On the Value of Information Structures in Stochastic Games (Daehyun Kim, Ichiro Obara, 2021)

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Content

Question of the Paper

- In this paper, the authors try to answer the following question: *"How do the information structures affect the size of the set of limit Perfect Public Equilibrium (PPE) payoffs in stochastic games with imperfect public monitoring?"*
- Rough idea:
  - In stochastic games with imperfect public monitoring, each player cannot observe their opponents' actions after each stage game.
  - Instead, players observe a public signal and the next state which are generated according to some *"distribution function"* related to players' actions and the current state after the stage game.
  - This paper compares the size of the set of equilibrium payoffs when some *"distribution function"* is more *"informative"* than others.

Answer from authors

- The authors introduce a novel notion called "*weighted garbling*" for comparing information structures.
- They shows that *if one information structure is a weighted garbling of another information structure, then the size of the set of limit PPE payoff of the former is weakly smaller than that of the latter.*

Importance and Application

- Stochastic games with imperfect public monitoring are important tools for studying long-term relationships between economic agents when actions are not observable.
- e.g. Repeated partnership, Cournot oligopolists, principal-agent problem, etc. can be studied under stochastic game framework.
- The set of payoffs that can be supported by PPE under fixed information structure have been well studied in the last century.
- However, It is also important to study how the changes in information structures affect the equilibrium payoff set.
- The notion weighted garbling allows comparison of larger class of information structures. For example, some information structures are not comparable in Blackwell sense[1] are comparable by weighted garbling.

[1] Check "The Economics of Uncertainty and Information" Chapter 4 by Jean-Jacques Laffont if you are interested.

Environment of Stochastic Game

- S is a finite set of states.
- $I = \{1, 2, ...N\}$ is a finte set of players.
- A_i is a finite action set of player i.
- $A = \prod_{i \in I} A_i$ is the set of action profiles.
- Each period k starts with a state $s^k \in S$. After action profile $a \in A$ is chosen, the next state s^{k+1} is drawn according to $q(\cdot|s^k, a) \in \Delta(S)$.
- $u_i : A \times S \to \mathbb{R}$ is player i's utility.
- **Idea**: At different states, players play different games. The outcome of the game in the current period affects what game to be play in the next period. So, when S is a singleton, it is a repeated game.

Information Structures

- At each period k, players cannot observe their opponents' actions.
- Instead, players observe a public signal $y \in Y$, where $|Y| < \infty$.
- y is drawn according to $f(\cdot|s^k, s^{k+1}, a) \in \Delta(Y)$.
- For examples, 2 partners cannot see whether their partner's actions e.g. works or shirks. However, they can observe a public signal depending on their actions e.g. the profit of their company.
- An **Information Structure** is a pair $\pi = (f, Y)$.
- π and q together defines $p \in \Delta(S \times Y)$ defined by

$$p(s^{k+1}, y|s^k, a) = f(y|s^k, s^{k+1}, a)q(s^{k+1}|s^k, a)$$

for all $(s^{k+1}, y) \in S \times Y$.

Perfect Public Equilibrium (PPE)

- A history of player i at period k is

$$h_i^k = (s^0, a_i^0, y^0, ..., s^{k-1}, a_i^{k-1}, y^{k-1}, s^k, a_i^k, y^k)$$

- A public history at period k is

$$h^k = (s^0, y^0, ..., s^{k-1}, y^{k-1}, s^k, y^k)$$

- A strategy of player i is a mapping from the set of all possible histories to A_i (or $\Delta(A_i)$ for behavioral strategies).
- A strategy of player i is **public** if player i's private action does not affect the outcome of the strategy.
- A strategy profile is a **PPE** if it is a Nash equilibrium of the continuation game after any public histories.[2]

[2] Check Fudenberg, Levine and Maskin (1994) if you are interested.

Weighted Garbling

For simplicity, I focus on special case: $|S| = 1$ i.e. repeated game.The notations presented previously still applies except ignoring the states.[3]

Definition

An information structure $\pi = (f, Y)$ is a **weighted garbling** of $\pi' = (f', Y')$ if there exists $\phi : Y' \to \Delta(Y)$ and for each $y' \in Y'$, a constant $\gamma^{y'} \geq 0$ such that for all $a \in A$,

$$f(y|a) = \sum_{y' \in Y'} \gamma^{y'} \phi(y|y') f'(y'|a), \quad \forall y \in Y,$$

where $\sum_{y' \in Y'} \gamma^{y'} f'(y'|a) = 1.$

[3]When there is no state transition, it is clear that $p = f, p' = f'$.

Weighted Garbling

- $\phi(\cdot|y')$ is the garbling to the public signals in Y by $y' \in Y$.
- $\gamma^{y'} f'(y'|a)$ is the weight put on the garbling by y'.
- Thus, for each $y \in Y$, $f(y|a)$ is the weighted average of garbling by public signals in Y'.
- So, we can say $\pi(f, Y)$ is less informative than $\pi'(f', Y')$.

Example of weighted garbling

Consider a repeated 2-player partnership problem.

- $A_i = \{\text{Work(w), Shirk(s)}\}$ for $i = 1, 2$.
- $Y = \{g, b\}$ and $Y' = \{g, b, n\}$ where g, b, n stand for 'good', 'bad' and 'no' signals respectively.
- $f(g|ww) = f(b|\neg ww) = 0.5; f(b|ww) = f(g|\neg ww) = 0.5$.
- $f'(g|ww) = f'(b|\neg ww) = 0.2; f'(b|ww) = f'(g|\neg ww) = 0.3$.
  So, no signal (n) is observed with probability 0.5.
- Let $\pi = (Y, f)$ and $\pi' = (Y', f')$.
- After some calculation, we can see π is a weighted garbling of π'.
- **Intuition**: Although no signal is observed half of the time under π', π' provides strictly more informative signals than π in another half of the time.

Main Result

Theorem

Suppose $\pi = (Y, f)$ *is a weighted garbling of* $\pi'(Y', f')$*. Then for each* $\lambda \in \Lambda$*,* $k(\lambda, \pi) \leq k(\lambda, \pi')$ *i.e.* $H(\pi) \subseteq H(\pi')$*.*

- $H(\pi)$ is the set of PPE payoff under information structure π when discount factor $\delta \to 1$.
- Roughly speaking, the theorem says an information structure that is a weighted garbling of another information structure has relatively fewer PPE payoffs when players are arbitrarily patient.

Discussion

1. Can similar notion of garbling be extended to the information structures in repeated game with private monitoring?
2. Consider a principal-agent setting: 1 manager and a group of workers.
    - Manager can choose different monitoring structures (possibly costly) to induce the workers to exert effort.
    - Is it reasonable for the manager to compare monitoring structures by weighted garbling criterion?
    - What is the effect on the equilibrium payoff of the manager when he/she chooses a monitoring structure that is a weighted garbling of another monitoring structures?