{E}\left[\tilde{v}|\tilde{v} < T, \tilde{s}_i\right]|\tilde{v} < T\right\} \\ &= \mathbb{E}\left[\tilde{\mu}_i|\tilde{v} < T\right] - \sigma_s\mathbb{E}\left\{h\left(\frac{T - \tilde{\mu}_i}{\sigma_s}\right)|\tilde{v} < T\right\}, \end{aligned}$$

such that:

$$\begin{aligned} &\mathbb{E}\left[P_{ND}|\tilde{v} < T\right] - \mathbb{E}\left[\tilde{v}|\tilde{v} < T\right] \\ \propto\ &\mathbb{E}\left\{h\left(\sigma_s^{-1}\left(T - \tilde{\mu}_i\right)\right)|\tilde{v} < T\right\} - \mathbb{E}\left[h\left(\sigma_s^{-1}\left(T - \int_0^1 \tilde{\mu}_i di - \frac{\tilde{z}}{\tau}\sigma_s^2\right)\right)|\tilde{v} < T\right]. \end{aligned}$$

Following the reasoning in the proof of part i., this has the sign of:

$$\begin{aligned} var\left[\tilde{\mu}_i|\tilde{v}\right] - var\left[\int_0^1 \tilde{\mu}_i di + \frac{\tilde{z}}{\tau}\sigma_s^2|\tilde{v}\right] &= var\left[\tilde{\mu}_i|\tilde{v}\right] - var\left[\frac{\tilde{z}}{\tau}\sigma_s^2|\tilde{v}\right] \\ &= \left(\frac{\sigma_v^2}{\sigma_v^2+\sigma_\varepsilon^2}\right)^2 \sigma_\varepsilon^2 - \frac{1}{\tau^2}\left(\frac{\sigma_\varepsilon^2\sigma_v^2}{\sigma_v^2+\sigma_\varepsilon^2}\right)^2 \sigma_z^2 \\ &= \frac{\sigma_v^4\sigma_\varepsilon^2\left(\tau^2-\sigma_z^2\sigma_\varepsilon^2\right)}{\tau^2\left(\sigma_v^2+\sigma_\varepsilon^2\right)^2}, \end{aligned}$$

and is thus positive when $\sigma_z^2 < \frac{\tau^2}{\sigma_\varepsilon^2}$ and when $\sigma_z^2 > \frac{\tau^2}{\sigma_\varepsilon^2}$. □